

ALGEBARSKI IZRAZI (m©h)

kvadrat zbroja	$(a + b)^2 = a^2 + 2ab + b^2$
kvadrat razlike	$(a - b)^2 = a^2 - 2ab + b^2$
kub zbroja	$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
kub razlike	$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
kvadrat trinoma	$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$
kub trinoma	$(a + b + c)^3 = a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3ab^2 + 3ac^2 + 3b^2c + 3bc^2 + 6abc$
razlika kvadrata	$a^2 - b^2 = (a - b) \cdot (a + b)$, $a - b = (\sqrt{a} - \sqrt{b}) \cdot (\sqrt{a} + \sqrt{b})$
razlika kubova	$a^3 - b^3 = (a - b) \cdot (a^2 + ab + b^2)$
razlika bikvadrata	$a^4 - b^4 = (a^2 - b^2) \cdot (a^2 + b^2) = (a - b) \cdot (a + b) \cdot (a^2 + b^2)$
zbroj kvadrata	$a^2 + b^2 =$ nema rastava nad \mathbf{R} (skup realnih brojeva)
zbroj kubova	$a^3 + b^3 = (a + b) \cdot (a^2 - ab + b^2)$
zbroj bikvadrata	$a^4 + b^4 = (a^2 - ab\sqrt{2} + b^2) \cdot (a^2 + ab\sqrt{2} + b^2)$
generalizirane relacije	$a^n - b^n = (a - b) \cdot (a^{n-1} + a^{n-2} \cdot b + \dots + a \cdot b^{n-2} + b^{n-1})$
	$a^{2n+1} + b^{2n+1} = (a + b) \cdot (a^{2n} - a^{2n-1} \cdot b + \dots - a \cdot b^{2n-1} + b^{2n})$
Pozor!	$(-a - b)^2 = (a + b)^2$, $(-a - b)^3 = -(a + b)^3$

POTENCIJE

potencije broja	$a^n = a \cdot a \cdot a \cdot \dots \cdot a$ (a se množi n puta sam sa sobom)
potenciranje broja jedinicom	$a^1 = a$
potenciranje broja nulom	$a^0 = 1, a \neq 0$
recipročna vrijednost potencije	$a^{-n} = \frac{1}{a^n}$, $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$
množenje potencija jednakih baza	$a^n \cdot a^m = a^{n+m}$
dijeljenje potencija jednakih baza	$a^n : a^m = a^{n-m}$, $\frac{a^n}{a^m} = a^{n-m}$
potenciranje umnoška brojeva	$(a \cdot b)^n = a^n \cdot b^n$
množenje potencija jednakih eksponenata	$a^n \cdot b^n = (a \cdot b)^n$
potenciranje količnika brojeva	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$, $(a : b)^n = a^n : b^n$
dijeljenje potencija jednakih eksponenata	$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$, $a^n : b^n = (a : b)^n$
potenciranje potencije	$(a^n)^m = (a^m)^n = a^{n \cdot m}$
rastuća funkcija	$a > 1$, $x < y \Rightarrow a^x < a^y$
Primjer	$2^{3x+4} < 2^{x+10} \Rightarrow 3x+4 < x+10 \Rightarrow 2x < 6 \Rightarrow x < 3$
padajuća funkcija	$0 < a < 1$, $x < y \Rightarrow a^x > a^y$
Primjer	$\left(\frac{1}{2}\right)^{3x+4} < \left(\frac{1}{2}\right)^{x+10} \Rightarrow 3x+4 > x+10 \Rightarrow 2x > 6 \Rightarrow x > 3$
jednakost potencija	$a^x = a^y \Rightarrow x = y$, $a^x = b^x \Rightarrow a = b$
Primjeri	$0.1^x = 100 \Rightarrow 10^{-x} = 10^2 \Rightarrow -x = 2 \Rightarrow x = -2$
	$8^x = 4^{x+1} \Rightarrow (2^3)^x = (2^2)^{x+1} \Rightarrow 2^{3x} = 2^{2x+2} \Rightarrow 3x = 2x+2 \Rightarrow x = 2$
nejednakost potencija	$n > 0$, $a > b > 0 \Rightarrow a^n > b^n$
	$n > 0$, $a > b > 0 \Rightarrow \frac{1}{a^n} < \frac{1}{b^n}$
potenciranje korijena	$(\sqrt[n]{a})^m = \sqrt[n]{a^m}$, $(\sqrt[n]{a^p})^m = \sqrt[n]{a^{p \cdot m}}$

samo potpuno jednake potencije možemo zbrajati i oduzimati

$$a^n \pm a^m = ? \quad , \quad a^n \pm b^n = ? \quad , \quad a^n + a^n = 2a^n$$

veza potencije i korijena
predznaci kod množenja

$$a^{\frac{1}{n}} = \sqrt[n]{a} \quad , \quad a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

$$+ a \cdot (+ b) = + ab \quad , \quad - a \cdot (- b) = + ab$$

$$+ a \cdot (- b) = - ab \quad , \quad - a \cdot (+ b) = - ab$$

KORIJENI

veza korijena i potencije

$$\sqrt[n]{a} = a^{\frac{1}{n}} \quad , \quad \sqrt[n]{a^m} = a^{\frac{m}{n}}$$

korjenovanje umnoška brojeva

$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

množenje korijena istih eksponenata

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$$

množenje korijena istih radikanada

$$\sqrt[n]{a} \cdot \sqrt[m]{a} = \sqrt[n \cdot m]{a^{n+m}}$$

množenje korijena

$$\sqrt[n]{a} \cdot \sqrt[m]{b} = \sqrt[n \cdot m]{a^m \cdot b^n}$$

korjenovanje količnika brojeva

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}} \quad , \quad \sqrt[n]{a:b} = \sqrt[n]{a} : \sqrt[n]{b}$$

dijeljenje korijena istih eksponenata

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}} \quad , \quad \sqrt[n]{a} : \sqrt[n]{b} = \sqrt[n]{a:b}$$

dijeljenje korijena istih radikanada

$$\frac{\sqrt[n]{a}}{\sqrt[m]{a}} = \sqrt[n \cdot m]{a^{m-n}} \quad , \quad \sqrt[n]{a} : \sqrt[m]{a} = \sqrt[n \cdot m]{a^{m-n}}$$

dijeljenje korijena

$$\frac{\sqrt[n]{a}}{\sqrt[m]{b}} = \sqrt[n \cdot m]{\frac{a^m}{b^n}} \quad , \quad \sqrt[n]{a} : \sqrt[m]{b} = \sqrt[n \cdot m]{a^m : b^n}$$

proširivanje korijena

$$\sqrt[n]{a^m} = \sqrt[n \cdot p]{a^{m \cdot p}}$$

skaćivanje korijena

$$\sqrt[n \cdot p]{a^{m \cdot p}} = \sqrt[n]{a^m} \quad , \quad \sqrt[n]{a^n} = 1 \quad , \quad \sqrt{a^2} = |a|$$

korjenovanje korijena

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[m \cdot n]{a} = \sqrt[n \cdot m]{a} \quad , \quad \sqrt[n]{a \cdot \sqrt[m]{b}} = \sqrt[n \cdot m]{a^m \cdot b}$$

djelomično korjenovanje

$$\sqrt[n]{a^n \cdot b} = a \cdot \sqrt[n]{b} \quad , \quad \sqrt[n]{a^{p \cdot n} \cdot b} = a^p \cdot \sqrt[n]{b}$$

unošenje pod korijen

$$a \cdot \sqrt[n]{b} = \sqrt[n]{a^n \cdot b} \quad , \quad a^p \cdot \sqrt[n]{b} = \sqrt[n]{a^{p \cdot n} \cdot b}$$

racionalizacija nazivnika

$$\frac{a}{\sqrt{b}} = \frac{a}{\sqrt{b}} \cdot \frac{\sqrt{b}}{\sqrt{b}} = \frac{a \cdot \sqrt{b}}{b} \quad , \quad \frac{a}{\sqrt[n]{b}} = \frac{a}{\sqrt[n]{b}} \cdot \frac{\sqrt[n]{b^{n-1}}}{\sqrt[n]{b^{n-1}}} = \frac{a \cdot \sqrt[n]{b^{n-1}}}{b}$$

$$\frac{a}{\sqrt{b \pm \sqrt{c}}} = \frac{a}{\sqrt{b \pm \sqrt{c}}} \cdot \frac{\sqrt{b \mp \sqrt{c}}}{\sqrt{b \mp \sqrt{c}}} = \frac{a \cdot (\sqrt{b \mp \sqrt{c}})}{b - c}$$

$$\frac{a}{\sqrt[3]{b \pm \sqrt[3]{c}}} = \frac{a}{\sqrt[3]{b \pm \sqrt[3]{c}}} \cdot \frac{\sqrt[3]{b^2 \mp \sqrt[3]{b \cdot c} + \sqrt[3]{c^2}}}{\sqrt[3]{b^2 \mp \sqrt[3]{b \cdot c} + \sqrt[3]{c^2}}} = \frac{a \cdot (\sqrt[3]{b^2 \mp \sqrt[3]{b \cdot c} + \sqrt[3]{c^2}})}{b \pm c}$$

$$\frac{m}{a\sqrt{b \pm c}\sqrt{d}} = \frac{m}{a\sqrt{b \pm c}\sqrt{d}} \cdot \frac{a\sqrt{b \mp c}\sqrt{d}}{a\sqrt{b \mp c}\sqrt{d}} = \frac{m \cdot (a\sqrt{b \mp c}\sqrt{d})}{a^2 b - c^2 d}$$

potencije korijena

$$(\sqrt{a})^2 = a, a \geq 0 \quad , \quad (2^n \sqrt{a})^{2^n} = a, a \geq 0 \quad , \quad (\sqrt[3]{a})^3 = a \quad , \quad (2^{n-1} \sqrt{a})^{2^{n-1}} = a \quad , \quad \sqrt{a^2} = |a|$$

važne formule

$$\sqrt{a+\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} + \sqrt{\frac{a-\sqrt{a^2-b}}{2}} \quad , \quad \sqrt{a-\sqrt{b}} = \sqrt{\frac{a+\sqrt{a^2-b}}{2}} - \sqrt{\frac{a-\sqrt{a^2-b}}{2}}$$

nejednakost korijena

$$a > b > 0 \Rightarrow \sqrt[n]{a} > \sqrt[n]{b}$$

samo potpuno jednake korijene možemo zbrajati i oduzimati

$$\sqrt[n]{a} \pm \sqrt[m]{a} = ? \quad , \quad \sqrt[n]{a} \pm \sqrt[n]{b} = ? \quad , \quad \sqrt[n]{a} + \sqrt[n]{a} = 2 \cdot \sqrt[n]{a}.$$

LOGARITMI

definicija logaritma

$$\log_b a = c \Leftrightarrow b^c = a \quad , \quad b > 0 \quad , \quad b \neq 1 \quad , \quad a > 0$$

definicija logaritma

$\log_b a$ je broj kojim treba potencirati b da se dobije a

$$b^c = a \Leftrightarrow c = \log_b a = c \quad \text{ili} \quad b^{\log_b a} = a$$

svojstva

$$\log_b b = 1 \quad , \quad \log_b 1 = 0$$

$$\log_b 0 = -\infty \quad , \quad b > 1 \quad , \quad \log_b 0 = +\infty \quad , \quad 0 < b < 1$$

Primjeri

$$5^{\log_5 2} = 2 \quad , \quad \log_3 3 = 1 \quad , \quad 25^{\log_5 2} = (5^2)^{\log_5 2} = (5^{\log_5 2})^2 = 2^2 = 4$$

dekadski logaritam (Briggsov)

$$\log_{10} a = \lg a = \log a$$

prirodni logaritam (Neperov)

$$\log_e a = \ln a \quad , \quad e = 2.718281828\dots$$

logaritam umnoška

$$\log_b (a \cdot c) = \log_b a + \log_b c$$

Primjeri

$$\log_2 6 = \log_2 (2 \cdot 3) = \log_2 2 + \log_2 3 = 1 + \log_2 3$$

$$\log_2 (8 \cdot x) = \log_2 8 + \log_2 x = \log_2 2^3 + \log_2 x = 3 \cdot \log_2 2 + \log_2 x = 3 + \log_2 x$$

logaritam kvocijenta

$$\log_b \frac{a}{c} = \log_b a - \log_b c$$

Primjeri

$$\log \frac{100}{x} = \log 100 - \log x = 2 - \log x$$

$$\log_3 \left(\frac{2}{3} \right) = \log_3 2 - \log_3 3 = \log_3 2 - 1$$

logaritam potencije

$$\log_b a^n = n \cdot \log_b a \quad , \quad \log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \log_{b^n} a^n = \log_b a$$

$$\log_b b^n = n \quad , \quad \log_{\frac{b}{m}} a = \frac{n}{m} \cdot \log_b a \quad , \quad \log_{\frac{1}{b}} a = \log_b a$$

Primjer

$$\log_2 32 = \log_2 2^5 = 5 \cdot \log_2 2 = 5 \cdot 1 = 5$$

logaritam korijena

$$\log_b \sqrt[n]{a} = \frac{1}{n} \cdot \log_b a \quad , \quad \log_b \sqrt[n]{a^m} = \frac{m}{n} \cdot \log_b a$$

Primjer

$$\log_2 \sqrt[3]{2} = \frac{1}{3} \cdot \log_2 2 = \frac{1}{3} \cdot 1 = \frac{1}{3}$$

formule za promjenu baze

$$\log_b a = \frac{\log_c a}{\log_c b} = \log_b c \cdot \log_c a \quad , \quad \frac{\log_b c}{\log_{ab} c} = 1 + \log_b a$$

Primjer

$$\log_{100} 2 = \frac{\log 2}{\log 100} = \frac{\log 2}{2}$$

recipročnost logaritama

$$\log_b a = \frac{1}{\log_a b}$$

veza s dekadskim logaritmima

$$\log_b a = \frac{\log a}{\log b}$$

Primjer

$$\log_{100} 1000 = \frac{\log 1000}{\log 100} = \frac{3}{2} = 1.5$$

rastuća funkcija

$$b > 1, x < y \Rightarrow \log_b x < \log_b y$$

Primjer

$$\left. \begin{array}{l} \log_2(2x+4) < \log_2(x+6) \Rightarrow 2x+4 < x+6 \Rightarrow x < 2 \\ 2x+4 > 0 \text{ i } x+6 > 0 \Rightarrow x > -2 \end{array} \right\} \Rightarrow x \in \langle -2, 2 \rangle$$

padajuća funkcija

$$0 < b < 1, x < y \Rightarrow \log_b x > \log_b y$$

Primjeri

$$\left. \begin{array}{l} \log_{\frac{1}{2}}(2x+6) < \log_{\frac{1}{2}}(x+8) \Rightarrow 2x+6 > x+8 \Rightarrow x > 2 \\ 2x+6 > 0 \text{ i } x+8 > 0 \Rightarrow x > -3 \end{array} \right\} \Rightarrow x > 2$$

$$\left. \begin{array}{l} \log_{\frac{1}{2}}(2x+5) > \log_{\frac{1}{2}}(x+8) \Rightarrow 2x+5 < x+8 \Rightarrow x < 3 \\ 2x+5 > 0 \text{ i } x+8 > 0 \Rightarrow x > -\frac{5}{2} \end{array} \right\} \Rightarrow x \in \langle -\frac{5}{2}, 3 \rangle$$

injektivnost

$$\log_b x = \log_b y \Rightarrow x = y$$

Primjeri

$$\left. \begin{array}{l} \log(3x+2) = \log(2x+7) \Rightarrow 3x+2 = 2x+7 \Rightarrow x = 5 \\ 3x+2 > 0 \text{ i } 2x+7 > 0 \Rightarrow x > -\frac{2}{3} \end{array} \right\} \Rightarrow x = 5$$

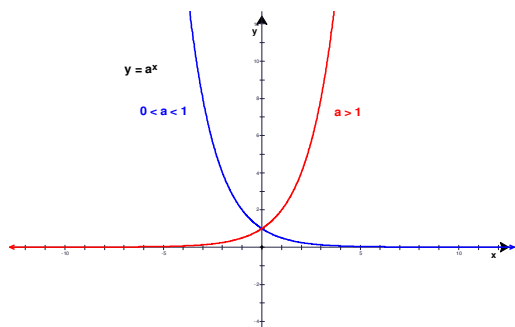
$$\left. \begin{array}{l} \log 2 + \log(x-3) = 1 \Rightarrow \log 2 \cdot (x-3) = \log 10 \Rightarrow \\ \Rightarrow 2 \cdot (x-3) = 10 \Rightarrow 2x-6 = 10 \Rightarrow 2x = 16 \Rightarrow x = 8 \\ x-3 > 0 \Rightarrow x > 3 \end{array} \right\} \Rightarrow x = 8$$

množenje istih logaritama

$$\log_b a \cdot \log_b a = (\log_b a)^2 = \log_b^2 a$$

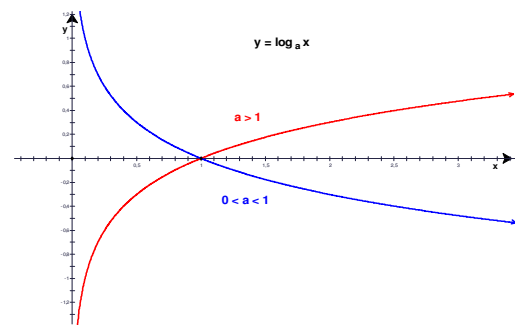
EKSPONENCIJALNA FUNKCIJA

$$f: \mathbb{R} \rightarrow \mathbb{R}^+, f(x) = a^x, a > 0, a \neq 1$$



LOGARITAMSKA FUNKCIJA

$$f: \mathbb{R}^+ \rightarrow \mathbb{R}, f(x) = \log_a x, a > 0, a \neq 1$$



René Descartes (1596.–1650.)
lat. "potencia" – moć, snaga
današnja oznaka potječe od njega



Leonhard Euler (1707.–1783.)
uveo eksponencijalnu funkciju u
današnjem smislu