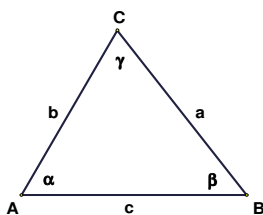


TROKUT (mCh)

stranice i kutovi



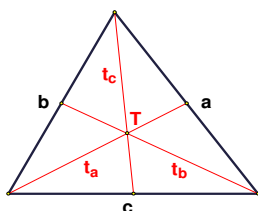
svaka stranica trokuta manja je od zbroja ostalih dviju stranica

$$a < b + c, \quad b < c + a, \quad c < a + b$$

$$\alpha + \beta + \gamma = 180^\circ$$

težišnica

dužina koja spaja vrh trokuta s polovištem nasuprotne stranice
težišnice trokuta sijeku se u jednoj točki koju zovemo težište, T
težište dijeli svaku težišnicu u omjeru 2 : 1 računajući (gledano) od vrha
duljine težišnica

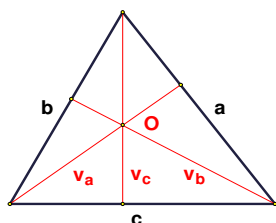


$$t_a = \frac{1}{2} \cdot \sqrt{2 \cdot (b^2 + c^2) - a^2} = \frac{1}{2} \cdot \sqrt{b^2 + c^2 + 2 \cdot b \cdot c \cdot \cos \alpha}$$

$$t_b = \frac{1}{2} \cdot \sqrt{2 \cdot (c^2 + a^2) - b^2} = \frac{1}{2} \cdot \sqrt{c^2 + a^2 + 2 \cdot c \cdot a \cdot \cos \beta}$$

$$t_c = \frac{1}{2} \cdot \sqrt{2 \cdot (a^2 + b^2) - c^2} = \frac{1}{2} \cdot \sqrt{a^2 + b^2 + 2 \cdot a \cdot b \cdot \cos \gamma}$$

visina



okomica spuštena iz vrha trokuta na suprotnu stranicu
visine trokuta sijeku se u jednoj točki koju zovemo ortocentar, O

$$P = \frac{1}{\sqrt{\left(\frac{1}{v_a} + \frac{1}{v_b} + \frac{1}{v_c}\right) \cdot \left(\frac{1}{v_b} + \frac{1}{v_c} - \frac{1}{v_a}\right) \cdot \left(\frac{1}{v_c} + \frac{1}{v_a} - \frac{1}{v_b}\right) \cdot \left(\frac{1}{v_a} + \frac{1}{v_b} - \frac{1}{v_c}\right)}}$$

$$v_a = \frac{2}{a} \cdot \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}, \quad v_b = \frac{2}{b} \cdot \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}$$

$$v_c = \frac{2}{c} \cdot \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}, \quad s = \frac{a+b+c}{2}$$

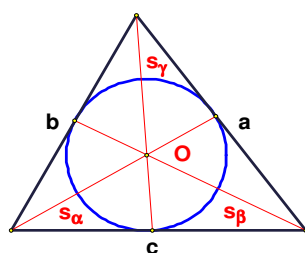
stranice trokuta odnose se kao recipročne vrijednosti visina trokuta

$$a:b:c = \frac{1}{v_a} : \frac{1}{v_b} : \frac{1}{v_c}$$

simetrala kuta

pravac koji raspolavlja unutarnji kut trokuta
simetrale kutova sijeku se u jednoj točki koja je središte trokutu upisane kružnice

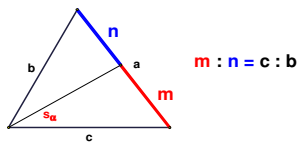
duljine odsječaka simetrala kutova unutar trokuta



$$s_\alpha = \frac{\sqrt{b \cdot c \cdot [(b+c)^2 - a^2]}}{b+c} = \frac{1}{b+c} \cdot 2 \cdot b \cdot c \cdot \cos \frac{\alpha}{2} = \frac{2 \cdot \sqrt{b \cdot c \cdot s \cdot (s-a)}}{b+c}$$

$$s_\beta = \frac{\sqrt{a \cdot c \cdot [(a+c)^2 - b^2]}}{a+c} = \frac{1}{a+c} \cdot 2 \cdot a \cdot c \cdot \cos \frac{\beta}{2} = \frac{2 \cdot \sqrt{c \cdot a \cdot s \cdot (s-b)}}{c+a}$$

$$s_\gamma = \frac{\sqrt{a \cdot b \cdot [(a+b)^2 - c^2]}}{a+b} = \frac{1}{a+b} \cdot 2 \cdot a \cdot b \cdot \cos \frac{\gamma}{2} = \frac{2 \cdot \sqrt{a \cdot b \cdot s \cdot (s-c)}}{a+b}$$



simetrala kuta dijeli nasuprotnu stranicu u omjeru preostale dvije stranice

$$m : n = c : b$$

kosinusov poučak

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha, \quad b^2 = c^2 + a^2 - 2 \cdot c \cdot a \cdot \cos \beta, \quad c^2 = a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \gamma$$

$$\cos \alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}, \quad \cos \beta = \frac{c^2 + a^2 - b^2}{2 \cdot c \cdot a}, \quad \cos \gamma = \frac{a^2 + b^2 - c^2}{2 \cdot a \cdot b}$$

sinusov poučak

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma} = 2 \cdot R, \quad R - \text{polumjer opisane kružnice}$$

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta}, \quad \frac{a}{\sin \alpha} = \frac{c}{\sin \gamma}, \quad \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}, \quad a = 2 \cdot R \cdot \sin \alpha, \quad b = 2 \cdot R \cdot \sin \beta, \quad c = 2 \cdot R \cdot \sin \gamma$$

tangensov poučak

$$(a+b):(a-b) = \operatorname{tg} \frac{\alpha+\beta}{2} : \operatorname{tg} \frac{\alpha-\beta}{2}, \quad (b+c):(b-c) = \operatorname{tg} \frac{\beta+\gamma}{2} : \operatorname{tg} \frac{\beta-\gamma}{2}$$

$$(c+a):(c-a) = \operatorname{tg} \frac{\gamma+\alpha}{2} : \operatorname{tg} \frac{\gamma-\alpha}{2}$$

polumjer upisane kružnice

$$r = (s-a) \cdot \operatorname{tg} \frac{\alpha}{2} = (s-b) \cdot \operatorname{tg} \frac{\beta}{2} = (s-c) \cdot \operatorname{tg} \frac{\gamma}{2}, \quad s = \frac{a+b+c}{2}$$

$$r = \frac{s}{4 \cdot \cos \frac{\alpha}{2} \cdot \cos \frac{\beta}{2} \cdot \cos \frac{\gamma}{2}}, \quad s = \frac{a+b+c}{2}$$

polumjer opisane kružnice

$$R = \frac{a}{2 \cdot \sin \alpha} = \frac{b}{2 \cdot \sin \beta} = \frac{c}{2 \cdot \sin \gamma}$$

$$R = \frac{a \cdot b \cdot c}{4 \cdot P}, \quad P - \text{površina trokuta}$$

kutovi u trokutu ($\alpha + \beta + \gamma = 180^\circ$)

$$\sin \frac{\alpha}{2} = \sqrt{\frac{(s-b) \cdot (s-c)}{b \cdot c}}, \quad \sin \frac{\beta}{2} = \sqrt{\frac{(s-a) \cdot (s-c)}{a \cdot c}}, \quad \sin \frac{\gamma}{2} = \sqrt{\frac{(s-a) \cdot (s-b)}{a \cdot b}}$$

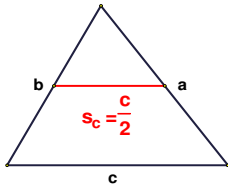
$$\cos \frac{\alpha}{2} = \sqrt{\frac{s \cdot (s-a)}{b \cdot c}}, \quad \cos \frac{\beta}{2} = \sqrt{\frac{s \cdot (s-b)}{a \cdot c}}, \quad \cos \frac{\gamma}{2} = \sqrt{\frac{s \cdot (s-c)}{a \cdot b}}$$

$$\operatorname{tg} \frac{\alpha}{2} = \sqrt{\frac{(s-b) \cdot (s-c)}{s \cdot (s-a)}} = \frac{r}{s-a}, \quad \operatorname{tg} \frac{\beta}{2} = \sqrt{\frac{(s-a) \cdot (s-c)}{s \cdot (s-b)}} = \frac{r}{s-b}$$

$$\operatorname{tg} \frac{\gamma}{2} = \sqrt{\frac{(s-a) \cdot (s-b)}{s \cdot (s-c)}} = \frac{r}{s-c}$$

$$r - \text{polumjer upisane kružnice}, \quad s = \frac{a+b+c}{2}$$

srednjica



dužina koja spaja polovišta dviju stranica paralelna (usporedna) je s trećom stranicom jednaka je polovini njezine duljine

$$s_a = \frac{a}{2}, s_b = \frac{b}{2}, s_c = \frac{c}{2}$$

površine

$$P = \frac{1}{2} a \cdot v_a = \frac{1}{2} b \cdot v_b = \frac{1}{2} c \cdot v_c = \frac{1}{2} a \cdot b \cdot \sin \gamma = \frac{1}{2} a \cdot c \cdot \sin \beta = \frac{1}{2} b \cdot c \cdot \sin \alpha$$

$$P = \frac{1}{2} \frac{a^2 \cdot \sin \beta \cdot \sin \gamma}{\sin \alpha} = \frac{1}{2} \frac{b^2 \cdot \sin \alpha \cdot \sin \gamma}{\sin \beta} = \frac{1}{2} \frac{c^2 \cdot \sin \alpha \cdot \sin \beta}{\sin \gamma}$$

$$P = \frac{a \cdot b \cdot c}{4 \cdot R}, \text{ R – polumjer opisane kružnice}$$

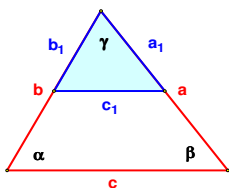
$$P = r \cdot s, \text{ r – polumjer upisane kružnice, } s = \frac{a+b+c}{2}$$

$$P = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)}, \quad P = \frac{1}{4} \cdot \sqrt{(a^2 + b^2 + c^2)^2 - 2 \cdot (a^4 + b^4 + c^4)}$$

poučci o sukladnosti trokuta

- poučak S – S – S (stranica – stranica – stranica)
dva su trokuta sukladna ako se podudaraju u sve tri stranice
- poučak S – KS – S (stranica – kut – stranica)
dva su trokuta sukladna ako se podudaraju u dvije stranice i kutu između njih
- poučak K – S – K (kut – stranica – kut)
dva su trokuta sukladna ako se podudaraju u jednoj stranici i oba kuta na toj stranici
- poučak S – S – K (stranica – stranica – kut nasuprot većoj stranici)
dva su trokuta sukladna ako se podudaraju u dvije stranice i kutu nasuprot većoj stranici

sličnost



dva su trokuta slična ako su im odgovarajući kutovi međusobno jednaki i ako za njihove stranice vrijedi

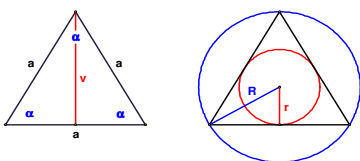
$$\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1} = k, \quad k > 0, \text{ koeficijent sličnosti}$$

za slične trokute vrijedi

$$\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1} = \frac{O}{O_1} = \frac{v}{v_1} = \frac{t}{t_1} = \frac{s}{s_1} = \frac{r}{r_1} = \frac{R}{R_1} = k, \quad \frac{P}{P_1} = k^2$$

pri čemu je O opseg, v visina, t težišnica, s odsječak simetrale kuta unutar trokuta, r polumjer upisane kružnice, R polumjer opisane kružnice, P površina trokuta

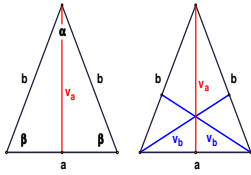
jednakostranični trokut



$$\alpha = 60^\circ, \quad O = 3 \cdot a, \quad v = \frac{a \cdot \sqrt{3}}{2}, \quad P = \frac{a^2 \cdot \sqrt{3}}{4}$$

$$r = \frac{a \cdot \sqrt{3}}{6}, \quad R = \frac{a \cdot \sqrt{3}}{3}, \quad R = 2 \cdot r$$

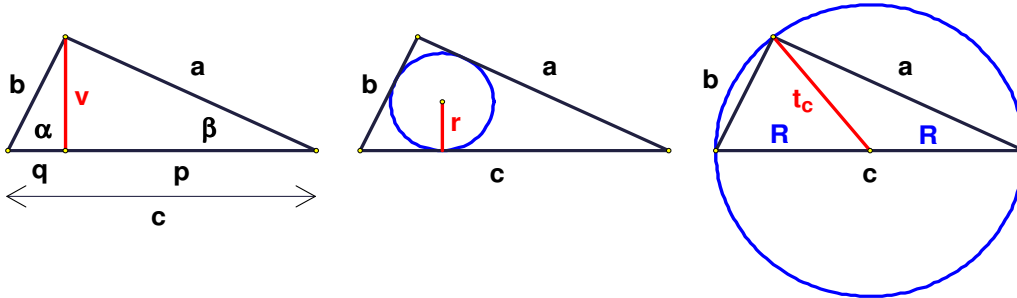
jednakokrani trokut



$$O = a + 2 \cdot b, \quad P = \frac{a \cdot v_a}{2} = \frac{b \cdot v_b}{2}$$

$$P = \frac{1}{2} \cdot a \cdot \sqrt{b^2 - \frac{a^2}{4}}, \quad a \cdot v_a = b \cdot v_b$$

pravokutni trokut



$$p + q = c, \quad \alpha + \beta = 90^\circ$$

$$a^2 = p \cdot c, \quad b^2 = q \cdot c,$$

$$c^2 = a^2 + b^2, \quad v^2 = p \cdot q$$

$$c \cdot v = a \cdot b$$

$$r = \frac{a + b - c}{2}$$

r – polumjer upisane kruznice

$$R = t_c = \frac{c}{2}$$

R – polumjer opisane kruznice

t_c – težišnica na hipotenuzu

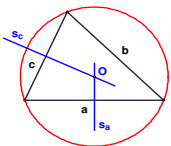
površine

$$P = \frac{a \cdot b}{2} = \frac{c \cdot v}{2} = \frac{1}{2} \cdot a \cdot c \cdot \sin \beta = \frac{1}{2} \cdot b \cdot c \cdot \sin \alpha$$

$$P = \frac{1}{2} \cdot a^2 \cdot \operatorname{tg} \beta = \frac{1}{2} \cdot b^2 \cdot \operatorname{tg} \alpha = \frac{1}{4} \cdot c^2 \cdot \sin 2\alpha$$

četiri karakteristične (značajne) točke trokuta

1. središte opisane kruznice trokuta

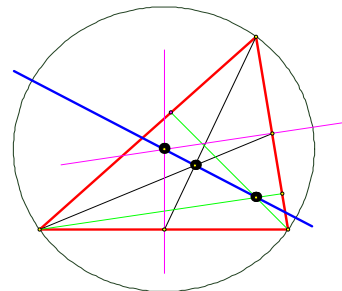


središte O opisane kruznice je u presjeku simetrala stranica (dovoljne su dvije simetrale)

2. središte upisane kruznice trokuta
3. težište trokuta (sjecište težišnica)
4. ortocentar (sjecište visina) trokuta



$$e^i \cdot \pi + 1 = 0$$



Eulerov pravac: središte opisane kruznice, težište i ortocentar bilo kojeg trokuta leže na jednom pravcu