

Zadnjih deset zadataka

I.

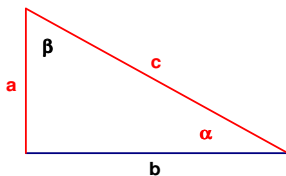
Zadatak 170 (Petra, srednja škola)

Odredi duljinu visine na hipotenuzu pravokutnog trokuta ako je poznato: $b = 223.5 \text{ cm}$, $\beta = 38^\circ 30'$.

Rješenje 170

Ponovimo!

Pravokutan trokut



Sinus šiljastog kuta pravokutnog trokuta jednak je omjeru duljine katete nasuprot toga kuta i duljine hipotenuze.

$$\sin \alpha = \frac{a}{c}, \quad \alpha + \beta = 90^\circ.$$

$$\beta = 38^\circ 30'$$

$$b = 223.5 \text{ cm}$$

$$v_c = ?$$

Nademo kut α :

$$\left. \begin{array}{l} \beta = 38^\circ 30' \\ \alpha + \beta = 90^\circ \end{array} \right\} \Rightarrow \left. \begin{array}{l} \beta = 38^\circ 30' \\ \alpha = 90^\circ - \beta \end{array} \right\} \Rightarrow \alpha = 90^\circ - 38^\circ 30' \Rightarrow \alpha = 89^\circ 60' - 38^\circ 30' \Rightarrow \alpha = 51^\circ 30'.$$

Sa slika vidi se:

$$\sin \alpha = \frac{v_c}{b} \Rightarrow \sin \alpha = \frac{v_c}{b} \cdot b \Rightarrow v_c = b \cdot \sin \alpha \Rightarrow v_c = 223.5 \text{ cm} \cdot \sin 51^\circ 30' \Rightarrow v_c = 174.91 \text{ cm}.$$

Vježba 170

Odredi duljinu visine na hipotenuzu pravokutnog trokuta ako je poznato: $b = 40 \text{ cm}$, $\beta = 60^\circ$.

Rezultat: 20 cm.

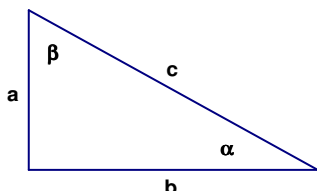
II.

Zadatak 169 (Petra, srednja škola)

Odredi preostale elemente pravokutnog trokuta ako je: $\alpha = 57^\circ 30'$, $P = 44.8 \text{ cm}^2$.

Rješenje 169

Ponovimo!



Pravokutan trokut

$$\alpha + \beta = 90^0, \quad P = \frac{1}{4} \cdot c^2 \cdot \sin 2\alpha.$$

$$\sin \alpha = \frac{a}{c}, \quad \cos \alpha = \frac{b}{c}.$$

$$O = a + b + c.$$

$$\alpha = 57^0 30'$$

$$P = 44.8 \text{ cm}^2$$

Najprije izračunamo kut β :

$$\beta, a, b, c, O = ?$$

$$\left. \begin{array}{l} \alpha = 57^0 30' \\ \alpha + \beta = 90^0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \alpha = 57^0 30' \\ \beta = 90^0 - \alpha \end{array} \right\} \Rightarrow \beta = 90^0 - 57^0 30' \Rightarrow \beta = 89^0 60' - 57^0 30' \Rightarrow \beta = 32^0 30'.$$

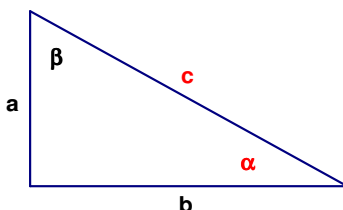
Iz formule za površinu pravokutnog trokuta dobije se duljina hipotenuze c :

$$P = \frac{1}{4} \cdot c^2 \cdot \sin 2\alpha \Rightarrow P = \frac{1}{4} \cdot c^2 \cdot \sin 2\alpha \quad / \cdot \frac{4}{\sin 2\alpha} \Rightarrow c^2 = \frac{4 \cdot P}{\sin 2\alpha} \quad / \sqrt{\quad} \Rightarrow c = \sqrt{\frac{4 \cdot P}{\sin 2\alpha}} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \alpha = 57^0 30' \\ 2 \cdot \alpha = 2 \cdot 57^0 30' \end{array} \right] \Rightarrow \left[2 \cdot \alpha = 114^0 60' \right] \Rightarrow \left[2 \cdot \alpha = 115^0 \right] \Rightarrow$$

$$\Rightarrow c = \sqrt{\frac{4 \cdot 44.8 \text{ cm}^2}{\sin 115^0}} \Rightarrow c = 14.06 \text{ cm}.$$

Računamo duljine kateta a i b pomoću funkcija sinus i kosinus:



$$\left. \begin{array}{l} \sin \alpha = \frac{a}{c} \\ \cos \alpha = \frac{b}{c} \end{array} \right\} \Rightarrow \left. \begin{array}{l} \sin \alpha = \frac{a}{c} \quad / \cdot c \\ \cos \alpha = \frac{b}{c} \quad / \cdot c \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = c \cdot \sin \alpha \\ b = c \cdot \cos \alpha \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = 14.06 \text{ cm} \cdot \sin 57^0 30' \\ b = 14.06 \text{ cm} \cdot \cos 57^0 30' \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = 11.86 \text{ cm} \\ b = 7.55 \text{ cm} \end{array} \right\}.$$

Opseg pravokutnog trokuta iznosi:

$$\left. \begin{array}{l} a = 11.86 \text{ cm} \\ b = 7.55 \text{ cm} \\ c = 14.06 \text{ cm} \\ O = a + b + c \end{array} \right\} \Rightarrow O = 11.86 \text{ cm} + 7.55 \text{ cm} + 14.06 \text{ cm} \Rightarrow O = 33.47 \text{ cm}.$$

Vježba 169

Odredi duljinu hipotenuze pravokutnog trokuta ako je: $\alpha = 45^\circ$, $P = 8 \text{ cm}^2$.

Rezultat: $4 \cdot \sqrt{2} \text{ cm}$.

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III.

Zadatak 049 (Malena, gimnazija)

Riješi u skupu \mathbb{R} nejednadžbu i skup rješenja skiciraj na brojevnom pravcu: $\frac{1}{x} > 5$.

Rješenje 049

Ponovimo!

$$a > b, c > 0 \Rightarrow a \cdot c > b \cdot c, \quad a > b, c < 0 \Rightarrow a \cdot c < b \cdot c, \quad a \in \mathbb{R} \setminus \{0\} \Rightarrow a^2 > 0.$$

$$a \cdot b > 0 \Leftrightarrow \left. \begin{array}{l} a > 0 \\ b > 0 \end{array} \right\} \text{ ili } \left. \begin{array}{l} a < 0 \\ b < 0 \end{array} \right\}, \quad \frac{a}{b} > 0 \Leftrightarrow \left. \begin{array}{l} a > 0 \\ b > 0 \end{array} \right\} \text{ ili } \left. \begin{array}{l} a < 0 \\ b < 0 \end{array} \right\}.$$

1. inačica

Uočimo da x ne smije biti jednak nuli:

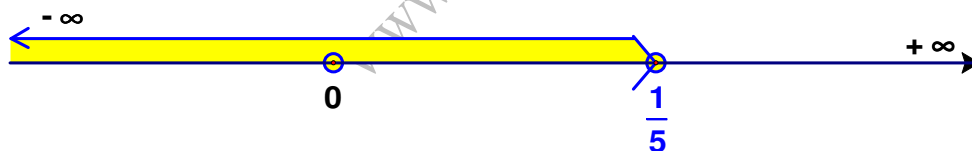
$$x \neq 0. \\ \frac{1}{x} > 5 \Rightarrow \frac{1}{x} - 5 > 0 \Rightarrow \frac{1 - 5 \cdot x}{x} > 0.$$

Ova nejednakost ispunjena je u dva slučaja.

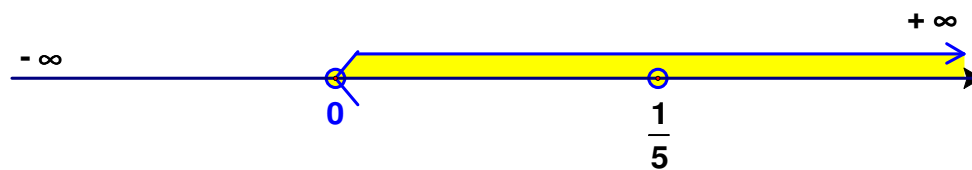
1. slučaj

$$\left. \begin{array}{l} \frac{1 - 5 \cdot x}{x} > 0 \\ x > 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 1 - 5 \cdot x > 0 \\ x > 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} -5 \cdot x > -1 \\ x > 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} -5 \cdot x > -1 / : (-5) \\ x > 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x < \frac{1}{5} \\ x > 0 \end{array} \right\} \Rightarrow \\ \Rightarrow \left[\begin{array}{l} \text{tražimo presjek,} \\ \text{zajednički dio} \end{array} \right] \Rightarrow x \in \left\langle 0, \frac{1}{5} \right\rangle \Rightarrow 0 < x < \frac{1}{5}.$$

$$x < \frac{1}{5}$$



$$x > 0$$

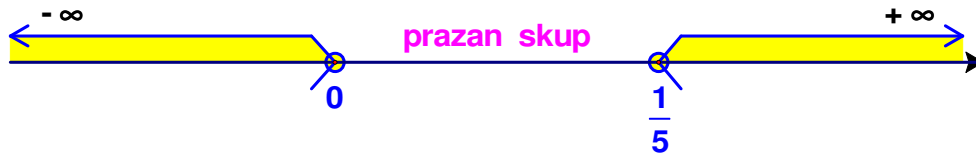


$$x \in \left\langle 0, \frac{1}{5} \right\rangle \Rightarrow 0 < x < \frac{1}{5}$$



2. slučaj

$$\left. \begin{aligned} \frac{1-5 \cdot x}{x} > 0 &\Rightarrow \begin{cases} 1-5 \cdot x < 0 \\ x < 0 \end{cases} \Rightarrow \begin{cases} -5 \cdot x < -1 \\ x < 0 \end{cases} \Rightarrow \begin{cases} -5 \cdot x < -1 /: (-5) \\ x < 0 \end{cases} \Rightarrow \begin{cases} x > \frac{1}{5} \\ x < 0 \end{cases} \Rightarrow \\ \Rightarrow \left[\begin{array}{l} \text{tražimo presjek,} \\ \text{zajednički dio} \end{array} \right] \Rightarrow \emptyset, \text{ prazan skup.} \end{aligned}$$



Rješenje je, dakle,

$$x \in \left\langle 0, \frac{1}{5} \right\rangle.$$

2. inačica

Uočimo da x ne smije biti jednak nuli:

$$x \neq 0.$$

$$\frac{1}{x} > 5 \Rightarrow \left[\begin{array}{l} \text{množimo nejednakost} \\ \text{pozitivnim brojem } x^2 \end{array} \right] \Rightarrow \frac{1}{x} > 5 /: x^2 \Rightarrow x > 5 \cdot x^2 \Rightarrow x - 5 \cdot x^2 > 0 \Rightarrow \\ \Rightarrow x \cdot (1 - 5 \cdot x) > 0.$$

Ova nejednakost ispunjena je u dva slučaja.

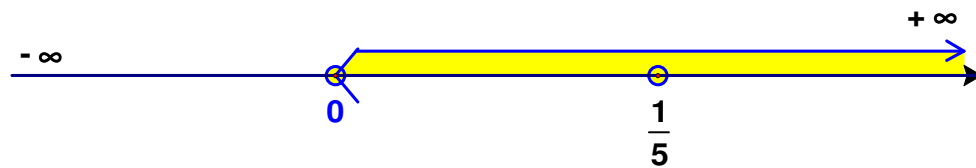
1. slučaj

$$\left. \begin{aligned} x \cdot (1 - 5 \cdot x) > 0 &\Rightarrow \begin{cases} 1 - 5 \cdot x > 0 \\ x > 0 \end{cases} \Rightarrow \begin{cases} -5 \cdot x > -1 \\ x > 0 \end{cases} \Rightarrow \begin{cases} -5 \cdot x > -1 /: (-5) \\ x > 0 \end{cases} \Rightarrow \begin{cases} x < \frac{1}{5} \\ x > 0 \end{cases} \Rightarrow \\ \Rightarrow \left[\begin{array}{l} \text{tražimo presjek,} \\ \text{zajednički dio} \end{array} \right] \Rightarrow x \in \left\langle 0, \frac{1}{5} \right\rangle \Rightarrow 0 < x < \frac{1}{5}. \end{aligned}$$

$$x < \frac{1}{5}$$



$$x > 0$$



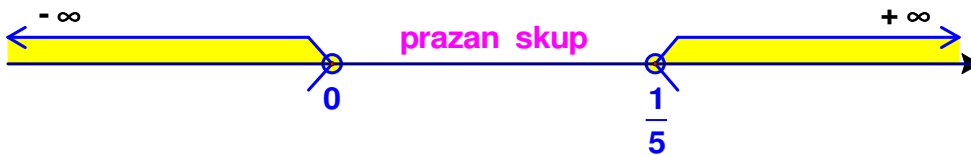
$$x \in \left\langle 0, \frac{1}{5} \right\rangle \Rightarrow 0 < x < \frac{1}{5}$$



2. slučaj

$$x \cdot (1 - 5 \cdot x) > 0 \Rightarrow \left. \begin{array}{l} 1 - 5 \cdot x < 0 \\ x < 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} -5 \cdot x < -1 \\ x < 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} -5 \cdot x < -1 \quad /: (-5) \\ x < 0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x > \frac{1}{5} \\ x < 0 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{tražimo presjek,} \\ \text{zajednički dio} \end{array} \right] \Rightarrow \emptyset, \text{ prazan skup.}$$



Rješenje je, dakle,

$$x \in \left\langle 0, \frac{1}{5} \right\rangle.$$

Vježba 049

Riješi u skupu \mathbb{R} nejednadžbu i skup rješenja skiciraj na brojevnom pravcu: $\frac{1}{x} > 3$.

Rezultat: $x \in \left\langle 0, \frac{1}{3} \right\rangle.$

IV.

Zadatak 029 (Mirza, elektrotehnička škola)

Riješi iracionalnu nejednadžbu: $\sqrt{x^2 + 3 \cdot x + 3} < 2 \cdot x + 1$.

Rješenje 029

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2, \quad (\sqrt[n]{a})^n = a.$$

$$\left. \begin{array}{l} f(x) \geq 0 \\ \text{Iracionalna nejednadžba } \sqrt{f(x)} < g(x) \Leftrightarrow g(x) > 0 \\ f(x) < (g(x))^2 \end{array} \right\} \text{ sustav nejednadžbi}$$

Općenito:

$$\left. \begin{array}{l} f(x) \geq 0 \\ \text{Iracionalna nejednadžba } \sqrt[2 \cdot n]{f(x)} < g(x) \Leftrightarrow g(x) > 0 \\ f(x) < (g(x))^{2 \cdot n} \end{array} \right\} \text{ sustav nejednadžbi}$$

Iracionalna nejednadžba je nejednadžba u kojoj se nepoznanica x pojavljuje pod znakom korijena (odnosno s nekim racionalnim eksponentom).

Funkcija $f(x) = \sqrt{x}$ definirana je na skupu nenegativnih realnih brojeva, $R^+ \cup \{0\}$. To znači da negativni realni brojevi ne mogu biti rješenja iracionalne jednadžbe $\sqrt{x} = a$. To vrijedi za sve korijene parnog eksponenta.

Postavimo uvjete zadane iracionalne nejednadžbe:

$$\left. \begin{array}{l} x^2 + 3 \cdot x + 3 \geq 0 \quad \text{1. uvjet} \\ \sqrt{x^2 + 3 \cdot x + 3} < 2 \cdot x + 1 \Leftrightarrow 2 \cdot x + 1 > 0 \quad \text{2. uvjet} \\ x^2 + 3 \cdot x + 3 < (2 \cdot x + 1)^2 \quad \text{3. uvjet} \end{array} \right\}$$

1. uvjet

$$x^2 + 3 \cdot x + 3 \geq 0.$$

Najprije riješimo kvadratnu jednadžbu:

$$\left. \begin{array}{l} x^2 + 3 \cdot x + 3 = 0 \Rightarrow \left. \begin{array}{l} x^2 + 3 \cdot x + 3 = 0 \\ a = 1, b = 3, c = 3 \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = 1, b = 3, c = 3 \\ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array} \right\} \Rightarrow \\ \Rightarrow x_{1,2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} \Rightarrow x_{1,2} = \frac{-3 \pm \sqrt{9 - 12}}{2} \Rightarrow x_{1,2} = \frac{-3 \pm \sqrt{-3}}{2} \Rightarrow \left. \begin{array}{l} \text{nema realnih rješenja} \\ \text{nema nultočka} \end{array} \right\}$$

Odaberemo bilo koji realan broj x , na primjer $x = 0$ i uvrstimo ga u $x^2 + 3 \cdot x + 3$:

$$0^2 + 3 \cdot 0 + 3 = 0 + 0 + 3 = 3 > 0.$$

Rezultat je pozitivan, što znači da je $x^2 + 3 \cdot x + 3$ pozitivan na cijelom intervalu $x \in \langle -\infty, +\infty \rangle$.

2.uvjet

$$2 \cdot x + 1 > 0 \Rightarrow 2 \cdot x > -1 \quad / : 2 \Rightarrow x > -\frac{1}{2} \Rightarrow x \in \left\langle -\frac{1}{2}, +\infty \right\rangle.$$

3.uvjet

$$\begin{aligned} x^2 + 3 \cdot x + 3 < (2 \cdot x + 1)^2 &\Rightarrow x^2 + 3 \cdot x + 3 < 4 \cdot x^2 + 4 \cdot x + 1 \Rightarrow x^2 + 3 \cdot x + 3 - 4 \cdot x^2 - 4 \cdot x - 1 < 0 \Rightarrow \\ &\Rightarrow -3 \cdot x^2 - x + 2 < 0 \Rightarrow -3 \cdot x^2 - x + 2 < 0 \quad / \cdot (-1) \Rightarrow 3 \cdot x^2 + x - 2 > 0. \end{aligned}$$

Najprije riješimo kvadratnu jednadžbu:

$$\begin{aligned} 3 \cdot x^2 + x - 2 = 0 &\Rightarrow \left. \begin{array}{l} 3 \cdot x^2 + x - 2 = 0 \\ a = 3, b = 1, c = -2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = 3, b = 1, c = -2 \\ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array} \right\} \Rightarrow \\ \Rightarrow x_{1,2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 3 \cdot (-2)}}{2 \cdot 3} &\Rightarrow x_{1,2} = \frac{-1 \pm \sqrt{1 + 24}}{6} \Rightarrow x_{1,2} = \frac{-1 \pm \sqrt{25}}{6} \Rightarrow x_{1,2} = \frac{-1 \pm 5}{6} \Rightarrow \\ &\Rightarrow \left. \begin{array}{l} x_1 = \frac{-1 + 5}{6} \\ x_2 = \frac{-1 - 5}{6} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_1 = \frac{4}{6} \\ x_2 = -\frac{6}{6} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_1 = \frac{2}{3} \\ x_2 = -1 \end{array} \right\}. \end{aligned}$$

Nultočke -1 i $\frac{2}{3}$ ucrtamo na x osi. U točkama -1 i $\frac{2}{3}$ vrijedi jednakost = pa one nisu rješenje naše stroge nejednakosti $>$. Zato ih nismo popunili (kružići su prazni).

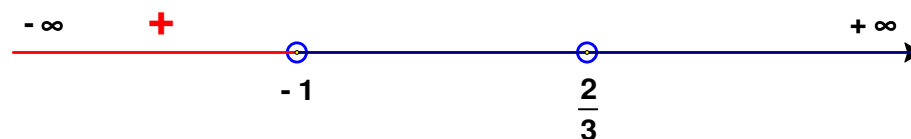


- Odaberemo x između $-\infty$ i -1 , na primjer, $x = -2$ i uvrstimo ga u $3 \cdot x^2 + x - 2$:

$$3 \cdot (-2)^2 + (-2) - 2 = 3 \cdot 4 - 2 - 2 = 12 - 2 - 2 = 8 > 0.$$

Rezultat je pozitivan, što znači da je $3 \cdot x^2 + x - 2$ pozitivan na cijelom intervalu $\langle -\infty, -1 \rangle$.

Upišimo simbol **+** iznad tog intervala.



- Odaberemo x veći od -1 , a manji od $\frac{2}{3}$, na primjer, $x = 0$ i uvrstimo ga u $3 \cdot x^2 + x - 2$:

$$3 \cdot 0^2 + 0 - 2 = 3 \cdot 0 + 0 - 2 = 0 + 0 - 2 = -2 < 0.$$

Rezultat je negativan, što znači da je $3 \cdot x^2 + x - 2$ negativan na cijelom intervalu $\langle -1, \frac{2}{3} \rangle$.

Upišimo simbol **-** iznad tog intervala.

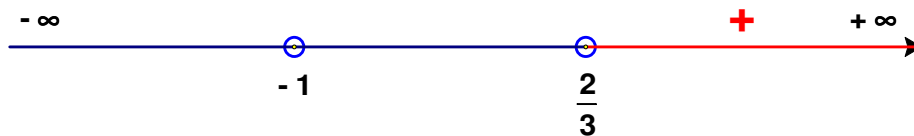


- Odaberemo x veći od $\frac{2}{3}$, na primjer $x = 1$ i uvrstimo ga u $3 \cdot x^2 + x - 2$:

$$3 \cdot 1^2 + 1 - 2 = 3 \cdot 1 + 1 - 2 = 3 + 1 - 2 = 2 > 0.$$

Rezultat je pozitivan, što znači da je $3 \cdot x^2 + x - 2$ pozitivan na cijelom intervalu $\left\langle \frac{2}{3}, +\infty \right\rangle$.

Upišimo simbol **+** iznad tog intervala.



Dakle, nejednadžba $3 \cdot x^2 + x - 2 > 0$ vrijedi za $x \in \langle -\infty, -1 \rangle \cup \left\langle \frac{2}{3}, +\infty \right\rangle$.



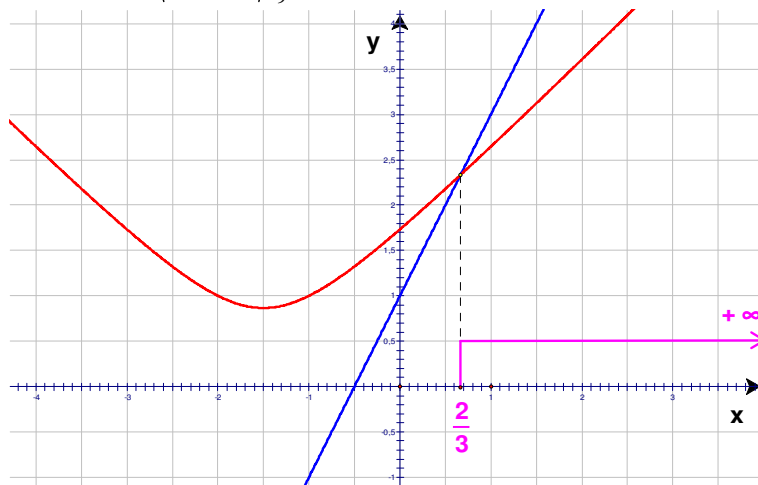
Rješenje zadane iracionalne nejednadžbe $\sqrt{x^2 + 3 \cdot x + 3} < 2 \cdot x + 1$ mora ispuniti sva tri uvjeta:

1. uvjet $x \in \langle -\infty, +\infty \rangle$

2. uvjet $x \in \left\langle -\frac{1}{2}, +\infty \right\rangle$

3. uvjet $x \in \langle -\infty, -1 \rangle \cup \left\langle \frac{2}{3}, +\infty \right\rangle$

$$\Rightarrow \left[\begin{array}{l} \text{presjek sva tri rješenja,} \\ \text{zajednički dio sva tri uvjeta} \end{array} \right] \Rightarrow x \in \left\langle \frac{2}{3}, +\infty \right\rangle.$$



Vježba 029

Riješi iracionalnu nejednadžbu: $\sqrt{x^2 + 3 \cdot x + 3} > 2 \cdot x + 1$.

Rezultat: $x \in \left\langle -\infty, \frac{2}{3} \right\rangle$.

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Zadatak 167 (YOYO, srednja škola)

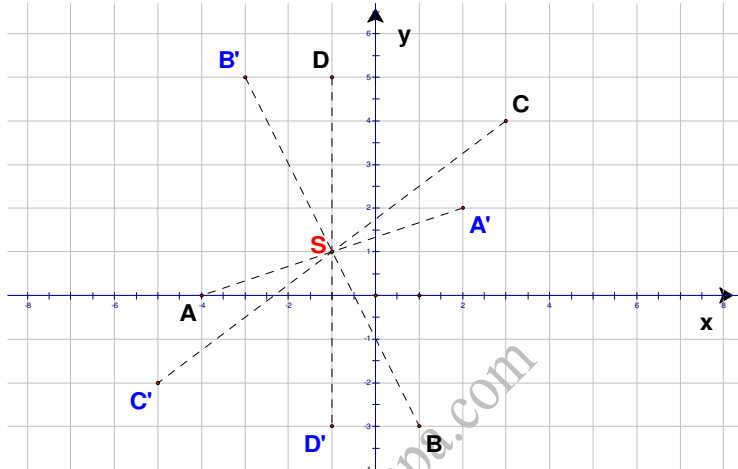
Odredi koordinate točkama koje su simetrične točkama $A(-4, 0)$, $B(1, -3)$, $C(3, 4)$ i $D(-1, 5)$ s obzirom na točku $S(-1, 1)$.

Rješenje 167

Ponovimo!

Ako točka $P(x, y)$ raspolažlja dužinu \overline{AB} , $A(x_1, y_1)$, $B(x_2, y_2)$, tada je:

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}.$$



Neka je točka A' simetrična točki A s obzirom na točku S . Dakle, točka S je polovište dužine $\overline{AA'}$ pa slijedi:

$$\left. \begin{array}{l} A(x_1, y_1) = A(-4, 0) \\ S(x, y) = S(-1, 1) \\ A'(x_2, y_2) = A'(? , ?) \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{x_1 + x_2}{2} = x \\ \frac{y_1 + y_2}{2} = y \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{-4 + x_2}{2} = -1 \\ \frac{0 + y_2}{2} = 1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{-4 + x_2}{2} = -1 \quad / \cdot 2 \\ \frac{0 + y_2}{2} = 1 \quad / \cdot 2 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} -4 + x_2 = -2 \\ y_2 = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_2 = -2 + 4 \\ y_2 = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_2 = 2 \\ y_2 = 2 \end{array} \right\} \Rightarrow A'(2, 2).$$

Neka je točka B' simetrična točki B s obzirom na točku S . Dakle, točka S je polovište dužine $\overline{BB'}$ pa slijedi:

$$\left. \begin{array}{l} B(x_1, y_1) = B(1, -3) \\ S(x, y) = S(-1, 1) \\ B'(x_2, y_2) = B'(? , ?) \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{x_1 + x_2}{2} = x \\ \frac{y_1 + y_2}{2} = y \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{1 + x_2}{2} = -1 \\ \frac{-3 + y_2}{2} = 1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{1 + x_2}{2} = -1 \quad / \cdot 2 \\ \frac{-3 + y_2}{2} = 1 \quad / \cdot 2 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} 1 + x_2 = -2 \\ -3 + y_2 = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_2 = -2 - 1 \\ y_2 = 2 + 3 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_2 = -3 \\ y_2 = 5 \end{array} \right\} \Rightarrow B'(-3, 5).$$

Neka je točka C' simetrična točki C s obzirom na točku S . Dakle, točka S je polovište dužine $\overline{CC'}$ pa slijedi:

$$\begin{aligned}
& \left. \begin{array}{l} C(x_1, y_1) = C(3, 4) \\ S(x, y) = S(-1, 1) \\ C'(x_2, y_2) = C'(? , ?) \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{x_1 + x_2}{2} = x \\ \frac{y_1 + y_2}{2} = y \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{3 + x_2}{2} = -1 \\ \frac{4 + y_2}{2} = 1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{3 + x_2}{2} = -1 \quad / \cdot 2 \\ \frac{4 + y_2}{2} = 1 \quad / \cdot 2 \end{array} \right\} \Rightarrow \\
& \Rightarrow \left. \begin{array}{l} 3 + x_2 = -2 \\ 4 + y_2 = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_2 = -2 - 3 \\ y_2 = 2 - 4 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_2 = -5 \\ y_2 = -2 \end{array} \right\} \Rightarrow C'(-5, -2).
\end{aligned}$$

Neka je točka D' simetrična točki D s obzirom na točku S . Dakle, točka S je polovište dužine $\overline{DD'}$ pa slijedi:

$$\begin{aligned}
& \left. \begin{array}{l} D(x_1, y_1) = D(-1, 5) \\ S(x, y) = S(-1, 1) \\ D'(x_2, y_2) = D'(? , ?) \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{x_1 + x_2}{2} = x \\ \frac{y_1 + y_2}{2} = y \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{-1 + x_2}{2} = -1 \\ \frac{5 + y_2}{2} = 1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{-1 + x_2}{2} = -1 \quad / \cdot 2 \\ \frac{5 + y_2}{2} = 1 \quad / \cdot 2 \end{array} \right\} \Rightarrow \\
& \Rightarrow \left. \begin{array}{l} -1 + x_2 = -2 \\ 5 + y_2 = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_2 = -2 + 1 \\ y_2 = 2 - 5 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_2 = -1 \\ y_2 = -3 \end{array} \right\} \Rightarrow D'(-1, -3).
\end{aligned}$$

Vježba 167

Odredi koordinate točke koja je simetrična točki $A(2, 2)$ s obzirom na točku $S(-1, 1)$.

Rezultat: $A'(-4, 0)$.

VI.

Zadatak 168 (YOYO, srednja škola)

Kolike su duljine srednjica trokuta ABC ako su vrhovi trokuta točke: $A(-3, 1)$, $B(3, -5)$, $C(5, 7)$?

Rješenje 168

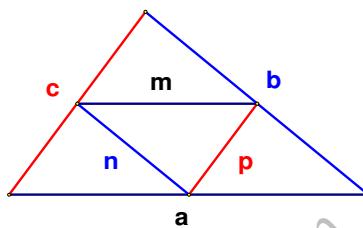
Ponovimo!

Ako točka $P(x, y)$ raspolavlja dužinu \overline{AB} , $A(x_1, y_1)$, $B(x_2, y_2)$, tada je:

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}.$$

Srednjice trokuta

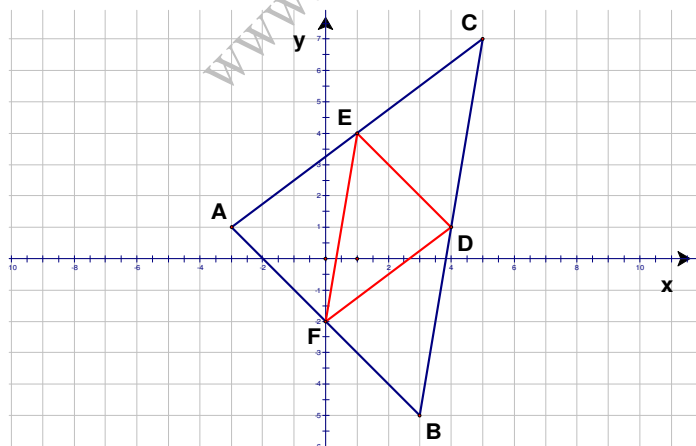
Dužine koje spajaju polovišta stranica trokuta zovu se srednjice trokuta. Svaki trokut ima tri srednjice. Svaka srednjica trokuta usporedna je sa suprotnom stranicom trokuta, a duljina joj je jednaka polovici duljine te stranice.



$$a \parallel m, \quad a = 2 \cdot m, \quad b \parallel n, \quad b = 2 \cdot n, \quad c \parallel p, \quad c = 2 \cdot p.$$

Udaljenost točaka $A(x_A, y_A)$ i $B(x_B, y_B)$:

$$|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}.$$



1. inačica

Točka F je polovište dužine \overline{AB} pa njezine koordinate glase:

$$\left. \begin{array}{l} A(x_1, y_1) = A(-3, 1) \\ F(x, y) \\ B(x_2, y_2) = B(3, -5) \end{array} \right\} \Rightarrow \left[\begin{array}{l} x = \frac{x_1 + x_2}{2} \\ y = \frac{y_1 + y_2}{2} \end{array} \right] \Rightarrow \left. \begin{array}{l} x = \frac{-3 + 3}{2} \\ y = \frac{1 + (-5)}{2} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \frac{0}{2} \\ y = \frac{1 - 5}{2} \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} x = \frac{0}{2} \\ y = -\frac{4}{2} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = 0 \\ y = -2 \end{array} \right\} \Rightarrow F(0, -2).$$

Točka D je polovište dužine \overline{BC} pa njezine koordinate glase:

$$\left. \begin{array}{l} B(x_1, y_1) = B(3, -5) \\ D(x, y) \\ C(x_2, y_2) = C(5, 7) \end{array} \right\} \Rightarrow \left[\begin{array}{l} x = \frac{x_1 + x_2}{2} \\ y = \frac{y_1 + y_2}{2} \end{array} \right] \Rightarrow \left. \begin{array}{l} x = \frac{3+5}{2} \\ y = \frac{-5+7}{2} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \frac{8}{2} \\ y = \frac{2}{2} \end{array} \right\} \Rightarrow \\ \Rightarrow \left. \begin{array}{l} x = 4 \\ y = 1 \end{array} \right\} \Rightarrow D(4, 1).$$

Točka E je polovište dužine \overline{CA} pa njezine koordinate glase:

$$\left. \begin{array}{l} C(x_1, y_1) = C(5, 7) \\ E(x, y) \\ A(x_2, y_2) = A(-3, 1) \end{array} \right\} \Rightarrow \left[\begin{array}{l} x = \frac{x_1 + x_2}{2} \\ y = \frac{y_1 + y_2}{2} \end{array} \right] \Rightarrow \left. \begin{array}{l} x = \frac{5+(-3)}{2} \\ y = \frac{7+1}{2} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \frac{5-3}{2} \\ y = \frac{8}{2} \end{array} \right\} \Rightarrow \\ \Rightarrow \left. \begin{array}{l} x = \frac{2}{2} \\ y = \frac{8}{2} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = 1 \\ y = 4 \end{array} \right\} \Rightarrow E(1, 4).$$

Duljine srednjica trokuta iznose:

$$\left. \begin{array}{l} F(x_1, y_1) = F(0, -2) \\ \bullet D(x_2, y_2) = D(4, 1) \\ |FD| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{array} \right\} \Rightarrow |FD| = \sqrt{(4-0)^2 + (1-(-2))^2} \Rightarrow \\ \Rightarrow |FD| = \sqrt{(4-0)^2 + (1+2)^2} \Rightarrow |FD| = \sqrt{4^2 + 3^2} \Rightarrow |FD| = \sqrt{16+9} \Rightarrow \\ \Rightarrow |FD| = \sqrt{25} \Rightarrow |FD| = 5.$$

$$\left. \begin{array}{l} D(x_1, y_1) = D(4, 1) \\ \bullet E(x_2, y_2) = E(1, 4) \\ |DE| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{array} \right\} \Rightarrow |DE| = \sqrt{(1-4)^2 + (4-1)^2} \Rightarrow \\ \Rightarrow |DE| = \sqrt{(-3)^2 + 3^2} \Rightarrow |DE| = \sqrt{9+9} \Rightarrow |DE| = \sqrt{18} \Rightarrow \left[\begin{array}{l} \text{djelomično} \\ \text{korjenovanje} \end{array} \right] \Rightarrow \\ \Rightarrow |DE| = \sqrt{9 \cdot 2} \Rightarrow |DE| = 3 \cdot \sqrt{2}.$$

$$\begin{aligned}
 & \left. \begin{array}{l} E(x_1, y_1) = E(1, 4) \\ \bullet F(x_2, y_2) = F(0, -2) \\ |EF| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{array} \right\} \Rightarrow |EF| = \sqrt{(0-1)^2 + (-2-4)^2} \Rightarrow \\
 & \Rightarrow |EF| = \sqrt{(-1)^2 + (-6)^2} \Rightarrow |EF| = \sqrt{1+36} \Rightarrow |EF| = \sqrt{37}.
 \end{aligned}$$

2. inačica

Budući da je duljina srednjice trokuta jednaka polovici duljine nasuprotne stranice trokuta, slijedi (gledaj sliku):

$$\begin{aligned}
 & \left. \begin{array}{l} C(x_1, y_1) = C(5, 7) \\ \bullet A(x_2, y_2) = A(-3, 1) \\ |FD| = \frac{1}{2} \cdot |CA| \end{array} \right\} \Rightarrow \left. \begin{array}{l} C(x_1, y_1) = C(5, 7) \\ A(x_2, y_2) = A(-3, 1) \\ |FD| = \frac{1}{2} \cdot \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{array} \right\} \Rightarrow \\
 & \Rightarrow |FD| = \sqrt{(-3-5)^2 + (1-7)^2} \Rightarrow |FD| = \frac{1}{2} \cdot \sqrt{(-8)^2 + (-6)^2} \Rightarrow |FD| = \frac{1}{2} \cdot \sqrt{64+36} \Rightarrow \\
 & \Rightarrow |FD| = \frac{1}{2} \cdot \sqrt{100} \Rightarrow |FD| = \frac{1}{2} \cdot 10 \Rightarrow |FD| = 5.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{array}{l} A(x_1, y_1) = A(-3, 1) \\ \bullet B(x_2, y_2) = B(3, -5) \\ |DE| = \frac{1}{2} \cdot |AB| \end{array} \right\} \Rightarrow \left. \begin{array}{l} A(x_1, y_1) = A(-3, 1) \\ B(x_2, y_2) = B(3, -5) \\ |DE| = \frac{1}{2} \cdot \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{array} \right\} \Rightarrow \\
 & \Rightarrow |DE| = \sqrt{(3-(-3))^2 + (-5-1)^2} \Rightarrow |DE| = \frac{1}{2} \cdot \sqrt{(3+3)^2 + (-6)^2} \Rightarrow |DE| = \frac{1}{2} \cdot \sqrt{6^2 + (-6)^2} \Rightarrow \\
 & \Rightarrow |DE| = \frac{1}{2} \cdot \sqrt{36+36} \Rightarrow |DE| = \frac{1}{2} \cdot \sqrt{72} \Rightarrow \left[\begin{array}{l} \text{djelomično} \\ \text{korjenovanje} \end{array} \right] \Rightarrow |DE| = \frac{1}{2} \cdot \sqrt{36 \cdot 2} \Rightarrow \\
 & \Rightarrow |DE| = \frac{1}{2} \cdot 6 \cdot \sqrt{2} \Rightarrow |DE| = 3 \cdot \sqrt{2}.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \begin{array}{l} B(x_1, y_1) = B(3, -5) \\ \bullet C(x_2, y_2) = C(5, 7) \\ |EF| = \frac{1}{2} \cdot |BC| \end{array} \right\} \Rightarrow \left. \begin{array}{l} B(x_1, y_1) = B(3, -5) \\ C(x_2, y_2) = C(5, 7) \\ |EF| = \frac{1}{2} \cdot \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{array} \right\} \Rightarrow \\
 & \Rightarrow |EF| = \sqrt{(5-3)^2 + (7-(-5))^2} \Rightarrow |EF| = \frac{1}{2} \cdot \sqrt{2^2 + (7+5)^2} \Rightarrow |EF| = \frac{1}{2} \cdot \sqrt{2^2 + 12^2} \Rightarrow \\
 & \Rightarrow |EF| = \frac{1}{2} \cdot \sqrt{4+144} \Rightarrow |EF| = \frac{1}{2} \cdot \sqrt{148} \Rightarrow \left[\begin{array}{l} \text{djelomično} \\ \text{korjenovanje} \end{array} \right] \Rightarrow |EF| = \frac{1}{2} \cdot \sqrt{4 \cdot 37} \Rightarrow
 \end{aligned}$$

$$\Rightarrow |EF| = \frac{1}{2} \cdot 2 \cdot \sqrt{37} \Rightarrow |EF| = \frac{1}{2} \cdot 2 \cdot \sqrt{37} \Rightarrow |EF| = \sqrt{37}.$$

Vježba 168

Koliki je zbroj duljina srednjica trokuta ABC ako su vrhovi trokuta točke: A(-3, 1), B(3, -5), C(5, 7)?

Rezultat: $5 + 3 \cdot \sqrt{2} + \sqrt{37}$.

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VII.

Zadatak 064 (YOYO, srednja škola)

Na osi ordinata odredi točku koja je od točke $A(3, 2)$ udaljena 5.

Rješenje 064

Ponovimo!

Udaljenost točaka $A(x_A, y_A)$ i $B(x_B, y_B)$:

$$|AB| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}.$$
$$(\sqrt{a})^2 = a.$$

Točka koju tražimo leži na ordinati (y osi) pa njezine koordinate glase:

$$B(0, y).$$

Budući da udaljenost između točaka A i B iznosi 5, slijedi:

$$\left. \begin{array}{l} A(x_A, y_A) = A(3, 2), \quad B(x_B, y_B) = B(0, y) \\ |AB| = 5 \end{array} \right\} \Rightarrow \left. \begin{array}{l} A(x_A, y_A) = A(3, 2), \quad B(x_B, y_B) = B(0, y) \\ \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2} = 5 \end{array} \right\} \Rightarrow$$

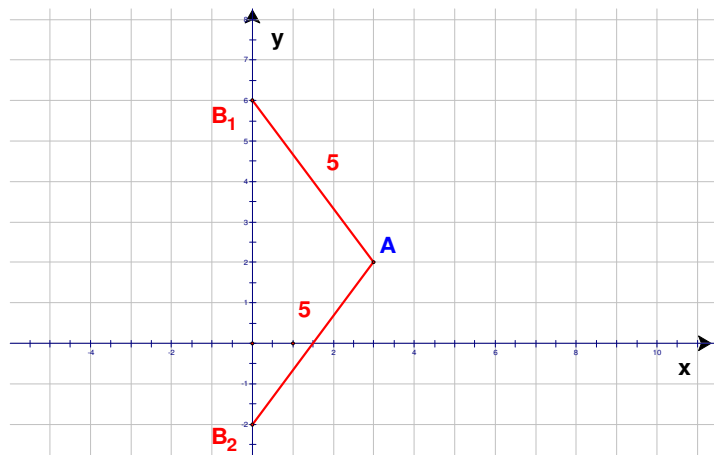
$$\Rightarrow \sqrt{(0-3)^2 + (y-2)^2} = 5 \Rightarrow \sqrt{(-3)^2 + (y-2)^2} = 5 \Rightarrow \sqrt{9+(y-2)^2} = 5 \quad / \sqrt{\quad} \Rightarrow$$

$$\Rightarrow \left(\sqrt{9+(y-2)^2} \right)^2 = 5^2 \Rightarrow 9+(y-2)^2 = 25 \Rightarrow (y-2)^2 = 25-9 \Rightarrow (y-2)^2 = 16 \quad / \sqrt{\quad} \Rightarrow$$

$$\Rightarrow y-2 = \pm\sqrt{16} \Rightarrow y-2 = \pm 4 \Rightarrow \left. \begin{array}{l} y-2=4 \\ y-2=-4 \end{array} \right\} \Rightarrow \left. \begin{array}{l} y=4+2 \\ y=-4+2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} y_1=6 \\ y_2=-2 \end{array} \right\}.$$

Postoje dvije točke koje ispunjavaju zadani uvjet:

$$B_1(0, 6), \quad B_2(0, -2).$$



Vježba 064

Na osi apscisa odredi točku koja je od točke $A(2, 3)$ udaljena 5.

Rezultat: $B_1(6, 0), \quad B_2(-2, 0).$

VIII.

Zadatak 065 (YOYO, srednja škola)

Odredi nepoznatu koordinatu točke C tako da ona pripada pravcu AB:
 $A(-1, 4)$, $B(2, 2)$, $C(x, -4)$.

Rješenje 065

Ponovimo!
Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x , $x \geq 0$, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj $-x$ koji je pozitivan. Za svaki x , $x < 0$, je $|x| = -x$.

Pravac točkama $A(x_1, y_1)$, $B(x_2, y_2)$, $x_1 \neq x_2$, ima jednadžbu:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1).$$

Pravac točkama $A(x_1, y_1)$, $B(x_2, y_2)$, $x_1 \neq x_2$, ima koeficijent smjera:

$$k = \frac{y_2 - y_1}{x_2 - x_1}.$$

Površina trokuta ABC, $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ dana je formulom:

$$P = \frac{1}{2} \cdot |x_1 \cdot (y_2 - y_3) + x_2 \cdot (y_3 - y_1) + x_3 \cdot (y_1 - y_2)|.$$

1. inačica

Odredimo jednadžbu pravca točkama A i B.

$$\left. \begin{array}{l} A(x_1, y_1) = A(-1, 4) \\ B(x_2, y_2) = A(2, 2) \\ y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1) \end{array} \right\} \Rightarrow y - 4 = \frac{2 - 4}{2 - (-1)} \cdot (x - (-1)) \Rightarrow y - 4 = \frac{2 - 4}{2 + 1} \cdot (x + 1) \Rightarrow$$

$$\Rightarrow y - 4 = -\frac{2}{3} \cdot (x + 1) \Rightarrow y - 4 = -\frac{2}{3} \cdot x - \frac{2}{3} \Rightarrow y = -\frac{2}{3} \cdot x - \frac{2}{3} + 4 \Rightarrow y = -\frac{2}{3} \cdot x + \frac{10}{3}.$$

Budući da točka C pripada tom pravcu, njezine koordinate uvrstimo u jednadžbu pravca i dobije se apscisa x .

$$\left. \begin{array}{l} C(x, y) = C(x, -4) \\ y = -\frac{2}{3} \cdot x + \frac{10}{3} \end{array} \right\} \Rightarrow -4 = -\frac{2}{3} \cdot x + \frac{10}{3} \Rightarrow -4 = -\frac{2}{3} \cdot x + \frac{10}{3} \quad /: 3 \Rightarrow -12 = -2 \cdot x + 10 \Rightarrow$$
$$\Rightarrow 2 \cdot x = 10 + 12 \Rightarrow 2 \cdot x = 22 \Rightarrow 2 \cdot x = 22 \quad /: 2 \Rightarrow x = 11.$$

2. inačica

Točka C leži na pravcu AB pa koeficijent smjera pravca AC mora biti jednak koeficijentu smjera pravca BC.

$$\left. \begin{array}{l}
 A(x_1, y_1) = A(-1, 4) \\
 \bullet C(x_2, y_2) = C(x, -4) \\
 k_{AC} = \frac{y_2 - y_1}{x_2 - x_1}
 \end{array} \right\} \Rightarrow k_{AC} = \frac{-4-4}{x-(-1)} \Rightarrow k_{AC} = \frac{-8}{x+1}$$

$$\left. \begin{array}{l}
 B(x_1, y_1) = B(2, 2) \\
 \bullet C(x_2, y_2) = C(x, -4) \\
 k_{BC} = \frac{y_2 - y_1}{x_2 - x_1}
 \end{array} \right\} \Rightarrow k_{BC} = \frac{-4-2}{x-2} \Rightarrow k_{BC} = \frac{-6}{x-2}$$

Apscisa x točke C iznosi:

$$\begin{aligned}
 k_{AC} = k_{BC} &\Rightarrow \frac{-8}{x+1} = \frac{-6}{x-2} \Rightarrow -6 \cdot (x+1) = -8 \cdot (x-2) \Rightarrow -6 \cdot x - 6 = -8 \cdot x + 16 \Rightarrow \\
 &\Rightarrow -6 \cdot x + 8 \cdot x = 16 + 6 \Rightarrow 2 \cdot x = 22 \Rightarrow 2 \cdot x = 22 \quad /: 2 \Rightarrow x = 11.
 \end{aligned}$$

3. inačica

Tri točke A, B i C u ravnini, općenito, tvore trokut ABC. Ako točke leže na istom pravcu (ako su kolinearne), površina trokuta ABC jednaka je nuli. Zato apscisa x točke C iznosi:

$$\left. \begin{array}{l}
 A(x_1, y_1) = A(-1, 4) \\
 B(x_2, y_2) = B(2, 2) \\
 C(x_3, y_3) = C(x, -4) \\
 \frac{1}{2} \cdot |x_1 \cdot (y_2 - y_3) + x_2 \cdot (y_3 - y_1) + x_3 \cdot (y_1 - y_2)| = 0 \\
 \Rightarrow \frac{1}{2} \cdot |-1 \cdot (2 - (-4)) + 2 \cdot (-4 - 4) + x \cdot (4 - 2)| = 0 \quad /: 2 \Rightarrow
 \end{array} \right\} \Rightarrow$$

$$\Rightarrow |-1 \cdot (2 + 4) + 2 \cdot (-4 - 4) + x \cdot (4 - 2)| = 0 \Rightarrow |-1 \cdot 6 + 2 \cdot (-8) + 2 \cdot x| = 0 \Rightarrow$$

$$\Rightarrow |-6 - 16 + 2 \cdot x| = 0 \Rightarrow |-22 + 2 \cdot x| = 0 \Rightarrow -22 + 2 \cdot x = 0 \Rightarrow 2 \cdot x = 22 \Rightarrow 2 \cdot x = 22 \quad /: 2 \Rightarrow x = 11.$$

Vježba 065

Odredi nepoznatu koordinatu točke A tako da ona pripada pravcu BC:

B(-1, 4), C(2, 2), A(x, -4).

Rezultat: 11.

IX.

Zadatak 050 (Mirza, elektrotehnička škola)

Nacrtaj graf: $y = -2^{-|x|}$.

Rješenje 050

Ponovimo!

$$a^{-n} = \frac{1}{a^n}, \quad a^0 = 1, \quad a^1 = a.$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x , $x \geq 0$, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj $-x$ koji je pozitivan. Za svaki x , $x < 0$, je $|x| = -x$.

Ili ovako:

Apsolutna vrijednost realnog broja x definiira se

$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0. \\ -x, & x < 0 \end{cases}$$

Ako je broj x pozitivan broj, onda ga prepisemo: $|x| = x$, $|7| = 7$.

Ako je broj x negativan broj, onda ga pišemo s minusom: $|x| = -x$, $|-4| = -(-4) = 4$.

Izračunamo vrijednosti funkcije

$$f(x) = -2^{-|x|} \Rightarrow f(x) = -\frac{1}{2^{|x|}}$$

za, na primjer,

$$x = -3, x = -2, x = -1, x = 0, x = 1, x = 2, x = 3.$$

$$x = -3 \Rightarrow y = -\frac{1}{2^{|-3|}} \Rightarrow y = -\frac{1}{2^3} \Rightarrow y = -\frac{1}{8}.$$

$$x = -2 \Rightarrow y = -\frac{1}{2^{|-2|}} \Rightarrow y = -\frac{1}{2^2} \Rightarrow y = -\frac{1}{4}.$$

$$x = -1 \Rightarrow y = -\frac{1}{2^{|-1|}} \Rightarrow y = -\frac{1}{2^1} \Rightarrow y = -\frac{1}{2}.$$

$$x = 0 \Rightarrow y = -\frac{1}{2^{|0|}} \Rightarrow y = -\frac{1}{2^0} \Rightarrow y = -\frac{1}{1} \Rightarrow y = -1.$$

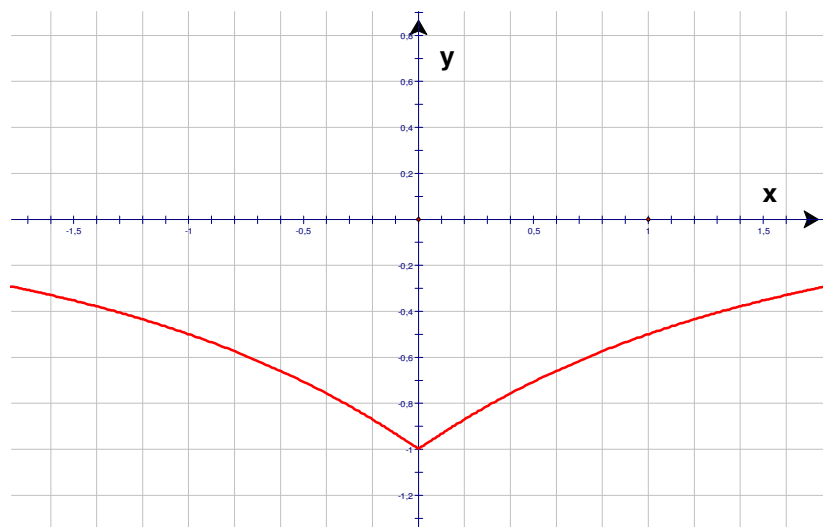
$$x = 1 \Rightarrow y = -\frac{1}{2^{|1|}} \Rightarrow y = -\frac{1}{2^1} \Rightarrow y = -\frac{1}{2}.$$

$$x = 2 \Rightarrow y = -\frac{1}{2^{|2|}} \Rightarrow y = -\frac{1}{2^2} \Rightarrow y = -\frac{1}{4}.$$

$$x = 3 \Rightarrow y = -\frac{1}{2^{|3|}} \Rightarrow y = -\frac{1}{2^3} \Rightarrow y = -\frac{1}{8}.$$

Tablica zadanih argumenata x i njima pripadnih vrijednosti izgleda ovako:

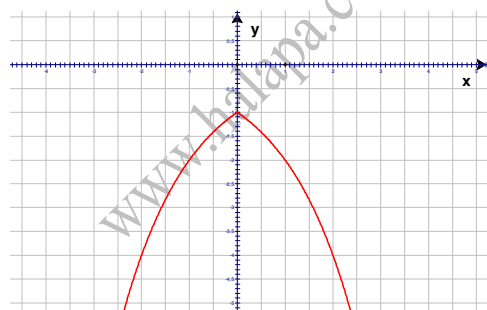
x	-3	-2	-1	0	1	2	3
y	$-\frac{1}{8}$	$-\frac{1}{4}$	$-\frac{1}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{8}$



Vježba 050

Nacrtaj graf: $y = -2|x|$.

Rezultat:





Zadatak 187 (Mirza, elektrotehnička škola)

Riješi jednadžbu: $\sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{0.125}}} = 4 \cdot \sqrt[3]{2}$.

Rješenje 187

Ponovimo!

$$a^{-n} = \frac{1}{a^n}, \quad \sqrt[n]{a^m} = a^{\frac{m}{n}}, \quad (a^n)^m = a^{n \cdot m}, \quad (a \cdot b)^n = a^n \cdot b^n.$$

$$a^n \cdot a^m = a^{n+m}.$$

$$a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x), \quad (\sqrt{a})^2 = a, \quad (\sqrt[3]{a})^3 = a, \quad (\sqrt[n]{a})^n = a.$$

1. inačica

$$\sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{0.125}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{\frac{125}{1000}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{\frac{1}{8}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow$$

$$\Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{\frac{1}{2^3}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{2^{-3}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot 2^{-\frac{3}{x}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow$$

$$\Rightarrow \sqrt{2^x \cdot \sqrt[3]{(2^2)^x \cdot 2^{-\frac{3}{x}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{2^{2 \cdot x} \cdot 2^{-\frac{3}{x}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{2^{2 \cdot x - \frac{3}{x}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow$$

$$\Rightarrow \sqrt{2^x \cdot \sqrt[3]{2^{\frac{2 \cdot x^2 - 3}{x}}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^x \cdot 2^{\frac{2 \cdot x^2 - 3}{3 \cdot x}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^{x + \frac{2 \cdot x^2 - 3}{3 \cdot x}}} = 4 \cdot \sqrt[3]{2} \Rightarrow$$

$$\Rightarrow \sqrt{2^{\frac{3 \cdot x^2 + 2 \cdot x^2 - 3}{3 \cdot x}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^{\frac{5 \cdot x^2 - 3}{3 \cdot x}}} = 4 \cdot \sqrt[3]{2} \Rightarrow 2^{\frac{5 \cdot x^2 - 3}{6 \cdot x}} = 2^{2 \cdot 2^{\frac{1}{3}}} \Rightarrow$$

$$\Rightarrow 2^{\frac{5 \cdot x^2 - 3}{6 \cdot x}} = 2^{2 + \frac{1}{3}} \Rightarrow 2^{\frac{5 \cdot x^2 - 3}{6 \cdot x}} = 2^{\frac{7}{3}} \Rightarrow \frac{5 \cdot x^2 - 3}{6 \cdot x} = \frac{7}{3} \quad / \cdot 6 \cdot x \Rightarrow 5 \cdot x^2 - 3 = 14 \cdot x \Rightarrow$$

$$\Rightarrow 5 \cdot x^2 - 14 \cdot x - 3 = 0 \Rightarrow \left. \begin{array}{l} 5 \cdot x^2 - 14 \cdot x - 3 = 0 \\ a = 5, b = -14, c = -3 \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = 5, b = -14, c = -3 \\ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array} \right\} \Rightarrow$$

$$\Rightarrow x_{1,2} = \frac{14 \pm \sqrt{196 - 4 \cdot 5 \cdot (-3)}}{2 \cdot 5} \Rightarrow x_{1,2} = \frac{14 \pm \sqrt{196 + 60}}{10} \Rightarrow x_{1,2} = \frac{14 \pm \sqrt{256}}{10} \Rightarrow$$

$$\Rightarrow x_{1,2} = \frac{14 \pm 16}{10} \Rightarrow \left. \begin{array}{l} x_1 = \frac{14 + 16}{10} \\ x_2 = \frac{14 - 16}{10} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_1 = \frac{30}{10} \\ x_2 = -\frac{2}{10} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_1 = 3 \\ x_2 = -\frac{1}{5} \end{array} \right\}.$$

2. inačica

$$\sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{0.125}}} = 4 \cdot \sqrt[3]{2} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{\frac{125}{1000}}}} = 2^2 \cdot 2^{\frac{1}{3}} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{\frac{1}{8}}}} = 2^{2 + \frac{1}{3}} \Rightarrow$$

$$\begin{aligned}
&\Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{\frac{1}{2^3}}}} = 2^{\frac{7}{3}} \Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{2^{-3}}}} = 2^{\frac{7}{3}} \Rightarrow \left[\text{kvadriramo} \right. \\
&\quad \left. \text{jednadžbu} \right] \Rightarrow \\
&\Rightarrow \sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{2^{-3}}}} = 2^{\frac{7}{3}} / 2 \Rightarrow \left(\sqrt{2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{2^{-3}}}} \right)^2 = \left(2^{\frac{7}{3}} \right)^2 \Rightarrow \\
&\Rightarrow 2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{2^{-3}}} = 2^{\frac{14}{3}} \Rightarrow \left[\text{kubiramo} \right. \\
&\quad \left. \text{jednadžbu} \right] \Rightarrow 2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{2^{-3}}} = 2^{\frac{14}{3}} / 3 \Rightarrow \\
&\Rightarrow \left(2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{2^{-3}}} \right)^3 = \left(2^{\frac{14}{3}} \right)^3 \Rightarrow (2^x)^3 \cdot \left(\sqrt[3]{4^x \cdot x \sqrt{2^{-3}}} \right)^3 = \left(2^{\frac{14}{3}} \right)^3 \Rightarrow \\
&\Rightarrow 2^{3 \cdot x} \cdot 4^x \cdot x \sqrt{2^{-3}} = 2^{14} \Rightarrow 2^{3 \cdot x} \cdot (2^2)^x \cdot x \sqrt{2^{-3}} = 2^{14} \Rightarrow 2^{3 \cdot x} \cdot 2^{2 \cdot x} \cdot x \sqrt{2^{-3}} = 2^{14} \Rightarrow \\
&\Rightarrow 2^{3 \cdot x + 2 \cdot x} \cdot x \sqrt{2^{-3}} = 2^{14} \Rightarrow 2^{5 \cdot x} \cdot x \sqrt{2^{-3}} = 2^{14} \Rightarrow 2^{5 \cdot x} \cdot 2^{-\frac{3}{2}} = 2^{14} \Rightarrow 2^{5 \cdot x - \frac{3}{2}} = 2^{14} \Rightarrow \\
&\quad \Rightarrow 5 \cdot x - \frac{3}{2} = 14 \Rightarrow 5 \cdot x - \frac{3}{2} = 14 / \cdot x \Rightarrow 5 \cdot x^2 - 3 = 14 \cdot x \Rightarrow \\
&\Rightarrow 5 \cdot x^2 - 14 \cdot x - 3 = 0 \Rightarrow \left. \begin{array}{l} 5 \cdot x^2 - 14 \cdot x - 3 = 0 \\ a = 5, b = -14, c = -3 \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = 5, b = -14, c = -3 \\ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array} \right\} \Rightarrow \\
&\Rightarrow x_{1,2} = \frac{14 \pm \sqrt{196 - 4 \cdot 5 \cdot (-3)}}{2 \cdot 5} \Rightarrow x_{1,2} = \frac{14 \pm \sqrt{196 + 60}}{10} \Rightarrow x_{1,2} = \frac{14 \pm \sqrt{256}}{10} \Rightarrow \\
&\Rightarrow x_{1,2} = \frac{14 \pm 16}{10} \Rightarrow \left. \begin{array}{l} x_1 = \frac{14 + 16}{10} \\ x_2 = \frac{14 - 16}{10} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_1 = \frac{30}{10} \\ x_2 = -\frac{2}{10} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x_1 = 3 \\ x_2 = -\frac{1}{5} \end{array} \right\}.
\end{aligned}$$

Vježba 187

Riješi jednadžbu: $2^x \cdot \sqrt[3]{4^x \cdot x \sqrt{0.125}} = 16 \cdot \sqrt[3]{4}$.

Rezultat: $x_1 = 3, x_2 = -\frac{1}{5}$.