Zadatak 121 (Marino i Medax, srednja škola)

Koliko je x + y ako vrijedi: $\begin{cases} |x| + x + y = 5 \\ x + |y| - y = 10 \end{cases}$?

$A.\ 1 \qquad B.\ 2 \qquad C.\ 3 \qquad D.\ 4 \qquad E.\ 5$

Rješenje 121

Ponovimo!

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x \ge 0$, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, $x < 0$, je $|x| = -x$.

Postoje četiri slučaja!

Prvi slučaj

$$\left.\begin{matrix} x \ge 0 \\ y \ge 0 \end{matrix}\right\} \Rightarrow \begin{bmatrix} |x| + x + y = 5 \\ x + |y| - y = 10 \end{bmatrix} \Rightarrow \left.\begin{matrix} x + x + y = 5 \\ x + y - y = 10 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2 \cdot x + y = 5 \\ x + y - y = 10 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} 2 \cdot x + y = 5 \\ x = 10 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{zamjene} \end{bmatrix} \Rightarrow \left.\begin{matrix} 2 \cdot 10 + y = 5 \\ x = 10 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 20 + y = 5 \\ x = 10 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} y = 5 - 20 \\ x = 10 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = -15 \\ x = 10 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{nema rješenja zbog} \\ y \ge 0 \end{bmatrix}.$$

Drugi slučaj

$$\left.\begin{matrix} x \ge 0 \\ y < 0 \end{matrix}\right\} \Rightarrow \begin{bmatrix} |x| + x + y = 5 \\ x + |y| - y = 10 \end{bmatrix} \Rightarrow \left.\begin{matrix} x + x + y = 5 \\ x - y - y = 10 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2 \cdot x + y = 5 \\ x - 2 \cdot y = 10 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{bmatrix} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} 2 \cdot x + y = 5 \ / \cdot 2 \\ x - 2 \cdot y = 10 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 4 \cdot 10 + 2 \cdot y = 10 \\ x - 2 \cdot y = 10 \end{matrix}\right\} \Rightarrow 5 \cdot x = 20 \Rightarrow 5 \cdot x = 20 \ /: 5 \Rightarrow x = 4.$$

Računamo y.

$$\left.\begin{matrix} 2 \cdot x + y = 5 \\ x = 4 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{zamjene} \end{bmatrix} \Rightarrow 2 \cdot 4 + y = 5 \Rightarrow 8 + y = 5 \Rightarrow y = 5 - 8 \Rightarrow y = -3.$$

Dakle, rješenje je

$$(x,\ y) = (4,\ -3)$$

pa je

$$x + y = 4 + (-3) = 1.$$

Odgovor je pod A

Treća slučaj

$$\left.\begin{matrix} x < 0 \\ y \ge 0 \end{matrix}\right\} \Rightarrow \begin{bmatrix} |x| + x + y = 5 \\ x + |y| - y = 10 \end{bmatrix} \Rightarrow \left.\begin{matrix} -x + x + y = 5 \\ x + y - y = 10 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} -x + x + y = 5 \\ x + y - y = 10 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} y = 5 \\ x = 10 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{nema rješenja zbog} \\ x < 0 \end{bmatrix}.$$

Četvrti slučaj

$$\left.\begin{array}{l} x<0 \\ y<0 \end{array}\right\} \Rightarrow \left[\begin{array}{l} |x|+x+y=5 \\ x+|y|-y=10 \end{array}\right] \Rightarrow \left.\begin{array}{l} -x+x+y=5 \\ x-y-y=10 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x+x+y=5 \\ x-2\cdot y=10 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} y=5 \\ x-2\cdot y=10 \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{nema rješenja zbog} \\ y<0 \end{array}\right].$$

Vježba 121

Koliko je x + y ako vrijedi: $\left\{\begin{array}{l} |x|+x+y-5=0 \\ x+|y|-y-10=0 \end{array}\right.$?

$A.\ 1 \qquad B.\ 2 \qquad C.\ 3 \qquad D.\ 4 \qquad E.\ 5$

Rezultat: A.

Zadatak 122 (Tomislav, srednja škola)

Ako je $a+b=5,\ b+c=7,\ c+a=6,$ koliko je $a\cdot b\cdot c$?

Rješenje 122

Ponovimo!

$$\left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a+c=b+d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left.\begin{array}{l} a+b=5 \\ b+c=7 \\ c+a=6 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a+b=5 \\ b=7-c \\ a=6-c \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow (6-c)+(7-c)=5 \Rightarrow 6-c+7-c=5 \Rightarrow$$

$$\Rightarrow -c-c=5-6-7 \Rightarrow -2\cdot c=-8 \Rightarrow -2\cdot c=-8\ /:(-2) \Rightarrow c=4.$$

Računamo a i b.

$$\left.\begin{array}{l} a=6-c \\ b=7-c \end{array}\right\} \Rightarrow [c=4] \Rightarrow \left.\begin{array}{l} a=6-4 \\ b=7-4 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=2 \\ b=3 \end{array}\right\}.$$

Konačno je

$$a\cdot b\cdot c=2\cdot 3\cdot 4 \Rightarrow a\cdot b\cdot c=24.$$

2.inačica

$$\left.\begin{array}{l} a+b=5 \\ b+c=7 \\ c+a=6 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow a+b+b+c+c+a=5+7+6 \Rightarrow 2\cdot a+2\cdot b+2\cdot c=18 \Rightarrow$$

$$\Rightarrow 2\cdot(a+b+c)=18 \Rightarrow 2\cdot(a+b+c)=18\ /:2 \Rightarrow a+b+c=9.$$

Računamo a.

$$\left.\begin{array}{l} b+c=7 \\ a+b+c=9 \end{array}\right\} \Rightarrow \left.\begin{array}{l} b+c=7 \\ a+(b+c)=9 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow a+7=9 \Rightarrow a=9-7 \Rightarrow a=2.$$

Računamo b i c.

$$\left.\begin{array}{l} a+b=5 \\ c+a=6 \end{array}\right\} \Rightarrow \left.\begin{array}{l} b=5-a \\ c=6-a \end{array}\right\} \Rightarrow [a=2] \Rightarrow \left.\begin{array}{l} b=5-2 \\ c=6-2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} b=3 \\ c=4 \end{array}\right\}.$$

Konačno je

$$a\cdot b\cdot c=2\cdot 3\cdot 4 \Rightarrow a\cdot b\cdot c=24.$$

Vježba 122

Ako je $a+b=3,\ b+c=5,\ c+a=4,$ koliko je $a\cdot b\cdot c$?

Rezultat: 6.

Zadatak 123 (Valentina, gimnazija)

Ako je $2\cdot x^2+5\cdot y^2+z^2-4\cdot x\cdot y+2\cdot x\cdot z+2\cdot y+1=0,$ koliko je $x+y+z$?

Rješenje 123

Ponovimo!

$$(a\cdot b)^n=a^n\cdot b^n \quad , \quad a^2-2\cdot a\cdot b+b^2=(a-b)^2 \quad , \quad a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

$$a^2+b^2=0 \Rightarrow \left.\begin{matrix} a=0 \\ b=0 \end{matrix}\right\}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Preoblikujemo zadanu jednadžbu.

$$2\cdot x^2+5\cdot y^2+z^2-4\cdot x\cdot y+2\cdot x\cdot z+2\cdot y+1=0 \Rightarrow$$

$$\Rightarrow x^2-4\cdot x\cdot y+4\cdot y^2+x^2+2\cdot x\cdot z+z^2+y^2+2\cdot y+1=0 \Rightarrow$$

$$\Rightarrow \left(x^2-4\cdot x\cdot y+4\cdot y^2\right)+\left(x^2+2\cdot x\cdot z+z^2\right)+\left(y^2+2\cdot y+1\right)=0 \Rightarrow$$

$$\Rightarrow (x-2\cdot y)^2+(x+z)^2+(y+1)^2=0 \Rightarrow \left.\begin{matrix} x-2\cdot y=0 \\ x+z=0 \\ y+1=0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=2\cdot y \\ x=-z \\ y=-1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=2\cdot(-1) \\ x=-z \\ y=-1 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x=-2 \\ x=-z \\ y=-1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=-2 \\ z=-x \\ y=-1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=-2 \\ z=-(-2) \\ y=-1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=-2 \\ z=2 \\ y=-1 \end{matrix}\right\}.$$

Sada je:

$$x+y+z=-2-1+2 \Rightarrow x+y+z=-2-1+2 \Rightarrow x+y+z=-1.$$

Vježba 123

Ako je $2\cdot x^2+5\cdot y^2+z^2-4\cdot x\cdot y+2\cdot x\cdot z+2\cdot y-8=0,$ koliko je $x+y+z$?

Rezultat: 3.

Zadatak 124 (Vegy, gimnazija)

Neka je $x^2=1+x.$ Ako je $x^{10}=a+b\cdot x,$ onda je $a+b$ jednako:

$A.\ 85 \qquad B.\ 86 \qquad C.\ 87 \qquad D.\ 88 \qquad E.\ 89$

Rješenje 124

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$\left(\sqrt{a}\right)^2=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^1=a \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$n=\frac{n}{1} \quad , \quad \left(\frac{a}{b}\right)^{n}=\frac{a^{n}}{b^{n}} \quad , \quad \left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a-c=b-d \quad , \quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Nađemo prvo rješenja kvadratne jednadžbe.

$$x^{2}=1+x \Rightarrow x^{2}-x-1=0 \Rightarrow \left.\begin{array}{l} x^{2}-x-1=0 \\ a=1\ ,\ b=-1\ ,\ c=-1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=1\ ,\ b=-1\ ,\ c=-1 \\ x_{1,2}=\dfrac{-b\pm\sqrt{b^{2}-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow x_{1,2}=\frac{-(-1)\pm\sqrt{(-1)^{2}-4\cdot 1\cdot(-1)}}{2\cdot 1} \Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+4}}{2} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{5}}{2} \Rightarrow \left.\begin{array}{l} x_{1}=\dfrac{1+\sqrt{5}}{2} \\ x_{2}=\dfrac{1-\sqrt{5}}{2} \end{array}\right\}.$$

Uvrstimo vrijednosti x_1 i x_2 u drugu jednadžbu

$$x^{10}=a+b\cdot x.$$

- $$\left.\begin{array}{l} x=\dfrac{1+\sqrt{5}}{2} \\ x^{10}=a+b\cdot x \end{array}\right\} \Rightarrow \left(\frac{1+\sqrt{5}}{2}\right)^{10}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\left(\frac{1+\sqrt{5}}{2}\right)^{2}\right)^{5}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \left(\frac{1+2\cdot\sqrt{5}+\left(\sqrt{5}\right)^{2}}{4}\right)^{5}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{1+2\cdot\sqrt{5}+5}{4}\right)^{5}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \left(\frac{6+2\cdot\sqrt{5}}{4}\right)^{5}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{2\cdot\left(3+\sqrt{5}\right)}{4}\right)^{5}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \left(\frac{2\cdot\left(3+\sqrt{5}\right)}{4}\right)^{5}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{3+\sqrt{5}}{2}\right)^{5}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \left(\frac{3+\sqrt{5}}{2}\right)^{4}\cdot\left(\frac{3+\sqrt{5}}{2}\right)^{1}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\left(\frac{3+\sqrt{5}}{2}\right)^2\right)^2 \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{9+6\cdot\sqrt{5}+\left(\sqrt{5}\right)^2}{4}\right)^2 \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{9+6\cdot\sqrt{5}+5}{4}\right)^2 \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{14+6\cdot\sqrt{5}}{4}\right)^2 \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \left(\frac{2\cdot\left(7+3\cdot\sqrt{5}\right)}{4}\right)^2 \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{2\cdot\left(7+3\cdot\sqrt{5}\right)}{4}\right)^2 \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \left(\frac{7+3\cdot\sqrt{5}}{2}\right)^2 \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{49+2\cdot7\cdot3\cdot\sqrt{5}+\left(3\cdot\sqrt{5}\right)^2}{4} \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{49+42\cdot\sqrt{5}+3^2\cdot\left(\sqrt{5}\right)^2}{4} \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{49+42\cdot\sqrt{5}+9\cdot5}{4} \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \frac{49+42\cdot\sqrt{5}+45}{4} \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{94+42\cdot\sqrt{5}}{4} \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \frac{2\cdot\left(47+21\cdot\sqrt{5}\right)}{4} \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{2\cdot\left(47+21\cdot\sqrt{5}\right)}{4} \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \frac{47+21\cdot\sqrt{5}}{2} \cdot \frac{3+\sqrt{5}}{2} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{\left(47+21\cdot\sqrt{5}\right)\cdot\left(3+\sqrt{5}\right)}{4} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{141+47\cdot\sqrt{5}+63\cdot\sqrt{5}+21\cdot\left(\sqrt{5}\right)^2}{4} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{141+47\cdot\sqrt{5}+63\cdot\sqrt{5}+21\cdot5}{4} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \frac{141+47\cdot\sqrt{5}+63\cdot\sqrt{5}+105}{4} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{246+110\cdot\sqrt{5}}{4} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \frac{2\cdot\left(123+55\cdot\sqrt{5}\right)}{4} = a + b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{2\cdot\left(123+55\cdot\sqrt{5}\right)}{4}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \frac{123+55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2}.$$

- $$\left.\begin{array}{l} x=\dfrac{1-\sqrt{5}}{2} \\ x^{10}=a+b\cdot x \end{array}\right\} \Rightarrow \left(\frac{1-\sqrt{5}}{2}\right)^{10}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\left(\frac{1-\sqrt{5}}{2}\right)^{2}\right)^{5}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \left(\frac{1-2\cdot\sqrt{5}+\left(\sqrt{5}\right)^{2}}{4}\right)^{5}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{1-2\cdot\sqrt{5}+5}{4}\right)^{5}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \left(\frac{6-2\cdot\sqrt{5}}{4}\right)^{5}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{2\cdot\left(3-\sqrt{5}\right)}{4}\right)^{5}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \left(\frac{2\cdot\left(3-\sqrt{5}\right)}{4}\right)^{5}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{3-\sqrt{5}}{2}\right)^{5}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \left(\frac{3-\sqrt{5}}{2}\right)^{4}\cdot\left(\frac{3-\sqrt{5}}{2}\right)^{1}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\left(\frac{3-\sqrt{5}}{2}\right)^{2}\right)^{2}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{9-6\cdot\sqrt{5}+\left(\sqrt{5}\right)^{2}}{4}\right)^{2}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{9-6\cdot\sqrt{5}+5}{4}\right)^{2}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{14-6\cdot\sqrt{5}}{4}\right)^{2}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \left(\frac{2\cdot\left(7-3\cdot\sqrt{5}\right)}{4}\right)^{2}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{2\cdot\left(7-3\cdot\sqrt{5}\right)}{4}\right)^{2}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \left(\frac{7-3\cdot\sqrt{5}}{2}\right)^{2}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{49-2\cdot7\cdot3\cdot\sqrt{5}+\left(3\cdot\sqrt{5}\right)^{2}}{4}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{49-42\cdot\sqrt{5}+3^2\cdot\left(\sqrt{5}\right)^2}{4}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{49-42\cdot\sqrt{5}+9\cdot 5}{4}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \frac{49-42\cdot\sqrt{5}+45}{4}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{94-42\cdot\sqrt{5}}{4}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \frac{2\cdot\left(47-21\cdot\sqrt{5}\right)}{4}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{2\cdot\left(47-21\cdot\sqrt{5}\right)}{4}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \frac{47-21\cdot\sqrt{5}}{2}\cdot\frac{3-\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{\left(47-21\cdot\sqrt{5}\right)\cdot\left(3-\sqrt{5}\right)}{4}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{141-47\cdot\sqrt{5}-63\cdot\sqrt{5}+21\cdot\left(\sqrt{5}\right)^2}{4}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{141-47\cdot\sqrt{5}-63\cdot\sqrt{5}+21\cdot 5}{4}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \frac{141-47\cdot\sqrt{5}-63\cdot\sqrt{5}+105}{4}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{246-110\cdot\sqrt{5}}{4}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \frac{2\cdot\left(123-55\cdot\sqrt{5}\right)}{4}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{2\cdot\left(123-55\cdot\sqrt{5}\right)}{4}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow \frac{123-55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{123}{2}-\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2}.$$

Dobili smo sustav jednadžbi:

$$\left.\begin{array}{l} \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2} \\ \frac{123}{2}-\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{od prve jednadžbe} \\ \text{oduzmemo drugu} \end{array}\right] \Rightarrow$$

$$\Rightarrow \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}-\left(\frac{123}{2}-\frac{55\cdot\sqrt{5}}{2}\right)=a+b\cdot\frac{1+\sqrt{5}}{2}-\left(a+b\cdot\frac{1-\sqrt{5}}{2}\right) \Rightarrow$$

$$\Rightarrow \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}-\frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2}-a-b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}-\frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2}-a-b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{55\cdot\sqrt{5}}{2}+\frac{55\cdot\sqrt{5}}{2}=b\cdot\frac{1+\sqrt{5}}{2}-b\cdot\frac{1-\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{55\cdot\sqrt{5}}{2}+\frac{55\cdot\sqrt{5}}{2}=b\cdot\frac{1+\sqrt{5}}{2}-b\cdot\frac{1-\sqrt{5}}{2} \;/\cdot 2 \Rightarrow$$

$$\Rightarrow 55\cdot\sqrt{5}+55\cdot\sqrt{5}=b\cdot\left(1+\sqrt{5}\right)-b\cdot\left(1-\sqrt{5}\right) \Rightarrow$$

$$\Rightarrow 110\cdot\sqrt{5}=b+b\cdot\sqrt{5}-b+b\cdot\sqrt{5} \Rightarrow 110\cdot\sqrt{5}=b+b\cdot\sqrt{5}-b+b\cdot\sqrt{5} \Rightarrow$$

$$\Rightarrow 110\cdot\sqrt{5}=b+b\cdot\sqrt{5}-b+b\cdot\sqrt{5} \Rightarrow 110\cdot\sqrt{5}=b\cdot\sqrt{5}+b\cdot\sqrt{5} \Rightarrow$$

$$\Rightarrow 110\cdot\sqrt{5}=2\cdot b\cdot\sqrt{5} \Rightarrow 2\cdot b\cdot\sqrt{5}=110\cdot\sqrt{5} \Rightarrow 2\cdot b\cdot\sqrt{5}=110\cdot\sqrt{5}\ /\cdot\frac{1}{2\cdot\sqrt{5}} \Rightarrow b=55.$$

Računamo nepoznanicu a.

$$\left.\begin{array}{l} b=55 \\ \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2} \end{array}\right\} \Rightarrow \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+55\cdot\frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+\frac{55}{1}\cdot\frac{1+\sqrt{5}}{2} \Rightarrow \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+\frac{55+55\cdot\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow a+\frac{55+55\cdot\sqrt{5}}{2}=\frac{123}{2}+\frac{55\cdot\sqrt{5}}{2} \Rightarrow a+\frac{55}{2}+\frac{55\cdot\sqrt{5}}{2}=\frac{123}{2}+\frac{55\cdot\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow a+\frac{55}{2}+\frac{55\cdot\sqrt{5}}{2}=\frac{123}{2}+\frac{55\cdot\sqrt{5}}{2} \Rightarrow a+\frac{55}{2}=\frac{123}{2} \Rightarrow a=\frac{123}{2}-\frac{55}{2} \Rightarrow a=\frac{123-55}{2} \Rightarrow$$

$$\Rightarrow a=\frac{68}{2} \Rightarrow a=\frac{68}{2} \Rightarrow a=34.$$

Rješenje zadatka glasi:

$$a+b=34+55 \Rightarrow a+b=89.$$

Odgovor je pod E.

Vježba 124

Neka je $x^2=1+x$. Ako je $x^{10}=a+b\cdot x$, onda je $b-a$ jednako:

A. 19 *B.* 20 *C.* 21 *D.* 22 *E.* 23

Rezultat: C.

Zadatak 125 (Karlo, gimnazija)

Riješi sustav jednadžbi: $\begin{cases} x^2-y\cdot z=y-x \\ y^2-x\cdot z=z-y. \\ z^2-x\cdot y=x-z \end{cases}$

Rješenje 125

Ponovimo!

$$\left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a+c=b+d \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad a^2+b^2=0 \Rightarrow a=b=0.$$

$$\left.\begin{array}{l} x^2-y\cdot z=y-x \\ y^2-x\cdot z=z-y \\ z^2-x\cdot y=x-z \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow$$

$$\Rightarrow x^2-y\cdot z+y^2-x\cdot z+z^2-x\cdot y=y-x+z-y+x-z \Rightarrow$$

$$\Rightarrow x^2 - y\cdot z + y^2 - x\cdot z + z^2 - x\cdot y = y - x + z - y + x - z \Rightarrow$$

$$\Rightarrow x^2 - y\cdot z + y^2 - x\cdot z + z^2 - x\cdot y = 0 \Rightarrow x^2 + y^2 + z^2 - x\cdot y - x\cdot z - y\cdot z = 0 \Rightarrow$$

$$\Rightarrow x^2 + y^2 + z^2 - x\cdot y - x\cdot z - y\cdot z = 0 \;/\cdot 2 \Rightarrow$$

$$\Rightarrow 2\cdot x^2 + 2\cdot y^2 + 2\cdot z^2 - 2\cdot x\cdot y - 2\cdot x\cdot z - 2\cdot y\cdot z = 0 \Rightarrow$$

$$\Rightarrow x^2 - 2\cdot x\cdot y + y^2 + x^2 - 2\cdot x\cdot z + z^2 + y^2 - 2\cdot y\cdot z + z^2 = 0 \Rightarrow$$

$$\Rightarrow \left(x^2 - 2\cdot x\cdot y + y^2\right) + \left(x^2 - 2\cdot x\cdot z + z^2\right) + \left(y^2 - 2\cdot y\cdot z + z^2\right) = 0 \Rightarrow$$

$$\Rightarrow (x-y)^2 + (x-z)^2 + (y-z)^2 = 0 \Rightarrow \left.\begin{matrix} x - y = 0 \\ x - z = 0 \\ y - z = 0 \end{matrix}\right\} \Rightarrow x = y = z.$$

Rješenje sustava je svaka trojka realnih brojeva oblika:

$$(a,\ a,\ a)\ ,\ a \in R.$$

Vježba 125

Riješi sustav jednadžbi: $\begin{cases} x^2 + x = y\cdot z + y \\ y^2 + y = x\cdot z + z \\ z^2 + z = x\cdot y + x \end{cases}.$

Rezultat: $(a,\ a,\ a)\ ,\ a \in R.$