

Zadatak 061 (Ivana, ekonomska škola)

Ako je $x + y = 5$, $y + z = 7$, $z + u = 9$, koliko je $u + x$?

Rješenje 061

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1. inačica

Zbrojimo sve tri jednačbe.

$$\left. \begin{array}{l} x + y = 5 \\ y + z = 7 \\ z + u = 9 \end{array} \right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednačbe} \end{array} \right] \Rightarrow x + y + y + z + z + u = 5 + 7 + 9 \Rightarrow x + 2 \cdot y + 2 \cdot z + u = 21 \Rightarrow \\ \Rightarrow x + 2 \cdot (y + z) + u = 21.$$

Tada je

$$\left. \begin{array}{l} x + 2 \cdot (y + z) + u = 21 \\ y + z = 7 \end{array} \right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{supstitucije} \end{array} \right] \Rightarrow x + 2 \cdot 7 + u = 21 \Rightarrow \\ \Rightarrow x + 14 + u = 21 \Rightarrow x + u = 21 - 14 \Rightarrow x + u = 7 \Rightarrow u + x = 7.$$

2. inačica

U sustavu jednačbi

$$\begin{cases} x + y = 5 \\ y + z = 7 \\ z + u = 9 \end{cases}$$

oduzmemo drugu jednačbu od prve jednačbe.

$$x + y - y - z = 5 - 7 \Rightarrow x + y - y - z = 5 - 7 \Rightarrow x - z = -2.$$

Sada je

$$\left. \begin{array}{l} x - z = -2 \\ z + u = 9 \end{array} \right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednačbe} \end{array} \right] \Rightarrow x - z + z + u = -2 + 9 \Rightarrow x - z + z + u = -2 + 9 \Rightarrow \\ \Rightarrow x + u = 7 \Rightarrow u + x = 7.$$

3. inačica

U sustavu jednačbi

$$\begin{cases} x + y = 5 \\ y + z = 7 \\ z + u = 9 \end{cases}$$

iz treće jednačbe izračunamo z i uvrstimo u drugu jednačbu.

$$\left. \begin{array}{l} x + y = 5 \\ y + z = 7 \\ z + u = 9 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x + y = 5 \\ y + z = 7 \\ z = 9 - u \end{array} \right\} \Rightarrow \left. \begin{array}{l} x + y = 5 \\ y + 9 - u = 7 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x + y = 5 \\ y = 7 - 9 + u \end{array} \right\} \Rightarrow \left. \begin{array}{l} x + y = 5 \\ y = -2 + u \end{array} \right\} \Rightarrow \left. \begin{array}{l} x + y = 5 \\ y = u - 2 \end{array} \right\} \Rightarrow \\ \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{supstitucije} \end{array} \right] \Rightarrow x + u - 2 = 5 \Rightarrow x + u = 5 + 2 \Rightarrow x + u = 7 \Rightarrow u + x = 7.$$

Vježba 061

Ako je $x + y = 5$, $y + z = 6$, $z + u = 7$, koliko je $u + x$?

Rezultat: 6.

Zadatak 062 (Jelena, gimnazija)

$$\text{Riješi sustav: } \begin{cases} x+y+z=3 \\ x^2+y^2+z^2=3 \\ x \cdot y \cdot z=1. \end{cases}$$

Rješenje 062

Ponovimo!

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c.$$

$$(a-b)^3 = a^3 - 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 - b^3, \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Kvadriramo prvu (linearnu) jednadžbu.

$$\begin{aligned} x+y+z=3 &\Rightarrow x+y+z=3 / ^2 \Rightarrow (x+y+z)^2 = 3^2 \Rightarrow \\ &\Rightarrow x^2 + y^2 + z^2 + 2 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y \cdot z = 9 \Rightarrow [x^2 + y^2 + z^2 = 3] \Rightarrow \\ &\Rightarrow 3 + 2 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y \cdot z = 9 \Rightarrow 2 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y \cdot z = 9 - 3 \Rightarrow 2 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y \cdot z = 6 \Rightarrow \\ &\Rightarrow 2 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y \cdot z = 6 / : 2 \Rightarrow x \cdot y + x \cdot z + y \cdot z = 3 \Rightarrow x \cdot (y+z) + y \cdot z = 3. \end{aligned}$$

Transformiramo prvu i treću jednadžbu sustava.

$$\left. \begin{array}{l} x+y+z=3 \\ x \cdot y \cdot z=1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} y+z=3-x \\ x \cdot y \cdot z=1 / : \frac{1}{x} \end{array} \right\} \Rightarrow \left. \begin{array}{l} y+z=3-x \\ y \cdot z = \frac{1}{x} \end{array} \right\}.$$

Sada je:

$$\begin{aligned} x \cdot (y+z) + y \cdot z = 3 &\Rightarrow \left[\begin{array}{l} y+z=3-x \\ y \cdot z = \frac{1}{x} \end{array} \right] \Rightarrow x \cdot (3-x) + \frac{1}{x} = 3 \Rightarrow 3 \cdot x - x^2 + \frac{1}{x} = 3 \Rightarrow \\ &\Rightarrow 3 \cdot x - x^2 + \frac{1}{x} - 3 = 0 \Rightarrow 3 \cdot x - x^2 + \frac{1}{x} - 3 = 0 / \cdot x \Rightarrow 3 \cdot x^2 - x^3 + 1 - 3 \cdot x = 0 \Rightarrow \\ &\Rightarrow -x^3 + 3 \cdot x^2 - 3 \cdot x + 1 = 0 \Rightarrow -x^3 + 3 \cdot x^2 - 3 \cdot x + 1 = 0 / \cdot (-1) \Rightarrow x^3 - 3 \cdot x^2 + 3 \cdot x - 1 = 0 \Rightarrow \\ &\Rightarrow (x-1)^3 = 0 \Rightarrow (x-1)^3 = 0 / \sqrt[3]{\quad} \Rightarrow x-1=0 \Rightarrow x=1. \end{aligned}$$

Uvrstimo $x = 1$ u prvu i treću jednadžbu sustava.

$$\left. \begin{array}{l} x+y+z=3 \\ x \cdot y \cdot z=1 \\ x=1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 1+y+z=3 \\ 1 \cdot y \cdot z=1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} y+z=3-1 \\ y \cdot z=1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} y+z=2 \\ y \cdot z=1 \end{array} \right\} \Rightarrow \left. \begin{array}{l} z=2-y \\ y \cdot z=1 \end{array} \right\} \Rightarrow$$

$$\begin{aligned} &\Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{supstitucije} \end{array} \right] \Rightarrow y \cdot (2-y) = 1 \Rightarrow 2 \cdot y - y^2 = 1 \Rightarrow -y^2 + 2 \cdot y - 1 = 0 \Rightarrow \\ &\Rightarrow -y^2 + 2 \cdot y - 1 = 0 / \cdot (-1) \Rightarrow y^2 - 2 \cdot y + 1 = 0 \Rightarrow (y-1)^2 = 0 \Rightarrow \\ &\Rightarrow (y-1)^2 = 0 / \sqrt{\quad} \Rightarrow y-1=0 \Rightarrow y=1. \end{aligned}$$

Računamo nepoznanicu z .

$$\left. \begin{array}{l} z = 2 - y \\ y = 1 \end{array} \right\} \Rightarrow z = 2 - 1 \Rightarrow z = 1.$$

Rješenje sustava je:

$$(x, y, z) = (1, 1, 1).$$

Vježba 062

$$\text{Riješi sustav: } \begin{cases} x + y + z = 3 \\ (x + y)^2 + z^2 = 3 + 2 \cdot x \cdot y \\ x \cdot y \cdot z = 1. \end{cases}$$

Rezultat: $(x, y, z) = (1, 1, 1).$

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