

PRAVAC (m©h)



René Descartes

udaljenost točkara $A(x_1, y_1)$ i $B(x_2, y_2)$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

koordinate polovišta $P(x, y)$ dužine \overline{AB} , $A(x_1, y_1)$, $B(x_2, y_2)$

$$P(x, y) = P\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

površina trokuta određena točkama $A(x_1, y_1)$, $B(x_2, y_2)$ i $C(x_3, y_3)$

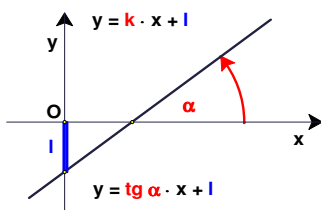
$$P = \frac{1}{2} \cdot |x_1 \cdot (y_2 - y_3) + x_2 \cdot (y_3 - y_1) + x_3 \cdot (y_1 - y_2)|$$

$$P = \frac{1}{2} \cdot \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

koordinate težišta trokuta s vrhovima $A(x_1, y_1)$, $B(x_2, y_2)$ i $C(x_3, y_3)$

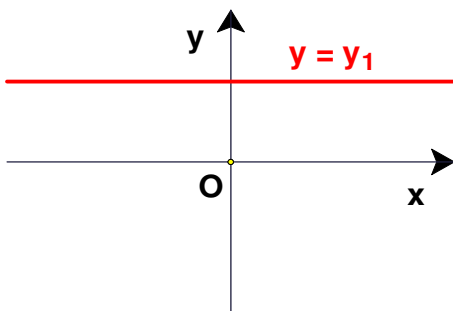
$$T(x, y) = T\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

eksplicitni oblik jednadžbe pravca

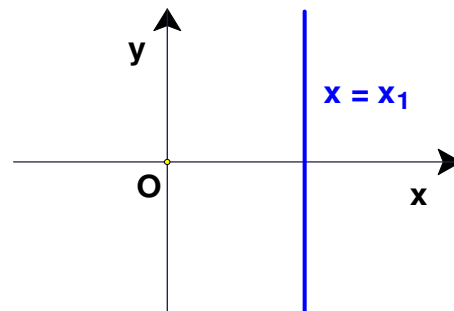


$y = k \cdot x + l$
 k – koeficijent smjera
 α – prikloni kut
 $k = \text{tg } \alpha$
 l – odsječak na osi y

jednadžba pravca usporednog
(paralelnog) s x -osi, $y = y_1$



jednadžba pravca usporednog
(paralelnog) s y -osi, $x = x_1$



implicitni oblik jednadžbe pravca

$$Ax + By + C = 0$$

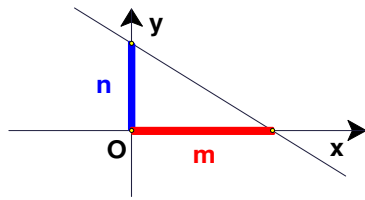
jednadžba pravca zadanog koeficijentom smjera i jednom točkom

$$\left. \begin{matrix} k \\ T(x_1, y_1) \end{matrix} \right\} \Rightarrow y - y_1 = k \cdot (x - x_1)$$

jednadžba pravca kroz dvije točke

$$\left. \begin{matrix} A(x_1, y_1) \\ B(x_2, y_2) \end{matrix} \right\} \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1), \quad k = \frac{y_2 - y_1}{x_2 - x_1}, \quad l = y_1 - \frac{y_2 - y_1}{x_2 - x_1} \cdot x_1$$

segmentni oblik jednadžbe pravca



$$\frac{x}{m} + \frac{y}{n} = 1$$

m – odsječak (segment) na x – osi

n – odsječak (segment) na y – osi

vektori smjera

$$y = k \cdot x + l \Rightarrow \text{vektori smjera tog pravca kolinearni su s vektorom } \vec{s} = \vec{i} + k \cdot \vec{j}$$

$$Ax + By + C = 0 \Rightarrow \text{vektori smjera tog pravca kolinearni su s vektorom } \vec{s} = B \cdot \vec{i} - A \cdot \vec{j}$$

jednadžba pravca zadanog točkom $T(x_0, y_0)$ i vektorom smjera $\vec{s} = A \cdot \vec{i} + B \cdot \vec{j}$

$$B \cdot (x - x_0) - A \cdot (y - y_0) = 0$$

okomica točkom $T(x_0, y_0)$ uz vektor smjera $\vec{s} = A \cdot \vec{i} + B \cdot \vec{j}$

$$A \cdot (x - x_0) + B \cdot (y - y_0) = 0$$

kut dva pravca

$$\left. \begin{array}{l} A_1x + B_1y + C_1 = 0 \\ A_2x + B_2y + C_2 = 0 \end{array} \right\} \Rightarrow \cos \varphi = \frac{|A_1 \cdot A_2 + B_1 \cdot B_2|}{\sqrt{A_1^2 + B_1^2} \cdot \sqrt{A_2^2 + B_2^2}}, \quad \left. \begin{array}{l} y = k_1x + l_1 \\ y = k_2x + l_2 \end{array} \right\} \Rightarrow \operatorname{tg} \varphi = \left| \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \right|$$

uvjet okomitosti

$$\left. \begin{array}{l} A_1x + B_1y + C_1 = 0 \\ A_2x + B_2y + C_2 = 0 \end{array} \right\} \Rightarrow A_1 \cdot A_2 + B_1 \cdot B_2 = 0, \quad \left. \begin{array}{l} y = k_1x + l_1 \\ y = k_2x + l_2 \end{array} \right\} \Rightarrow k_1 = -\frac{1}{k_2}$$

$$\left. \begin{array}{l} \frac{x}{m_1} + \frac{y}{n_1} = 1 \\ \frac{x}{m_2} + \frac{y}{n_2} = 1 \end{array} \right\} \Rightarrow m_1 \cdot m_2 + n_1 \cdot n_2 = 0$$

uvjet usporednosti (paralelnosti)

$$\left. \begin{array}{l} A_1x + B_1y + C_1 = 0 \\ A_2x + B_2y + C_2 = 0 \end{array} \right\} \Rightarrow \frac{A_1}{A_2} = \frac{B_1}{B_2}, \quad \left. \begin{array}{l} y = k_1x + l_1 \\ y = k_2x + l_2 \end{array} \right\} \Rightarrow k_1 = k_2, \quad \left. \begin{array}{l} \frac{x}{m_1} + \frac{y}{n_1} = 1 \\ \frac{x}{m_2} + \frac{y}{n_2} = 1 \end{array} \right\} \Rightarrow \frac{m_1}{m_2} = \frac{n_1}{n_2}$$

udaljenost točke od pravca

$$\left. \begin{array}{l} Ax + By + C = 0 \\ T(x_0, y_0) \end{array} \right\} \Rightarrow d = \frac{|A \cdot x_0 + B \cdot y_0 + C|}{\sqrt{A^2 + B^2}}$$

udaljenost para usporednih (paralelnih) pravaca

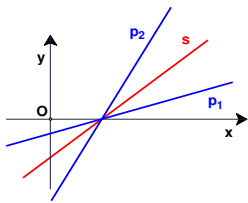
$$\left. \begin{array}{l} Ax + By + C_1 = 0 \\ Ax + By + C_2 = 0 \end{array} \right\} \Rightarrow d = \frac{|C_2 - C_1|}{\sqrt{A^2 + B^2}}$$

pramen pravaca

jednadžba svakog pravca koji prolazi kroz sjecište dvaju pravaca

$$\left. \begin{array}{l} A_1x + B_1y + C_1 = 0 \\ A_2x + B_2y + C_2 = 0 \end{array} \right\} \Rightarrow (A_1x + B_1y + C_1) + \lambda \cdot (A_2x + B_2y + C_2) = 0, \quad \lambda \in \mathbb{R}$$

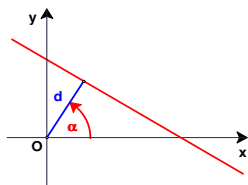
simetrala para pravaca



$$\left. \begin{aligned} A_1x + B_1y + C_1 &= 0 \\ A_2x + B_2y + C_2 &= 0 \end{aligned} \right\} \Rightarrow \frac{A_1x + B_1y + C_1}{\sqrt{A_1^2 + B_1^2}} = \pm \frac{A_2x + B_2y + C_2}{\sqrt{A_2^2 + B_2^2}}$$

normalni oblik jednadžbe pravca

$$\frac{Ax + By + C}{(-\text{sgn } C) \cdot \sqrt{A^2 + B^2}} = 0 \text{ ili } \frac{Ax + By + C}{\pm \sqrt{A^2 + B^2}} = 0 \text{ (predznak mora biti suprotan predznaku od C)}$$



$$x \cdot \cos \alpha + y \cdot \sin \alpha = d$$

d – udaljenost pravca od ishodišta

α – kut koji zatvara okomica d sa pozitivnim smjerom osi x

djelište dužine

ako točka $T(x, y)$ dijeli dužinu \overline{AB} u omjeru $\frac{|AT|}{|TB|} = -\lambda$, tada su koordinate djelišta

$$x = \frac{x_1 - \lambda \cdot x_2}{1 - \lambda}, \quad y = \frac{y_1 - \lambda \cdot y_2}{1 - \lambda}$$

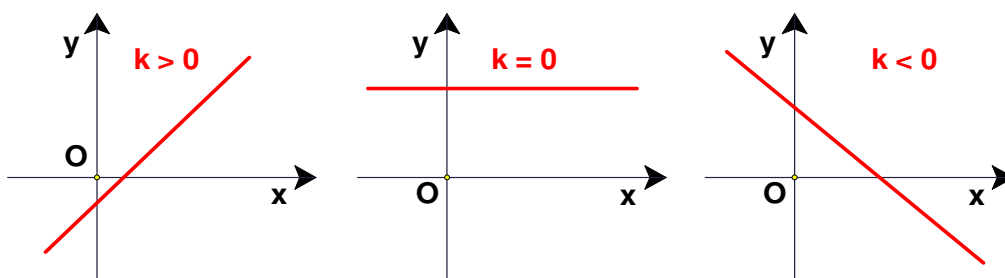
1. $\lambda > 0 \Rightarrow$ točka T je izvan dužine \overline{AB} 2. $\lambda < 0 \Rightarrow$ točka T je unutar dužine \overline{AB}

3. $\lambda = -1 \Rightarrow$ točka T raspolavlja dužinu \overline{AB} pa je $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$

linearna (afina) funkcija

neka su k i l realni brojevi, $k \neq 0$, tada se funkcija $f(x) = k \cdot x + l$ naziva linearna (afina) funkcija graf linearne (afine) funkcije je pravac $y = k \cdot x + l$

$k > 0 \Rightarrow$ funkcija raste, $k = 0 \Rightarrow$ konstantna je, $k < 0 \Rightarrow$ funkcija pada



crtanje

$$\left. \begin{aligned} f(x) &= 2 \cdot x - 3 \\ f(0) &= 2 \cdot 0 - 3 = -3 \\ f(1) &= 2 \cdot 1 - 3 = -1 \end{aligned} \right\} \Rightarrow \begin{array}{l} x | 0 | 1 | \\ y | -3 | -1 | \end{array}$$

$$\left. \begin{aligned} g(x) &= -2 \cdot x + 3 \\ g(0) &= -2 \cdot 0 + 3 = 3 \\ g(1) &= -2 \cdot 1 + 3 = 1 \end{aligned} \right\} \Rightarrow \begin{array}{l} x | 0 | 1 | \\ y | 3 | 1 | \end{array}$$

