

### Zadatak 121 (Marino i Medax, srednja škola)

Koliko je  $x + y$  ako vrijedi: 
$$\begin{cases} |x| + x + y = 5 \\ x + |y| - y = 10 \end{cases} ?$$

A. 1      B. 2      C. 3      D. 4      E. 5

### Rješenje 121

Ponovimo!

Za realni broj  $x$  njegova je apsolutna vrijednost (modul) broj  $|x|$  koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj  $x$  pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki  $x, x \geq 0$ , vrijedi  $|x| = x$ .

Ako je  $x$  negativan broj, njegova apsolutna vrijednost je suprotan broj  $-x$  koji je pozitivan. Za svaki  $x, x < 0$ , je  $|x| = -x$ .

Postoje četiri slučaja!

Prvi slučaj

$$\begin{aligned} \left. \begin{array}{l} x \geq 0 \\ y \geq 0 \end{array} \right\} &\Rightarrow \left[ \begin{array}{l} |x| + x + y = 5 \\ x + |y| - y = 10 \end{array} \right] \Rightarrow \left. \begin{array}{l} x + x + y = 5 \\ x + y - y = 10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 2 \cdot x + y = 5 \\ x + y - y = 10 \end{array} \right\} \Rightarrow \\ \Rightarrow \left. \begin{array}{l} 2 \cdot x + y = 5 \\ x = 10 \end{array} \right\} &\Rightarrow \left[ \begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array} \right] \Rightarrow \left. \begin{array}{l} 2 \cdot 10 + y = 5 \\ x = 10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 20 + y = 5 \\ x = 10 \end{array} \right\} \Rightarrow \\ \Rightarrow \left. \begin{array}{l} y = 5 - 20 \\ x = 10 \end{array} \right\} &\Rightarrow \left. \begin{array}{l} y = -15 \\ x = 10 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{nema rješenja zbog} \\ y \geq 0 \end{array} \right]. \end{aligned}$$

Drugi slučaj

$$\begin{aligned} \left. \begin{array}{l} x \geq 0 \\ y < 0 \end{array} \right\} &\Rightarrow \left[ \begin{array}{l} |x| + x + y = 5 \\ x + |y| - y = 10 \end{array} \right] \Rightarrow \left. \begin{array}{l} x + x + y = 5 \\ x - y - y = 10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 2 \cdot x + y = 5 \\ x - 2 \cdot y = 10 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array} \right] \Rightarrow \\ \Rightarrow \left. \begin{array}{l} 2 \cdot x + y = 5 / \cdot 2 \\ x - 2 \cdot y = 10 \end{array} \right\} &\Rightarrow \left. \begin{array}{l} 4 \cdot 10 + 2 \cdot y = 10 \\ x - 2 \cdot y = 10 \end{array} \right\} \Rightarrow 5 \cdot x = 20 \Rightarrow 5 \cdot x = 20 / : 5 \Rightarrow x = 4. \end{aligned}$$

Računamo  $y$ .

$$\left. \begin{array}{l} 2 \cdot x + y = 5 \\ x = 4 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array} \right] \Rightarrow 2 \cdot 4 + y = 5 \Rightarrow 8 + y = 5 \Rightarrow y = 5 - 8 \Rightarrow y = -3.$$

Dakle, rješenje je

$$(x, y) = (4, -3)$$

pa je

$$x + y = 4 + (-3) = 1.$$

Odgovor je pod A

Treća slučaj

$$\begin{aligned} \left. \begin{array}{l} x < 0 \\ y \geq 0 \end{array} \right\} &\Rightarrow \left[ \begin{array}{l} |x| + x + y = 5 \\ x + |y| - y = 10 \end{array} \right] \Rightarrow \left. \begin{array}{l} -x + x + y = 5 \\ x + y - y = 10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} -x + x + y = 5 \\ x + y - y = 10 \end{array} \right\} \Rightarrow \\ \Rightarrow \left. \begin{array}{l} y = 5 \\ x = 10 \end{array} \right\} &\Rightarrow \left[ \begin{array}{l} \text{nema rješenja zbog} \\ x < 0 \end{array} \right]. \end{aligned}$$

Četvrti slučaj

$$\left. \begin{array}{l} x < 0 \\ y < 0 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} |x| + x + y = 5 \\ x + |y| - y = 10 \end{array} \right] \Rightarrow \left. \begin{array}{l} -x + x + y = 5 \\ x - y - y = 10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} -x + x + y = 5 \\ x - 2 \cdot y = 10 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} y = 5 \\ x - 2 \cdot y = 10 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{nema rješenja zbog} \\ y < 0 \end{array} \right].$$

### Vježba 121

Koliko je  $x + y$  ako vrijedi:  $\begin{cases} |x| + x + y - 5 = 0 \\ x + |y| - y - 10 = 0 \end{cases}$  ?

A. 1      B. 2      C. 3      D. 4      E. 5

**Rezultat:** A.

### Zadatak 122 (Tomislav, srednja škola)

Ako je  $a + b = 5$ ,  $b + c = 7$ ,  $c + a = 6$ , koliko je  $a \cdot b \cdot c$ ?

### Rješenje 122

Ponovimo!

$$\left. \begin{array}{l} a = b \\ c = d \end{array} \right\} \Rightarrow a + c = b + d.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1. inačica

$$\left. \begin{array}{l} a + b = 5 \\ b + c = 7 \\ c + a = 6 \end{array} \right\} \Rightarrow \left. \begin{array}{l} a + b = 5 \\ b = 7 - c \\ a = 6 - c \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array} \right] \Rightarrow (6 - c) + (7 - c) = 5 \Rightarrow 6 - c + 7 - c = 5 \Rightarrow$$

$$\Rightarrow -c - c = 5 - 6 - 7 \Rightarrow -2 \cdot c = -8 \Rightarrow -2 \cdot c = -8 \quad /: (-2) \Rightarrow c = 4.$$

Računamo a i b.

$$\left. \begin{array}{l} a = 6 - c \\ b = 7 - c \end{array} \right\} \Rightarrow [c = 4] \Rightarrow \left. \begin{array}{l} a = 6 - 4 \\ b = 7 - 4 \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = 2 \\ b = 3 \end{array} \right\}.$$

Konačno je

$$a \cdot b \cdot c = 2 \cdot 3 \cdot 4 \Rightarrow a \cdot b \cdot c = 24.$$

2. inačica

$$\left. \begin{array}{l} a + b = 5 \\ b + c = 7 \\ c + a = 6 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array} \right] \Rightarrow a + b + b + c + c + a = 5 + 7 + 6 \Rightarrow 2 \cdot a + 2 \cdot b + 2 \cdot c = 18 \Rightarrow$$

$$\Rightarrow 2 \cdot (a + b + c) = 18 \Rightarrow 2 \cdot (a + b + c) = 18 \quad /: 2 \Rightarrow a + b + c = 9.$$

Računamo a.

$$\left. \begin{array}{l} b + c = 7 \\ a + b + c = 9 \end{array} \right\} \Rightarrow \left. \begin{array}{l} b + c = 7 \\ a + (b + c) = 9 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array} \right] \Rightarrow a + 7 = 9 \Rightarrow a = 9 - 7 \Rightarrow a = 2.$$

Računamo b i c.

$$\left. \begin{array}{l} a + b = 5 \\ c + a = 6 \end{array} \right\} \Rightarrow \left. \begin{array}{l} b = 5 - a \\ c = 6 - a \end{array} \right\} \Rightarrow [a = 2] \Rightarrow \left. \begin{array}{l} b = 5 - 2 \\ c = 6 - 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} b = 3 \\ c = 4 \end{array} \right\}.$$

Konačno je

$$a \cdot b \cdot c = 2 \cdot 3 \cdot 4 \Rightarrow a \cdot b \cdot c = 24.$$

### Vježba 122

Ako je  $a+b=3$ ,  $b+c=5$ ,  $c+a=4$ , koliko je  $a \cdot b \cdot c$ ?

**Rezultat:** 6.

### Zadatak 123 (Valentina, gimnazija)

Ako je  $2 \cdot x^2 + 5 \cdot y^2 + z^2 - 4 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y + 1 = 0$ , koliko je  $x + y + z$ ?

### Rješenje 123

Ponovimo!

$$(a \cdot b)^n = a^n \cdot b^n, \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2, \quad a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2.$$

$$a^2 + b^2 = 0 \Rightarrow \left. \begin{array}{l} a=0 \\ b=0 \end{array} \right\}$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Preoblikujemo zadanu jednačbu.

$$\begin{aligned} 2 \cdot x^2 + 5 \cdot y^2 + z^2 - 4 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y + 1 &= 0 \Rightarrow \\ \Rightarrow x^2 - 4 \cdot x \cdot y + 4 \cdot y^2 + x^2 + 2 \cdot x \cdot z + z^2 + y^2 + 2 \cdot y + 1 &= 0 \Rightarrow \\ \Rightarrow (x^2 - 4 \cdot x \cdot y + 4 \cdot y^2) + (x^2 + 2 \cdot x \cdot z + z^2) + (y^2 + 2 \cdot y + 1) &= 0 \Rightarrow \\ \Rightarrow (x-2 \cdot y)^2 + (x+z)^2 + (y+1)^2 = 0 \Rightarrow \left. \begin{array}{l} x-2 \cdot y=0 \\ x+z=0 \\ y+1=0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x=2 \cdot y \\ x=-z \\ y=-1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x=2 \cdot (-1) \\ x=-z \\ y=-1 \end{array} \right\} \Rightarrow \\ \left. \begin{array}{l} x=-2 \\ x=-z \\ y=-1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x=-2 \\ z=-x \\ y=-1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x=-2 \\ z=-(-2) \\ y=-1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x=-2 \\ z=2 \\ y=-1 \end{array} \right\}. \end{aligned}$$

Sada je:

$$x + y + z = -2 - 1 + 2 \Rightarrow x + y + z = -2 - 1 + 2 \Rightarrow x + y + z = -1.$$

### Vježba 123

Ako je  $2 \cdot x^2 + 5 \cdot y^2 + z^2 - 4 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y - 8 = 0$ , koliko je  $x + y + z$ ?

**Rezultat:** 3.

### Zadatak 124 (Vegy, gimnazija)

Neka je  $x^2 = 1+x$ . Ako je  $x^{10} = a+b \cdot x$ , onda je  $a+b$  jednako:

- A. 85      B. 86      C. 87      D. 88      E. 89

### Rješenje 124

Ponovimo!

$$(a^n)^m = a^{n \cdot m}, \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2, \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

$$(\sqrt{a})^2 = a, \quad a^n \cdot a^m = a^{n+m}, \quad a^1 = a, \quad (a \cdot b)^n = a^n \cdot b^n, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

$$n = \frac{n}{1}, \quad \left\{ \begin{array}{l} \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \\ a = b \\ c = d \end{array} \right\} \Rightarrow a - c = b - d, \quad \frac{a}{n} - \frac{b}{n} = \frac{a-b}{n}.$$

$$\frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Nađemo prvo rješenja kvadratne jednadžbe.

$$\left. \begin{array}{l} x^2 = 1+x \Rightarrow x^2 - x - 1 = 0 \Rightarrow x^2 - x - 1 = 0 \\ a = 1, b = -1, c = -1 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} a = 1, b = -1, c = -1 \\ x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array} \right\} \Rightarrow x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 1 \cdot (-1)}}{2 \cdot 1} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} \Rightarrow x_{1,2} = \frac{1 \pm \sqrt{5}}{2} \Rightarrow \begin{cases} x_1 = \frac{1+\sqrt{5}}{2} \\ x_2 = \frac{1-\sqrt{5}}{2} \end{cases} \end{array} \right\}.$$

Uvrstimo vrijednosti  $x_1$  i  $x_2$  u drugu jednadžbu

$$x^{10} = a + b \cdot x.$$

$$\bullet \left. \begin{array}{l} x = \frac{1+\sqrt{5}}{2} \\ x^{10} = a + b \cdot x \end{array} \right\} \Rightarrow \left( \frac{1+\sqrt{5}}{2} \right)^{10} = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \left( \frac{1+\sqrt{5}}{2} \right)^2 \right)^5 = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \left( \frac{1+2 \cdot \sqrt{5} + (\sqrt{5})^2}{4} \right)^5 = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{1+2 \cdot \sqrt{5} + 5}{4} \right)^5 = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \left( \frac{6+2 \cdot \sqrt{5}}{4} \right)^5 = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{2 \cdot (3+\sqrt{5})}{4} \right)^5 = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \left( \frac{2 \cdot (3+\sqrt{5})}{4} \right)^5 = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{3+\sqrt{5}}{2} \right)^5 = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \left( \frac{3+\sqrt{5}}{2} \right)^4 \cdot \left( \frac{3+\sqrt{5}}{2} \right)^1 = a + b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow$$

$$\begin{aligned}
&\Rightarrow \left( \left( \frac{3+\sqrt{5}}{2} \right)^2 \right)^2 \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \left( \frac{9+6\cdot\sqrt{5}+(\sqrt{5})^2}{4} \right)^2 \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \left( \frac{9+6\cdot\sqrt{5}+5}{4} \right)^2 \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \left( \frac{14+6\cdot\sqrt{5}}{4} \right)^2 \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \left( \frac{2\cdot(7+3\cdot\sqrt{5})}{4} \right)^2 \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \left( \frac{2\cdot(7+3\cdot\sqrt{5})}{4} \right)^2 \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \left( \frac{7+3\cdot\sqrt{5}}{2} \right)^2 \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{49+2\cdot 7\cdot 3\cdot\sqrt{5}+(3\cdot\sqrt{5})^2}{4} \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{49+42\cdot\sqrt{5}+3^2\cdot(\sqrt{5})^2}{4} \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{49+42\cdot\sqrt{5}+9\cdot 5}{4} \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \frac{49+42\cdot\sqrt{5}+45}{4} \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{94+42\cdot\sqrt{5}}{4} \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \frac{2\cdot(47+21\cdot\sqrt{5})}{4} \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{2\cdot(47+21\cdot\sqrt{5})}{4} \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \frac{47+21\cdot\sqrt{5}}{2} \cdot \frac{3+\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{(47+21\cdot\sqrt{5})\cdot(3+\sqrt{5})}{4} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{141+47\cdot\sqrt{5}+63\cdot\sqrt{5}+21\cdot(\sqrt{5})^2}{4} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{141+47\cdot\sqrt{5}+63\cdot\sqrt{5}+21\cdot 5}{4} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \frac{141+47\cdot\sqrt{5}+63\cdot\sqrt{5}+105}{4} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \\
&\Rightarrow \frac{246+110\cdot\sqrt{5}}{4} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow \frac{2\cdot(123+55\cdot\sqrt{5})}{4} = a+b \cdot \frac{1+\sqrt{5}}{2} \Rightarrow
\end{aligned}$$

$$\Rightarrow \frac{2 \cdot (123 + 55 \cdot \sqrt{5})}{4} = a + b \cdot \frac{1 + \sqrt{5}}{2} \Rightarrow \frac{123 + 55 \cdot \sqrt{5}}{2} = a + b \cdot \frac{1 + \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{123}{2} + \frac{55 \cdot \sqrt{5}}{2} = a + b \cdot \frac{1 + \sqrt{5}}{2}.$$

$$\bullet \left. \begin{array}{l} x = \frac{1 - \sqrt{5}}{2} \\ x^{10} = a + b \cdot x \end{array} \right\} \Rightarrow \left( \frac{1 - \sqrt{5}}{2} \right)^{10} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \left( \frac{1 - \sqrt{5}}{2} \right)^2 \right)^5 = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow \left( \frac{1 - 2 \cdot \sqrt{5} + (\sqrt{5})^2}{4} \right)^5 = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{1 - 2 \cdot \sqrt{5} + 5}{4} \right)^5 = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow \left( \frac{6 - 2 \cdot \sqrt{5}}{4} \right)^5 = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{2 \cdot (3 - \sqrt{5})}{4} \right)^5 = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow \left( \frac{2 \cdot (3 - \sqrt{5})}{4} \right)^5 = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{3 - \sqrt{5}}{2} \right)^5 = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow \left( \frac{3 - \sqrt{5}}{2} \right)^4 \cdot \left( \frac{3 - \sqrt{5}}{2} \right)^1 = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \left( \frac{3 - \sqrt{5}}{2} \right)^2 \right)^2 \cdot \frac{3 - \sqrt{5}}{2} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{9 - 6 \cdot \sqrt{5} + (\sqrt{5})^2}{4} \right)^2 \cdot \frac{3 - \sqrt{5}}{2} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{9 - 6 \cdot \sqrt{5} + 5}{4} \right)^2 \cdot \frac{3 - \sqrt{5}}{2} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{14 - 6 \cdot \sqrt{5}}{4} \right)^2 \cdot \frac{3 - \sqrt{5}}{2} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow \left( \frac{2 \cdot (7 - 3 \cdot \sqrt{5})}{4} \right)^2 \cdot \frac{3 - \sqrt{5}}{2} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \left( \frac{2 \cdot (7 - 3 \cdot \sqrt{5})}{4} \right)^2 \cdot \frac{3 - \sqrt{5}}{2} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow \left( \frac{7 - 3 \cdot \sqrt{5}}{2} \right)^2 \cdot \frac{3 - \sqrt{5}}{2} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow \frac{49 - 2 \cdot 7 \cdot 3 \cdot \sqrt{5} + (3 \cdot \sqrt{5})^2}{4} \cdot \frac{3 - \sqrt{5}}{2} = a + b \cdot \frac{1 - \sqrt{5}}{2} \Rightarrow$$

$$\begin{aligned} &\Rightarrow \frac{49-42\cdot\sqrt{5}+3^2\cdot(\sqrt{5})^2}{4} \cdot \frac{3-\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{49-42\cdot\sqrt{5}+9\cdot 5}{4} \cdot \frac{3-\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \frac{49-42\cdot\sqrt{5}+45}{4} \cdot \frac{3-\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{94-42\cdot\sqrt{5}}{4} \cdot \frac{3-\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \frac{2\cdot(47-21\cdot\sqrt{5})}{4} \cdot \frac{3-\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{2\cdot(47-21\cdot\sqrt{5})}{4} \cdot \frac{3-\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \frac{47-21\cdot\sqrt{5}}{2} \cdot \frac{3-\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{(47-21\cdot\sqrt{5})\cdot(3-\sqrt{5})}{4} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{141-47\cdot\sqrt{5}-63\cdot\sqrt{5}+21\cdot(\sqrt{5})^2}{4} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{141-47\cdot\sqrt{5}-63\cdot\sqrt{5}+21\cdot 5}{4} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \frac{141-47\cdot\sqrt{5}-63\cdot\sqrt{5}+105}{4} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{246-110\cdot\sqrt{5}}{4} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \frac{2\cdot(123-55\cdot\sqrt{5})}{4} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{2\cdot(123-55\cdot\sqrt{5})}{4} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \frac{123-55\cdot\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{123-55\cdot\sqrt{5}}{2} = a+b \cdot \frac{1-\sqrt{5}}{2}. \end{aligned}$$

Dobili smo sustav jednadžbi:

$$\left. \begin{aligned} \frac{123}{2} + \frac{55\cdot\sqrt{5}}{2} &= a+b \cdot \frac{1+\sqrt{5}}{2} \\ \frac{123}{2} - \frac{55\cdot\sqrt{5}}{2} &= a+b \cdot \frac{1-\sqrt{5}}{2} \end{aligned} \right\} \Rightarrow \left[ \begin{array}{l} \text{od prve jednadžbe} \\ \text{oduzmemo drugu} \end{array} \right] \Rightarrow$$

$$\begin{aligned} &\Rightarrow \frac{123}{2} + \frac{55\cdot\sqrt{5}}{2} - \left( \frac{123}{2} - \frac{55\cdot\sqrt{5}}{2} \right) = a+b \cdot \frac{1+\sqrt{5}}{2} - \left( a+b \cdot \frac{1-\sqrt{5}}{2} \right) \Rightarrow \\ &\Rightarrow \frac{123}{2} + \frac{55\cdot\sqrt{5}}{2} - \frac{123}{2} + \frac{55\cdot\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} - a-b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{123}{2} + \frac{55\cdot\sqrt{5}}{2} - \frac{123}{2} + \frac{55\cdot\sqrt{5}}{2} = a+b \cdot \frac{1+\sqrt{5}}{2} - a-b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{55\cdot\sqrt{5}}{2} + \frac{55\cdot\sqrt{5}}{2} = b \cdot \frac{1+\sqrt{5}}{2} - b \cdot \frac{1-\sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{55\cdot\sqrt{5}}{2} + \frac{55\cdot\sqrt{5}}{2} = b \cdot \frac{1+\sqrt{5}}{2} - b \cdot \frac{1-\sqrt{5}}{2} \quad / \cdot 2 \Rightarrow \end{aligned}$$

$$\begin{aligned} &\Rightarrow 55 \cdot \sqrt{5} + 55 \cdot \sqrt{5} = b \cdot (1 + \sqrt{5}) - b \cdot (1 - \sqrt{5}) \Rightarrow \\ &\Rightarrow 110 \cdot \sqrt{5} = b + b \cdot \sqrt{5} - b + b \cdot \sqrt{5} \Rightarrow 110 \cdot \sqrt{5} = b + b \cdot \sqrt{5} - b + b \cdot \sqrt{5} \Rightarrow \\ &\Rightarrow 110 \cdot \sqrt{5} = b + b \cdot \sqrt{5} - b + b \cdot \sqrt{5} \Rightarrow 110 \cdot \sqrt{5} = b \cdot \sqrt{5} + b \cdot \sqrt{5} \Rightarrow \\ &\Rightarrow 110 \cdot \sqrt{5} = 2 \cdot b \cdot \sqrt{5} \Rightarrow 2 \cdot b \cdot \sqrt{5} = 110 \cdot \sqrt{5} \Rightarrow 2 \cdot b \cdot \sqrt{5} = 110 \cdot \sqrt{5} / \frac{1}{2 \cdot \sqrt{5}} \Rightarrow b = 55. \end{aligned}$$

Računamo nepoznanicu a.

$$\begin{aligned} &\left. \begin{array}{l} b = 55 \\ \frac{123}{2} + \frac{55 \cdot \sqrt{5}}{2} = a + b \cdot \frac{1 + \sqrt{5}}{2} \end{array} \right\} \Rightarrow \frac{123}{2} + \frac{55 \cdot \sqrt{5}}{2} = a + 55 \cdot \frac{1 + \sqrt{5}}{2} \Rightarrow \\ &\Rightarrow \frac{123}{2} + \frac{55 \cdot \sqrt{5}}{2} = a + \frac{55 \cdot 1 + \sqrt{5}}{2} \Rightarrow \frac{123}{2} + \frac{55 \cdot \sqrt{5}}{2} = a + \frac{55 + 55 \cdot \sqrt{5}}{2} \Rightarrow \\ &\Rightarrow a + \frac{55 + 55 \cdot \sqrt{5}}{2} = \frac{123}{2} + \frac{55 \cdot \sqrt{5}}{2} \Rightarrow a + \frac{55}{2} + \frac{55 \cdot \sqrt{5}}{2} = \frac{123}{2} + \frac{55 \cdot \sqrt{5}}{2} \Rightarrow \\ &\Rightarrow a + \frac{55}{2} + \frac{55 \cdot \sqrt{5}}{2} = \frac{123}{2} + \frac{55 \cdot \sqrt{5}}{2} \Rightarrow a + \frac{55}{2} = \frac{123}{2} \Rightarrow a = \frac{123}{2} - \frac{55}{2} \Rightarrow a = \frac{123 - 55}{2} \Rightarrow \\ &\Rightarrow a = \frac{68}{2} \Rightarrow a = \frac{68}{2} \Rightarrow a = 34. \end{aligned}$$

Rješenje zadatka glasi:

$$a + b = 34 + 55 \Rightarrow a + b = 89.$$

Odgovor je pod E.

### Vježba 124

Neka je  $x^2 = 1 + x$ . Ako je  $x^{10} = a + b \cdot x$ , onda je  $b - a$  jednako:

- A. 19      B. 20      C. 21      D. 22      E. 23

**Rezultat:** C.

### Zadatak 125 (Karlo, gimnazija)

$$\text{Riješi sustav jednačbi: } \begin{cases} x^2 - y \cdot z = y - x \\ y^2 - x \cdot z = z - y \\ z^2 - x \cdot y = x - z \end{cases}$$

### Rješenje 125

Ponovimo!

$$\left. \begin{array}{l} a = b \\ c = d \end{array} \right\} \Rightarrow a + c = b + d, \quad (a - b)^2 = a^2 - 2 \cdot a \cdot b + b^2, \quad a^2 + b^2 = 0 \Rightarrow a = b = 0.$$

$$\begin{aligned} &\left. \begin{array}{l} x^2 - y \cdot z = y - x \\ y^2 - x \cdot z = z - y \\ z^2 - x \cdot y = x - z \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{zbrojimo} \\ \text{jednačbe} \end{array} \right] \Rightarrow \\ &\Rightarrow x^2 - y \cdot z + y^2 - x \cdot z + z^2 - x \cdot y = y - x + z - y + x - z \Rightarrow \end{aligned}$$



$$\begin{aligned}
&\Rightarrow x^2 - y \cdot z + y^2 - x \cdot z + z^2 - x \cdot y = y - x + z - y + x - z \Rightarrow \\
&\Rightarrow x^2 - y \cdot z + y^2 - x \cdot z + z^2 - x \cdot y = 0 \Rightarrow x^2 + y^2 + z^2 - x \cdot y - x \cdot z - y \cdot z = 0 \Rightarrow \\
&\quad \Rightarrow x^2 + y^2 + z^2 - x \cdot y - x \cdot z - y \cdot z = 0 \quad / \cdot 2 \Rightarrow \\
&\quad \Rightarrow 2 \cdot x^2 + 2 \cdot y^2 + 2 \cdot z^2 - 2 \cdot x \cdot y - 2 \cdot x \cdot z - 2 \cdot y \cdot z = 0 \Rightarrow \\
&\quad \Rightarrow x^2 - 2 \cdot x \cdot y + y^2 + x^2 - 2 \cdot x \cdot z + z^2 + y^2 - 2 \cdot y \cdot z + z^2 = 0 \Rightarrow \\
&\quad \Rightarrow (x^2 - 2 \cdot x \cdot y + y^2) + (x^2 - 2 \cdot x \cdot z + z^2) + (y^2 - 2 \cdot y \cdot z + z^2) = 0 \Rightarrow \\
&\quad \Rightarrow (x-y)^2 + (x-z)^2 + (y-z)^2 = 0 \Rightarrow \left. \begin{array}{l} x-y=0 \\ x-z=0 \\ y-z=0 \end{array} \right\} \Rightarrow x=y=z.
\end{aligned}$$

Rješenje sustava je svaka trojka realnih brojeva oblika:

$$(a, a, a), \quad a \in R.$$

### Vježba 125

Riješi sustav jednačini:

$$\begin{cases} x^2 + x = y \cdot z + y \\ y^2 + y = x \cdot z + z \\ z^2 + z = x \cdot y + x \end{cases}$$

**Rezultat:**  $(a, a, a), \quad a \in R.$

### Zadatak 126 (Katarina, maturantica)

Zeleni čaj pakiran je u kutije od 20 g i 50 g. Kutija od 20 g košta 11.30 kn, a kutija od 50 g košta 25 kn. Veletrgovac je 5200 g čaja platio 2743 kn. Koliko je ukupno kutija čaja kupio?

A. 75      B. 107      C. 170      D. 354

### Rješenje 126

Ponovimo!

Sve jasno!

Neka je  $x$  broj kutija od 20 g, a  $y$  broj kutija od 50 g. Kutija od 20 g košta 11.30 kn, a kutija od 50 g košta 25 kn.

Veletrgovac je:

- kupio 5200 g čaja pa vrijedi jednačina

$$20 \cdot x + 50 \cdot y = 5200$$

- platio 2743 kn pa vrijedi jednačina

$$11.30 \cdot x + 25 \cdot y = 2743.$$

Dobijemo sustav!

$$\begin{aligned}
&\left. \begin{array}{l} 20 \cdot x + 50 \cdot y = 5200 \\ 11.30 \cdot x + 25 \cdot y = 2743 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenta} \end{array} \right] \Rightarrow \left. \begin{array}{l} 20 \cdot x + 50 \cdot y = 5200 \\ 11.30 \cdot x + 25 \cdot y = 2743 \cdot (-2) \end{array} \right\} \Rightarrow \\
&\Rightarrow \left. \begin{array}{l} 20 \cdot x + 50 \cdot y = 5200 \\ -22.60 \cdot x - 50 \cdot y = -5486 \end{array} \right\} \Rightarrow -2.6 \cdot x = -286 \Rightarrow -2.6 \cdot x = -286 \quad / : (-2.6) \Rightarrow x = 110.
\end{aligned}$$

Računamo  $y$ .

$$\begin{aligned}
 & \left. \begin{array}{l} x = 110 \\ 20 \cdot x + 50 \cdot y = 5200 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = 110 \\ 20 \cdot x + 50 \cdot y = 5200 \quad /: 10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = 110 \\ 2 \cdot x + 5 \cdot y = 520 \end{array} \right\} \Rightarrow \\
 & \Rightarrow 2 \cdot 110 + 5 \cdot y = 520 \Rightarrow 220 + 5 \cdot y = 520 \Rightarrow 5 \cdot y = 520 - 220 \Rightarrow 5 \cdot y = 300 \Rightarrow \\
 & \qquad \qquad \qquad \Rightarrow 5 \cdot y = 300 \quad /: 5 \Rightarrow y = 60.
 \end{aligned}$$

Veletrgovac je ukupno kupio 170 kutija čaja.

$$x + y = 110 + 60 = 170.$$

Odgovor je pod C.



### Vježba 126

Zeleni čaj pakiran je u kutije od 2 dag i 5 dag. Kutija od 2 dag košta 11.30 kn, a kutija od 5 dag košta 25 kn. Veletrgovac je 520 dag čaja platio 2743 kn. Koliko je ukupno kutija čaja kupio?

- A. 75      B. 107      C. 170      D. 354

**Rezultat:**      C.