Zadatak 161 (Tony, ekonomska škola)

Vrijednost razlomka $\frac{(m-n)^2+4\cdot m\cdot n}{(m+n)^2-4\cdot m\cdot n}$ za m = 111111, n= 7777777 jednaka je:

$$A)\ \frac{1}{49} \qquad B)\ \frac{16}{9} \qquad C)\ \frac{3}{4} \qquad D)\ \frac{1}{6}$$

Rješenje 161

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad (-a)^2=a^2.$$

$$\frac{(m-n)^2+4\cdot m\cdot n}{(m+n)^2-4\cdot m\cdot n}=\frac{m^2-2\cdot m\cdot n+n^2+4\cdot m\cdot n}{m^2+2\cdot m\cdot n+n^2-4\cdot m\cdot n}=\frac{m^2+2\cdot m\cdot n+n^2}{m^2-2\cdot m\cdot n+n^2}=\frac{(m+n)^2}{(m-n)^2}=$$

$$=\left(\frac{m+n}{m-n}\right)^2=\left(\frac{111111+777777}{111111-777777}\right)^2=\left(\frac{888888}{-666666}\right)^2=\left[\begin{array}{l}\text{kratimo razlomak}\\ \text{sa 111111}\end{array}\right]=\left(\frac{8}{-6}\right)^2=$$

$$=\left(\frac{8}{-6}\right)^2=\left(-\frac{4}{3}\right)^2=\frac{16}{9}.$$

Odgovor je pod B.

Vježba 161

Vrijednost razlomka $\frac{(m+n)^2-4\cdot m\cdot n}{(m-n)^2+4\cdot m\cdot n}$ za m = 111111, n= 7777777 jednaka je:

$$A)\ 49 \qquad B)\ \frac{9}{16} \qquad C)\ \frac{4}{3} \qquad D)\ 6$$

Rezultat: B.

Zadatak 162 (Tony, ekonomska škola)

Prikazati u obliku potencije s bazom 2: $2^8+4^5+8^3+16^2$.

Rješenje 162

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^1=a.$$

$$x\cdot a^n+y\cdot a^n=(x+y)\cdot a^n \quad , \quad a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$2^8+4^5+8^3+16^2=2^8+\left(2^2\right)^5+\left(2^3\right)^3+\left(2^4\right)^2=2^8+2^{10}+2^9+2^8=2\cdot 2^8+2^{10}+2^9=$$

$$=2^1\cdot 2^8+2^{10}+2^9=2^9+2^{10}+2^9=2\cdot 2^9+2^{10}=2^1\cdot 2^9+2^{10}=2^{10}+2^{10}=$$

$$=2\cdot 2^{10}=2^1\cdot 2^{10}=2^{11}.$$

2.inačica

$$2^8+4^5+8^3+16^2=2^8+\left(2^2\right)^5+\left(2^3\right)^3+\left(2^4\right)^2=2^8+2^{10}+2^9+2^8=$$

$$=2^8\cdot\left(1+2^2+2^1+1\right)=2^8\cdot(1+4+2+1)=2^8\cdot 8=2^8\cdot 2^3=2^{11}.$$

Vježba 162

Prikazati u obliku potencije s bazom 3: $5\cdot 9^5+4\cdot 27^3+8\cdot 3^9$.

Rezultat: 3^{12}.

Zadatak 163 (Tony, ekonomska škola)

Broj $\left(4^{-1}-5^{-1}\right)^{-2}$ jednak je:

$A)\ -9 \qquad B)\ 12 \qquad C)\ 41 \qquad D)\ 400$

Rješenje 163

Ponovimo!

$$a^{-n}=\frac{1}{a^n} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad \frac{n}{1}=n.$$

$$\left(4^{-1}-5^{-1}\right)^{-2}=\left(\frac{1}{4}-\frac{1}{5}\right)^{-2}=\left(\frac{5-4}{20}\right)^{-2}=\left(\frac{1}{20}\right)^{-2}=\left(\frac{20}{1}\right)^2=20^2=400.$$

Odgovor je pod D.

Vježba 163

Broj $\left(5^{-1}-4^{-1}\right)^{-2}$ jednak je:

$A)\ -9 \qquad B)\ 12 \qquad C)\ 41 \qquad D)\ 400$

Rezultat: D.

Zadatak 164 (Ana, srednja škola)

Ako je $5^m=3$, te $3^n=0.2$, koliko je $m\cdot n$?

Rješenje 164

Ponovimo!

$$a^{-n}=\frac{1}{a^n} \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad \log a^n=n\cdot\log a.$$

1.inačica

$$\left.\begin{array}{l}5^m=3\\3^n=0.2\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^m=3\\3^n=\frac{2}{10}\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^m=3\\3^n=\frac{2}{10}\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^m=3\\3^n=\frac{1}{5}\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^m=3\\3^n=5^{-1}\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda}\\\text{supstitucije}\end{array}\right]\Rightarrow$$

$$\Rightarrow\left(5^m\right)^n=5^{-1}\Rightarrow 5^{m\cdot n}=5^{-1}\Rightarrow m\cdot n=-1.$$

2.inačica

$$\left.\begin{array}{l}5^m=3\\3^n=0.2\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^m=3\\3^n=\frac{2}{10}\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^m=3\\3^n=\frac{2}{10}\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^m=3\\3^n=\frac{1}{5}\end{array}\right\}\Rightarrow\left.\begin{array}{l}5^m=3\\3^n=5^{-1}\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{logaritmiramo}\\\text{jednadžbe}\end{array}\right]\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}5^m=3\ /\ \log\\3^n=5^{-1}\ /\ \log\end{array}\right\}\Rightarrow\left.\begin{array}{l}\log 5^m=\log 3\\\log 3^n=\log 5^{-1}\end{array}\right\}\Rightarrow\left.\begin{array}{l}m\cdot\log 5=\log 3\\n\cdot\log 3=-\log 5\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{pomnožimo}\\\text{jednadžbe}\end{array}\right]\Rightarrow$$

$$\Rightarrow m\cdot\log 5\cdot n\cdot\log 3=-\log 3\cdot\log 5\Rightarrow m\cdot\log 5\cdot n\cdot\log 3=-\log 3\cdot\log 5\ /\cdot\frac{1}{\log 3\cdot\log 5}\Rightarrow m\cdot n=-1.$$

Vježba 164

Ako je $5^m = 3$, te $5 \cdot 3^n = 1$, koliko je $m \cdot n$?

Rezultat: -1.

Zadatak 165 (Ana, srednja škola)

Ako je $2^{10} \cdot 5^{12} = n \cdot 10^8$, onda je

$A)\ n = 2.5 \qquad B)\ n = 25 \qquad C)\ n = 250 \qquad D)\ n = 2500$

Rješenje 165

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad (a \cdot b)^n = a^n \cdot b^n \quad , \quad a^n : a^m = a^{n-m} \quad , \quad \log(a \cdot b) = \log a + \log b.$$

$$\log a^n = n \cdot \log a \quad , \quad \log \frac{a}{b} = \log a - \log b \quad , \quad \log 10 = 1 \quad , \quad \log 100 = 2.$$

$$\log f(x) = \log g(x) \ \Rightarrow\ f(x) = g(x).$$

1.inačica

$$2^{10} \cdot 5^{12} = n \cdot 10^8 \ \Rightarrow\ 2^{10} \cdot 5^{10} \cdot 5^2 = n \cdot 10^8 \ \Rightarrow\ (2 \cdot 5)^{10} \cdot 5^2 = n \cdot 10^8 \ \Rightarrow\ 10^{10} \cdot 25 = n \cdot 10^8 \ \Rightarrow$$

$$\Rightarrow\ 10^{10} \cdot 25 = n \cdot 10^8 \ /: 10^8 \ \Rightarrow\ 10^2 \cdot 25 = n \ \Rightarrow\ n = 10^2 \cdot 25 \ \Rightarrow\ n = 100 \cdot 25 \ \Rightarrow\ n = 2\,500.$$

Odgovor je pod D.

2.inačica

$$2^{10} \cdot 5^{12} = n \cdot 10^8 \ \Rightarrow\ \begin{bmatrix} \text{logaritmiramo} \\ \text{jednadžbu} \end{bmatrix} \ \Rightarrow\ 2^{10} \cdot 5^{12} = n \cdot 10^8 \ / \log \ \Rightarrow\ \log\left(2^{10} \cdot 5^{12}\right) = \log\left(n \cdot 10^8\right) \ \Rightarrow$$

$$\Rightarrow\ \log 2^{10} + \log 5^{12} = \log n + \log 10^8 \ \Rightarrow\ 10 \cdot \log 2 + 12 \cdot \log 5 = \log n + 8 \cdot \log 10 \ \Rightarrow$$

$$\Rightarrow\ 10 \cdot \log \frac{10}{5} + 12 \cdot \log 5 = \log n + 8 \cdot 1 \ \Rightarrow\ 10 \cdot (\log 10 - \log 5) + 12 \cdot \log 5 = \log n + 8 \ \Rightarrow$$

$$\Rightarrow\ 10 \cdot (1 - \log 5) + 12 \cdot \log 5 = \log n + 8 \ \Rightarrow\ 10 - 10 \cdot \log 5 + 12 \cdot \log 5 = \log n + 8 \ \Rightarrow$$

$$\Rightarrow\ 10 + 2 \cdot \log 5 = \log n + 8 \ \Rightarrow\ -\log n = 8 - 10 - 2 \cdot \log 5 \ \Rightarrow\ -\log n = -2 - 2 \cdot \log 5 \ \Rightarrow$$

$$\Rightarrow\ -\log n = -2 - 2 \cdot \log 5 \ / \cdot (-1) \ \Rightarrow\ \log n = 2 + 2 \cdot \log 5 \ \Rightarrow\ \log n = \log 100 + 2 \cdot \log 5 \ \Rightarrow$$

$$\Rightarrow\ \log n = \log 100 + \log 5^2 \ \Rightarrow\ \log n = \log 100 + \log 25 \ \Rightarrow\ \log n = \log(100 \cdot 25) \ \Rightarrow$$

$$\Rightarrow\ \log n = \log 2\,500 \ \Rightarrow\ n = 2\,500.$$

Odgovor je pod D.

Vježba 165

Ako je $2^9 \cdot 5^{11} = n \cdot 10^7$, onda je

$A)\ n = 2.5 \qquad B)\ n = 25 \qquad C)\ n = 250 \qquad D)\ n = 2500$

Rezultat: D.

Zadatak 166 (Željka, ekonomska škola)

Izračunaj: $\left(\sqrt{3} - 1\right)^2 \cdot \left(4 + 2 \cdot \sqrt{3}\right)$.

$A)\ 2 \qquad B)\ 4 \qquad C)\ 6 \qquad D)\ 8$

Rješenje 166

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$
$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n.$$

1.inačica

$$\left(\sqrt{3}-1\right)^2\cdot\left(4+2\cdot\sqrt{3}\right)=\left(\left(\sqrt{3}\right)^2-2\cdot\sqrt{3}+1\right)\cdot\left(4+2\cdot\sqrt{3}\right)=\left(3-2\cdot\sqrt{3}+1\right)\cdot\left(4+2\cdot\sqrt{3}\right)=$$
$$=\left(4-2\cdot\sqrt{3}\right)\cdot\left(4+2\cdot\sqrt{3}\right)=4^2-\left(2\cdot\sqrt{3}\right)^2=16-2^2\cdot\left(\sqrt{3}\right)^2=16-4\cdot 3=16-12=4.$$

Odgovor je pod B.

2.inačica

$$\left(\sqrt{3}-1\right)^2\cdot\left(4+2\cdot\sqrt{3}\right)=\left(\sqrt{3}-1\right)^2\cdot\left(3+2\cdot\sqrt{3}+1\right)=\left(\sqrt{3}-1\right)^2\cdot\left(\left(\sqrt{3}\right)^2+2\cdot\sqrt{3}+1\right)=$$
$$=\left(\sqrt{3}-1\right)^2\cdot\left(\sqrt{3}+1\right)^2=\left(\left(\sqrt{3}-1\right)\cdot\left(\sqrt{3}+1\right)\right)^2=\left(\left(\sqrt{3}\right)^2-1\right)^2=(3-1)^2=2^2=4.$$

Odgovor je pod B.

Vježba 166

Izračunaj: $\left(\sqrt{2}-1\right)^2\cdot\left(3+2\cdot\sqrt{2}\right).$

$A)\ 1 \qquad B)\ 2 \qquad C)\ 3 \qquad D)\ 4$

Rezultat: A.

Zadatak 167 (Željka, ekonomska škola)

Prikažite u obliku potencije s bazom 6:

$$2^{n-1}\cdot 3^{n+1}-2^{n+1}\cdot 3^{n-1}+6^{n-1}.$$

Rješenje 167

Ponovimo!

$$a^n\cdot a^m=a^{n+m} \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad a^1=a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$2^{n-1}\cdot 3^{n+1}-2^{n+1}\cdot 3^{n-1}+6^{n-1}=2^n\cdot 2^{-1}\cdot 3^n\cdot 3^1-2^n\cdot 2^1\cdot 3^n\cdot 3^{-1}+6^n\cdot 6^{-1}=$$
$$=2^n\cdot\frac{1}{2}\cdot 3^n\cdot 3-2^n\cdot 2\cdot 3^n\cdot\frac{1}{3}+6^n\cdot\frac{1}{6}=2^n\cdot 3^n\cdot\frac{3}{2}-2^n\cdot 3^n\cdot\frac{2}{3}+6^n\cdot\frac{1}{6}=$$
$$=(2\cdot 3)^n\cdot\frac{3}{2}-(2\cdot 3)^n\cdot\frac{2}{3}+6^n\cdot\frac{1}{6}=6^n\cdot\frac{3}{2}-6^n\cdot\frac{2}{3}+6^n\cdot\frac{1}{6}=6^n\cdot\left(\frac{3}{2}-\frac{2}{3}+\frac{1}{6}\right)=$$
$$=6^n\cdot\frac{9-4+1}{6}=6^n\cdot\frac{6}{6}=6^n\cdot\frac{6}{6}=6^n\cdot 1=6^n.$$

2.inačica

$$2^{n-1}\cdot 3^{n+1}-2^{n+1}\cdot 3^{n-1}+6^{n-1}=2^{n-1}\cdot 3^{n-1}\cdot 3^2-2^{n-1}\cdot 2^2\cdot 3^{n-1}+6^{n-1}=$$
$$=2^{n-1}\cdot 3^{n-1}\cdot 3^2-2^{n-1}\cdot 3^{n-1}\cdot 2^2+6^{n-1}=(2\cdot 3)^{n-1}\cdot 3^2-(2\cdot 3)^{n-1}\cdot 2^2+6^{n-1}=$$

$$= 6^{n-1} \cdot 3^2 - 6^{n-1} \cdot 2^2 + 6^{n-1} = 6^{n-1} \cdot \left(3^2 - 2^2 + 1\right) = 6^{n-1} \cdot (9 - 4 + 1) = 6^{n-1} \cdot 6 = 6^{n-1} \cdot 6^1 = 6^n.$$

Vježba 167

Prikažite u obliku potencije s bazom 6:

$$2^{n+1} \cdot 3^{n-1} - 2^{n-1} \cdot 3^{n+1} - 6^{n-1}.$$

Rezultat: -6^n.

Zadatak 168 (Zdravko, srednja škola)

Nejednakost $(a-b)^2 \le a^2 + b^2$ ispunjena je ako i samo ako je

$A)\ a \le b \qquad B)\ a - b \ge 0 \qquad C)\ a \cdot b \ge 0 \qquad D)\ a \ge b$

Rješenje 168

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2.$$

$$(a-b)^2 \le a^2 + b^2 \Rightarrow a^2 - 2 \cdot a \cdot b + b^2 \le a^2 + b^2 \Rightarrow a^2 - 2 \cdot a \cdot b + b^2 \le a^2 + b^2 \Rightarrow$$

$$\Rightarrow -2 \cdot a \cdot b \le 0 \Rightarrow -2 \cdot a \cdot b \le 0 \ /:(-2) \Rightarrow a \cdot b \ge 0.$$

Odgovor je pod C.

Vježba 168

Nejednakost $(a+b)^2 \ge a^2 + b^2$ ispunjena je ako i samo ako je

$A)\ a \le b \qquad B)\ a - b \ge 0 \qquad C)\ a \cdot b \ge 0 \qquad D)\ a \ge b$

Rezultat: C.

Zadatak 169 (Emy, gimnazija)

Ako je $a = 4^{n+1}$, $b = 25^{n-1}$ te $n \ge 1$, onda je umnožak $a \cdot b$ broj koji ima:

$A)\ 2 \cdot n$ znamenki $\qquad B)\ n+1$ znamenku $\qquad C)\ n+2$ znamenke $\qquad D)\ 2 \cdot n + 1$ znamenku

Rješenje 169

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^n \cdot b^n = (a \cdot b)^n.$$

Prirodni broj deset označavamo simbolom 10 i nazivamo ga dekadskom jedinicom prvog reda ili deseticom. Ostale su dekadske jedinice:

$$10^2 \ , \ 10^3 \ , \ 10^4 \ , \ 10^5 \ , \ 10^6 \ , \ \ldots \ , \ 10^n \ , \ \ldots$$

Prirodni broj množi se s dekadskom jedinicom tako da mu se zdesna dopiše toliko nula koliko ih ima dekadska jedinica. Na primjer,

$$57 \cdot 10^4 = 570\,000.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$a \cdot b = 4^{n+1} \cdot 25^{n-1} = \left(2^2\right)^{n+1} \cdot \left(5^2\right)^{n-1} = 2^{2 \cdot n + 2} \cdot 5^{2 \cdot n - 2} = 2^{2 \cdot n - 2} \cdot 2^4 \cdot 5^{2 \cdot n - 2} =$$

$$= 2^{2 \cdot n - 2} \cdot 5^{2 \cdot n - 2} \cdot 2^4 = (2 \cdot 5)^{2 \cdot n - 2} \cdot 2^4 = 10^{2 \cdot n - 2} \cdot 16 = 16 \cdot 10^{2 \cdot n - 2}.$$

Budući da broju 16 (koji ima 2 znamenke) treba nadopisati $2 \cdot n - 2$ nula, dobije se broj s

$$2 \cdot n - 2 + 2 = 2 \cdot n - 2 + 2 = 2 \cdot n$$

znamenki. Odgovor je pod A.

Vježba 169

Ako je $a = 4^{n+1}$, $b = 25^{n}$ te $n \geq 1$, onda je umnožak $a \cdot b$ broj koji ima :

$A)$ $2 \cdot n$ znamenki $B)$ $n+1$ znamenku $C)$ $n+2$ znamenke $D)$ $2 \cdot n+1$ znamenku

Rezultat: D.

Zadatak 170 (Emy, gimnazija)

Koliko znamenki ima broj $4^{5} \cdot 5^{13}$?

Rješenje 170

Ponovimo!

$$\left(a^{n}\right)^{m} = a^{n \cdot m} \quad , \quad a^{n} \cdot a^{m} = a^{n+m} \quad , \quad a^{n} \cdot b^{n} = (a \cdot b)^{n}.$$

Prirodni broj deset označavamo simbolom 10 i nazivamo ga dekadskom jedinicom prvog reda ili deseticom. Ostale su dekadske jedinice:

$$10^{2} \ , \ 10^{3} \ , \ 10^{4} \ , \ 10^{5} \ , \ 10^{6} \ , \ \ldots \ , \ 10^{n} \ , \ \ldots$$

Prirodni broj množi se s dekadskom jedinicom tako da mu se zdesna dopiše toliko nula koliko ih ima dekadska jedinica. Na primjer,

$$38 \cdot 10^{5} = 3800000.$$

$$4^{5} \cdot 5^{13} = \left(2^{2}\right)^{5} \cdot 5^{13} = 2^{10} \cdot 5^{13} = 2^{10} \cdot 5^{10} \cdot 5^{3} = (2 \cdot 5)^{10} \cdot 5^{3} = 10^{10} \cdot 5^{3} = 10^{10} \cdot 125 = 125 \cdot 10^{10}.$$

Budući da broju 125 (koji ima 3 znamenke) treba nadopisati 10 nula, dobije se broj s 13 znamenki.

Vježba 170

Koliko znamenki ima broj $4^{5} \cdot 5^{12}$?

Rezultat: 12 znamenki.

Zadatak 171 (Lucija, srednja škola)

Pojednostavni: $\sqrt{7+4 \cdot \sqrt{3}} + \sqrt{7-4 \cdot \sqrt{3}}$.

Rješenje 171

Ponovimo!

$$(a+b)^{2} = a^{2} + 2 \cdot a \cdot b + b^{2} \quad , \quad (a-b)^{2} = a^{2} - 2 \cdot a \cdot b + b^{2} \quad , \quad \left(\sqrt{a}\right)^{2} = a \quad , \quad \sqrt{a^{2}} = a \ , \ a \geq 0.$$

$$(a-b) \cdot (a+b) = a^{2} - b^{2} \quad , \quad a^{n} \cdot b^{n} = (a \cdot b)^{n} \quad , \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}.$$

1.inačica

Zadani izraz označimo sa x i kvadriramo ga.

$$x = \sqrt{7+4 \cdot \sqrt{3}} + \sqrt{7-4 \cdot \sqrt{3}} \Rightarrow x = \sqrt{7+4 \cdot \sqrt{3}} + \sqrt{7-4 \cdot \sqrt{3}} \ /^{2} \Rightarrow$$

$$\Rightarrow x^{2} = \left(\sqrt{7+4 \cdot \sqrt{3}} + \sqrt{7-4 \cdot \sqrt{3}}\right)^{2} \Rightarrow$$

$$\Rightarrow x^{2} = \left(\sqrt{7+4 \cdot \sqrt{3}}\right)^{2} + 2 \cdot \sqrt{7+4 \cdot \sqrt{3}} \cdot \sqrt{7-4 \cdot \sqrt{3}} + \left(\sqrt{7-4 \cdot \sqrt{3}}\right)^{2} \Rightarrow$$

$$\Rightarrow x^{2} = 7+4 \cdot \sqrt{3} + 2 \cdot \sqrt{\left(7+4 \cdot \sqrt{3}\right) \cdot \left(7-4 \cdot \sqrt{3}\right)} + 7 - 4 \cdot \sqrt{3} \Rightarrow$$

$$\Rightarrow x^{2} = 7+4 \cdot \sqrt{3} + 2 \cdot \sqrt{\left(7+4 \cdot \sqrt{3}\right) \cdot \left(7-4 \cdot \sqrt{3}\right)} + 7 - 4 \cdot \sqrt{3} \Rightarrow$$

$$\Rightarrow x^2=7+2\cdot\sqrt{\left(7+4\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)}+7 \Rightarrow x^2=14+2\cdot\sqrt{7^2-\left(4\cdot\sqrt{3}\right)^2} \Rightarrow$$

$$\Rightarrow x^2=14+2\cdot\sqrt{49-4^2\cdot\left(\sqrt{3}\right)^2} \Rightarrow x^2=14+2\cdot\sqrt{49-16\cdot 3} \Rightarrow x^2=14+2\cdot\sqrt{49-48} \Rightarrow$$

$$\Rightarrow x^2=14+2\cdot\sqrt{1} \Rightarrow x^2=14+2\cdot 1 \Rightarrow x^2=14+2 \Rightarrow x^2=16 \Rightarrow x^2=16 \;/\sqrt{\;} \Rightarrow$$

$$\Rightarrow x=\sqrt{16} \Rightarrow x=\sqrt{4^2} \Rightarrow x=4.$$

2.inačica

Radikande (izraze pod korijenima) transformiramo u kvadrat zbroja i kvadrat razlike.

$$\sqrt{7+4\cdot\sqrt{3}}+\sqrt{7-4\cdot\sqrt{3}}=\sqrt{4+2\cdot 2\cdot\sqrt{3}+3}+\sqrt{4-2\cdot 2\cdot\sqrt{3}+3}=$$

$$=\sqrt{2^2+2\cdot 2\cdot\sqrt{3}+\left(\sqrt{3}\right)^2}+\sqrt{2^2-2\cdot 2\cdot\sqrt{3}+\left(\sqrt{3}\right)^2}=\sqrt{\left(2+\sqrt{3}\right)^2}+\sqrt{\left(2-\sqrt{3}\right)^2}=$$

$$=2+\sqrt{3}+2-\sqrt{3}=2+\sqrt{3}+2-\sqrt{3}=4.$$

Vježba 171

Pojednostavni: $\sqrt{7+4\cdot\sqrt{3}}-\sqrt{7-4\cdot\sqrt{3}}.$

Rezultat: $2\cdot\sqrt{3}.$

Zadatak 172 (Lucija, srednja škola)

Izračunaj: $\sqrt{20+2\cdot\sqrt{19}}+\sqrt{20-2\cdot\sqrt{19}}.$

Rješenje 172

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \sqrt{a^2}=a \; , \; a\ge 0.$$

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad a^n\cdot b^n=(a\cdot b)^n \quad , \quad 1^n=1 \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}.$$

1.inačica

Zadani izraz označimo sa x i kvadriramo ga.

$$x=\sqrt{20+2\cdot\sqrt{19}}+\sqrt{20-2\cdot\sqrt{19}} \Rightarrow x=\sqrt{20+2\cdot\sqrt{19}}+\sqrt{20-2\cdot\sqrt{19}} \;/^2 \Rightarrow$$

$$\Rightarrow x^2=\left(\sqrt{20+2\cdot\sqrt{19}}+\sqrt{20-2\cdot\sqrt{19}}\right)^2 \Rightarrow$$

$$\Rightarrow x^2=\left(\sqrt{20+2\cdot\sqrt{19}}\right)^2+2\cdot\sqrt{20+2\cdot\sqrt{19}}\cdot\sqrt{20-2\cdot\sqrt{19}}+\left(\sqrt{20-2\cdot\sqrt{19}}\right)^2 \Rightarrow$$

$$\Rightarrow x^2=20+2\cdot\sqrt{19}+2\cdot\sqrt{\left(20+2\cdot\sqrt{19}\right)\cdot\left(20-2\cdot\sqrt{19}\right)}+20-2\cdot\sqrt{19} \Rightarrow$$

$$\Rightarrow x^2=20+2\cdot\sqrt{19}+2\cdot\sqrt{\left(20+2\cdot\sqrt{19}\right)\cdot\left(20-2\cdot\sqrt{19}\right)}+20-2\cdot\sqrt{19} \Rightarrow$$

$$\Rightarrow x^2=20+2\cdot\sqrt{\left(20+2\cdot\sqrt{19}\right)\cdot\left(20-2\cdot\sqrt{19}\right)}+20 \Rightarrow x^2=40+2\cdot\sqrt{20^2-\left(2\cdot\sqrt{19}\right)^2} \Rightarrow$$

$$\Rightarrow x^2=40+2\cdot\sqrt{400-2^2\cdot\left(\sqrt{19}\right)^2} \Rightarrow x^2=40+2\cdot\sqrt{400-4\cdot 19} \Rightarrow x^2=40+2\cdot\sqrt{400-76} \Rightarrow$$

$$\Rightarrow x^2=40+2\cdot\sqrt{324} \Rightarrow x^2=40+2\cdot\sqrt{18^2} \Rightarrow x^2=40+2\cdot 18 \Rightarrow x^2=40+36 \Rightarrow x^2=76 \Rightarrow$$

$$\Rightarrow x^2 = 76 \ /\sqrt{\ } \Rightarrow x = \sqrt{76} \Rightarrow \left[\begin{matrix}\text{djelomično}\\ \text{korjenovanje}\end{matrix}\right] \Rightarrow x = \sqrt{4\cdot 19} \Rightarrow x = 2\cdot\sqrt{19}.$$

2.inačica

Radikande (izraze pod korijenima) transformiramo u kvadrat zbroja i kvadrat razlike.

$$\sqrt{20+2\cdot\sqrt{19}} + \sqrt{20-2\cdot\sqrt{19}} = \sqrt{19+2\cdot\sqrt{19}+1} + \sqrt{19-2\cdot\sqrt{3}+1} =$$

$$= \sqrt{\left(\sqrt{19}\right)^2 + 2\cdot\sqrt{19}+1^2} + \sqrt{\left(\sqrt{19}\right)^2 - 2\cdot\sqrt{19}+1^2} = \sqrt{\left(\sqrt{19}+1\right)^2} + \sqrt{\left(\sqrt{19}-1\right)^2} =$$

$$= \sqrt{19}+1+\sqrt{19}-1 = \sqrt{19}+1+\sqrt{19}-1 = 2\cdot\sqrt{19}.$$

Vježba 172

Izračunaj: $\sqrt{20+2\cdot\sqrt{19}} - \sqrt{20-2\cdot\sqrt{19}}$.

Rezultat: 2.

Zadatak 173 (Mirela, ekonomska škola)

$$\text{Ako je } x = 333333333^2,\ y = 333333332\cdot 333333334, \text{ onda je:}$$

$$A)\ x-y=1 \qquad B)\ y=x+1 \qquad C)\ y-x=1 \qquad D)\ x=(y-1)^2$$

Rješenje 173

Ponovimo!

$$(a-b)\cdot(a+b) = a^2-b^2 \quad , \quad 1^n = 1.$$

$$y = 333333332\cdot 333333334 = (333333333-1)\cdot(333333333+1) = 333333333^2 - 1^2 =$$

$$= 333333333^2 - 1 = \left[x = 333333333^2\right] = x-1.$$

Tada je:

$$y = x-1 \Rightarrow -x+y=-1 \Rightarrow -x+y=-1 \ /\cdot(-1) \Rightarrow x-y=1.$$

Odgovor je pod A.

Vježba 173

$$\text{Ako je } x = 333333333^2,\ y = 333333331\cdot 333333335, \text{ onda je:}$$

$$A)\ x-y=4 \qquad B)\ y=x+4 \qquad C)\ y-x=4 \qquad D)\ x=(y-2)^2$$

Rezultat: A.

Zadatak 174 (Mirela, ekonomska škola)

$$\text{Ako je } x-y=3 \text{ i } x^2+y^2=5, \text{ onda je:}$$

$$A)\ x\cdot y=-5 \qquad B)\ x\cdot y=15 \qquad C)\ x\cdot y=-2 \qquad D)\ x\cdot y=25$$

Rješenje 174

Ponovimo!

$$(a+b)^2 = a^2+2\cdot a\cdot b+b^2 \quad , \quad a^2-2\cdot a\cdot b+b^2 = (a-b)^2 \quad , \quad -a+a=0.$$

1.inačica

Riješimo sustav linearne i kvadratne jednadžbe.

$$\left.\begin{matrix} x-y=3 \\ x^2+y^2=5 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x=3+y \\ x^2+y^2=5 \end{matrix}\right\} \Rightarrow \left[\begin{matrix}\text{metoda}\\ \text{supstitucije}\end{matrix}\right] \Rightarrow (3+y)^2+y^2=5 \Rightarrow$$

$$\Rightarrow 9+6\cdot y+y^2+y^2=5 \Rightarrow 2\cdot y^2+6\cdot y+9-5=0 \Rightarrow 2\cdot y^2+6\cdot y+4=0 \ /:2 \Rightarrow$$

$$\Rightarrow\ y^2+3\cdot y+2=0\ \Rightarrow\ \left.\begin{matrix} y^2+3\cdot y+2=0 \\ a=1\ ,\ b=3\ ,\ c=2\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} a=1\ ,\ b=3\ ,\ c=2 \\ y_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\}\ \Rightarrow$$

$$\Rightarrow\ y_{1,2}=\frac{-3\pm\sqrt{9-4\cdot 1\cdot 2}}{2\cdot 1}\ \Rightarrow\ y_{1,2}=\frac{-3\pm\sqrt{9-8}}{2}\ \Rightarrow\ y_{1,2}=\frac{-3\pm\sqrt{1}}{2}\ \Rightarrow\ y_{1,2}=\frac{-3\pm 1}{2}\ \Rightarrow$$

$$\Rightarrow\ \left.\begin{matrix} y_1=\dfrac{-3+1}{2} \\ y_2=\dfrac{-3-1}{2}\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} y_1=-\dfrac{2}{2} \\ y_2=-\dfrac{4}{2}\end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} y_1=-1 \\ y_2=-2\end{matrix}\right\}.$$

Računamo nepoznanicu x.

- $\left.\begin{matrix} y_1=-1 \\ x=3+y\end{matrix}\right\}\ \Rightarrow\ x=3+(-1)\ \Rightarrow\ x=3-1\ \Rightarrow\ x_1=2.$
- $\left.\begin{matrix} y_2=-2 \\ x=3+y\end{matrix}\right\}\ \Rightarrow\ x=3+(-2)\ \Rightarrow\ x=3-2\ \Rightarrow\ x_2=1.$

Tada je:

$$(x_1,\ y_1)=(2,\ -1)\ \Rightarrow\ x_1\cdot y_1=2\cdot(-1)\ \Rightarrow\ x_1\cdot y_1=-2.$$

$$(x_2,\ y_2)=(1,\ -2)\ \Rightarrow\ x_2\cdot y_2=1\cdot(-2)\ \Rightarrow\ x_2\cdot y_2=-2.$$

Odgovor je pod C.

2.inačica

Kvadriramo prvu (linearnu) jednadžbu i u dobiveni rezultat uvrstimo slobodni član druge (kvadratne) jednadžbe.

$$x-y=3\ \Rightarrow\ x-y=3\ /^2\ \Rightarrow\ (x-y)^2=3^2\ \Rightarrow\ x^2-2\cdot x\cdot y+y^2=9\ \Rightarrow\ x^2+y^2-2\cdot x\cdot y=9\ \Rightarrow$$

$$\Rightarrow\ \left[x^2+y^2=5\right]\ \Rightarrow\ 5-2\cdot x\cdot y=9\ \Rightarrow\ -2\cdot x\cdot y=9-5\ \Rightarrow\ -2\cdot x\cdot y=4\ \Rightarrow\ -2\cdot x\cdot y=4\ /:(-2)\ \Rightarrow$$

$$\Rightarrow\ x\cdot y=-2.$$

Odgovor je pod C.

3.inačica

Transformiramo drugu (kvadratnu) jednadžbu i u dobiveni rezultat uvrstimo slobodni član prve (linearne) jednadžbe.

$$x^2+y^2=5\ \Rightarrow\ x^2+y^2-2\cdot x\cdot y+2\cdot x\cdot y=5\ \Rightarrow\ x^2-2\cdot x\cdot y+y^2+2\cdot x\cdot y=5\ \Rightarrow$$

$$\Rightarrow\ \left(x^2-2\cdot x\cdot y+y^2\right)+2\cdot x\cdot y=5\ \Rightarrow\ (x-y)^2+2\cdot x\cdot y=5\ \Rightarrow\ [x-y=3]\ \Rightarrow$$

$$\Rightarrow\ 3^2+2\cdot x\cdot y=5\ \Rightarrow\ 9+2\cdot x\cdot y=5\ \Rightarrow\ 2\cdot x\cdot y=5-9\ \Rightarrow\ 2\cdot x\cdot y=-4\ \Rightarrow$$

$$\Rightarrow\ 2\cdot x\cdot y=-4\ /:2\ \Rightarrow\ x\cdot y=-2.$$

Odgovor je pod C.

Vježba 174

Ako je $x-y=1$ i $x^2+y^2=3$, onda je:

$A)\ x\cdot y=-1$ $\quad B)\ x\cdot y=1$ $\quad C)\ x\cdot y=-2$ $\quad D)\ x\cdot y=2$

Rezultat: B.

Zadatak 175 (Mirela, ekonomska škola)

Ako je $a \cdot (a-1) = \frac{1}{2}$, onda je $(2 \cdot a - 1)^2$ jednako:

$A)\ 1 \qquad B)\ 2 \qquad C)\ 3 \qquad D)\ 4$

Rješenje 175

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad -x + x = 0 \quad , \quad \left(\sqrt{x}\right)^2 = x \quad , \quad (x \cdot y)^n = x^n \cdot y^n .$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

Iz zadane jednadžbe (kvadratne jednadžbe) izračunamo a.

$$a \cdot (a-1) = \frac{1}{2} \Rightarrow a^2 - a = \frac{1}{2} \Rightarrow a^2 - a = \frac{1}{2} \, / \cdot 2 \Rightarrow 2 \cdot a^2 - 2 \cdot a = 1 \Rightarrow 2 \cdot a^2 - 2 \cdot a - 1 = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 2 \cdot a^2 - 2 \cdot a - 1 = 0 \\ a = 2 \ , \ b = -2 \ , \ c = -1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = 2 \ , \ b = -2 \ , \ c = -1 \\ a_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array}\right\} \Rightarrow$$

$$\Rightarrow a_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2} \Rightarrow a_{1,2} = \frac{2 \pm \sqrt{4+8}}{4} \Rightarrow a_{1,2} = \frac{2 \pm \sqrt{12}}{4} \Rightarrow a_{1,2} = \frac{2 \pm \sqrt{4 \cdot 3}}{4} \Rightarrow$$

$$\Rightarrow a_{1,2} = \frac{2 \pm 2 \cdot \sqrt{3}}{4} \Rightarrow a_{1,2} = \frac{2 \cdot \left(1 \pm \sqrt{3}\right)}{4} \Rightarrow a_{1,2} = \frac{2 \cdot \left(1 \pm \sqrt{3}\right)}{4} \Rightarrow a_{1,2} = \frac{1 \pm \sqrt{3}}{2} \Rightarrow \left.\begin{array}{l} a_1 = \dfrac{1+\sqrt{3}}{2} \\ a_2 = \dfrac{1-\sqrt{3}}{2} \end{array}\right\}.$$

Tada je:

- $$(2 \cdot a - 1)^2 = \left[a = \frac{1+\sqrt{3}}{2} \right] = \left(2 \cdot \frac{1+\sqrt{3}}{2} - 1\right)^2 = \left(2 \cdot \frac{1+\sqrt{3}}{2} - 1\right)^2 = \left(1 + \sqrt{3} - 1\right)^2 =$$
  $$= \left(1 + \sqrt{3} - 1\right)^2 = \left(\sqrt{3}\right)^2 = 3.$$
- $$(2 \cdot a - 1)^2 = \left[a = \frac{1-\sqrt{3}}{2} \right] = \left(2 \cdot \frac{1-\sqrt{3}}{2} - 1\right)^2 = \left(2 \cdot \frac{1-\sqrt{3}}{2} - 1\right)^2 = \left(1 - \sqrt{3} - 1\right)^2 =$$
  $$= \left(1 - \sqrt{3} - 1\right)^2 = \left(-\sqrt{3}\right)^2 = 3.$$

Odgovor je pod C.

2.inačica

$$a \cdot (a-1) = \frac{1}{2} \Rightarrow a^2 - a = \frac{1}{2}.$$

Tada je:

$$(2 \cdot a - 1)^2 = 4 \cdot a^2 - 4 \cdot a + 1 = 4 \cdot \left(a^2 - a\right) + 1 = \left[a^2 - a = \frac{1}{2} \right] = 4 \cdot \frac{1}{2} + 1 = 4 \cdot \frac{1}{2} + 1 = 2 + 1 = 3.$$

Odgovor je pod C.

3.inačica

$$a \cdot (a-1) = \frac{1}{2} \Rightarrow a^2 - a = \frac{1}{2} \Rightarrow a^2 - a = \frac{1}{2} \, / \cdot 4 \Rightarrow 4 \cdot a^2 - 4 \cdot a = 2.$$

Tada je:

$$(2\cdot a-1)^2=4\cdot a^2-4\cdot a+1=\left(4\cdot a^2-4\cdot a\right)+1=\left[\,4\cdot a^2-4\cdot a=2\,\right]=2+1=3.$$

Odgovor je pod C.

Vježba 175

Ako je $a\cdot(1-a)=-\frac{1}{2}$, onda je $(1-2\cdot a)^2$ jednako:

$A)\ 1 \qquad B)\ 2 \qquad C)\ 3 \qquad D)\ 4$

Rezultat: C.

Zadatak 176 (Jelena, gimnazija)

Pojednostavni izraz: $$\frac{\sqrt{\sqrt{2}-1}\cdot\sqrt[4]{3+2\cdot\sqrt{2}}+\sqrt[3]{(x+12)\cdot\sqrt{x}-6\cdot x-8}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt{\sqrt{2}+1}\cdot\sqrt[4]{3-2\cdot\sqrt{2}}}.$$

Rješenje 176

Ponovimo!

$$\sqrt[n]{a}\cdot\sqrt[m]{b}=\sqrt[n\cdot m]{a^m\cdot b^n}\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad(a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad\sqrt[3]{a^3}=a.$$

$$(a-b)^3=a^3-3\cdot a^2\cdot b+3\cdot a\cdot b^2-b^3\quad,\quad a\cdot\sqrt{b}=\sqrt{a^2\cdot b}\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}.$$

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad(a\cdot b)^n=a^n\cdot b^n\quad,\quad\left(a^n\right)^m=a^{n\cdot m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$\frac{\sqrt{\sqrt{2}-1}\cdot\sqrt[4]{3+2\cdot\sqrt{2}}+\sqrt[3]{(x+12)\cdot\sqrt{x}-6\cdot x-8}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt{\sqrt{2}+1}\cdot\sqrt[4]{3-2\cdot\sqrt{2}}}=\frac{\sqrt[4]{\left(\sqrt{2}-1\right)^2\cdot\left(3+2\cdot\sqrt{2}\right)}+\sqrt[3]{(x+12)\cdot\sqrt{x}-6\cdot x-8}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{\left(\sqrt{2}+1\right)^2\cdot\left(3-2\cdot\sqrt{2}\right)}}=$$

$$=\frac{\sqrt[4]{\left(\left(\sqrt{2}\right)^2-2\cdot\sqrt{2}+1\right)\cdot\left(3+2\cdot\sqrt{2}\right)}+\sqrt[3]{(x+12)\cdot\sqrt{x}-6\cdot x-8}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{\left(\left(\sqrt{2}\right)^2+2\cdot\sqrt{2}+1\right)\cdot\left(3-2\cdot\sqrt{2}\right)}}=$$

$$=\frac{\sqrt[4]{\left(2-2\cdot\sqrt{2}+1\right)\cdot\left(3+2\cdot\sqrt{2}\right)}+\sqrt[3]{(x+12)\cdot\sqrt{x}-6\cdot x-8}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{\left(2+2\cdot\sqrt{2}+1\right)\cdot\left(3-2\cdot\sqrt{2}\right)}}=$$

$$=\frac{\sqrt[4]{\left(3-2\cdot\sqrt{2}\right)\cdot\left(3+2\cdot\sqrt{2}\right)}+\sqrt[3]{x\cdot\sqrt{x}+12\cdot\sqrt{x}-6\cdot x-8}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{\left(3+2\cdot\sqrt{2}\right)\cdot\left(3-2\cdot\sqrt{2}\right)}}=$$

$$= \frac{\sqrt[4]{3^2-\left(2\cdot\sqrt{2}\right)^2}+\sqrt[3]{\sqrt{x^2}\cdot x-6\cdot x+12\cdot\sqrt{x}-8}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{3^2-\left(2\cdot\sqrt{2}\right)^2}} =$$

$$= \frac{\sqrt[4]{9-2^2\cdot\left(\sqrt{2}\right)^2}+\sqrt[3]{\sqrt{x}^3-3\cdot\left(\sqrt{x}\right)^2\cdot 2+3\cdot\sqrt{x}\cdot 2^2-2^3}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{9-2^2\cdot\left(\sqrt{2}\right)^2}} =$$

$$= \frac{\sqrt[4]{9-4\cdot 2}+\sqrt[3]{\left(\sqrt{x}\right)^3-3\cdot\left(\sqrt{x}\right)^2\cdot 2+3\cdot\sqrt{x}\cdot 2^2-2^3}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{9-4\cdot 2}} = \frac{\sqrt[4]{9-8}+\sqrt[3]{\left(\sqrt{x}-2\right)^3}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{9-8}} = \frac{\sqrt[4]{1}+\sqrt[3]{\left(\sqrt{x}-2\right)^3}}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt[4]{1}} =$$

$$= \frac{1+\sqrt{x}-2}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-1} = \frac{\sqrt{x}-1}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-1} = \frac{\sqrt{x}-1}{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\frac{1}{1}} = \frac{\sqrt{x}-1}{\frac{x-\sqrt{x}-\left(\sqrt{x}-1\right)}{\sqrt{x}-1}} = \frac{\sqrt{x}-1}{\frac{x-\sqrt{x}-\sqrt{x}+1}{\sqrt{x}-1}} =$$

$$= \frac{\sqrt{x}-1}{\frac{x-2\cdot\sqrt{x}+1}{\sqrt{x}-1}} = \frac{\frac{\sqrt{x}-1}{1}}{\frac{\left(\sqrt{x}\right)^2-2\cdot\sqrt{x}+1}{\sqrt{x}-1}} = \frac{\frac{\sqrt{x}-1}{1}}{\frac{\left(\sqrt{x}-1\right)^2}{\sqrt{x}-1}} = \frac{\left(\sqrt{x}-1\right)^2}{\left(\sqrt{x}-1\right)^2} = \frac{\left(\sqrt{x}-1\right)^2}{\left(\sqrt{x}-1\right)^2} = 1.$$

Vježba 176

Pojednostavni izraz: $\dfrac{\frac{x-\sqrt{x}}{\sqrt{x}-1}-\sqrt{\sqrt{2}+1}\cdot\sqrt[4]{3-2\cdot\sqrt{2}}}{\sqrt{\sqrt{2}-1}\cdot\sqrt[4]{3+2\cdot\sqrt{2}}+\sqrt[3]{\left(x+12\right)\cdot\sqrt{x}-6\cdot x-8}}.$

Rezultat: 1.

Zadatak 177 (Josip, srednja škola)

Racionalizirajte nazivnik u razlomku: $\dfrac{\sqrt[3]{2}-\sqrt[3]{3}}{\sqrt[3]{4}-\sqrt[3]{6}+\sqrt[3]{9}}.$

Rješenje 177

Ponovimo!

$$\left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m} \quad , \quad \sqrt[n]{a\cdot b} = \sqrt[n]{a}\cdot\sqrt[n]{b} \quad , \quad \sqrt[3]{a^3} = a \quad , \quad \left(a-b\right)\cdot\left(a+b\right) = a^2-b^2.$$

$$a^3+b^3 = \left(a+b\right)\cdot\left(a^2-a\cdot b+b^2\right).$$

$$\frac{\sqrt[3]{2}-\sqrt[3]{3}}{\sqrt[3]{4}-\sqrt[3]{6}+\sqrt[3]{9}} = \frac{\sqrt[3]{2}-\sqrt[3]{3}}{\sqrt[3]{2^2}-\sqrt[3]{2\cdot 3}+\sqrt[3]{3^2}} = \frac{\sqrt[3]{2}-\sqrt[3]{3}}{\left(\sqrt[3]{2}\right)^2-\sqrt[3]{2}\cdot\sqrt[3]{3}+\left(\sqrt[3]{3}\right)^2} = \left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right] =$$

$$= \frac{\sqrt[3]{2}-\sqrt[3]{3}}{\left(\sqrt[3]{2}\right)^2-\sqrt[3]{2}\cdot\sqrt[3]{3}+\left(\sqrt[3]{3}\right)^2}\cdot\frac{\sqrt[3]{2}+\sqrt[3]{3}}{\sqrt[3]{2}+\sqrt[3]{3}} = \frac{\left(\sqrt[3]{2}-\sqrt[3]{3}\right)\cdot\left(\sqrt[3]{2}+\sqrt[3]{3}\right)}{\left(\left(\sqrt[3]{2}\right)^2-\sqrt[3]{2}\cdot\sqrt[3]{3}+\left(\sqrt[3]{3}\right)^2\right)\cdot\left(\sqrt[3]{2}+\sqrt[3]{3}\right)} =$$

$$=\frac{\left(\sqrt[3]{2}\right)^2-\left(\sqrt[3]{3}\right)^2}{\left(\sqrt[3]{2}\right)^3+\left(\sqrt[3]{3}\right)^3}=\frac{\sqrt[3]{2^2}-\sqrt[3]{3^2}}{\sqrt[3]{2^3}+\sqrt[3]{3^3}}=\frac{\sqrt[3]{4}-\sqrt[3]{9}}{\sqrt[3]{2^3}+\sqrt[3]{3^3}}=\frac{\sqrt[3]{4}-\sqrt[3]{9}}{2+3}=\frac{\sqrt[3]{4}-\sqrt[3]{9}}{5}.$$

Vježba 177

Racionalizirajte nazivnik u razlomku: $\frac{\sqrt[3]{3}+\sqrt[3]{2}}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}$.

Rezultat: $\sqrt[3]{9}-\sqrt[3]{4}.$

Zadatak 178 (Josip, srednja škola)

Racionalizirajte nazivnik u razlomku: $\frac{3}{\sqrt[3]{4}-1}$.

Rješenje 178

Ponovimo!

$$\left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}\quad,\quad\sqrt[3]{a^3}=a\quad,\quad a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right).$$

$$\frac{3}{\sqrt[3]{4}-1}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{3}{\sqrt[3]{4}-1}\cdot\frac{\left(\sqrt[3]{4}\right)^2+\sqrt[3]{4}+1}{\left(\sqrt[3]{4}\right)^2+\sqrt[3]{4}+1}=\frac{3\cdot\left(\left(\sqrt[3]{4}\right)^2+\sqrt[3]{4}+1\right)}{\left(\sqrt[3]{4}-1\right)\cdot\left(\left(\sqrt[3]{4}\right)^2+\sqrt[3]{4}+1\right)}=$$

$$=\frac{3\cdot\left(\sqrt[3]{4^2}+\sqrt[3]{4}+1\right)}{\left(\sqrt[3]{4}\right)^3-1^3}=\frac{3\cdot\left(\sqrt[3]{16}+\sqrt[3]{4}+1\right)}{\sqrt[3]{4^3}-1}=\frac{3\cdot\left(\sqrt[3]{16}+\sqrt[3]{4}+1\right)}{\sqrt[3]{4^3}-1}=\frac{3\cdot\left(\sqrt[3]{16}+\sqrt[3]{4}+1\right)}{4-1}=$$

$$=\frac{3\cdot\left(\sqrt[3]{16}+\sqrt[3]{4}+1\right)}{3}=\frac{3\cdot\left(\sqrt[3]{16}+\sqrt[3]{4}+1\right)}{3}=\sqrt[3]{16}+\sqrt[3]{4}+1.$$

Vježba 178

Racionalizirajte nazivnik u razlomku: $\frac{5}{\sqrt[3]{4}+1}$.

Rezultat: $\sqrt[3]{16}-\sqrt[3]{4}+1.$

Zadatak 179 (Valentina, gimnazija)

Pojednostavnite izraz: $\sqrt{x+y+z+2\cdot\sqrt{x\cdot z+y\cdot z}}+\sqrt{x+y+z-2\cdot\sqrt{x\cdot z+y\cdot z}}\ ,\ x,\ y,\ z\geq 0.$

Rješenje 179

Ponovimo!

$$\left(\sqrt{a}\right)^2=a\quad,\quad(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$\sqrt{a^2}=a\ ,\ a\geq 0\quad,\quad\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}\quad,\quad -a+a=0.$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0.\end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x\geq 0$, vrijedi

$$|x| = x.$$

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, x < 0, je

$$|x| = -x.$$

Ili ovako:
Apsolutna vrijednost realnog broja x definira se

$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}.$$

Ako je broj x pozitivan broj, onda ga prepišemo: | x | = x, | 7 | = 7.
Ako je broj x negativan broj, onda ga pišemo s minusom: | x | = – x, | – 4 | = – (– 4) = 4.

$$\sqrt{x+y+z+2\cdot\sqrt{x\cdot z+y\cdot z}}+\sqrt{x+y+z-2\cdot\sqrt{x\cdot z+y\cdot z}} =$$

$$=\sqrt{x+y+2\cdot\sqrt{x\cdot z+y\cdot z}+z}+\sqrt{x+y-2\cdot\sqrt{x\cdot z+y\cdot z}+z} =$$

$$=\sqrt{\left(\sqrt{x+y}\right)^2+2\cdot\sqrt{(x+y)\cdot z}+\left(\sqrt{z}\right)^2}+\sqrt{\left(\sqrt{x+y}\right)^2-2\cdot\sqrt{(x+y)\cdot z}+\left(\sqrt{z}\right)^2} =$$

$$=\sqrt{\left(\sqrt{x+y}\right)^2+2\cdot\sqrt{x+y}\cdot\sqrt{z}+\left(\sqrt{z}\right)^2}+\sqrt{\left(\sqrt{x+y}\right)^2-2\cdot\sqrt{x+y}\cdot\sqrt{z}+\left(\sqrt{z}\right)^2} =$$

$$=\sqrt{\left(\sqrt{x+y}+\sqrt{z}\right)^2}+\sqrt{\left(\sqrt{x+y}-\sqrt{z}\right)^2}=\left|\sqrt{x+y}+\sqrt{z}\right|+\left|\sqrt{x+y}-\sqrt{z}\right| =$$

$$=\sqrt{x+y}+\sqrt{z}+\left|\sqrt{x+y}-\sqrt{z}\right|.$$

- Ako je

$$\sqrt{x+y}>\sqrt{z},$$

slijedi:

$$\sqrt{x+y+z+2\cdot\sqrt{x\cdot z+y\cdot z}}+\sqrt{x+y+z-2\cdot\sqrt{x\cdot z+y\cdot z}} =$$

$$=\sqrt{x+y}+\sqrt{z}+\left|\sqrt{x+y}-\sqrt{z}\right|=\sqrt{x+y}+\sqrt{z}+\sqrt{x+y}-\sqrt{z} =$$

$$=\sqrt{x+y}+\sqrt{z}+\sqrt{x+y}-\sqrt{z}=2\cdot\sqrt{x+y}.$$

- Ako je

$$\sqrt{x+y}<\sqrt{z},$$

slijedi:

$$\sqrt{x+y+z+2\cdot\sqrt{x\cdot z+y\cdot z}}+\sqrt{x+y+z-2\cdot\sqrt{x\cdot z+y\cdot z}} =$$

$$=\sqrt{x+y}+\sqrt{z}+\left|\sqrt{x+y}-\sqrt{z}\right|=\sqrt{x+y}+\sqrt{z}-\left(\sqrt{x+y}-\sqrt{z}\right) =$$

$$=\sqrt{x+y}+\sqrt{z}-\sqrt{x+y}+\sqrt{z}=\sqrt{x+y}+\sqrt{z}-\sqrt{x+y}+\sqrt{z}=2\cdot\sqrt{z}.$$

Vježba 179

Pojednostavnite izraz: $\sqrt{x+y+z+2\cdot\sqrt{x\cdot z+y\cdot z}}+\sqrt{x+y+z-2\cdot\sqrt{x\cdot z+y\cdot z}}$, $x,\ y,\ z\geq 0$.

Rezultat: $\sqrt{x+y}>\sqrt{z}\ \Rightarrow\ 2\cdot\sqrt{z}$, $\sqrt{x+y}<\sqrt{z}\ \Rightarrow\ 2\cdot\sqrt{x+y}$.

Zadatak 180 (Frane, gimnazija)

Oduzmi binarne brojeve:

$$\begin{array}{r} 1\ 0\ 0\ 1\ 0_{(2)} \\ -\quad 1\ 1\ 0\ 1_{(2)} \\ \hline \end{array}$$

Rješenje 180

Ponovimo!

Za bazu brojevnog sustava možemo uzeti bilo koji prirodni broj veći od 1. U sustavu s bazom b prirodni broj N zapisujemo kao

$$N = a_n a_{n-1} a_{n-2} \ \dots \ a_2 a_1 a_{0(b)}$$

pri čemu brojevi a_i, i = 0, 1, 2, 3, …, n mogu poprimati vrijednosti iz skupa $\{0, 1, 2, 3, ..., b-1\}$. Vrijednost prirodnog broja N zapisanog u sustavu s bazom b je

$$N = a_n a_{n-1} a_{n-2} \ \dots \ a_2 a_1 a_{0(b)} = a_n \cdot b^n + a_{n-1} \cdot b^{n-1} + a_{n-2} \cdot b^{n-2} + \ \dots \ + a_2 \cdot b^2 + a_1 \cdot b^1 + a_0 \cdot b^0 =$$

$$= a_n \cdot b^n + a_{n-1} \cdot b^{n-1} + a_{n-2} \cdot b^{n-2} + \ \dots \ + a_2 \cdot b^2 + a_1 \cdot b + a_0.$$

U binarnom sustavu (sustavu s bazom 2) postoje samo dvije različite znamenke: 0 i 1. U njemu također vrijedi zakon pozicije (mjesne vrijednosti!).

Tablica zbrajanja u binarnom sustavu glasi:

0 + 0 = 0	**0 + 1 = 1**
1 + 0 = 1	**1 + 1 = 10 (čitaj jedan nula)** **ili** **1 + 1 = 0 i prijenos 1** **Prijenos se prenosi u susjedni stupac s lijeve strane.**

Tablica oduzimanja u binarnom sustavu glasi:

0 – 0 = 0	**1 – 0 = 1**
1 – 1 = 0	**0 – 1 = 1 i '1 dalje'** **('1 dalje' oduzimamo od umanjenika u sljedećem stupcu nalijevo)**

1.inačica

Oduzima se kao i u dekadskom sustavu počevši od najnižih mjesnih vrijednosti.

U ovom primjeru govorimo.

$$\begin{array}{rrrrrr} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & & & & & 1 \end{array}$$

- u stupcu 'jedinica' – nula minus jedan je jedan i jedan dalje (0 – 1 = 1 i 1 dalje)

$$\begin{array}{rrrrrr} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & & & & 0 & 1 \end{array}$$

- u stupcu 'desetica' – jedan minus jedan (dalje) je nula, nula minus nula je nula (1 – 1 – 0 = 0)

$$\begin{array}{rrrrrr} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & & & 1 & 0 & 1 \end{array}$$

- u stupcu 'stotica' – nula minus jedan je jedan i jedan dalje (0 – 1 = 1 i 1 dalje)

$$\begin{array}{rrrrrr} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & & 0 & 1 & 0 & 1 \end{array}$$

- u stupcu 'tisućica' – nula minus jedan (dalje) je jedan (i jedan dalje), jedan minus jedan je nula (0 – 1 = 1 i 1 dalje, 1 – 1 = 0)

$$\begin{array}{rccccc} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & 0 & 0 & 1 & 0 & 1 \end{array}$$

- u stupcu 'deset tisućica' – jedan minus jedan (dalje) je nula (1 – 1 = 0)

Rezultat oduzimanja zadana dva broja je:

$$10010_{(2)} - 1101_{(2)} = 00101_{(2)} = 101_{(2)}$$

Ili ovako!
Oduzima se kao i u dekadskom sustavu počevši od najnižih mjesnih vrijednosti.
U ovom primjeru govorimo.

$$\begin{array}{rccccc} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & & & & & 1 \end{array}$$

- jedan do nula ne može pa je jedan do jedan nula jednako jedan i jedan pišemo dalje

$$\begin{array}{rccccc} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & & & & 0 & 1 \end{array}$$

- nula i jedan (dalje) je jedan i do jedan jednako je nula

$$\begin{array}{rccccc} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & & & 1 & 0 & 1 \end{array}$$

- jedan do nula ne može pa je jedan do jedan nula jednako jedan i jedan pišemo dalje

$$\begin{array}{rccccc} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline & & 0 & 1 & 0 & 1 \end{array}$$

- jedan i jedan (dalje) je jedan nula i do jedan nula je nula

Dakle, rezultat oduzimanja je $101_{(2)}$.

2.inačica

Binarni brojevi mogu se oduzimati pomoću dvojnoga komplementa. Dvojnim komplementom prikazuju se negativni brojevi u binarnom brojevnom sustavu pa se oduzimanje svodi na pribrajanje negativnog broja. Taj postupak provodi se u četiri koraka.

① korak

Umanjenik (broj od kojeg se oduzima) i umanjitelj (broj koji se oduzima) moraju imati isti broj znamenki pa se zato umanjitelju s lijeve strane dopišu nule (ako je potrebno) tako da umanjenik i umanjitelj imaju jednaki broj znamenki.

$$\begin{array}{rccccc} & 1 & 0 & 0 & 1 & 0 \\ - & & 1 & 1 & 0 & 1 \\ \hline \end{array} \qquad \begin{array}{rccccc} & 1 & 0 & 0 & 1 & 0 \\ - & 0 & 1 & 1 & 0 & 1 \\ \hline \end{array} \begin{array}{l} \to \text{umanjenik} \\ \to \text{umanjitelj} \\ \end{array}$$

② korak

Odredimo komplement umanjitelja tako da sve 0 u umanjitelju pretvorimo u 1, a 1 pretvorimo u 0. (umjesto 0 pišemo 1, a umjesto 1 pišemo 0).

$$0\,1\,1\,0\,1 \;\to\; 1\,0\,0\,1\,0$$

③ korak

Komplementu pribrojimo 1.

$$\begin{array}{rccccc} & 1 & 0 & 0 & 1 & 0 \\ + & & & & & 1 \\ \hline & 1 & 0 & 0 & 1 & 1 \end{array} \quad \to \quad \text{dvojni komplement}$$

Dobije se dvojni komplement.

④ korak

Umanjenik zbrojimo sa dvojnim komplementom i odbacimo krajnju jedinicu na lijevoj strani rezultata.

$$\begin{array}{cccccc} & 1 & 0 & 0 & 1 & 0 \\ + & 1 & 0 & 0 & 1 & 1 \\ \hline & & & & & 1 \end{array}$$

$$\begin{array}{cccccc} & & & 1 & & \\ & 1 & 0 & 0 & 1 & 0 \\ + & 1 & 0 & 0 & 1 & 1 \\ \hline & & & & 0 & 1 \end{array}$$

$$\begin{array}{cccccc} & 1 & 0 & 0 & 1 & 0 \\ + & 1 & 0 & 0 & 1 & 1 \\ \hline & & & 1 & 0 & 1 \end{array}$$

$$\begin{array}{cccccc} & 1 & 0 & 0 & 1 & 0 \\ + & 1 & 0 & 0 & 1 & 1 \\ \hline & & 0 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{ccccccc} & & 1 & 0 & 0 & 1 & 0 \\ + & & 1 & 0 & 0 & 1 & 1 \\ \hline & 1 & 0 & 0 & 1 & 0 & 1 \end{array}$$

Odbacimo krajnju lijevu jedinicu pa je rezultat oduzimanja binarnih brojeva broj $00101_{(2)}$ ili $101_{(2)}$.

Vježba 180

Oduzmi binarne brojeve:

$$\begin{array}{cccccc} & 1 & 1 & 0 & 1 & 1_{(2)} \\ - & & 1 & 1 & 1 & 0_{(2)} \\ \hline \end{array}$$

Rezultat: $1101_{(2)}$.