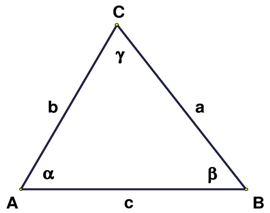
TROKUT (m©h)

stranice i kutovi



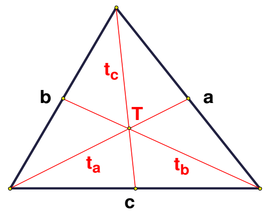
svaka stranica trokuta manja je od zbroja ostalih dviju stranica

$$a < b + c \quad , \quad b < c + a \quad , \quad c < a + b$$

$$\alpha + \beta + \gamma = 180°$$

težišnica

dužina koja spaja vrh trokuta s polovištem nasuprotne stranice
težišnice trokuta sijeku se u jednoj točki koju zovemo težište, T
težište dijeli svaku težišnicu u omjeru 2 : 1 računajući (gledano) od vrha trokuta

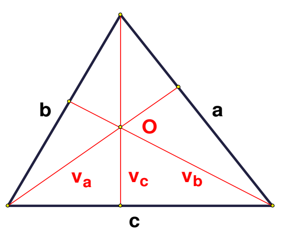


$$t_a = \frac{1}{2}\cdot\sqrt{2\cdot\left(b^2+c^2\right)-a^2} = \frac{1}{2}\cdot\sqrt{b^2+c^2+2\cdot b\cdot c\cdot\cos\alpha}$$

$$t_b = \frac{1}{2}\cdot\sqrt{2\cdot\left(c^2+a^2\right)-b^2} = \frac{1}{2}\cdot\sqrt{c^2+a^2+2\cdot c\cdot a\cdot\cos\beta}$$

$$t_c = \frac{1}{2}\cdot\sqrt{2\cdot\left(a^2+b^2\right)-c^2} = \frac{1}{2}\cdot\sqrt{a^2+b^2+2\cdot a\cdot b\cdot\cos\gamma}$$

visina



okomica spuštena iz vrha trokuta na suprotnu stranicu
visine trokuta sijeku se u jednoj točki koju zovemo ortocentar, O

$$P = \frac{1}{\sqrt{\left(\frac{1}{v_a}+\frac{1}{v_b}+\frac{1}{v_c}\right)\cdot\left(\frac{1}{v_b}+\frac{1}{v_c}-\frac{1}{v_a}\right)\cdot\left(\frac{1}{v_c}+\frac{1}{v_a}-\frac{1}{v_b}\right)\cdot\left(\frac{1}{v_a}+\frac{1}{v_b}-\frac{1}{v_c}\right)}}$$

$$v_a = \frac{2}{a}\cdot\sqrt{s\cdot(s-a)\cdot(s-b)\cdot(s-c)} \ , \ v_b = \frac{2}{b}\cdot\sqrt{s\cdot(s-a)\cdot(s-b)\cdot(s-c)}$$

$$v_c = \frac{2}{c}\cdot\sqrt{s\cdot(s-a)\cdot(s-b)\cdot(s-c)} \ , \ s = \frac{a+b+c}{2}$$

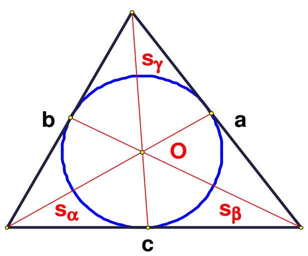
stranice trokuta odnose se kao recipročne vrijednosti visina trokuta

$$a:b:c = \frac{1}{v_a}:\frac{1}{v_b}:\frac{1}{v_c}$$

simetrala kuta

pravac koji raspolavlja unutarnji kut trokuta
simetrale kutova sijeku se u jednoj točki koja je središte trokutu upisane kružnice

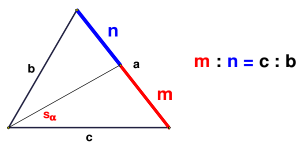
duljine odsječaka simetrala kutova unutar trokuta



$$s_\alpha = \frac{\sqrt{b\cdot c\cdot\left[(b+c)^2-a^2\right]}}{b+c} = \frac{1}{b+c}\cdot 2\cdot b\cdot c\cdot\cos\frac{\alpha}{2} = \frac{2\cdot\sqrt{b\cdot c\cdot s\cdot(s-a)}}{b+c}$$

$$s_\beta = \frac{\sqrt{a\cdot c\cdot\left[(a+c)^2-b^2\right]}}{a+c} = \frac{1}{a+c}\cdot 2\cdot a\cdot c\cdot\cos\frac{\beta}{2} = \frac{2\cdot\sqrt{c\cdot a\cdot s\cdot(s-b)}}{c+a}$$

$$s_\gamma = \frac{\sqrt{a\cdot b\cdot\left[(a+b)^2-c^2\right]}}{a+b} = \frac{1}{a+b}\cdot 2\cdot a\cdot b\cdot\cos\frac{\gamma}{2} = \frac{2\cdot\sqrt{a\cdot b\cdot s\cdot(s-c)}}{a+b}$$



simetrala kuta dijeli nasuprotnu stranicu u omjeru preostale dvije stranice

m : n = c : b

kosinusov poučak

$$a^2=b^2+c^2-2\cdot b\cdot c\cdot\cos\alpha\ ,\ b^2=c^2+a^2-2\cdot c\cdot a\cdot\cos\beta\ ,\ c^2=a^2+b^2-2\cdot a\cdot b\cdot\cos\gamma$$

$$\cos\alpha=\frac{b^2+c^2-a^2}{2\cdot b\cdot c}\ ,\ \cos\beta=\frac{c^2+a^2-b^2}{2\cdot c\cdot a}\ ,\ \cos\gamma=\frac{a^2+b^2-c^2}{2\cdot a\cdot b}$$

sinusov poučak

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}=\frac{c}{\sin\gamma}=2\cdot R,\ R-\text{polumjer opisane kružnice}$$

$$\frac{a}{\sin\alpha}=\frac{b}{\sin\beta}\ ,\ \frac{a}{\sin\alpha}=\frac{c}{\sin\gamma}\ ,\ \frac{b}{\sin\beta}=\frac{c}{\sin\gamma}\ ,\ a=2\cdot R\cdot\sin\alpha\ ,\ b=2\cdot R\cdot\sin\beta\ ,\ c=2\cdot R\cdot\sin\gamma$$

tangensov poučak

$$(a+b):(a-b)=tg\frac{\alpha+\beta}{2}:tg\frac{\alpha-\beta}{2}\ ,\ (b+c):(b-c)=tg\frac{\beta+\gamma}{2}:tg\frac{\beta-\gamma}{2}$$

$$(c+a):(c-a)=tg\frac{\gamma+\alpha}{2}:tg\frac{\gamma-\alpha}{2}$$

polumjer upisane kružnice

$$r=(s-a)\cdot tg\frac{\alpha}{2}=(s-b)\cdot tg\frac{\beta}{2}=(s-c)\cdot tg\frac{\gamma}{2}\ ,\ s=\frac{a+b+c}{2}$$

$$r=\frac{s}{4\cdot\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2}}\ ,\ s=\frac{a+b+c}{2}$$

polumjer opisane kružnice

$$R=\frac{a}{2\cdot\sin\alpha}=\frac{b}{2\cdot\sin\beta}=\frac{c}{2\cdot\sin\gamma}$$

$$R=\frac{a\cdot b\cdot c}{4\cdot P},\ \text{P}-\text{površina trokuta}$$

kutovi u trokutu (α + β + γ = 180°)

$$\sin\frac{\alpha}{2}=\sqrt{\frac{(s-b)\cdot(s-c)}{b\cdot c}}\ ,\ \sin\frac{\beta}{2}=\sqrt{\frac{(s-a)\cdot(s-c)}{a\cdot c}}\ ,\ \sin\frac{\gamma}{2}=\sqrt{\frac{(s-a)\cdot(s-b)}{a\cdot b}}$$

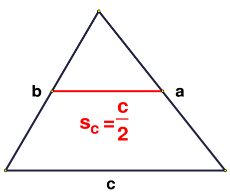
$$\cos\frac{\alpha}{2}=\sqrt{\frac{s\cdot(s-a)}{b\cdot c}}\ ,\ \cos\frac{\beta}{2}=\sqrt{\frac{s\cdot(s-b)}{a\cdot c}}\ ,\ \cos\frac{\gamma}{2}=\sqrt{\frac{s\cdot(s-c)}{a\cdot b}}$$

$$tg\frac{\alpha}{2}=\sqrt{\frac{(s-b)\cdot(s-c)}{s\cdot(s-a)}}=\frac{r}{s-a}\ ,\ tg\frac{\beta}{2}=\sqrt{\frac{(s-a)\cdot(s-c)}{s\cdot(s-b)}}=\frac{r}{s-b}$$

$$tg\frac{\gamma}{2}=\sqrt{\frac{(s-a)\cdot(s-b)}{s\cdot(s-c)}}=\frac{r}{s-c}$$

r-polumjer upisane kružnice , $s=\dfrac{a+b+c}{2}$

srednjica



dužina koja spaja polovišta dviju stranica
paralelna (usporedna) je s trećom stranicom
jednaka je polovini njezine duljine

$$s_a = \frac{a}{2} \ , \ s_b = \frac{b}{2} \ , \ s_c = \frac{c}{2}$$

površine

$$P = \frac{1}{2} a \cdot v_a = \frac{1}{2} b \cdot v_b = \frac{1}{2} c \cdot v_c = \frac{1}{2} a \cdot b \cdot \sin\gamma = \frac{1}{2} a \cdot c \cdot \sin\beta = \frac{1}{2} b \cdot c \cdot \sin\alpha$$

$$P = \frac{1}{2} \frac{a^2 \cdot \sin\beta \cdot \sin\gamma}{\sin\alpha} = \frac{1}{2} \frac{b^2 \cdot \sin\alpha \cdot \sin\gamma}{\sin\beta} = \frac{1}{2} \frac{c^2 \cdot \sin\alpha \cdot \sin\beta}{\sin\gamma}$$

$$P = \frac{a \cdot b \cdot c}{4 \cdot R}, \ \text{R} - \text{polumjer opisane kružnice}$$

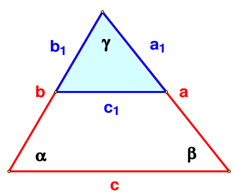
$$\text{P} = \text{r} \cdot \text{s}, \ \text{r} - \text{polumjer upisane kružnice}, \ \text{s} = \frac{\text{a+b+c}}{2}$$

$$P = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)} \ , \ P = \frac{1}{4} \cdot \sqrt{\left(a^2 + b^2 + c^2\right)^2 - 2 \cdot \left(a^4 + b^4 + c^4\right)}$$

poučci o sukladnosti trokuta

- poučak S – S – S (stranica – stranica – stranica)
  dva su trokuta sukladna ako se podudaraju u sve tri stranice
- poučak S – KS – S (stranica – kut – stranica)
  dva su trokuta sukladna ako se podudaraju u dvije stranice i kutu između njih
- poučak K – S – K (kut – stranica – kut)
  dva su trokuta sukladna ako se podudaraju u jednoj stranici i oba kuta na toj stranici
- poučak S – S – K (stranica – stranica – kut nasuprot većoj stranici)
  dva su trokuta sukladna ako se podudaraju u dvije stranice i kutu nasuprot većoj stranici

sličnost



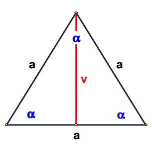
dva su trokuta slična ako su im odgovarajući kutovi međusobno jednaki i ako za njihove stranice vrijedi

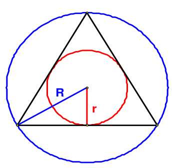
$$\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1} = k, \ k > 0, \ \text{koeficijent sličnosti}$$

za slične trokute vrijedi

$$\frac{a}{a_1} = \frac{b}{b_1} = \frac{c}{c_1} = \frac{O}{O_1} = \frac{v}{v_1} = \frac{t}{t_1} = \frac{s}{s_1} = \frac{r}{r_1} = \frac{R}{R_1} = k \ , \ \frac{P}{P_1} = k^2$$

pri čemu je O opseg, v visina, t težišnica, s odsječak simetrale kuta unutar trokuta, r polumjer upisane kružnice, R polumjer opisane kružnice, P površina trokuta





jednakostranični trokut

$$\alpha = 60^0 \ , \ O = 3 \cdot a \ , \ v = \frac{a \cdot \sqrt{3}}{2} \ , \ P = \frac{a^2 \cdot \sqrt{3}}{4}$$

$$r = \frac{a \cdot \sqrt{3}}{6} \ , \ R = \frac{a \cdot \sqrt{3}}{3} \ , \ R = 2 \cdot r$$

jednakokračni trokut