PRAVAC (m©h)

René Descartes

udaljenost točaka $A(x_1, y_1)$ i $B(x_2, y_2)$

$$|AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

koordinate polovišta $P(x, y)$ dužine $\overline{AB}$, $A(x_1, y_1)$, $B(x_2, y_2)$

$$P(x, y) = P\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

površina trokuta određena točkama $A(x_1, y_1)$, $B(x_2, y_2)$ i $C(x_3, y_3)$

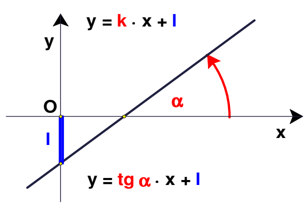
$$P = \frac{1}{2} \cdot |x_1 \cdot (y_2 - y_3) + x_2 \cdot (y_3 - y_1) + x_3 \cdot (y_1 - y_2)|$$

$$P = \frac{1}{2} \cdot | \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} |$$

koordinate težišta trokuta s vrhovima $A(x_1, y_1)$, $B(x_2, y_2)$ i $C(x_3, y_3)$

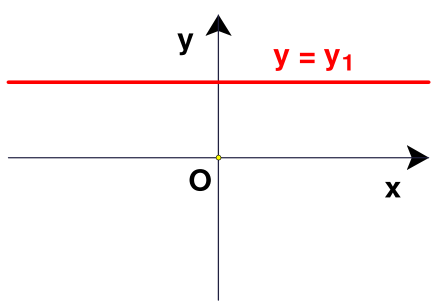
$$T(x, y) = T\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

eksplicitni oblik jednadžbe pravca

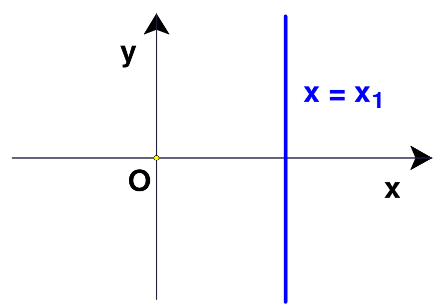


$y = k \cdot x + l$
k – koeficijent smjera
α – prikloni kut
k = tg α
l – odsječak na osi y

jednadžba pravca usporednog (paralelnog) s x-osi, $y = y_1$



jednadžba pravca usporednog (paralelnog) s y-osi, $x = x_1$



implicitni oblik jednadžbe pravca

$$Ax + By + C = 0$$

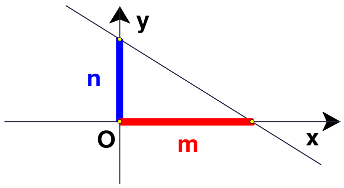
jednadžba pravca zadanog koeficijentom smjera i jednom točkom

$$\left.\begin{array}{c} k \\ T(x_1, y_1) \end{array}\right\} \Rightarrow y - y_1 = k \cdot (x - x_1)$$

jednadžba pravca kroz dvije točke

$$\left.\begin{array}{c} A(x_1, y_1) \\ B(x_2, y_2) \end{array}\right\} \Rightarrow y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) \ , \ k = \frac{y_2 - y_1}{x_2 - x_1} \ , \ l = y_1 - \frac{y_2 - y_1}{x_2 - x_1} \cdot x_1$$

segmentni oblik jednadžbe pravca



$$\frac{x}{m}+\frac{y}{n}=1$$

m – odsječak (segment) na x – osi

n – odsječak (segment) na y – osi

vektori smjera

$y=k\cdot x+l \Rightarrow$ vektori smjera tog pravca kolinearni su s vektorom $\vec{s}=\vec{i}+k\cdot\vec{j}$

$Ax+By+C=0 \Rightarrow$ vektori smjera tog pravca kolinearni su s vektorom $\vec{s}=B\cdot\vec{i}-A\cdot\vec{j}$

jednadžba pravca zadanog točkom $T(x_0, y_0)$ i vektorom smjera $\vec{s}=A\cdot\vec{i}+B\cdot\vec{j}$

$$B\cdot(x-x_0)-A\cdot(y-y_0)=0$$

okomica točkom $T(x_0, y_0)$ uz vektor smjera $\vec{s}=A\cdot\vec{i}+B\cdot\vec{j}$

$$A\cdot(x-x_0)+B\cdot(y-y_0)=0$$

kut dva pravca

$$\left.\begin{matrix}A_1x+B_1y+C_1=0\\A_2x+B_2y+C_2=0\end{matrix}\right\}\Rightarrow \cos\varphi=\frac{|A_1\cdot A_2+B_1\cdot B_2|}{\sqrt{A_1^2+B_1^2}\cdot\sqrt{A_2^2+B_2^2}} \;,\; \left.\begin{matrix}y=k_1x+l_1\\y=k_2x+l_2\end{matrix}\right\}\Rightarrow tg\,\varphi=\left|\frac{k_2-k_1}{1+k_1\cdot k_2}\right|$$

uvjet okomitosti

$$\left.\begin{matrix}A_1x+B_1y+C_1=0\\A_2x+B_2y+C_2=0\end{matrix}\right\}\Rightarrow A_1\cdot A_2+B_1\cdot B_2=0 \;,\; \left.\begin{matrix}y=k_1x+l_1\\y=k_2x+l_2\end{matrix}\right\}\Rightarrow k_1=-\frac{1}{k_2}$$

$$\left.\begin{matrix}\frac{x}{m_1}+\frac{y}{n_1}=1\\\frac{x}{m_1}+\frac{y}{n_1}=1\end{matrix}\right\}\Rightarrow m_1\cdot m_2+n_1\cdot n_2=0$$

uvjet usporednosti (paralelnosti)

$$\left.\begin{matrix}A_1x+B_1y+C_1=0\\A_2x+B_2y+C_2=0\end{matrix}\right\}\Rightarrow \frac{A_1}{A_2}=\frac{B_1}{B_2} \;,\; \left.\begin{matrix}y=k_1x+l_1\\y=k_2x+l_2\end{matrix}\right\}\Rightarrow k_1=k_2 \;,\; \left.\begin{matrix}\frac{x}{m_1}+\frac{y}{n_1}=1\\\frac{x}{m_1}+\frac{y}{n_1}=1\end{matrix}\right\}\Rightarrow \frac{m_1}{m_2}=\frac{n_1}{n_2}$$

udaljenost točke od pravca

$$\left.\begin{matrix}Ax+By+C=0\\T(x_0,y_0)\end{matrix}\right\}\Rightarrow d=\frac{|A\cdot x_0+B\cdot y_0+C|}{\sqrt{A^2+B^2}}$$

udaljenost para usporednih (paralelnih) pravaca

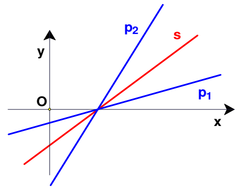
$$\left.\begin{matrix}Ax+By+C_1=0\\Ax+By+C_2=0\end{matrix}\right\}\Rightarrow d=\frac{|C_2-C_1|}{\sqrt{A^2+B^2}}$$

pramen pravaca

jednadžba svakog pravca koji prolazi kroz sjecište dvaju pravaca

$$\left.\begin{matrix}A_1x+B_1y+C_1=0\\A_2x+B_2y+C_2=0\end{matrix}\right\}\Rightarrow (A_1x+B_1y+C_1)+\lambda\cdot(A_2x+B_2y+C_2)=0 \;,\; \lambda\in R$$

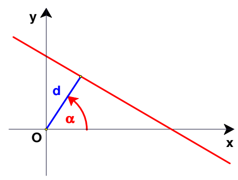
simetrala para pravaca



$$\left.\begin{aligned} A_1x+B_1y+C_1=0 \\ A_2x+B_2y+C_2=0 \end{aligned}\right\} \Rightarrow \frac{A_1x+B_1y+C_1}{\sqrt{A_1^2+B_1^2}}=\pm\frac{A_2x+B_2y+C_2}{\sqrt{A_2^2+B_2^2}}$$

normalni oblik jednadžbe pravca

$$\frac{Ax+By+C}{(-\operatorname{sgn} C)\cdot\sqrt{A^2+B^2}}=0 \text{ ili } \frac{Ax+By+C}{\pm\sqrt{A^2+B^2}}=0 \text{ (predznak mora biti suprotan predznaku od C)}$$



$$x\cdot\cos\alpha+y\cdot\sin\alpha=d$$

d – udaljenost pravca od ishodišta

α – kut koji zatvara okomica d sa pozitivnim smjerom osi x

djelište dužine

ako točka T(x, y) dijeli dužinu $\overline{AB}$ u omjeru $\frac{|AT|}{|TB|}=-\lambda$, tada su koordinate djelišta

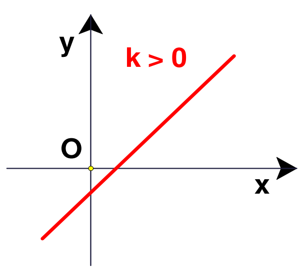
$$x=\frac{x_1-\lambda\cdot x_2}{1-\lambda}\ ,\ y=\frac{y_1-\lambda\cdot y_2}{1-\lambda}$$

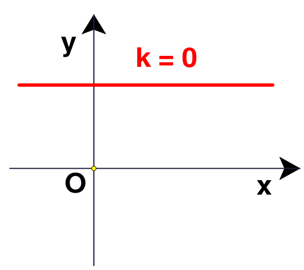
1. $\lambda>0$ => točka T je izvan dužine $\overline{AB}$
2. $\lambda<0$ => točka T je unutar dužine $\overline{AB}$
3. $\lambda=-1$ => točka T raspolavlja dužinu $\overline{AB}$ pa je $x=\frac{x_1+x_2}{2}$, $y=\frac{y_1+y_2}{2}$

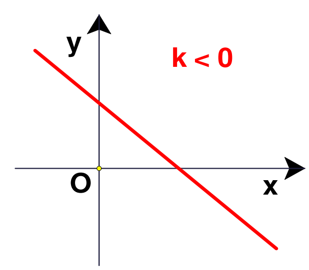
linearna (afina) funkcija

neka su k i l realni brojevi, $k\neq 0$, tada se funkcija f(x) = k · x + l naziva linearna (afina) funkcija
graf linearne (afine) funkcije je pravac y = k · x + l

$k>0 \Rightarrow$ funkcija raste , $k=0 \Rightarrow$ konstantna je , $k<0 \Rightarrow$ funkcija pada







crtanje

$$\left.\begin{aligned} f(x)&=2\cdot x-3 \\ f(0)&=2\cdot 0-3=-3 \\ f(1)&=2\cdot 1-3=-1 \end{aligned}\right\} \Rightarrow$$

x	0	1
y	−3	−1

$$\left.\begin{aligned} g(x)&=-2\cdot x+3 \\ g(0)&=-2\cdot 0+3=3 \\ g(1)&=-2\cdot 1+3=1 \end{aligned}\right\} \Rightarrow$$

x	0	1
y	3	1