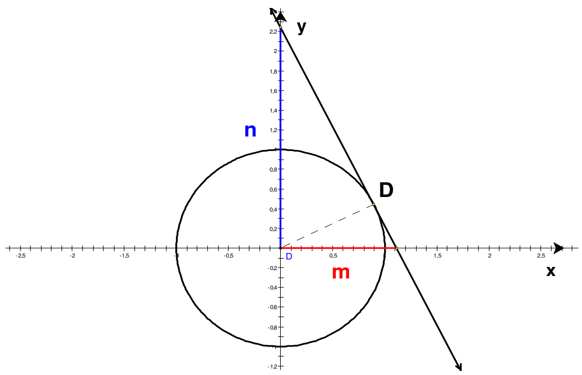
Zadatak 061 (Maturant, gimnazija)

Odredite jednadžbu tangente na luk kružnice $x^2 + y^2 = 1$ u prvom kvadrantu, tako da odsječak na osi y bude dvostruko veći od odsječka na osi x.

Rješenje 061



Napišimo jednadžbu tangente (pravca) u segmentnom obliku:

$$\frac{x}{m}+\frac{y}{n}=1.$$

Iz uvjeta zadatka je

$$\left.\begin{array}{l} n=2\cdot m \\ \frac{x}{m}+\frac{y}{n}=1 \end{array}\right\} \Rightarrow \frac{x}{m}+\frac{y}{2\cdot m}=1 \;/\cdot 2\cdot m \Rightarrow 2\cdot x+y=2\cdot m \Rightarrow$$

$$\Rightarrow y=-2\cdot x+2\cdot m \Rightarrow \left.\begin{array}{l} k=-2 \\ l=2\cdot m \end{array}\right\}.$$

Uvjet da pravac $y = k \cdot x + l$ bude tangenta kružnice $x^2 + y^2 = r^2$ glasi:

$$r^2\cdot\left(k^2+1\right)=l^2.$$

Zato je:

$$\left.\begin{array}{l} x^2+y^2=1 \\ k=-2\;,\;l=2\cdot m \end{array}\right\} \Rightarrow \left.\begin{array}{l} r^2\cdot\left(k^2+1\right)=l^2 \\ r^2=1\;,\;k=-2\;,\;l=2\cdot m \end{array}\right\} \Rightarrow 1\cdot(4+1)=4\cdot m^2 \Rightarrow 4\cdot m^2=5 \Rightarrow$$

$$\Rightarrow m^2=\frac{5}{4}\;/\sqrt{} \Rightarrow m=\pm\frac{\sqrt{5}}{2}.$$

Budući da je riječ o prvom kvadrantu jednadžba tangente je:

$$\left.\begin{array}{l} k=-2 \\ l=2\cdot m \end{array}\right\} \Rightarrow \left.\begin{array}{l} k=-2 \\ l=2\cdot\frac{\sqrt{5}}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} k=-2 \\ l=\sqrt{5} \end{array}\right\} \Rightarrow y=-2\cdot x+\sqrt{5}.$$

Vježba 061

Odredite jednadžbu tangente na luk kružnice $x^2 + y^2 = 1$ u trećem kvadrantu, tako da odsječak na osi y bude dvostruko veći od odsječka na osi x.

Rezultat: $y=-2\cdot x-\sqrt{5}.$

Zadatak 062 (Maturant, gimnazija)

Koliko iznosi površina dijela lika $x^2 + y^2 \le 2 \cdot x + 2 \cdot y$ koji se nalazi u prvom kvadrantu?

Rješenje 062

$$x^2+y^2\le 2\cdot x+2\cdot y \Rightarrow x^2-2\cdot x+y^2-2\cdot y\le 0 \Rightarrow x^2-2\cdot x+1-1+y^2-2\cdot y+1-1\le 0 \Rightarrow$$

$$\Rightarrow (x-1)^2+(y-1)^2\le 2 \Rightarrow S(1,1)\;,\;r=\sqrt{2}.$$

Dobili smo krug sa središtem S(1, 1) i polumjerom $r=\sqrt{2}.$

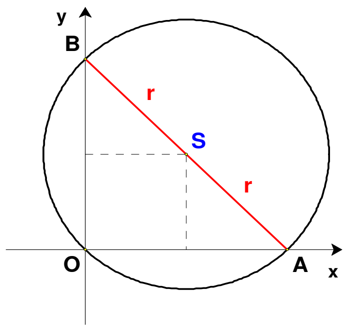
Odredimo presjek kružnice $(x - 1)^2 + (y - 1)^2 = 2$ i osi x:

$$\left.\begin{array}{l} (x-1)^2+(y-1)^2=2 \\ y=0 \end{array}\right\} \Rightarrow (x-1)^2+(0-1)^2=2 \Rightarrow (x-1)^2+1=2 \Rightarrow (x-1)^2=1\;/\sqrt{} \Rightarrow$$

$$\Rightarrow x-1=\pm 1 \Rightarrow \left.\begin{array}{l} x_1=1+1 \\ x_2=-1+1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1=2 \\ x_2=0 \end{array}\right\}.$$

Analogno nađemo presjek kružnice $(x-1)^2+(y-1)^2=2$ i osi y:

$$\left.\begin{array}{l}(x-1)^2+(y-1)^2=2\\ x=0\end{array}\right\} \Rightarrow (0-1)^2+(y-1)^2=2 \Rightarrow 1+(y-1)^2=2 \Rightarrow (y-1)^2=1 \,/\sqrt{\ } \Rightarrow$$



$$\Rightarrow y-1=\pm 1 \Rightarrow \left.\begin{array}{l}y_1=1+1\\ y_2=-1+1\end{array}\right\} \Rightarrow \left.\begin{array}{l}y_1=2\\ y_2=0\end{array}\right\}.$$

Trokut OAB je pravokutan i jednakokračan:

$$|OA|=|OB|=2\ ,\ r=|SA|=|SB|=\sqrt{2}.$$

Sa slike vidi se da je površina dijela lika koji se nalazi u prvom kvadrantu jednaka:

$$P=\frac{1}{2}\cdot r^2\cdot\pi+\frac{|OA|\cdot|OB|}{2} \Rightarrow P=\frac{1}{2}\cdot 2\cdot\pi+\frac{2\cdot 2}{2} \Rightarrow P=\pi+2.$$

Vježba 062

Koliko iznosi površina dijela lika $x^2+y^2\le 2\cdot x+2\cdot y$ koji se ne nalazi u prvom kvadrantu?

Rezultat: $P=\pi-2.$

Zadatak 063 (Maturant, gimnazija)

Kako glasi jednadžba hiperbole čija je jedna asimptota $y=\frac{1}{2}\cdot x,$ te koja dira pravac $x+y+1=0$?

Rješenje 063

Iz jednadžbe asimptote hiperbole dobije se:

$$\left.\begin{array}{l}y=\frac{b}{a}\cdot x\\ y=\frac{1}{2}\cdot x\end{array}\right\} \Rightarrow \frac{b}{a}=\frac{1}{2} \Rightarrow a=2\cdot b.$$

Uvjet da pravac $y=k\cdot x+l$ bude tangenta hiperbole $b^2\cdot x^2-a^2\cdot y^2=a^2\cdot b^2$ glasi:

$$a^2\cdot k^2-b^2=l^2.$$

Zato je:

$$x+y+1=0 \Rightarrow y=-x-1 \Rightarrow \left.\begin{array}{l}k=-1\ ,\ l=-1\ ,\ a=2\cdot b\\ a^2\cdot k^2-b^2=l^2\end{array}\right\} \Rightarrow 4\cdot b^2\cdot 1-b^2=1 \Rightarrow 4\cdot b^2-b^2=1 \Rightarrow$$

$$\Rightarrow 3\cdot b^2=1 \Rightarrow b^2=\frac{1}{3} \Rightarrow \left.\begin{array}{l}a=2\cdot b\\ b^2=\frac{1}{3}\end{array}\right\} \Rightarrow a^2=4\cdot b^2 \Rightarrow a^2=4\cdot\frac{1}{3} \Rightarrow a^2=\frac{4}{3}.$$

Jednadžba hiperbole je:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \Rightarrow \frac{x^2}{\frac{4}{3}}-\frac{y^2}{\frac{1}{3}}=1 \Rightarrow \frac{3}{4}\cdot x^2-3\cdot y^2=1.$$

Vježba 063

Kako glasi jednadžba hiperbole čija je jedna asimptota $y=-\frac{1}{2}\cdot x,$ te koja dira pravac $x+y+1=0$?

Rezultat: $\frac{3}{4}\cdot x^2-3\cdot y^2=1.$

Zadatak 064 (3A, TUPŠ)

Odredite jednadžbe tangenata na elipsu $9 \cdot x^2 + 16 \cdot y^2 = 144$ koje su paralelne (usporedne) pravcu $5 \cdot x - 33 \cdot y - 20 = 0$.

Rješenje 064

Prvo odredimo kanonski oblik jednadžbe dane elipse:

$$9 \cdot x^2 + 16 \cdot y^2 = 144 \; / :144 \Rightarrow \frac{9 \cdot x^2}{144} + \frac{16 \cdot y^2}{144} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow \left. \begin{array}{l} a^2 = 16 \\ b^2 = 9 \end{array} \right\}.$$

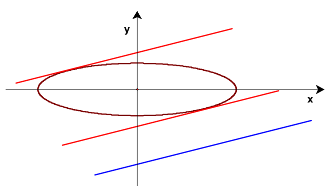
Zadanom pravcu nađemo koeficijent smjera k_1:

$$5 \cdot x - 33 \cdot y - 20 = 0 \Rightarrow -33 \cdot y = -5 \cdot x + 20 \; / \cdot \frac{-1}{33} \Rightarrow y = \frac{5}{33} \cdot x - \frac{20}{33} \Rightarrow k_1 = \frac{5}{33}.$$

Budući da su tangente paralelne (usporedne) sa tim pravcem, njihov koeficijent smjera k_2 glasi:

$$\left. \begin{array}{l} k_1 = \frac{5}{33} \\ k_2 = k_1 - \text{uvjet paralelnosti} \end{array} \right\} \Rightarrow k_2 = \frac{5}{33}.$$

Uvrštavajući sve poznate podatke u uvjet dodira pravca i elipse dobije se:



$$\left. \begin{array}{l} a^2 = 16 \; , \; b^2 = 9 \; , \; k_2 = k = \frac{5}{33} \\ a^2 \cdot k^2 + b^2 = l^2 - \text{uvjet dodira} \end{array} \right\} \Rightarrow 16 \cdot \left(\frac{5}{33} \right)^2 + 9 = l^2 \Rightarrow$$

$$\Rightarrow l^2 = 16 \cdot \frac{25}{1089} + 9 \Rightarrow l^2 = \frac{400}{1089} + 9 \Rightarrow$$

$$\Rightarrow l^2 = \frac{400 + 9801}{1089} \Rightarrow l^2 = \frac{10201}{1089} \; / \sqrt{\;} \Rightarrow l_{1,2} = \pm \sqrt{\frac{10201}{1089}} \Rightarrow l_{1,2} = \pm \frac{101}{33}.$$

Tražene tangente su:

$$y = \frac{5}{33} \cdot x + \frac{101}{33} \quad , \quad y = \frac{5}{33} \cdot x - \frac{101}{33}.$$

Vježba 064

Odredite jednadžbe tangenata na elipsu $9 \cdot x^2 + 16 \cdot y^2 = 144$ koje su paralelne (usporedne) pravcu $y = - x + 4$.

Rezultat: $y = - x + 5 \quad , \quad y = - x - 5.$

Zadatak 065 (3A, TUPŠ)

Odredite jednadžbe tangenata na elipsu $9 \cdot x^2 + 16 \cdot y^2 = 144$ koje su okomite na pravac $2 \cdot x - 3 \cdot \sqrt{2} \cdot y = 10.$

Rješenje 065

Prvo odredimo kanonski oblik jednadžbe dane elipse:

$$9 \cdot x^2 + 16 \cdot y^2 = 144 \; / :144 \Rightarrow \frac{9 \cdot x^2}{144} + \frac{16 \cdot y^2}{144} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{9} = 1 \Rightarrow \left. \begin{array}{l} a^2 = 16 \\ b^2 = 9 \end{array} \right\}.$$

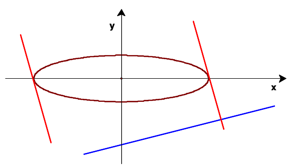
Zadanom pravcu nađemo koeficijent smjera k_1:

$$2 \cdot x - 3 \cdot \sqrt{2} \cdot y = 10 \Rightarrow -3 \cdot \sqrt{2} \cdot y = -2 \cdot x + 10 \; / \cdot \frac{-1}{3 \cdot \sqrt{2}} \Rightarrow y = \frac{2}{3 \cdot \sqrt{2}} \cdot x - \frac{10}{3 \cdot \sqrt{2}} \Rightarrow k_1 = \frac{2}{3 \cdot \sqrt{2}}.$$

Budući da su tangente okomite na taj pravac, njihov koeficijent smjera k_2 glasi:

$$\left.\begin{array}{l} k_1 = \dfrac{2}{3\cdot\sqrt{2}} \\ k_2 = -\dfrac{1}{k_1} - \text{uvjet okomitosti} \end{array}\right\} \Rightarrow k_2 = -\frac{3\cdot\sqrt{2}}{2}.$$

Uvrštavajući sve poznate podatke u uvjet dodira pravca i elipse dobije se:



$$\left.\begin{array}{l} a^2 = 16\ ,\ b^2 = 9\ ,\ k_2 = k = -\dfrac{3\cdot\sqrt{2}}{2} \\ a^2\cdot k^2 + b^2 = l^2 - \text{uvjet dodira} \end{array}\right\} \Rightarrow 16\cdot\left(-\frac{3\cdot\sqrt{2}}{2}\right)^2 + 9 = l^2 \Rightarrow$$

$$\Rightarrow l^2 = 16\cdot\frac{9\cdot 2}{4} + 9 \Rightarrow l^2 = 72 + 9 \Rightarrow l^2 = 81\ /\sqrt{\ } \Rightarrow l_{1,2} = \pm 9.$$

Tražene tangente su:

$$y = -\frac{3\cdot\sqrt{2}}{2}\cdot x + 9 \qquad , \qquad y = -\frac{3\cdot\sqrt{2}}{2}\cdot x - 9.$$

Vježba 065

Odredite jednadžbe tangenata na elipsu $25 \cdot x^2 + 36 \cdot y^2 = 900$ koje su okomite na pravac $x + y - 2 = 0$.

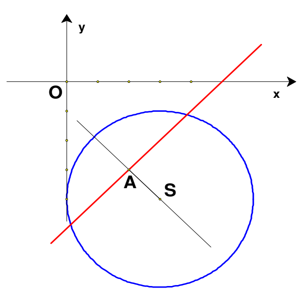
Rezultat: $y = x + \sqrt{61}\ ,\ y = x - \sqrt{61}.$

Zadatak 066 (3A, TUPŠ)

Odredite sekantu kružnice koja odsijeca tetivu čije polovište je točka A ako je A(2, – 3), $(x - 3)^2 + (y + 4)^2 = 9$.

Rješenje 066

Sekanta je okomita na spojnicu središta S kružnice i točke A, i prolazi točkom A. Središte dane kružnice je:



$$\left.\begin{array}{l} (x-p)^2 + (y-q)^2 = r^2 \\ (x-3)^2 + (y+4)^2 = 9 \end{array}\right\} \Rightarrow \left.\begin{array}{l} S(p, q) \\ S(3, -4) \end{array}\right\}.$$

Odredimo koeficijent smjera k_1 pravca SA:

$$\left.\begin{array}{l} S(3, -4)\ ,\ A(2, -3) \\ k_1 = \dfrac{y_2 - y_1}{x_2 - x_1} \end{array}\right\} \Rightarrow k_1 = \frac{-3+4}{2-3} \Rightarrow k_1 = -1.$$

Budući da je sekanta okomita na pravac SA, njezin koeficijent smjera k_2 glasi:

$$\left.\begin{array}{l} k_1 = -1 \\ k_2 = -\dfrac{1}{k_1} - \text{uvjet okomitosti} \end{array}\right\} \Rightarrow k_2 = 1.$$

Jednadžba sekante iznosi:

$$\left.\begin{array}{l} k_2 = k = 1\ ,\ A(2, -3) \\ y - y_1 = k\cdot(x - x_1) \end{array}\right\} \Rightarrow y + 3 = 1\cdot(x - 2) \Rightarrow y + 3 = x - 2 \Rightarrow y = x - 2 - 3 \Rightarrow y = x - 5.$$

Vježba 066

Odredite sekantu kružnice koja odsijeca tetivu čije polovište je točka A ako je A(1, 3), $x^2 + y^2 = 25$.

Rezultat: $y = -\frac{1}{3}\cdot x + \frac{10}{3}.$

Zadatak 067 (Carmen, ekonomska škola)

Za koje vrijednosti koeficijenta k pravac $y = k \cdot x$ mimoilazi kružnicu $x^2 + y^2 - 6 \cdot x - 6 \cdot y + 9 = 0$?

Rješenje 067

Ponovimo!

Uvjet da pravac $y = k \cdot x + l$ mimoilazi kružnicu $(x - p)^2 + (y - q)^2 = r^2$ glasi:

$$r^2 \cdot \left(1 + k^2\right) - (q - k \cdot p - l)^2 < 0.$$

Najprije iz opće jednadžbe kružnice $x^2 + y^2 - 2 \cdot p \cdot x - 2 \cdot q \cdot y + c = 0$ napišemo središnju ili centralnu jednadžbu $(x - p)^2 + (y - q)^2 = r^2$:

$$\left.\begin{array}{l} x^2 + y^2 - 6 \cdot x - 6 \cdot y + 9 = 0 \\ x^2 + y^2 - 2 \cdot p \cdot x - 2 \cdot q \cdot y + c = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -2 \cdot p = -6 \\ -2 \cdot q = -6 \\ c = 9 \end{array}\right\} \Rightarrow \left.\begin{array}{l} p = 3 \\ q = 3 \\ c = 9 \end{array}\right\} \Rightarrow r = \sqrt{p^2 + q^2 - c} \Rightarrow$$

$$\Rightarrow r = \sqrt{3^2 + 3^2 - 9} \Rightarrow r = \sqrt{9 + 9 - 9} \Rightarrow r = \sqrt{9} \Rightarrow r = 3.$$

Računamo koeficijent k:

$$\left.\begin{array}{l} p = 3 \ , \ q = 3 \ , \ r = 3 \\ y = k \cdot x \Rightarrow k = k \ , \ l = 0 \\ r^2 \cdot \left(1 + k^2\right) - (q - k \cdot p - l)^2 < 0 \end{array}\right\} \Rightarrow 3^2 \cdot \left(1 + k^2\right) - (3 - k \cdot 3 - 0)^2 < 0 \Rightarrow 9 \cdot \left(1 + k^2\right) - (3 - 3 \cdot k)^2 < 0 \Rightarrow$$

$$\Rightarrow 9 + 9 \cdot k^2 - \left(9 - 18 \cdot k + 9 \cdot k^2\right) < 0 \Rightarrow 9 + 9 \cdot k^2 - 9 + 18 \cdot k - 9 \cdot k^2 < 0 \Rightarrow 18 \cdot k < 0 \ /:18 \Rightarrow$$

$$\Rightarrow k < 0 \Rightarrow k \in \langle -\infty, 0 \rangle.$$

Vježba 067

Za koju vrijednost koeficijenta k pravac $y = k \cdot x$ dodiruje kružnicu $x^2 + y^2 - 6 \cdot x - 6 \cdot y + 9 = 0$?

Rezultat: $k = 0$.

Zadatak 068 (Carmen, ekonomska škola)

Ako točke A(6, 4) i B(– 8, 3) leže na elipsi nađite njezinu jednadžbu.

Rješenje 068

Budući da točke A i B leže na elipsi, njihove koordinate uvrstit ćemo u jednadžbu elipse i dobiti sustav od dvije kvadratne jednadžbe sa dvije nepoznanice:

$$\left.\begin{array}{l} A(6, 4) \ , \ \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \\ B(-8, 3) \ , \ \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \dfrac{6^2}{a^2} + \dfrac{4^2}{b^2} = 1 \\ \dfrac{(-8)^2}{a^2} + \dfrac{3^2}{b^2} = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \dfrac{36}{a^2} + \dfrac{16}{b^2} = 1 \\ \dfrac{64}{a^2} + \dfrac{9}{b^2} = 1 \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{supstitucije} \\ u = \dfrac{1}{a^2} \ , \ v = \dfrac{1}{b^2} \end{array}\right] \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 36 \cdot u + 16 \cdot v = 1 \\ 64 \cdot u + 9 \cdot v = 1 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow \left.\begin{array}{l} 36 \cdot u + 16 \cdot v = 1 \ / \cdot (-9) \\ 64 \cdot u + 9 \cdot v = 1 \ / \cdot 16 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -324 \cdot u - 144 \cdot v = -9 \\ 1024 \cdot u + 144 \cdot v = 16 \end{array}\right\} \Rightarrow$$

$$\Rightarrow 700 \cdot u = 7 \Rightarrow u = \frac{7}{700} \Rightarrow u = \frac{1}{100} \Rightarrow \left.\begin{array}{l} u = \dfrac{1}{100} \\ 64 \cdot u + 9 \cdot v = 1 \end{array}\right\} \Rightarrow 64 \cdot \frac{1}{100} + 9 \cdot v = 1 \Rightarrow \frac{64}{100} + 9 \cdot v = 1 \Rightarrow$$

$$\Rightarrow 9 \cdot v = 1 - \frac{64}{100} \Rightarrow 9 \cdot v = \frac{100 - 64}{100} \Rightarrow 9 \cdot v = \frac{36}{100} \ /:9 \Rightarrow v = \frac{4}{100} \Rightarrow v = \frac{1}{25}.$$

Vraćamo se na supstitucije:

$$\left.\begin{array}{l} u=\frac{1}{a^2}\ ,\ u=\frac{1}{100} \\ v=\frac{1}{b^2}\ ,\ v=\frac{1}{25} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{1}{a^2}=\frac{1}{100} \\ \frac{1}{b^2}=\frac{1}{25} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a^2=100 \\ b^2=25 \end{array}\right\} \Rightarrow \frac{x^2}{100}+\frac{y^2}{25}=1\ /\cdot 100 \Rightarrow x^2+4\cdot y^2=100.$$

Vježba 068

Ako točke A(– 5, 1) i B(7, – 5) leže na hiperboli nađite njezinu jednadžbu.

Rezultat: $x^2 - y^2 = 24$.

Zadatak 069 (Carmen, ekonomska škola)

Za koje vrijednosti koeficijenta $k \in R$ pravac y = k · x dodiruje kružnicu $(x - 10)^2 + (y - 10)^2 = 100$?

Rješenje 069

Ponovimo!

$$(a-b)^2 = a^2 - 2\cdot a\cdot b + b^2.$$

Uvjet da pravac y = k · x + l bude tangenta kružnice $(x - p)^2 + (y - q)^2 = r^2$ glasi:

$$r^2\cdot\left(1+k^2\right) = (q-k\cdot p-l)^2.$$

$$\left.\begin{array}{l} y=k\cdot x \\ (x-10)^2+(y-10)^2=100 \end{array}\right\} \Rightarrow \left.\begin{array}{l} y=k\cdot x+0 \\ (x-10)^2+(y-10)^2=100 \end{array}\right\} \Rightarrow \left.\begin{array}{l} l=0 \\ p=10\ ,\ q=10\ ,\ r^2=100 \end{array}\right\} \Rightarrow$$

$$\Rightarrow 100\cdot\left(1+k^2\right)=(10-k\cdot 10-0)^2 \Rightarrow 100+100\cdot k^2=(10-10\cdot k)^2 \Rightarrow$$

$$\Rightarrow 100+100\cdot k^2=100-200\cdot k+100\cdot k^2 \Rightarrow 200\cdot k=0 \Rightarrow k=0.$$

Vježba 069

Za koje vrijednosti koeficijenta $k \in R$ pravac y = k · x dodiruje kružnicu $(x - 5)^2 + (y - 5)^2 = 25$?

Rezultat: k = 0.

Zadatak 070 (Ivan, pomorska škola)

Krajevi promjera kružnice su točke A(0, – 1) i B(4, – 5). Nađite jednadžbu kružnice.

Rješenje 070

Ponovimo!

Promjer kružnice je dužina koja spaja dvije točke kružnice, a prolazi središtem kružnice.
Polumjer kružnice je dužina koja spaja središte kružnice sa bilo kojom točkom kružnice.
Ako točka P(x, y) raspolavlja dužinu $\overline{AB}$, $A\left(x_1, y_1\right)$, $B\left(x_2, y_2\right)$, tada je:

$$x=\frac{x_1+x_2}{2}\quad ,\quad y=\frac{y_1+y_2}{2}.$$

Međusobna udaljenost točaka $A\left(x_1, y_1\right)$ i $B\left(x_2, y_2\right)$ u ravnini je:

$$|AB|=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}.$$

Budući da je dužina $\overline{AB}$ promjer kružnice, središte S(p, q) kružnice je polovište dužine $\overline{AB}$:

$$\left.\begin{array}{l} A(0,-1)=A\left(x_1, y_1\right) \\ B(4,-5)=B\left(x_2, y_2\right) \\ S(p, q)=S\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \end{array}\right\} \Rightarrow S(p, q)=S\left(\frac{0+4}{2}, \frac{-1-5}{2}\right)=S(2,-3).$$

Polumjer kružnice možemo izračunati na dva načina.

1.inačica

Polumjer kružnice je dužina koja spaja središte kružnice sa bilo kojom točkom kružnice. Na primjer, za točke S i A duljina polumjera iznosi:

$$\left.\begin{array}{l} S(2,-3)=S(x_1, y_1) \\ A(0,-1)=A(x_2, y_2) \\ r=|SA|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \end{array}\right\} \Rightarrow r=\sqrt{(0-2)^2+(-1+3)^2} \Rightarrow r=\sqrt{(-2)^2+2^2} \Rightarrow$$

$$\Rightarrow r=\sqrt{4+4} \Rightarrow r=\sqrt{8} \Rightarrow r^2=8.$$

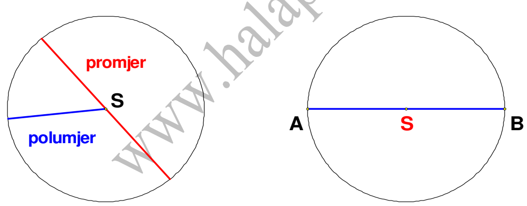
2.inačica

Poznato je središte S kružnice i jedna točka koja pripada kružnici, na primjet, točka A. Uvrstit ćemo koordinate točaka S i A u središnju jednadžbu kružnice i izračunati r:

$$\left.\begin{array}{l} S(p, q)=S(2,-3) \\ A(x, y)=A(0,-1) \\ (x-p)^2+(y-q)^2=r^2 \end{array}\right\} \Rightarrow (0-2)^2+(-1+3)^2=r^2 \Rightarrow (-2)^2+2^2=r^2 \Rightarrow 4+4=r^2 \Rightarrow r^2=8.$$

Jednadžba kružnice glasi:

$$\left.\begin{array}{l} S(p, q)=S(2,-3) \\ r^2=8 \\ (x-p)^2+(y-q)^2=r^2 \end{array}\right\} \Rightarrow (x-2)^2+(y+3)^2=8.$$



Vježba 070

Krajevi promjera kružnice su točke A(1, 0) i B(5, – 4). Nađite jednadžbu kružnice.

Rezultat: $(x-3)^2+(y+2)^2=8.$

Zadatak 071 (Tomislav, tehnička škola)

Kolika je udaljenost pravca $y=-\frac{4}{3}\cdot x+6$ od tjemena parabole $y = x^2 - 4 \cdot x + 14$?

Rješenje 071

Ponovimo!

Graf kvadratne funkcije $f(x) = a \cdot x^2 + b \cdot x + c$ je parabola $y = a \cdot x^2 + b \cdot x + c$ s tjemenom u točki:

$$T\left(-\frac{b}{2\cdot a}, \frac{4\cdot a\cdot c-b^2}{4\cdot a}\right).$$

Udaljenost točke $T(x_0, y_0)$ od pravca $A \cdot x + B \cdot y + C = 0$ iznosi:

$$d=\frac{|A\cdot x_0+B\cdot y_0+C|}{\sqrt{A^2+B^2}}.$$

Najprije odredimo tjeme zadane parabole:

$$\left.\begin{array}{l} y = x^2 - 4 \cdot x + 14 \\ a = 1 \ , \ b = -4 \ , \ c = 14 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = 1 \ , \ b = -4 \ , \ c = 14 \\ T\left(-\frac{b}{2 \cdot a}, \frac{4 \cdot a \cdot c - b^2}{4 \cdot a}\right) \end{array}\right\} \Rightarrow T\left(-\frac{-4}{2 \cdot 1}, \frac{4 \cdot 1 \cdot 14 - 16}{4 \cdot 1}\right) \Rightarrow$$

$$\Rightarrow T\left(\frac{4}{2}, \frac{40}{4}\right) \Rightarrow T(2, 10).$$

Udaljenost pravca od tjemena parabole je:

$$y = -\frac{4}{3} \cdot x + 6 \ / \cdot 3 \Rightarrow 3 \cdot y = -4 \cdot x + 18 \Rightarrow \left.\begin{array}{l} 4 \cdot x + 3 \cdot y - 18 = 0 \\ A = 4 \ , \ B = 3 \ , \ C = -18 \end{array}\right\} \Rightarrow \left.\begin{array}{l} T(x_0, y_0) = T\left(2, \frac{10}{3}\right) \\ A = 4 \ , \ B = 3 \ , \ C = -18 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[d = \frac{|A \cdot x_0 + B \cdot y_0 + C|}{\sqrt{A^2 + B^2}}\right] \Rightarrow d = \frac{|4 \cdot 2 + 3 \cdot 10 - 18|}{\sqrt{4^2 + 3^2}} \Rightarrow d = \frac{|8 + 30 - 18|}{\sqrt{16 + 9}} \Rightarrow d = \frac{|20|}{\sqrt{25}} \Rightarrow d = \frac{20}{5} = 4.$$

Vježba 071

Kolika je udaljenost pravca $6 \cdot x + 8 \cdot y + 1 = 0$ od tjemena parabole $y = x^2 - 4 \cdot x + 14$?

Rezultat: $d = 9.3.$

Zadatak 072 (Tony, gimnazija)

Pravac $y = 2 \cdot x - 14$ dodiruje elipsu $b^2 \cdot x^2 + a^2 \cdot y^2 = a^2 \cdot b^2$ čiji je linearni ekscentricitet $e = 2$. Nađite poluos b.

Rješenje 072

Ponovimo!

Osna jednadžba elipse

$$b^2 \cdot x^2 + a^2 \cdot y^2 = a^2 \cdot b^2 \quad \text{ili} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

a) linearni ekscentricitet

$$e^2 = a^2 - b^2,$$

b) uvjet da pravac $y = k \cdot x + l$ bude tangenta elipse

$$a^2 \cdot k^2 + b^2 = l^2.$$

Iz uvjeta tangencionalnosti i formule za linearni ekscentricitet postavimo sustav jednadžbi:

$$\left.\begin{array}{l} y = 2 \cdot x - 14 \\ e = 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} k = 2 \ , \ l = -14 \\ e = 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a^2 \cdot 2^2 + b^2 = (-14)^2 \\ 2^2 = a^2 - b^2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 4 \cdot a^2 + b^2 = 196 \\ a^2 - b^2 = 4 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow 5 \cdot a^2 = 200 \ /:5 \Rightarrow a^2 = 40 \Rightarrow \left.\begin{array}{l} a^2 = 40 \\ a^2 - b^2 = 4 \end{array}\right\} \Rightarrow 40 - b^2 = 4 \Rightarrow$$

$$\Rightarrow b^2 = 40 - 4 \Rightarrow b^2 = 36 \ / \sqrt{\ } \Rightarrow b = \pm\sqrt{36} \Rightarrow \left.\begin{array}{l} b_1 = -6 \ \text{nema smisla} \\ b_2 = 6 \end{array}\right\} \Rightarrow b = 6.$$

Vježba 072

Pravac $y = 2 \cdot x - 14$ dodiruje elipsu $b^2 \cdot x^2 + a^2 \cdot y^2 = a^2 \cdot b^2$ čiji je linearni ekscentricitet $e = 2$. Nađite poluos a.

Rezultat: $2 \cdot \sqrt{10}.$

Zadatak 073 (Marijana, maturantica)

Nađi razliku velike i male poluosi elipse $x^2 + 9 \cdot y^2 - 2 \cdot x = 8$.

Rješenje 073

Ponovimo!

$$a^2-2\cdot a\cdot b+b^2=(a-b)^2.$$

Translacija elipse
Jednadžba elipse kojoj su koordinate središta S(p, q), a osi 2a i 2b (velika i mala os) usporedne (paralelne) s koordinatnim osima glasi:

$$\frac{(x-p)^2}{a^2}+\frac{(y-q)^2}{b^2}=1.$$

$$x^2+9\cdot y^2-2\cdot x=8 \Rightarrow x^2-2\cdot x+9\cdot y^2=8 \Rightarrow x^2-2\cdot x+1-1+9\cdot y^2=8 \Rightarrow$$

$$\Rightarrow \left(x^2-2\cdot x+1\right)-1+9\cdot y^2=8 \Rightarrow (x-1)^2+9\cdot y^2=8+1 \Rightarrow (x-1)^2+9\cdot y^2=9 \;/:9 \Rightarrow$$

$$\Rightarrow \frac{(x-1)^2}{9}+\frac{9\cdot y^2}{9}=1 \Rightarrow \frac{(x-1)^2}{9}+\frac{y^2}{1}=1 \Rightarrow \left.\begin{array}{l} a^2=9 \;/\sqrt{} \\ b^2=1 \;/\sqrt{} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=3 \\ b=1 \end{array}\right\} \Rightarrow a-b=3-1 \Rightarrow a-b=2.$$

Vježba 073

Nađi razliku velike i male poluosi elipse $x^2+16\cdot y^2-2\cdot x=15$.

Rezultat: 3.

Zadatak 074 (Tajana, EBB)

Provjeriti jesu li pravci $p_1 \ldots y=-3\cdot x+23$ i $p_2 \ldots 3\cdot x-y+5=0$ tangente kružnice $k \ldots (x-3)^2+(y-4)^2=10$.

Rješenje 074

Ponovimo!

Uvjet dodira pravca i kružnice
Kružnica $k \ldots (x-p)^2+(y-q)^2=r^2$ i pravac $p \ldots y=k\cdot x+l$ dodiruju se ako i samo ako vrijedi

$$r^2\cdot\left(1+k^2\right)=(k\cdot p-q+l)^2.$$

Provjeravamo je li pravac $p_1 \ldots y=-3\cdot x+23$ tangenta kružnice $k \ldots (x-3)^2+(y-4)^2=10$.

$$\left.\begin{array}{l} y=-3\cdot x+23 \\ y=k\cdot x+l \end{array}\right\} \Rightarrow \left.\begin{array}{l} k=-3 \\ l=23 \end{array}\right\}.$$

$$\left.\begin{array}{l} (x-3)^2+(y-4)^2=10 \\ (x-p)^2+(y-q)^2=r^2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} p=3\;,\; q=4 \\ r^2=10 \end{array}\right\}.$$

Ispitujemo uvjet dodira:

$$\left.\begin{array}{l} k=-3\;,\; l=23\;,\; p=3\;,\; q=4\;,\; r^2=10 \\ r^2\cdot\left(1+k^2\right)=(k\cdot p-q+l)^2 \end{array}\right\} \Rightarrow 10\cdot\left(1+(-3)^2\right)=(-3\cdot 3-4+23)^2 \Rightarrow$$

$$\Rightarrow 10\cdot(1+9)=(-9-4+23)^2 \Rightarrow 10\cdot 10=10^2 \Rightarrow 100=100.$$

Pravac p_1 je tangenta kružnice k.

Provjeravamo je li pravac $p_2 \ldots 3\cdot x-y+5=0$ tangenta kružnice $k \ldots (x-3)^2+(y-4)^2=10$.

$$\left.\begin{array}{l} 3\cdot x-y+5=0 \\ y=k\cdot x+l \end{array}\right\} \Rightarrow \left.\begin{array}{l} -y=-3\cdot x-5 \;/\cdot(-1) \\ y=k\cdot x+l \end{array}\right\} \Rightarrow \left.\begin{array}{l} y=3\cdot x+5 \\ y=k\cdot x+l \end{array}\right\} \Rightarrow \left.\begin{array}{l} k=3 \\ l=5 \end{array}\right\}.$$

$$\left.\begin{array}{l} (x-3)^2+(y-4)^2=10 \\ (x-p)^2+(y-q)^2=r^2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} p=3\;,\; q=4 \\ r^2=10 \end{array}\right\}.$$

Ispitujemo uvjet dodira:

$$\left.\begin{array}{l} k=3 \;,\; l=5 \;,\; p=3 \;,\; q=4 \;,\; r^2=10 \\ r^2\cdot\left(1+k^2\right)=\left(k\cdot p-q+l\right)^2 \end{array}\right\} \Rightarrow 10\cdot\left(1+3^2\right)=\left(3\cdot 3-4+5\right)^2 \Rightarrow$$

$$\Rightarrow 10\cdot(1+9)=(9-4+5)^2 \Rightarrow 10\cdot 10=10^2 \Rightarrow 100=100.$$

Pravac p_2 je tangenta kružnice k.

Vježba 074

Provjeriti je li pravac p … $3\cdot x+4\cdot y+15=0$ tangenta kružnice k … $(x-2)^2+(y-1)^2=25$.

Rezultat: Pravac p je tangenta kružnice k.

Zadatak 075 (Tina, gimnazija)

Kako glasi jednadžba kružnice opisane trokutu što ga s koordinatnim osima zatvara pravac $3\cdot x-4\cdot y=24$?

Rješenje 075

Ponovimo!

Jednadžbu pravca oblika

$$\frac{x}{m}+\frac{y}{n}=1$$

nazivamo segmentnim oblikom jednadžbe pravca. Točke M(m, 0) i N(0, n) su presjeci tog pravca s koordinatnim osima. Brojevi m i n se nazivaju odsječci ili segmenti na osima.

Talesov poučak: Svaki obodni kut nad promjerom kružnice je pravi.

Polovište hipotenuze pravokutnog trokuta je središte opisane kružnice tom trokutu.

Koordinate polovišta P dužine $\overline{AB}$, $A(x_1, y_1)$, $B(x_2, y_2)$, su

$$P\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

Ako je S(p, q) središte kružnice, a r njezin polumjer, tada središnja jednadžba kružnice glasi

$$(x-p)^2+(y-q)^2=r^2.$$

Najprije odredimo točke koje su presjeci zadanog pravca s koordinatnim osima:

$$3\cdot x-4\cdot y=24 \Rightarrow 3\cdot x-4\cdot y=24 \;/:24 \Rightarrow \frac{3\cdot x}{24}-\frac{4\cdot y}{24}=1 \Rightarrow \frac{x}{8}+\frac{y}{-6}=1 \Rightarrow \left.\begin{array}{l} M(8, 0) \\ N(0, -6) \end{array}\right\}.$$

Budući da je trokut što ga s koordinatnim osima zatvara dani pravac pravokutan trokut, središte opisane kružnice je u polovištu hipotenuze $\overline{MN}$:

$$\left.\begin{array}{l} M(x_1, y_1)=M(8, 0) \;,\; N(x_2, y_2)=N(0, -6) \\ S(p, q)=S\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) \end{array}\right\} \Rightarrow S(p, q)=S\left(\frac{8+0}{2}, \frac{0-6}{2}\right) \Rightarrow S(p, q)=S(4, -3).$$

Jednadžba kružnice glasi:

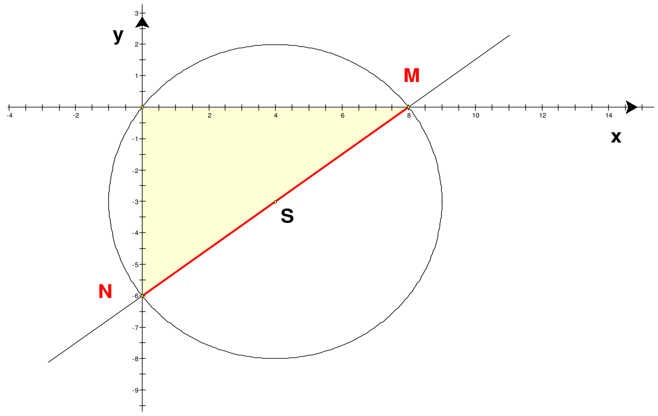
$$\left.\begin{array}{l} S(p, q)=S(4, -3) \\ (x-p)^2+(y-q)^2=r^2 \end{array}\right\} \Rightarrow (x-4)^2+(y+3)^2=r^2.$$

Polumjer r odredit ćemo tako da uvrstimo koordinate točke M (ili N) u jednadžbu kružnice:

$$\left.\begin{array}{l} M(x, y)=M(8, 0) \\ (x-4)^2+(y+3)^2=r^2 \end{array}\right\} \Rightarrow (8-4)^2+(0+3)^2=r^2 \Rightarrow r^2=4^2+3^2 \Rightarrow r^2=16+9 \Rightarrow r^2=25.$$

Jednadžba kružnice je:

$$(x-4)^2+(y+3)^2=25.$$



Vježba 075

Kako glasi jednadžba kružnice opisane trokutu što ga s koordinatnim osima zatvara pravac $y = \frac{3}{4} \cdot x - 6$?

Rezultat: $(x-4)^2 + (y+3)^2 = 25.$

Zadatak 076 (Lijepa Željka, srednja škola)

Odredi presjek pravca p i hiperbole ako je zadano: $2 \cdot x^2 - 3 \cdot y^2 = 50$, $y = x - 3$.

Rješenje 076

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Tražiti presjek hiperbole i pravca znači tražiti njihove zajedničke točke (ako postoje), tj. točke koje zadovoljavaju i jednadžbu hiperbole i jednadžbu pravca. Drugim riječima, ako je hiperbola dana jednadžbom $a^2 \cdot x^2 - b^2 \cdot y^2 = a^2 \cdot b^2$, a pravac jednadžbom $A \cdot x + B \cdot y + C = 0$, koordinate njihovih sjecišta su rješenja sustava:

$$\begin{cases} a^2 \cdot x^2 - b^2 \cdot y^2 = a^2 \cdot b^2 \\ A \cdot x + B \cdot y + C = 0. \end{cases}$$

Iz jednadžbe pravca izračunamo y i uvrstimo ga u jednadžbu hiperbole:

$$\left.\begin{array}{l} 2 \cdot x^2 - 3 \cdot y^2 = 50 \\ y = x - 3 \end{array}\right\} \Rightarrow 2 \cdot x^2 - 3 \cdot (x-3)^2 = 50 \Rightarrow 2 \cdot x^2 - 3 \cdot \left(x^2 - 6 \cdot x + 9\right) = 50 \Rightarrow$$

$$\Rightarrow 2 \cdot x^2 - 3 \cdot x^2 + 18 \cdot x - 27 = 50 \Rightarrow 2 \cdot x^2 - 3 \cdot x^2 + 18 \cdot x - 27 - 50 = 0 \Rightarrow$$

$$\Rightarrow -x^2 + 18 \cdot x - 77 = 0 \;/ \cdot (-1) \Rightarrow x^2 - 18 \cdot x + 77 = 0 \Rightarrow \left.\begin{array}{l} x^2 - 18 \cdot x + 77 = 0 \\ a = 1 \;,\; b = -18 \;,\; c = 77 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a = 1 \;,\; b = -18 \;,\; c = 77 \\ x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array}\right\} \Rightarrow x_{1,2} = \frac{18 \pm \sqrt{324 - 4 \cdot 1 \cdot 77}}{2 \cdot 1} \Rightarrow x_{1,2} = \frac{18 \pm \sqrt{324 - 308}}{2} \Rightarrow$$

$$\Rightarrow x_{1,2} = \frac{18 \pm \sqrt{16}}{2} \Rightarrow x_{1,2} = \frac{18 \pm 4}{2} \Rightarrow \left.\begin{array}{l} x_1 = \dfrac{18+4}{2} \\ x_2 = \dfrac{18-4}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = \dfrac{22}{2} \\ x_2 = \dfrac{14}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = 11 \\ x_2 = 7 \end{array}\right\}.$$

Sada računamo y:

- $\left.\begin{matrix} y = x - 3 \\ x_1 = 11 \end{matrix}\right\} \Rightarrow y_1 = 11 - 3 \Rightarrow y_1 = 8.$
- $\left.\begin{matrix} y = x - 3 \\ x_2 = 7 \end{matrix}\right\} \Rightarrow y_2 = 7 - 3 \Rightarrow y_2 = 4.$

Presjek pravca i hiperbole su točke:

$$A(11, 8) \;,\; B(7, 4).$$

Vježba 076

Odredi presjek pravca p i hiperbole ako je zadano: $2 \cdot x^2 - 3 \cdot y^2 = 50$, $x - y - 3 = 0$.

Rezultat: $A(11, 8) \;,\; B(7, 4).$

Zadatak 077 (Ana, gimnazija)

Kružnica prolazi točkom T(5, 1) i dira pravce $x - 2 \cdot y + 6 = 0$ i $x - 2 \cdot y - 4 = 0$. Kako glasi jednadžba te kružnice?

Rješenje 077

Ponovimo!

$$a^2 - b^2 = (a - b) \cdot (a + b) \quad , \quad (a - b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$
$$a \cdot b = 0 \;\Leftrightarrow\; a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Ako je S(p, q) središte kružnice, a r njezin polumjer, tada središnja jednadžba kružnice glasi:

$$(x - p)^2 + (y - q)^2 = r^2.$$

Kružnica k ... $(x - p)^2 + (y - q)^2 = r^2$ i pravac t ... $y = k \cdot x + l$ se dodiruju ako i samo ako vrijedi:

$$r^2 \cdot \left(1 + k^2\right) = (k \cdot p - q + l)^2.$$

Napišimo eksplicitne jednadžbe pravaca:

$x - 2 \cdot y + 6 = 0$ $-2 \cdot y = -x - 6$ $-2 \cdot y = -x - 6 \;/: (-2)$ $y = \frac{1}{2} \cdot x + 3$ $k = \frac{1}{2} \;,\; l = 3$	$x - 2 \cdot y - 4 = 0$ $-2 \cdot y = -x + 4$ $-2 \cdot y = -x + 4 \;/: (-2)$ $y = \frac{1}{2} \cdot x - 2$ $k = \frac{1}{2} \;,\; l = -2$

Uvrštavajući k i l od oba pravca u uvjet dodira kružnice i pravca imamo:

- $\left.\begin{matrix} k = \frac{1}{2} \;,\; l = 3 \\ r^2 \cdot \left(1 + k^2\right) = (k \cdot p - q + l)^2 \end{matrix}\right\} \Rightarrow r^2 \cdot \left(1 + \left(\frac{1}{2}\right)^2\right) = \left(\frac{1}{2} \cdot p - q + 3\right)^2 \Rightarrow$

$$\Rightarrow r^2 \cdot \left(1 + \frac{1}{4}\right) = \left(\frac{1}{2} \cdot p - q + 3\right)^2 \Rightarrow \frac{5}{4} \cdot r^2 = \left(\frac{1}{2} \cdot p - q + 3\right)^2.$$

- $\left.\begin{matrix} k = \frac{1}{2} \;,\; l = -2 \\ r^2 \cdot \left(1 + k^2\right) = (k \cdot p - q + l)^2 \end{matrix}\right\} \Rightarrow r^2 \cdot \left(1 + \left(\frac{1}{2}\right)^2\right) = \left(\frac{1}{2} \cdot p - q - 2\right)^2 \Rightarrow$

$$\Rightarrow\ r^2\cdot\left(1+\frac{1}{4}\right)=\left(\frac{1}{2}\cdot p-q-2\right)^2\ \Rightarrow\ \frac{5}{4}\cdot r^2=\left(\frac{1}{2}\cdot p-q-2\right)^2.$$

Sada se dobije:

$$\left.\begin{array}{l}\frac{5}{4}\cdot r^2=\left(\frac{1}{2}\cdot p-q+3\right)^2\\ \frac{5}{4}\cdot r^2=\left(\frac{1}{2}\cdot p-q-2\right)^2\end{array}\right\}\Rightarrow\ \left(\frac{1}{2}\cdot p-q+3\right)^2=\left(\frac{1}{2}\cdot p-q-2\right)^2.$$

Ako izraz s desne strane prebacimo na lijevu stranu, dobivamo razliku kvadrata koja u faktoriziranom obliku izgleda ovako:

$$\left(\frac{1}{2}\cdot p-q+3\right)^2-\left(\frac{1}{2}\cdot p-q-2\right)^2=0\ \Rightarrow\ \left(\frac{1}{2}\cdot p-q+3-\left(\frac{1}{2}\cdot p-q-2\right)\right)\cdot\left(\frac{1}{2}\cdot p-q+3+\left(\frac{1}{2}\cdot p-q-2\right)\right)=0\ \Rightarrow$$

$$\Rightarrow\ \left(\frac{1}{2}\cdot p-q+3-\frac{1}{2}\cdot p+q+2\right)\cdot\left(\frac{1}{2}\cdot p-q+3+\frac{1}{2}\cdot p-q-2\right)=0\ \Rightarrow$$

$$\Rightarrow\ \left(\frac{1}{2}\cdot p-q+3-\frac{1}{2}\cdot p+q+2\right)\cdot\left(\frac{1}{2}\cdot p-q+3+\frac{1}{2}\cdot p-q-2\right)=0\ \Rightarrow\ 5\cdot(p-2\cdot q+1)=0\ \Rightarrow\ p-2\cdot q+1=0\ \Rightarrow$$

$$\Rightarrow\ p=2\cdot q-1.$$

Iz p = 2 · q – 1 dobivamo:

$$\left.\begin{array}{l}p=2\cdot q-1\\ \frac{5}{4}\cdot r^2=\left(\frac{1}{2}\cdot p-q+3\right)^2\end{array}\right\}\Rightarrow\ \frac{5}{4}\cdot r^2=\left(\frac{1}{2}\cdot(2\cdot q-1)-q+3\right)^2\ \Rightarrow\ \frac{5}{4}\cdot r^2=\left(q-\frac{1}{2}-q+3\right)^2\ \Rightarrow$$

$$\Rightarrow\ \frac{5}{4}\cdot r^2=\left(\frac{5}{2}\right)^2\ \Rightarrow\ \frac{5}{4}\cdot r^2=\frac{25}{4}\ /\cdot\frac{4}{5}\ \Rightarrow\ r^2=5.$$

Iskoristimo i činjenicu da točka T(5, 1) pripada kružnici tako što ćemo koordinate točke uvrstiti u jednadžbu kružnice:

$$\left.\begin{array}{l}T(x,\ y)=T(5,\ 1)\\ (x-p)^2+(y-q)^2=r^2\end{array}\right\}\Rightarrow\ (5-p)^2+(1-q)^2=r^2.$$

Dalje računamo:

$$\left.\begin{array}{l}p=2\cdot q-1\ ,\ r^2=5\\ (5-p)^2+(1-q)^2=r^2\end{array}\right\}\Rightarrow\ (5-(2\cdot q-1))^2+(1-q)^2=5\ \Rightarrow\ (5-2\cdot q+1)^2+(1-q)^2=5\ \Rightarrow$$

$$\Rightarrow\ (6-2\cdot q)^2+(1-q)^2=5\ \Rightarrow\ 36-24\cdot q+4\cdot q^2+1-2\cdot q+q^2=5\ \Rightarrow$$

$$\Rightarrow\ 36-24\cdot q+4\cdot q^2+1-2\cdot q+q^2-5=0\ \Rightarrow\ 5\cdot q^2-26\cdot q+32=0\ \Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l}5\cdot q^2-26\cdot q+32=0\\ a=5\ ,\ b=-26\ ,\ c=32\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}a=5\ ,\ b=-26\ ,\ c=32\\ q_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\}\Rightarrow\ q_{1,2}=\frac{26\pm\sqrt{676-4\cdot5\cdot32}}{2\cdot5}\ \Rightarrow$$

$$\Rightarrow\ q_{1,2}=\frac{26\pm\sqrt{676-640}}{10}\ \Rightarrow\ q_{1,2}=\frac{26\pm\sqrt{36}}{10}\ \Rightarrow\ q_{1,2}=\frac{26\pm6}{10}\ \Rightarrow\ \left.\begin{array}{l}q_1=\frac{26+6}{10}\\ q_2=\frac{26-6}{10}\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} q_1 = \frac{32}{10} \\ q_2 = \frac{20}{10} \end{array}\right\} \Rightarrow \left.\begin{array}{l} q_1 = \frac{16}{5} \\ q_2 = 2 \end{array}\right\}.$$

Vraćamo se na zamjenu (supstituciju) p = 2 · q – 1 pa računamo p_1 i p_2.

- $\left.\begin{array}{l} q_1 = \frac{16}{5} \\ p_1 = 2 \cdot q_1 - 1 \end{array}\right\} \Rightarrow p_1 = 2 \cdot \frac{16}{5} - 1 \Rightarrow p_1 = \frac{32}{5} - 1 \Rightarrow p_1 = \frac{27}{5}.$
- $\left.\begin{array}{l} q_2 = 2 \\ p_2 = 2 \cdot q_2 - 1 \end{array}\right\} \Rightarrow p_2 = 2 \cdot 2 - 1 \Rightarrow p_2 = 4 - 1 \Rightarrow p_2 = 3.$

Jednadžbe kružnica su (postoje dvije kružnice):

- $\left.\begin{array}{l} p_1 = \frac{27}{5} \;,\; q_1 = \frac{16}{5} \;,\; r^2 = 5 \\ (x-p)^2 + (y-q)^2 = r^2 \end{array}\right\} \Rightarrow \left(x - \frac{27}{5}\right)^2 + \left(y - \frac{16}{5}\right)^2 = 5.$
- $\left.\begin{array}{l} p_2 = 3 \;,\; q_2 = 2 \;,\; r^2 = 5 \\ (x-p)^2 + (y-q)^2 = r^2 \end{array}\right\} \Rightarrow (x-3)^2 + (y-2)^2 = 5.$

Vježba 077

Kružnica prolazi točkom T(0, – 4) i dira pravce x – 3 · y + 8 = 0 i 3 · x + y – 6 = 0. Kako glasi jednadžba te kružnice?

Rezultat: $(x + 9)^2 + (y + 17)^2 = 250$, $(x + 1)^2 + (y + 1)^2 = 10$.

Zadatak 078 (Anita, gimnazija)

Odredi jednadžbu kružnice koja dira obje koordinatne osi i pravac 4 · x – 3 · y + 12 = 0.

Rješenje 078

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Ako je S(p, q) središte kružnice, a r njezin polumjer, tada središnja jednadžba kružnice glasi:

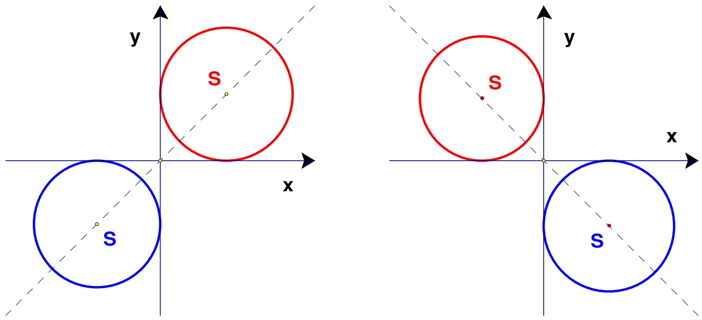
$$(x-p)^2 + (y-q)^2 = r^2.$$

Kružnica k ... $(x – p)^2 + (y – q)^2 = r^2$ i pravac t ... y = k · x + l dodiruju se ako i samo ako vrijedi:

$$r^2 \cdot \left(1 + k^2\right) = (k \cdot p - q + l)^2.$$

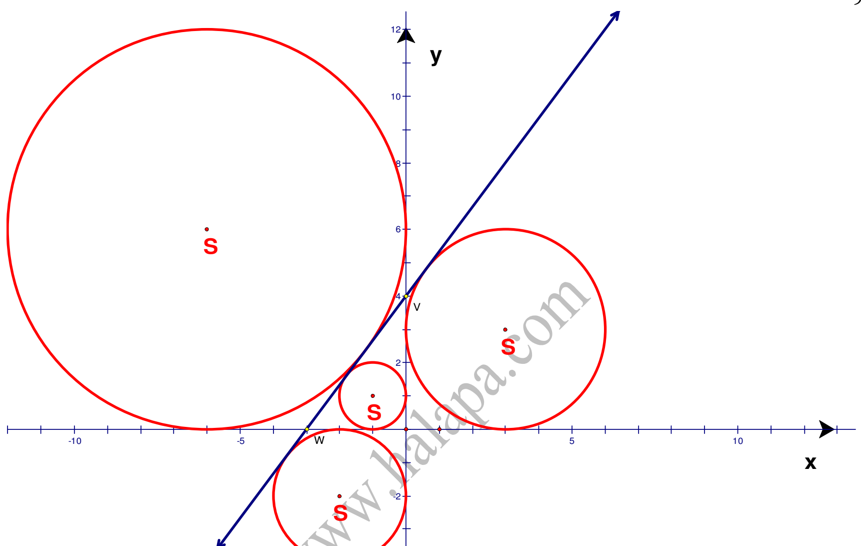
Kada kružnica dira obje koordinatne osi, tada se njezino središte S nalazi:
ili na pravcu y = x (simetrala I. i III. kvadranta),
ili na pravcu y = – x (simetrala II. i IV. kvadranta).
Budući da je r = | p | = | q |, jednadžba kružnice ima sljedeća 4 oblika:

$$(x-r)^2 + (y-r)^2 = r^2 \quad \text{središte u I. kvadrantu}$$
$$(x+r)^2 + (y-r)^2 = r^2 \quad \text{središte u II. kvadrantu}$$
$$(x+r)^2 + (y+r)^2 = r^2 \quad \text{središte u III. kvadrantu}$$
$$(x-r)^2 + (y+r)^2 = r^2 \quad \text{središte u IV. kvadrantu}$$



Napišimo eksplicitnu jednadžbu zadanog pravca:

$$4\cdot x-3\cdot y+12=0\ \Rightarrow\ -3\cdot y=-4\cdot x-12\ /:(-3)\ \Rightarrow\ y=\frac{4}{3}\cdot x+4\ \Rightarrow\ \left.\begin{matrix} k=\frac{4}{3} \\ l=4 \end{matrix}\right\}.$$



Sa slike vidi se da središte kružnice može biti u prvom, drugom i trećem kvadrantu koordinatnog sustava.

- Neka je središte kružnice u prvom kvadrantu.

Uvrštavajući k i l u uvjet dodira kružnice i pravca imamo:

$$\left.\begin{matrix} (x-r)^2+(y-r)^2=r^2\ ,\ p=r\ ,\ q=r \\ k=\frac{4}{3}\ ,\ l=4 \\ r^2\cdot\left(1+k^2\right)=(k\cdot p-q+l)^2 \end{matrix}\right\}\Rightarrow\ r^2\cdot\left(1+\left(\frac{4}{3}\right)^2\right)=\left(\frac{4}{3}\cdot r-r+4\right)^2\ \Rightarrow$$

$$\Rightarrow\ r^2\cdot\left(1+\frac{16}{9}\right)=\left(\frac{1}{3}\cdot r+4\right)^2\ \Rightarrow\ \frac{25}{9}\cdot r^2=\frac{1}{9}\cdot r^2+\frac{8}{3}\cdot r+16\ \Rightarrow\ \frac{25}{9}\cdot r^2=\frac{1}{9}\cdot r^2+\frac{8}{3}\cdot r+16\ /\cdot 9\ \Rightarrow$$

$$\Rightarrow\ 25\cdot r^2=r^2+24\cdot r+144\ \Rightarrow\ 25\cdot r^2-r^2-24\cdot r-144=0\ \Rightarrow\ 24\cdot r^2-24\cdot r-144=0\ /:24\ \Rightarrow$$

$$\Rightarrow\ r^2-r-6=0\ \Rightarrow\ \left.\begin{matrix} r^2-r-6=0 \\ a=1\ ,\ b=-1\ ,\ c=-6 \end{matrix}\right\}\Rightarrow\ \left.\begin{matrix} a=1\ ,\ b=-1\ ,\ c=-6 \\ r_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{matrix}\right\}\Rightarrow$$

$$\Rightarrow r_{1,2}=\frac{1\pm\sqrt{1-4\cdot1\cdot(-6)}}{2\cdot1}\Rightarrow r_{1,2}=\frac{1\pm\sqrt{1+24}}{2}\Rightarrow r_{1,2}=\frac{1\pm\sqrt{25}}{2}\Rightarrow r_{1,2}=\frac{1\pm5}{2}\Rightarrow \left.\begin{matrix} r_1=\frac{1+5}{2} \\ r_2=\frac{1-5}{2}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} r_1=\frac{6}{2} \\ r_2=-\frac{4}{2}\end{matrix}\right\}\Rightarrow \left.\begin{matrix} r_1=3 \\ r_2=-2 \text{ nema smisla}\end{matrix}\right\}\Rightarrow r=3\Rightarrow \left.\begin{matrix} p=3 \\ q=3\end{matrix}\right\}.$$

Jednadžba kružnice glasi:

$$(x-3)^2+(y-3)^2=9.$$

- Neka je središte kružnice u drugom kvadrantu.

Uvrštavajući k i l u uvjet dodira kružnice i pravca imamo:

$$\left.\begin{matrix} (x+r)^2+(y-r)^2=r^2 \ , \ p=-r \ , \ q=r \\ k=\frac{4}{3} \ , \ l=4 \\ r^2\cdot\left(1+k^2\right)=(k\cdot p-q+l)^2\end{matrix}\right\}\Rightarrow r^2\cdot\left(1+\left(\frac{4}{3}\right)^2\right)=\left(-\frac{4}{3}\cdot r-r+4\right)^2\Rightarrow$$

$$\Rightarrow r^2\cdot\left(1+\frac{16}{9}\right)=\left(-\frac{7}{3}\cdot r+4\right)^2\Rightarrow\frac{25}{9}\cdot r^2=\left(4-\frac{7}{3}\cdot r\right)^2\Rightarrow\frac{25}{9}\cdot r^2=16-\frac{56}{3}\cdot r+\frac{49}{9}\cdot r^2\Rightarrow$$

$$\Rightarrow\frac{25}{9}\cdot r^2=16-\frac{56}{3}\cdot r+\frac{49}{9}\cdot r^2 \ /\cdot 9\Rightarrow 25\cdot r^2=144-168\cdot r+49\cdot r^2\Rightarrow$$

$$\Rightarrow 25\cdot r^2-144+168\cdot r-49\cdot r^2=0\Rightarrow -24\cdot r^2+168\cdot r-144=0 \ /:(-24)\Rightarrow$$

$$\Rightarrow r^2-7\cdot r+6=0\Rightarrow \left.\begin{matrix} r^2-7\cdot r+6=0 \\ a=1 \ , \ b=-7 \ , \ c=6\end{matrix}\right\}\Rightarrow \left.\begin{matrix} a=1 \ , \ b=-7 \ , \ c=6 \\ r_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow r_{1,2}=\frac{7\pm\sqrt{49-4\cdot1\cdot6}}{2\cdot1}\Rightarrow r_{1,2}=\frac{7\pm\sqrt{49-24}}{2}\Rightarrow r_{1,2}=\frac{7\pm\sqrt{25}}{2}\Rightarrow r_{1,2}=\frac{7\pm5}{2}\Rightarrow \left.\begin{matrix} r_1=\frac{7+5}{2} \\ r_2=\frac{7-5}{2}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} r_1=\frac{12}{2} \\ r_2=\frac{2}{2}\end{matrix}\right\}\Rightarrow \left.\begin{matrix} r_1=6 \\ r_2=1\end{matrix}\right\}\Rightarrow \begin{bmatrix}\text{postoje dvije} \\ \text{kružnice}\end{bmatrix}\Rightarrow \left.\begin{matrix} r_1=6 \ , \ p_1=-6 \ , \ q_1=6 \\ r_2=1 \ , \ p_2=-1 \ , \ q_2=1\end{matrix}\right\}.$$

Jednadžbe kružnica glase:

$$(x+6)^2+(y-6)^2=36 \quad , \quad (x+1)^2+(y-1)^2=1.$$

- Neka je središte kružnice u trećem kvadrantu.

Uvrštavajući k i l u uvjet dodira kružnice i pravca imamo:

$$\left.\begin{array}{l}(x+r)^2+(y+r)^2=r^2 \ , \ p=-r \ , \ q=-r \\ k=\frac{4}{3} \ , \ l=4 \\ r^2\cdot\left(1+k^2\right)=(k\cdot p-q+l)^2\end{array}\right\} \Rightarrow r^2\cdot\left(1+\left(\frac{4}{3}\right)^2\right)=\left(-\frac{4}{3}\cdot r+r+4\right)^2 \Rightarrow$$

$$\Rightarrow r^2\cdot\left(1+\frac{16}{9}\right)=\left(-\frac{1}{3}\cdot r+4\right)^2 \Rightarrow \frac{25}{9}\cdot r^2=\left(4-\frac{1}{3}\cdot r\right)^2 \Rightarrow \frac{25}{9}\cdot r^2=16-\frac{8}{3}\cdot r+\frac{1}{9}\cdot r^2 \Rightarrow$$

$$\Rightarrow \frac{25}{9}\cdot r^2=16-\frac{8}{3}\cdot r+\frac{1}{9}\cdot r^2 \ /\cdot 9 \Rightarrow 25\cdot r^2=144-24\cdot r+r^2 \Rightarrow$$

$$\Rightarrow 25\cdot r^2-144+24\cdot r-r^2=0 \Rightarrow 24\cdot r^2+24\cdot r-144=0 \ /:24 \Rightarrow$$

$$\Rightarrow r^2+r-6=0 \Rightarrow \left.\begin{array}{l}r^2+r-6=0 \\ a=1 \ , \ b=1 \ , \ c=-6\end{array}\right\} \Rightarrow \left.\begin{array}{l}a=1 \ , \ b=1 \ , \ c=-6 \\ r_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\} \Rightarrow$$

$$\Rightarrow r_{1,2}=\frac{-1\pm\sqrt{1-4\cdot 1\cdot(-6)}}{2\cdot 1} \Rightarrow r_{1,2}=\frac{-1\pm\sqrt{1+24}}{2} \Rightarrow r_{1,2}=\frac{-1\pm\sqrt{25}}{2} \Rightarrow r_{1,2}=\frac{-1\pm 5}{2} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}r_1=\frac{-1+5}{2} \\ r_2=\frac{-1-5}{2}\end{array}\right\} \Rightarrow \left.\begin{array}{l}r_1=\frac{4}{2} \\ r_2=-\frac{6}{2}\end{array}\right\} \Rightarrow \left.\begin{array}{l}r_1=2 \\ r_2=-3 \text{ nema smisla}\end{array}\right\} \Rightarrow r=2 \Rightarrow \left.\begin{array}{l}p=-2 \\ q=-2\end{array}\right\}.$$

Jednadžba kružnice glasi:

$$(x+2)^2+(y+2)^2=4.$$

Vježba 078

Odredi jednadžbu kružnice koja dira obje koordinatne osi i pravac $0.4 \cdot x - 0.3 \cdot y + 1.2 = 0$.

Rezultat: $(x-3)^2+(y-3)^2=9$, $(x+6)^2+(y-6)^2=36$, $(x+1)^2+(y-1)^2=1$
$(x+2)^2+(y+2)^2=4$.

Zadatak 079 (Ana, gimnazija)

Napiši jednadžbu kružnice koja dira obje koordinatne osi i prolazi točkom T(– 2, 9).

Rješenje 079

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

Ako je S(p, q) središte kružnice, a r njezin polumjer, tada središnja jednadžba kružnice glasi:

$$(x-p)^2+(y-q)^2=r^2.$$

Kružnica dira obje koordinatne osi pa je njezino središte jednako udaljeno od x – osi i od y – osi:

$$\left.\begin{array}{l}S(p, q) \\ p=q=r\end{array}\right\} \Rightarrow S(r, r).$$

Budući da je točka T(– 2, 9), kojom prolazi kružnica, u drugom kvadrantu (T(x, y), x < 0, y > 0), slijedi:

$$\left.\begin{array}{l}S(p, q) \\ p=-r \ , \ q=r\end{array}\right\} \Rightarrow S(-r, r).$$

Jednadžba tražene kružnice glasi:

$$\left(x+r\right)^2+\left(y-r\right)^2=r^2.$$

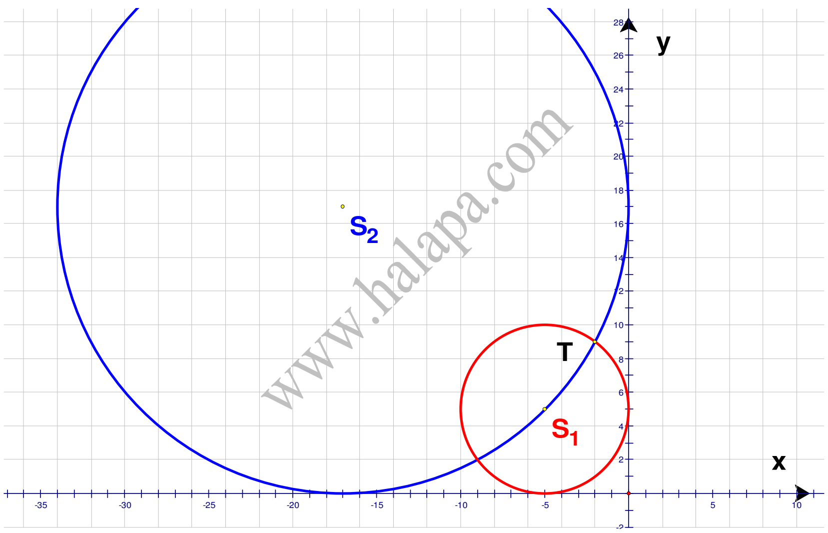
Budući da točka T pripada kružnici, koordinate točke uvrstit ćemo u jednadžbu kružnice:

$$\left.\begin{array}{l} T\left(x,\ y\right)=T\left(-2,\ 9\right) \\ \left(x+r\right)^2+\left(y-r\right)^2=r^2 \end{array}\right\} \Rightarrow \left(-2+r\right)^2+\left(9-r\right)^2=r^2 \Rightarrow 4-4\cdot r+r^2+81-18\cdot r+r^2=r^2 \Rightarrow$$

$$\Rightarrow 4-4\cdot r+r^2+81-18\cdot r+r^2=r^2 \Rightarrow r^2-22\cdot r+85=0 \Rightarrow \left.\begin{array}{l} r^2-22\cdot r+85=0 \\ a=1\ ,\ b=-22\ ,\ c=85 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=1\ ,\ b=-22\ ,\ c=85 \\ r_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow r_{1,2}=\frac{22\pm\sqrt{484-4\cdot 1\cdot 85}}{2\cdot 1} \Rightarrow r_{1,2}=\frac{22\pm\sqrt{484-340}}{2} \Rightarrow$$

$$\Rightarrow r_{1,2}=\frac{22\pm\sqrt{144}}{2} \Rightarrow r_{1,2}=\frac{22\pm 12}{2} \Rightarrow \left.\begin{array}{l} r_1=\dfrac{22+12}{2} \\ r_2=\dfrac{22-12}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} r_1=\dfrac{34}{2} \\ r_2=\dfrac{10}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} r_1=17 \\ r_2=5 \end{array}\right\}.$$



Postoje dvije kružnice:

- $$\left.\begin{array}{l} r=17\ ,\ p=-17\ ,\ q=17 \\ \left(x-p\right)^2+\left(y-q\right)^2=r^2 \end{array}\right\} \Rightarrow \left(x+17\right)^2+\left(y-17\right)^2=289.$$
- $$\left.\begin{array}{l} r=5\ ,\ p=-5\ ,\ q=5 \\ \left(x-p\right)^2+\left(y-q\right)^2=r^2 \end{array}\right\} \Rightarrow \left(x+5\right)^2+\left(y-5\right)^2=25.$$

Vježba 079

Napiši jednadžbu kružnice koja dira obje koordinatne osi i prolazi točkom T(2, 9).

Rezultat: $(x-17)^2+(y-17)^2=289$, $(x-5)^2+(y-5)^2=25$.

Zadatak 080 (3A, TUPŠ)

Napiši jednadžbu elipse ako je

$$a-b=2\quad ,\quad p=1.$$

Rješenje 080

Ponovimo!

Osna jednadžba elipse

$$b^2 \cdot x^2 + a^2 \cdot y^2 = a^2 \cdot b^2 \quad \text{ili} \quad \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

Tetiva elipse koja prolazi fokusom i okomita je na veliku os naziva se parametar elipse i njezina duljina se označava s $2 \cdot p$ $(p > 0)$:

$$p = \frac{b^2}{a}.$$

Broj p nazivamo poluparametar elipse.
Rješavamo sustav jednadžbi:

$$\left.\begin{array}{l} a - b = 2 \\ p = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = 2 + b \\ \frac{b^2}{a} = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = 2 + b \\ b^2 = a \end{array}\right\} \Rightarrow b^2 = 2 + b \Rightarrow b^2 - b - 2 = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} b^2 - b - 2 = 0 \\ a = 1 \ , \ b = -1 \ , \ c = -2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a = 1 \ , \ b = -1 \ , \ c = -2 \\ b_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{array}\right\} \Rightarrow b_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-2)}}{2 \cdot 1} \Rightarrow$$

$$\Rightarrow b_{1,2} = \frac{1 \pm \sqrt{1 + 8}}{2} \Rightarrow b_{1,2} = \frac{1 \pm \sqrt{9}}{2} \Rightarrow b_{1,2} = \frac{1 \pm 3}{2} \Rightarrow \left.\begin{array}{l} b_1 = \frac{1 + 3}{2} \\ b_2 = \frac{1 - 3}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} b_1 = \frac{4}{2} \\ b_2 = \frac{-2}{2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} b_1 = 2 \\ b_2 = -1 \ \text{nema smisla} \end{array}\right\}.$$

Mala poluos b elipse iznosi:

$$b = 2.$$

Velika poluos a elipse iznosi:

$$\left.\begin{array}{l} b = 2 \\ a = 2 + b \end{array}\right\} \Rightarrow a = 2 + 2 \Rightarrow a = 4.$$

Jednadžba elipse glasi:

$$\left.\begin{array}{l} a = 4 \ , \ b = 2 \\ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \end{array}\right\} \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{2^2} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1 \Rightarrow \frac{x^2}{16} + \frac{y^2}{4} = 1 \ / \cdot 16 \Rightarrow x^2 + 4 \cdot y^2 = 16.$$

Vježba 080

Napiši jednadžbu elipse ako je

$$a - b = 4 \quad , \quad p = 2.$$

Rezultat: $x^2 + 4 \cdot y^2 = 64.$