

Zadatak 781 (4B, TUPŠ)Rastavi na faktore: $(3 \cdot x - 2 \cdot y)^2 + 24 \cdot x \cdot y$.**Rješenje 781**

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2, \quad (a \cdot b)^n = a^n \cdot b^n, \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

$$a^2 = a \cdot a.$$

$$\begin{aligned} (3 \cdot x - 2 \cdot y)^2 + 24 \cdot x \cdot y &= (3 \cdot x)^2 - 2 \cdot 3 \cdot x \cdot 2 \cdot y + (2 \cdot y)^2 + 24 \cdot x \cdot y = \\ &= 9 \cdot x^2 - 12 \cdot x \cdot y + 4 \cdot y^2 + 24 \cdot x \cdot y = 9 \cdot x^2 + 12 \cdot x \cdot y + 4 \cdot y^2 = \\ &= (3 \cdot x + 2 \cdot y)^2 = (3 \cdot x + 2 \cdot y) \cdot (3 \cdot x + 2 \cdot y). \end{aligned}$$

Vježba 781Rastavi na faktore: $24 \cdot x \cdot y + (3 \cdot x - 2 \cdot y)^2$.**Rezultat:** $(3 \cdot x + 2 \cdot y) \cdot (3 \cdot x + 2 \cdot y)$.**Zadatak 782 (4B, TUPŠ)**Rastavi na faktore: $40 \cdot x \cdot y - (2 \cdot x + 5 \cdot y)^2$.**Rješenje 782**

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2, \quad (a \cdot b)^n = a^n \cdot b^n, \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

$$a^2 = a \cdot a.$$

Zakon distribucije množenja prema zbrajanju:

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$\begin{aligned} 40 \cdot x \cdot y - (2 \cdot x + 5 \cdot y)^2 &= 40 \cdot x \cdot y - \left((2 \cdot x)^2 + 2 \cdot 2 \cdot x \cdot 5 \cdot y + (5 \cdot y)^2 \right) = \\ &= 40 \cdot x \cdot y - \left(4 \cdot x^2 + 20 \cdot x \cdot y + 25 \cdot y^2 \right) = 40 \cdot x \cdot y - 4 \cdot x^2 - 20 \cdot x \cdot y - 25 \cdot y^2 = \\ &= -4 \cdot x^2 + 20 \cdot x \cdot y - 25 \cdot y^2 = -\left(4 \cdot x^2 - 20 \cdot x \cdot y + 25 \cdot y^2 \right) = -(2 \cdot x - 5 \cdot y)^2 = \\ &= -(2 \cdot x - 5 \cdot y) \cdot (2 \cdot x - 5 \cdot y). \end{aligned}$$

Vježba 782Rastavi na faktore: $24 \cdot x \cdot y - (2 \cdot x + 3 \cdot y)^2$.**Rezultat:** $-(2 \cdot x - 3 \cdot y) \cdot (2 \cdot x - 3 \cdot y)$.**Zadatak 783 (Miroslav, gimnazija)**Ako je $\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{1}{2}$, onda je $\frac{a}{b} - \frac{b}{a}$ jednako:

A. $\frac{1}{4}$ B. 2 C. $\frac{1}{2}$ D. 8

Rješenje 783

Ponovimo!

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \frac{a-c}{b-d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

$$\frac{a \cdot c}{b \cdot d} = \frac{a \cdot c}{b \cdot d} \quad , \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad , \quad n = \frac{n}{1}.$$

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

1. inačica

Preoblikujemo zadanu jednakost.

$$\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{1}{2} \Rightarrow \frac{(a+b)^2 - (a-b)^2}{(a-b) \cdot (a+b)} = \frac{1}{2} \Rightarrow \frac{((a+b) - (a-b)) \cdot ((a+b) + (a-b))}{a^2 - b^2} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{(a+b-a+b) \cdot (a+b+a-b)}{a^2 - b^2} = \frac{1}{2} \Rightarrow \frac{(a+b-a+b) \cdot (a+b+a-b)}{a^2 - b^2} = \frac{1}{2} \Rightarrow$$

$$\Rightarrow \frac{2 \cdot b \cdot 2 \cdot a}{a^2 - b^2} = \frac{1}{2} \Rightarrow \frac{4 \cdot a \cdot b}{a^2 - b^2} = \frac{1}{2} \Rightarrow \frac{4 \cdot a \cdot b}{a^2 - b^2} = \frac{1}{2} \cdot \frac{1}{4} \Rightarrow \frac{a \cdot b}{a^2 - b^2} = \frac{1}{8}.$$

Tada je

$$\frac{a}{b} - \frac{b}{a} = \frac{a^2 - b^2}{a \cdot b} = \left(\frac{a \cdot b}{a^2 - b^2} \right)^{-1} = \left[\frac{a \cdot b}{a^2 - b^2} = \frac{1}{8} \right]^{-1} = \left(\frac{1}{8} \right)^{-1} = 8.$$

Odgovor je pod D.

2. inačica

Preoblikujemo zadanu jednakost.

$$\frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{1}{2} \Rightarrow \frac{a+b}{a-b} - \frac{a-b}{a+b} = \frac{1}{2} \cdot 2 \cdot (a-b) \cdot (a+b) \Rightarrow$$

$$\Rightarrow 2 \cdot (a+b)^2 - 2 \cdot (a-b)^2 = (a-b) \cdot (a+b) \Rightarrow$$

$$\Rightarrow 2 \cdot (a^2 + 2 \cdot a \cdot b + b^2) - 2 \cdot (a^2 - 2 \cdot a \cdot b + b^2) = a^2 - b^2 \Rightarrow$$

$$\Rightarrow 2 \cdot a^2 + 4 \cdot a \cdot b + 2 \cdot b^2 - 2 \cdot a^2 + 4 \cdot a \cdot b - 2 \cdot b^2 = a^2 - b^2 \Rightarrow$$

$$\Rightarrow 2 \cdot a^2 + 4 \cdot a \cdot b + 2 \cdot b^2 - 2 \cdot a^2 + 4 \cdot a \cdot b - 2 \cdot b^2 = a^2 - b^2 \Rightarrow 4 \cdot a \cdot b + 4 \cdot a \cdot b = a^2 - b^2 \Rightarrow$$

$$\Rightarrow 8 \cdot a \cdot b = a^2 - b^2.$$

Tada je

$$\frac{a}{b} - \frac{b}{a} = \frac{a^2 - b^2}{a \cdot b} = \left[a^2 - b^2 = 8 \cdot a \cdot b \right] = \frac{8 \cdot a \cdot b}{a \cdot b} = \frac{8 \cdot a \cdot b}{a \cdot b} = 8.$$

Odgovor je pod D.

Vježba 783

Ako je $\frac{a+b}{a-b} + \frac{b-a}{a+b} = \frac{1}{2}$, onda je $\frac{a}{b} - \frac{b}{a}$ jednako:

- A. $\frac{1}{4}$ B. 2 C. $\frac{1}{2}$ D. 8

Rezultat: D.

Zadatak 784 (Miroslav, gimnazija)

Izraz $\left(\frac{b}{1-\frac{a}{b}} + \frac{a}{1-\frac{a}{b}} \right) \cdot \frac{1}{1-\frac{a^2}{b^2}}$ za $a \neq b \neq 0$ identičan je razlomku:

- A. $\frac{b^2}{b-a}$ B. $\frac{a^2}{a-b}$ C. $\frac{1}{a^2-b^2}$ D. $\frac{1}{a \cdot b}$

Rješenje 784

Ponovimo!

$$n = \frac{n}{1}, \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad a^1 = a, \quad a^n \cdot a^m = a^{n+m}, \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}, \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned} \left(\frac{b}{1-\frac{a}{b}} + \frac{a}{1-\frac{a}{b}} \right) \cdot \frac{1}{1-\frac{a^2}{b^2}} &= \left(\frac{b}{1-\frac{a}{b}} + \frac{a}{1-\frac{a}{b}} \right) \cdot \frac{1}{1-\frac{a^2}{b^2}} = \left(\frac{b}{b-a} + \frac{a}{a-b} \right) \cdot \frac{1}{\frac{b^2-a^2}{b^2}} = \\ &= \left(\frac{b}{b-a} + \frac{a}{a-b} \right) \cdot \frac{1}{\frac{b^2-a^2}{b^2}} = \left(\frac{b^2}{b-a} + \frac{a^2}{a-b} \right) \cdot \frac{b^2}{b^2-a^2} = \left(\frac{b^2}{b-a} + \frac{a^2}{-(b-a)} \right) \cdot \frac{b^2}{b^2-a^2} = \\ &= \left(\frac{b^2}{b-a} - \frac{a^2}{b-a} \right) \cdot \frac{b^2}{b^2-a^2} = \frac{b^2-a^2}{b-a} \cdot \frac{b^2}{b^2-a^2} = \frac{b^2-a^2}{b-a} \cdot \frac{b^2}{b^2-a^2} = \\ &= \frac{1}{b-a} \cdot \frac{b^2}{1} = \frac{b^2}{b-a}. \end{aligned}$$

Odgovor je pod A.

Vježba 784

Izraz $\left(\frac{b}{\frac{a}{b}-1} + \frac{a}{\frac{b}{a}-1}\right) \cdot \frac{1}{\frac{a^2}{b^2}-1}$ za $a \neq b \neq 0$ identičan je razlomku:

A. $\frac{b^2}{b-a}$ B. $\frac{a^2}{a-b}$ C. $\frac{1}{a^2-b^2}$ D. $\frac{1}{a \cdot b}$

Rezultat: A.

Zadatak 785 (4B, TUPŠ)

Izračunajte: $(4 \cdot x^2 + 1) \cdot (1 - 2 \cdot x) \cdot (1 + 2 \cdot x) - (x^2 - 1) \cdot (x^2 + 1)$.

Rješenje 785

Ponovimo!

$$(a-b) \cdot (a+b) = a^2 - b^2, \quad (a \cdot b)^n = a^n \cdot b^n, \quad (a^n)^m = a^{n \cdot m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$\begin{aligned} & (4 \cdot x^2 + 1) \cdot (1 - 2 \cdot x) \cdot (1 + 2 \cdot x) - (x^2 - 1) \cdot (x^2 + 1) = \\ & = (4 \cdot x^2 + 1) \cdot (1^2 - (2 \cdot x)^2) - \left((x^2)^2 - 1 \right) = (4 \cdot x^2 + 1) \cdot (1 - 4 \cdot x^2) - (x^4 - 1) = \\ & = (1 + 4 \cdot x^2) \cdot (1 - 4 \cdot x^2) - x^4 + 1 = 1^2 - (4 \cdot x^2)^2 - x^4 + 1 = 1 - 16 \cdot x^4 - x^4 + 1 = 2 - 17 \cdot x^4. \end{aligned}$$

Vježba 785

Izračunajte: $(4 \cdot x^2 + 1) \cdot (1 - 2 \cdot x) \cdot (1 + 2 \cdot x) + (1 - x^2) \cdot (x^2 + 1)$.

Rezultat: $2 - 17 \cdot x^4$.

Zadatak 786 (Iva i Ivan, bivši srednjoškolci ☺)

Ako je $\frac{a+b}{c} = 3$ i $\frac{a+1}{b} = 2$, koliko je $b-c$?

A. -3 B. $-\frac{1}{3}$ C. $\frac{1}{3}$ D. 3

Rješenje 786

Ponovimo!

$$\left. \begin{array}{l} a = b \\ c = d \end{array} \right\} a + c = b + d.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1. inačica

$$\left. \begin{array}{l} \frac{a+b}{c} = 3 \\ \frac{a+1}{b} = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{a+b}{c} = 3 \cdot c \\ \frac{a+1}{b} = 2 \cdot b \end{array} \right\} \Rightarrow \left. \begin{array}{l} a+b = 3 \cdot c \\ a+1 = 2 \cdot b \end{array} \right\} \Rightarrow \left. \begin{array}{l} a+b = 3 \cdot c \\ a = 2 \cdot b - 1 \end{array} \right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array} \right] \Rightarrow$$

$$\Rightarrow 2 \cdot b - 1 + b = 3 \cdot c \Rightarrow 2 \cdot b + b - 3 \cdot c = 1 \Rightarrow 3 \cdot b - 3 \cdot c = 1 \Rightarrow 3 \cdot (b - c) = 1 \Rightarrow$$

$$\Rightarrow 3 \cdot (b - c) = 1 \cdot \frac{1}{3} \Rightarrow b - c = \frac{1}{3}.$$

Odgovor je pod C.

2. inačica

$$\left. \begin{array}{l} \frac{a+b}{c} = 3 \\ \frac{a+1}{b} = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{a+b}{c} = 3 \cdot c \\ \frac{a+1}{b} = 2 \cdot b \end{array} \right\} \Rightarrow \left. \begin{array}{l} a+b = 3 \cdot c \\ a+1 = 2 \cdot b \end{array} \right\} \Rightarrow \left. \begin{array}{l} a+b = 3 \cdot c \\ a - 2 \cdot b = -1 \end{array} \right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array} \right] \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} a+b = 3 \cdot c \\ a - 2 \cdot b = -1 \cdot (-1) \end{array} \right\} \Rightarrow \left. \begin{array}{l} a+b = 3 \cdot c \\ -a + 2 \cdot b = 1 \end{array} \right\} \Rightarrow 3 \cdot b = 3 \cdot c + 1 \Rightarrow 3 \cdot b - 3 \cdot c = 1 \Rightarrow$$

$$\Rightarrow 3 \cdot (b - c) = 1 \Rightarrow 3 \cdot (b - c) = 1 \cdot \frac{1}{3} \Rightarrow b - c = \frac{1}{3}.$$

Odgovor je pod C.

3. inačica

$$\left. \begin{array}{l} \frac{a+b}{c} = 3 \\ \frac{a+1}{b} = 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} \frac{a+b}{c} = 3 \cdot c \\ \frac{a+1}{b} = 2 \cdot b \end{array} \right\} \Rightarrow \left. \begin{array}{l} a+b = 3 \cdot c \\ a+1 = 2 \cdot b \end{array} \right\} \Rightarrow \left. \begin{array}{l} a = 3 \cdot c - b \\ a = 2 \cdot b - 1 \end{array} \right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{komparacije} \end{array} \right] \Rightarrow$$

$$\Rightarrow 3 \cdot c - b = 2 \cdot b - 1 \Rightarrow 2 \cdot b - 1 = 3 \cdot c - b \Rightarrow 2 \cdot b - 3 \cdot c + b = 1 \Rightarrow 3 \cdot b - 3 \cdot c = 1 \Rightarrow$$

$$\Rightarrow 3 \cdot (b - c) = 1 \Rightarrow 3 \cdot (b - c) = 1 \cdot \frac{1}{3} \Rightarrow b - c = \frac{1}{3}.$$

Odgovor je pod C.

Vježba 786

Ako je $\frac{a+b}{c} = 3$ i $\frac{a+1}{b} = 2$, koliko je $c-b$?

- A. -3 B. $-\frac{1}{3}$ C. $\frac{1}{3}$ D. 3

Rezultat: B.

Zadatak 787 (Iva, maturantica)

Provedite računске operacije u izrazu $\left(\frac{1}{3 \cdot a - b} - \frac{1}{3 \cdot a + b} \right) \cdot (9 \cdot a^2 - b^2)$ i pojednostavnite ga

do kraja za sve a, b za koje je taj izraz definiran.

Rješenje 787

Ponovimo!

$$\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad (a \cdot b)^n = a^n \cdot b^n, \quad a^2 - b^2 = (a - b) \cdot (a + b).$$

$$\frac{n}{1} = n.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

1. inačica

$$\begin{aligned} & \left(\frac{1}{3 \cdot a - b} - \frac{1}{3 \cdot a + b} \right) \cdot (9 \cdot a^2 - b^2) = \frac{3 \cdot a + b - (3 \cdot a - b)}{(3 \cdot a - b) \cdot (3 \cdot a + b)} \cdot \frac{9 \cdot a^2 - b^2}{1} = \\ & = \frac{3 \cdot a + b - 3 \cdot a + b}{(3 \cdot a - b) \cdot (3 \cdot a + b)} \cdot \frac{9 \cdot a^2 - b^2}{1} = \frac{3 \cdot a + b - 3 \cdot a + b}{(3 \cdot a - b) \cdot (3 \cdot a + b)} \cdot \frac{9 \cdot a^2 - b^2}{1} = \frac{2 \cdot b}{(3 \cdot a)^2 - b^2} \cdot \frac{9 \cdot a^2 - b^2}{1} = \\ & = \frac{2 \cdot b}{9 \cdot a^2 - b^2} \cdot \frac{9 \cdot a^2 - b^2}{1} = \frac{2 \cdot b}{9 \cdot a^2 - b^2} \cdot \frac{9 \cdot a^2 - b^2}{1} = \frac{2 \cdot b}{1} = 2 \cdot b. \end{aligned}$$

2. inačica

$$\begin{aligned} & \left(\frac{1}{3 \cdot a - b} - \frac{1}{3 \cdot a + b} \right) \cdot (9 \cdot a^2 - b^2) = \left(\frac{1}{3 \cdot a - b} - \frac{1}{3 \cdot a + b} \right) \cdot ((3 \cdot a)^2 - b^2) = \\ & = \left(\frac{1}{3 \cdot a - b} - \frac{1}{3 \cdot a + b} \right) \cdot (3 \cdot a - b) \cdot (3 \cdot a + b) = \\ & = \frac{1}{3 \cdot a - b} \cdot (3 \cdot a - b) \cdot (3 \cdot a + b) - \frac{1}{3 \cdot a + b} \cdot (3 \cdot a - b) \cdot (3 \cdot a + b) = \\ & = \frac{1}{3 \cdot a - b} \cdot (3 \cdot a - b) \cdot (3 \cdot a + b) - \frac{1}{3 \cdot a + b} \cdot (3 \cdot a - b) \cdot (3 \cdot a + b) = 3 \cdot a + b - (3 \cdot a - b) = \\ & = 3 \cdot a + b - 3 \cdot a + b = 3 \cdot a + b - 3 \cdot a + b = 2 \cdot b. \end{aligned}$$

Vježba 787

Provedite računске operacije u izrazu $(9 \cdot a^2 - b^2) \cdot \left(\frac{1}{3 \cdot a - b} - \frac{1}{3 \cdot a + b} \right)$ i pojednostavnite ga do kraja za sve a, b za koje je taj izraz definiran.

Rezultat: $2 \cdot b$.

Zadatak 788 (Luka, građevinska škola)

Pojednostavnite: $\frac{e^{3 \cdot x} - 1}{e^x - 1}$.

Rješenje 788

Ponovimo!

$$(a^n)^m = a^{n \cdot m}.$$

$$a^3 - b^3 = (a-b) \cdot (a^2 + a \cdot b + b^2), \quad a^3 + b^3 = (a+b) \cdot (a^2 - a \cdot b + b^2).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i

jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned} \frac{e^{3 \cdot x} - 1}{e^x - 1} &= \frac{(e^x)^3 - 1}{e^x - 1} = \frac{(e^x - 1) \cdot \left((e^x)^2 + e^x \cdot 1 + 1 \right)}{e^x - 1} = \frac{(e^x - 1) \cdot (e^{2 \cdot x} + e^x + 1)}{e^x - 1} = \\ &= \frac{(e^x - 1) \cdot (e^{2 \cdot x} + e^x + 1)}{e^x - 1} = e^{2 \cdot x} + e^x + 1. \end{aligned}$$

Vježba 788

Pojednostavnite: $\frac{e^{3 \cdot x} + 1}{e^x + 1}$.

Rezultat: $e^{2 \cdot x} - e^x + 1$.

Zadatak 789 (Đurđica, ekonomska škola)

Rastavi na faktore: $x^3 + 3 \cdot x^2 - 4 \cdot x$.

Rješenje 789

Ponovimo!

$$a^1 = a, \quad a^n : a^m = a^{n-m}, \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1. inačica

$$\begin{aligned} x^3 + 3 \cdot x^2 - 4 \cdot x &= x \cdot (x^2 + 3 \cdot x - 4) = x \cdot (x^2 - x + 4 \cdot x - 4) = \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] = \\ &= x \cdot \left((x^2 - x) + (4 \cdot x - 4) \right) = x \cdot (x \cdot (x-1) + 4 \cdot (x-1)) = x \cdot (x-1) \cdot (x+4). \end{aligned}$$

2. inačica

$$\begin{aligned} x^3 + 3 \cdot x^2 - 4 \cdot x &= x \cdot (x^2 + 3 \cdot x - 4) = x \cdot (x^2 + 3 \cdot x - 1 - 3) = x \cdot (x^2 - 1 + 3 \cdot x - 3) = \\ &= \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] = x \cdot \left((x^2 - 1) + (3 \cdot x - 3) \right) = x \cdot ((x-1) \cdot (x+1) + 3 \cdot (x-1)) = \\ &= x \cdot (x-1) \cdot (x+1+3) = x \cdot (x-1) \cdot (x+4). \end{aligned}$$

Vježba 789

Rastavi na faktore: $x^3 - 3 \cdot x^2 - 4 \cdot x$.

Rezultat: $x \cdot (x+1) \cdot (x-4)$.

Zadatak 790 (Ivan, tehnička škola)

Izračunajte vrijednost razlomka $\frac{x+2 \cdot y}{x-2 \cdot y}$, ako je $x^2 + 4 \cdot y^2 = 5 \cdot x \cdot y$ i $0 < x < y$.

Rješenje 790

Ponovimo!

$$a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2, \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2, \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n.$$

$$\left. \begin{array}{l} a=b \\ c=d \end{array} \right\} \Rightarrow \frac{a}{c} = \frac{b}{d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Zadanu jednakost preoblikujemo na dva načina.

$$\begin{aligned} \left. \begin{array}{l} x^2 + 4 \cdot y^2 = 5 \cdot x \cdot y \\ x^2 + 4 \cdot y^2 = 5 \cdot x \cdot y \end{array} \right\} &\Rightarrow \left. \begin{array}{l} x^2 + 4 \cdot y^2 = 9 \cdot x \cdot y - 4 \cdot x \cdot y \\ x^2 + 4 \cdot y^2 = x \cdot y + 4 \cdot x \cdot y \end{array} \right\} \Rightarrow \\ \Rightarrow \left. \begin{array}{l} x^2 + 4 \cdot x \cdot y + 4 \cdot y^2 = 9 \cdot x \cdot y \\ x^2 - 4 \cdot x \cdot y + 4 \cdot y^2 = x \cdot y \end{array} \right\} &\Rightarrow \left. \begin{array}{l} (x+2 \cdot y)^2 = 9 \cdot x \cdot y \\ (x-2 \cdot y)^2 = x \cdot y \end{array} \right\} \Rightarrow \left[\begin{array}{l} \text{podijelimo} \\ \text{jednakosti} \end{array} \right] \Rightarrow \\ \Rightarrow \frac{(x+2 \cdot y)^2}{(x-2 \cdot y)^2} = \frac{9 \cdot x \cdot y}{x \cdot y} &\Rightarrow \left(\frac{x+2 \cdot y}{x-2 \cdot y} \right)^2 = \frac{9 \cdot x \cdot y}{x \cdot y} \Rightarrow \left(\frac{x+2 \cdot y}{x-2 \cdot y} \right)^2 = 9 \Rightarrow \\ \Rightarrow \left(\frac{x+2 \cdot y}{x-2 \cdot y} \right)^2 = 9 \quad / \sqrt{} &\Rightarrow \frac{x+2 \cdot y}{x-2 \cdot y} = \pm \sqrt{9} \Rightarrow \frac{x+2 \cdot y}{x-2 \cdot y} = \pm 3. \end{aligned}$$

Vježba 790

Izračunajte vrijednost razlomka $\frac{x-2 \cdot y}{x+2 \cdot y}$, ako je $x^2 + 4 \cdot y^2 = 5 \cdot x \cdot y$ i $0 < x < y$.

Rezultat: $\pm \frac{1}{3}$.

Zadatak 791 (Neven, gimnazija)

Ako je $\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b} = 0$, tada je $\frac{a}{(b-c)^2} + \frac{b}{(c-a)^2} + \frac{c}{(a-b)^2} = 0$. Dokazati!

Rješenje 791

Ponovimo!

$$a^1 = a, \quad a^n \cdot a^m = a^{n+m}, \quad a^n : a^m = a^{n-m}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$\frac{0}{n} = 0, \quad n \neq 0.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

Iz

$$\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b} = 0$$

dobiju se tri jednakosti:

$$\begin{aligned} \bullet \quad \frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b} = 0 &\Rightarrow \frac{a}{b-c} = -\frac{b}{c-a} - \frac{c}{a-b} \Rightarrow \frac{a}{b-c} = \frac{-b \cdot (a-b) - c \cdot (c-a)}{(c-a) \cdot (a-b)} \Rightarrow \\ &\Rightarrow \frac{a}{b-c} = \frac{-a \cdot b + b^2 - c^2 + a \cdot c}{(c-a) \cdot (a-b)} \Rightarrow \frac{a}{b-c} = \frac{-a \cdot b + b^2 + b \cdot c + a \cdot c - b \cdot c - c^2}{(c-a) \cdot (a-b)} \Rightarrow \\ &\Rightarrow \frac{a}{b-c} = \frac{-a \cdot b + b^2 + b \cdot c + a \cdot c - b \cdot c - c^2}{(c-a) \cdot (a-b)} \Rightarrow \frac{a}{b-c} = \frac{b \cdot (-a+b+c) - c \cdot (-a+b+c)}{(c-a) \cdot (a-b)} \Rightarrow \\ &\Rightarrow \frac{a}{b-c} = \frac{(-a+b+c) \cdot (b-c)}{(c-a) \cdot (a-b)} \Rightarrow \frac{a}{b-c} = \frac{(-a+b+c) \cdot (b-c)}{(c-a) \cdot (a-b)} \cdot \frac{1}{b-c} \Rightarrow \end{aligned}$$

$$\Rightarrow \frac{a}{(b-c)^2} = \frac{-a+b+c}{(c-a) \cdot (a-b)}$$

$$\begin{aligned} \bullet \quad \frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b} = 0 &\Rightarrow \frac{b}{c-a} = -\frac{a}{b-c} - \frac{c}{a-b} \Rightarrow \frac{b}{c-a} = \frac{-a \cdot (a-b) - c \cdot (b-c)}{(b-c) \cdot (a-b)} \Rightarrow \\ &\Rightarrow \frac{b}{c-a} = \frac{-a^2 + a \cdot b - b \cdot c + c^2}{(b-c) \cdot (a-b)} \Rightarrow \frac{b}{c-a} = \frac{a \cdot c - b \cdot c + c^2 - a^2 + a \cdot b - a \cdot c}{(b-c) \cdot (a-b)} \Rightarrow \\ &\Rightarrow \frac{b}{c-a} = \frac{a \cdot c - b \cdot c + c^2 - a^2 + a \cdot b - a \cdot c}{(b-c) \cdot (a-b)} \Rightarrow \frac{b}{c-a} = \frac{c \cdot (a-b+c) - a \cdot (a-b+c)}{(b-c) \cdot (a-b)} \Rightarrow \\ &\Rightarrow \frac{b}{c-a} = \frac{(a-b+c) \cdot (c-a)}{(b-c) \cdot (a-b)} \Rightarrow \frac{b}{c-a} = \frac{(a-b+c) \cdot (c-a)}{(b-c) \cdot (a-b)} \cdot \frac{1}{c-a} \Rightarrow \end{aligned}$$

$$\Rightarrow \frac{b}{(c-a)^2} = \frac{a-b+c}{(b-c) \cdot (a-b)}$$

$$\begin{aligned} \bullet \quad \frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b} = 0 &\Rightarrow \frac{c}{a-b} = -\frac{a}{b-c} - \frac{b}{c-a} \Rightarrow \frac{c}{a-b} = \frac{-a \cdot (c-a) - b \cdot (b-c)}{(b-c) \cdot (c-a)} \Rightarrow \\ &\Rightarrow \frac{c}{a-b} = \frac{-a \cdot c + a^2 - b^2 + b \cdot c}{(b-c) \cdot (c-a)} \Rightarrow \frac{c}{a-b} = \frac{a^2 + a \cdot b - a \cdot c - a \cdot b - b^2 + b \cdot c}{(b-c) \cdot (c-a)} \Rightarrow \\ &\Rightarrow \frac{c}{a-b} = \frac{a^2 + a \cdot b - a \cdot c - a \cdot b - b^2 + b \cdot c}{(b-c) \cdot (c-a)} \Rightarrow \frac{c}{a-b} = \frac{a \cdot (a+b-c) - b \cdot (a+b-c)}{(b-c) \cdot (c-a)} \Rightarrow \\ &\Rightarrow \frac{c}{a-b} = \frac{(a+b-c) \cdot (a-b)}{(b-c) \cdot (c-a)} \Rightarrow \frac{c}{a-b} = \frac{(a+b-c) \cdot (a-b)}{(b-c) \cdot (c-a)} \cdot \frac{1}{a-b} \Rightarrow \end{aligned}$$

$$\Rightarrow \frac{c}{(a-b)^2} = \frac{a+b-c}{(b-c) \cdot (c-a)}$$

Sada je:

$$\begin{aligned}
\frac{a}{(b-c)^2} + \frac{b}{(c-a)^2} + \frac{c}{(a-b)^2} &= \left[\begin{array}{l} \frac{a}{(b-c)^2} = \frac{-a+b+c}{(c-a)\cdot(a-b)} \\ \frac{b}{(c-a)^2} = \frac{a-b+c}{(b-c)\cdot(a-b)} \\ \frac{c}{(a-b)^2} = \frac{a+b-c}{(b-c)\cdot(c-a)} \end{array} \right] = \\
&= \frac{-a+b+c}{(c-a)\cdot(a-b)} + \frac{a-b+c}{(b-c)\cdot(a-b)} + \frac{a+b-c}{(b-c)\cdot(c-a)} = \\
&= \frac{(b-c)\cdot(-a+b+c) + (c-a)\cdot(a-b+c) + (a-b)\cdot(a+b-c)}{(c-a)\cdot(a-b)\cdot(b-c)} = \\
&= \frac{-a\cdot b + b^2 + b\cdot c + a\cdot c - b\cdot c - c^2 + a\cdot c - b\cdot c + c^2 - a^2 + a\cdot b - a\cdot c + a^2 + a\cdot b - a\cdot c - a\cdot b - b^2 + b\cdot c}{(c-a)\cdot(a-b)\cdot(b-c)} = \\
&= \frac{-a\cdot b + b^2 + b\cdot c + a\cdot c - b\cdot c - c^2 + a\cdot c - b\cdot c + c^2 - a^2 + a\cdot b - a\cdot c + a^2 + a\cdot b - a\cdot c - a\cdot b - b^2 + b\cdot c}{(c-a)\cdot(a-b)\cdot(b-c)} = \\
&= \frac{0}{(c-a)\cdot(a-b)\cdot(b-c)} = 0.
\end{aligned}$$

Dokaz gotov.

Vježba 791

Ako je $\frac{a}{b-c} - \frac{b}{a-c} - \frac{c}{b-a} = 0$, tada je $\frac{a}{(b-c)^2} + \frac{b}{(c-a)^2} + \frac{c}{(a-b)^2} = 0$. Dokazati!

Rezultat: Dokaz analogan.

Zadatak 792 (Zlatko, srednja škola)

Izračunati: $\left(\frac{1}{2+2\cdot\sqrt{a}} + \frac{1}{2-2\cdot\sqrt{a}} - \frac{a^2+1}{1-a^2} \right) \cdot \left(1 + \frac{1}{a} \right)$.

Rješenje 792

Ponovimo!

$$a^1 = a, \quad a^n \cdot a^m = a^{n+m}, \quad a^n : a^m = a^{n-m}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a\cdot d + b\cdot c}{b\cdot d}.$$

$$\frac{a}{b} - \frac{c}{d} = \frac{a\cdot d - b\cdot c}{b\cdot d}, \quad a^2 - b^2 = (a-b)\cdot(a+b), \quad (\sqrt{a})^2 = a, \quad \frac{a}{n} - \frac{b}{n} = \frac{a-b}{n}.$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a\cdot c}{b\cdot d}, \quad n = \frac{n}{1}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c, \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

1. inačica

$$\begin{aligned}
 & \left(\frac{1}{2+2\sqrt{a}} + \frac{1}{2-2\sqrt{a}} - \frac{a^2+1}{1-a^2} \right) \cdot \left(1 + \frac{1}{a} \right) = \left(\frac{1}{2\cdot(1+\sqrt{a})} + \frac{1}{2\cdot(1-\sqrt{a})} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \left(\frac{1}{1} + \frac{1}{a} \right) = \\
 & = \left(\frac{1-\sqrt{a}+1+\sqrt{a}}{2\cdot(1+\sqrt{a})\cdot(1-\sqrt{a})} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \left(\frac{1-\sqrt{a}+1+\sqrt{a}}{2\cdot(1-(\sqrt{a})^2)} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \\
 & = \left(\frac{2}{2\cdot(1-a)} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \left(\frac{2}{2\cdot(1-a)} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \\
 & = \left(\frac{1}{1-a} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \frac{1+a-(a^2+1)}{(1-a)\cdot(1+a)} \cdot \frac{a+1}{a} = \frac{1+a-a^2-1}{(1-a)\cdot(1+a)} \cdot \frac{a+1}{a} = \\
 & = \frac{1+a-a^2-1}{1-a} \cdot \frac{1}{a} = \frac{a-a^2}{a\cdot(1-a)} = \frac{a\cdot(1-a)}{a\cdot(1-a)} = \frac{a\cdot(1-a)}{a\cdot(1-a)} = 1.
 \end{aligned}$$

2. inačica

$$\begin{aligned}
 & \left(\frac{1}{2+2\sqrt{a}} + \frac{1}{2-2\sqrt{a}} - \frac{a^2+1}{1-a^2} \right) \cdot \left(1 + \frac{1}{a} \right) = \left(\frac{1}{2\cdot(1+\sqrt{a})} + \frac{1}{2\cdot(1-\sqrt{a})} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \left(\frac{1}{1} + \frac{1}{a} \right) = \\
 & = \left(\frac{1-\sqrt{a}+1+\sqrt{a}}{2\cdot(1+\sqrt{a})\cdot(1-\sqrt{a})} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \left(\frac{1-\sqrt{a}+1+\sqrt{a}}{2\cdot(1-(\sqrt{a})^2)} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \\
 & = \left(\frac{2}{2\cdot(1-a)} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \left(\frac{2}{2\cdot(1-a)} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \\
 & = \left(\frac{1}{1-a} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \frac{a+1}{a\cdot(1-a)} - \frac{(a^2+1)\cdot(a+1)}{a\cdot(1-a)\cdot(1+a)} = \\
 & = \left(\frac{1}{1-a} - \frac{a^2+1}{(1-a)\cdot(1+a)} \right) \cdot \frac{a+1}{a} = \frac{a+1}{a\cdot(1-a)} - \frac{(a^2+1)\cdot(a+1)}{a\cdot(1-a)\cdot(1+a)} = \frac{a+1}{a\cdot(1-a)} - \frac{a^2+1}{a\cdot(1-a)} = \\
 & = \frac{a+1}{a\cdot(1-a)} - \frac{a^2+1}{a\cdot(1-a)} = \frac{a+1-(a^2+1)}{a\cdot(1-a)} = \frac{a+1-a^2-1}{a\cdot(1-a)} = \frac{a+1-a^2-1}{a\cdot(1-a)} = \frac{a-a^2}{a\cdot(1-a)} = \\
 & = \frac{a\cdot(1-a)}{a\cdot(1-a)} = \frac{a\cdot(1-a)}{a\cdot(1-a)} = 1.
 \end{aligned}$$

Vježba 792

Izračunati: $\left(\frac{1}{2+2\cdot\sqrt{a}} + \frac{1}{2-2\cdot\sqrt{a}} + \frac{a^2+1}{a^2-1} \right) \cdot \left(1 + \frac{1}{a} \right)$.

Rezultat: 1.

Zadatak 793 (Igor, srednja škola)

Izraz $\frac{1}{1-b} : \frac{a}{b} - \frac{1}{a-a\cdot b}$ napišite kao jedan do kraja skraćen razlomak za sve a, b za koje je taj izraz definiran.

Rješenje 793

Ponovimo!

$$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}, \quad \frac{a}{n} - \frac{b}{n} = \frac{a-b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1. inačica

$$\begin{aligned} \frac{1}{1-b} : \frac{a}{b} - \frac{1}{a-a\cdot b} &= \frac{1}{1-b} \cdot \frac{b}{a} - \frac{1}{a \cdot (1-b)} = \frac{b}{a \cdot (1-b)} - \frac{1}{a \cdot (1-b)} = \frac{b-1}{a \cdot (1-b)} = \\ &= \frac{-(1-b)}{a \cdot (1-b)} = -\frac{1-b}{a \cdot (1-b)} = -\frac{1-b}{a \cdot (1-b)} = -\frac{1}{a}. \end{aligned}$$

2. inačica

$$\begin{aligned} \frac{1}{1-b} : \frac{a}{b} - \frac{1}{a-a\cdot b} &= \frac{1}{1-b} \cdot \frac{b}{a} - \frac{1}{a \cdot (1-b)} = \frac{1}{1-b} \cdot \left(\frac{b}{a} - \frac{1}{a} \right) = \frac{1}{1-b} \cdot \frac{b-1}{a} = \\ &= \frac{1}{1-b} \cdot \frac{-(1-b)}{a} = \frac{1}{1-b} \cdot \frac{-(1-b)}{a} = -\frac{1}{a}. \end{aligned}$$

Vježba 793

Izraz $\frac{1}{1-b} : \frac{a}{b} + \frac{1}{a \cdot b - a}$ napišite kao jedan do kraja skraćen razlomak za sve a, b za koje je taj izraz definiran.

Rezultat: $-\frac{1}{a}$.

Zadatak 794 (Blek, srednja škola)

Skrti razlomak: $\frac{a^2-1}{5+5\cdot a}$ ($a \neq -1$).

Rješenje 794

Ponovimo!

$$a^2 - b^2 = (a-b) \cdot (a+b).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$\frac{a^2 - 1}{5 + 5 \cdot a} = \frac{(a-1) \cdot (a+1)}{5 \cdot (1+a)} = \frac{(a-1) \cdot (a+1)}{5 \cdot (a+1)} = \frac{(a-1) \cdot \cancel{(a+1)}}{5 \cdot \cancel{(a+1)}} = \frac{a-1}{5}.$$

Vježba 794

Skrati razlomak: $\frac{a^2 - 1}{3 + 3 \cdot a} \quad (a \neq -1).$

Rezultat: $\frac{a-1}{3}.$

Zadatak 795 (TNT, gimnazija)

Izvedi operacije: $\left[\frac{(a+b)^3}{3 \cdot a \cdot b} - a - b \right]^n : \left[\frac{(a-b)^2}{a \cdot b} + 1 \right]^n.$

Rješenje 795

Ponovimo!

$$n = \frac{n}{1}, \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$(a+b)^3 = a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3, \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

$$a^n : b^n = (a : b)^n, \quad \frac{a}{b} : \frac{c}{d} = \frac{a \cdot d}{b \cdot c} = \frac{a \cdot d}{b \cdot c}, \quad a^3 + b^3 = (a+b) \cdot (a^2 - a \cdot b + b^2).$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned} & \left[\frac{(a+b)^3}{3 \cdot a \cdot b} - a - b \right]^n : \left[\frac{(a-b)^2}{a \cdot b} + 1 \right]^n = \left[\frac{(a+b)^3}{3 \cdot a \cdot b} - \frac{a}{1} - \frac{b}{1} \right]^n : \left[\frac{(a-b)^2}{a \cdot b} + \frac{1}{1} \right]^n = \\ & = \left[\frac{(a+b)^3 - 3 \cdot a^2 \cdot b - 3 \cdot a \cdot b^2}{3 \cdot a \cdot b} \right]^n : \left[\frac{(a-b)^2 + a \cdot b}{a \cdot b} \right]^n = \\ & = \left[\frac{a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3 - 3 \cdot a^2 \cdot b - 3 \cdot a \cdot b^2}{3 \cdot a \cdot b} \right]^n : \left[\frac{a^2 - 2 \cdot a \cdot b + b^2 + a \cdot b}{a \cdot b} \right]^n = \end{aligned}$$

$$\begin{aligned}
&= \left[\frac{a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3 - 3 \cdot a^2 \cdot b - 3 \cdot a \cdot b^2}{3 \cdot a \cdot b} \right]^n : \left[\frac{a^2 - a \cdot b + b^2}{a \cdot b} \right]^n = \\
&= \left[\frac{a^3 + b^3}{3 \cdot a \cdot b} \right]^n : \left[\frac{a^2 - a \cdot b + b^2}{a \cdot b} \right]^n = \left[\frac{a^3 + b^3}{3 \cdot a \cdot b} : \frac{a^2 - a \cdot b + b^2}{a \cdot b} \right]^n = \\
&= \left[\frac{a^3 + b^3}{3 \cdot a \cdot b} \cdot \frac{a \cdot b}{a^2 - a \cdot b + b^2} \right]^n = \left[\frac{(a+b) \cdot (a^2 - a \cdot b + b^2)}{3 \cdot a \cdot b} \cdot \frac{a \cdot b}{a^2 - a \cdot b + b^2} \right]^n = \\
&= \left[\frac{(a+b) \cdot (a^2 - a \cdot b + b^2)}{3 \cdot a \cdot b} \cdot \frac{a \cdot b}{a^2 - a \cdot b + b^2} \right]^n = \left[\frac{a+b}{3} \cdot \frac{1}{1} \right]^n = \left[\frac{a+b}{3} \right]^n .
\end{aligned}$$

Vježba 795

Izvedi operacije: $\left[\frac{(a+b)^3}{3 \cdot a \cdot b} - a - b \right]^n : \left[1 + \frac{(a-b)^2}{a \cdot b} \right]^n$.

Rezultat: $\left[\frac{a+b}{3} \right]^n$.

Zadatak 796 (Ivan, gimnazija)

Neka su x, y i z realni brojevi takvi da je $x + y + z = x \cdot y \cdot z$. Dokažite da vrijedi identitet $x \cdot (1 - y^2) \cdot (1 - z^2) + y \cdot (1 - z^2) \cdot (1 - x^2) + z \cdot (1 - x^2) \cdot (1 - y^2) = 4 \cdot x \cdot y \cdot z$.

Rješenje 796

Ponovimo!

$$a^1 = a, \quad a^n \cdot a^m = a^{n+m}, \quad a^n : a^m = a^{n-m}.$$

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Preoblikujemo lijevu stranu jednakosti.

$$\begin{aligned}
&x \cdot (1 - y^2) \cdot (1 - z^2) + y \cdot (1 - z^2) \cdot (1 - x^2) + z \cdot (1 - x^2) \cdot (1 - y^2) = \\
&= x \cdot (1 - z^2 - y^2 + y^2 \cdot z^2) + y \cdot (1 - x^2 - z^2 + z^2 \cdot x^2) + z \cdot (1 - y^2 - x^2 + x^2 \cdot y^2) = \\
&= x - x \cdot z^2 - x \cdot y^2 + x \cdot y^2 \cdot z^2 + y - y \cdot x^2 - y \cdot z^2 + y \cdot z^2 \cdot x^2 + z - z \cdot y^2 - z \cdot x^2 + z \cdot x^2 \cdot y^2 = \\
&= \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] = (x + y + z) + (z \cdot x^2 \cdot y^2 - x^2 \cdot y - x \cdot y^2) + (x \cdot y^2 \cdot z^2 - z^2 \cdot y - z \cdot y^2) + \\
&\quad + (y \cdot z^2 \cdot x^2 - x \cdot z^2 - z \cdot x^2) =
\end{aligned}$$

$$\begin{aligned}
&= (x+y+z) + x \cdot y \cdot (z \cdot x \cdot y - x - y) + y \cdot z \cdot (x \cdot y \cdot z - z - y) + x \cdot z \cdot (y \cdot z \cdot x - z - x) = \\
&= (x+y+z) + x \cdot y \cdot (x \cdot y \cdot z - x - y) + y \cdot z \cdot (x \cdot y \cdot z - y - z) + z \cdot x \cdot (x \cdot y \cdot z - x - z) = \\
&= \begin{bmatrix} \text{uvjet} \\ x+y+z = x \cdot y \cdot z \\ z = x \cdot y \cdot z - x - y \\ x = x \cdot y \cdot z - y - z \\ y = x \cdot y \cdot z - x - z \end{bmatrix} = x \cdot y \cdot z + x \cdot y \cdot z + y \cdot z \cdot x + z \cdot x \cdot y = \\
&= x \cdot y \cdot z + x \cdot y \cdot z + x \cdot y \cdot z + x \cdot y \cdot z = 4 \cdot x \cdot y \cdot z.
\end{aligned}$$

Vježba 796

Neka su x, y i z realni brojevi takvi da je $x + y + z = x \cdot y \cdot z$. Dokažite da vrijedi identitet $x \cdot (y^2 - 1) \cdot (z^2 - 1) + y \cdot (z^2 - 1) \cdot (x^2 - 1) + z \cdot (x^2 - 1) \cdot (y^2 - 1) = 4 \cdot x \cdot y \cdot z$.

Rezultat: Dokaz analogan.

Zadatak 797 (Luka, ekonomska škola)

Izračunaj: $\frac{x+2}{x \cdot y^2 + x \cdot y} + \frac{y-1}{y^2 + y} - \frac{2 \cdot x - y}{2 \cdot x \cdot y + 2 \cdot x}$.

Rješenje 797

Ponovimo!

$$\begin{aligned}
a^1 &= a, & a^n : a^m &= a^{n-m}, & a^n \cdot a^m &= a^{n+m}, \\
\frac{a}{b} + \frac{c}{d} &= \frac{a \cdot d + b \cdot c}{b \cdot d}, & \frac{a}{b} - \frac{c}{d} &= \frac{a \cdot d - b \cdot c}{b \cdot d}.
\end{aligned}$$

Zakon distribucije množenja prema zbrajanju,

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$\begin{aligned}
&\frac{x+2}{x \cdot y^2 + x \cdot y} + \frac{y-1}{y^2 + y} - \frac{2 \cdot x - y}{2 \cdot x \cdot y + 2 \cdot x} = \left[\begin{array}{l} \text{svaki nazivnik,} \\ \text{ako je moguće,} \\ \text{rastavimo na faktore} \end{array} \right] = \\
&= \frac{x+2}{x \cdot y \cdot (y+1)} + \frac{y-1}{y \cdot (y+1)} - \frac{2 \cdot x - y}{2 \cdot x \cdot (y+1)} = \left[\begin{array}{l} \text{nađemo najmanji} \\ \text{zajednički nazivnik} \end{array} \right] = \\
&= \frac{2 \cdot (x+2) + 2 \cdot x \cdot (y-1) - y \cdot (2 \cdot x - y)}{2 \cdot x \cdot y \cdot (y+1)} = \frac{2 \cdot x + 4 + 2 \cdot x \cdot y - 2 \cdot x - 2 \cdot x \cdot y + y^2}{2 \cdot x \cdot y \cdot (y+1)} = \\
&= \frac{2 \cdot x + 4 + 2 \cdot x \cdot y - 2 \cdot x - 2 \cdot x \cdot y + y^2}{2 \cdot x \cdot y \cdot (y+1)} = \frac{4 + y^2}{2 \cdot x \cdot y \cdot (y+1)} = \frac{y^2 + 4}{2 \cdot x \cdot y \cdot (y+1)}.
\end{aligned}$$

Vježba 797

Izračunaj: $\frac{x+2}{x \cdot y^2 + x \cdot y} - \frac{1-y}{y^2 + y} + \frac{y-2 \cdot x}{2 \cdot x \cdot y + 2 \cdot x}$.

Rezultat: $\frac{y^2 + 4}{2 \cdot x \cdot y \cdot (y+1)}$.

Zadatak 798 (Luka, ekonomska škola)

Izračunaj: $\frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{x^2+x-2}$.

Rješenje 798

Ponovimo!

$$a^1 = a, \quad a^n : a^m = a^{n-m}, \quad a^n \cdot a^m = a^{n+m}.$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1. inačica

$$\begin{aligned} \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{x^2+x-2} &= \left[\begin{array}{l} \text{svaki nazivnik,} \\ \text{ako je moguće,} \\ \text{rastavimo na faktore} \end{array} \right] = \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{x^2+2 \cdot x-x-2} = \\ &= \left[\begin{array}{l} \text{u zadnjem nazivniku primijenimo} \\ \text{metodu grupiranja} \end{array} \right] = \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{(x^2+2 \cdot x)+(-x-2)} = \\ &= \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{x \cdot (x+2) - (x+2)} = \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{(x+2) \cdot (x-1)} = \\ &= \left[\begin{array}{l} \text{nađemo najmanji} \\ \text{zajednički nazivnik} \end{array} \right] = \frac{x+2-x-1-1}{(x+2) \cdot (x-1)} = \frac{x+2-x-1-1}{(x+2) \cdot (x-1)} = \frac{2 \cdot x}{(x+2) \cdot (x-1)}. \end{aligned}$$

2. inačica

$$\begin{aligned} \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{x^2+x-2} &= \left[\begin{array}{l} \text{svaki nazivnik,} \\ \text{ako je moguće,} \\ \text{rastavimo na faktore} \end{array} \right] = \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{x^2-1+x-1} = \\ &= \left[\begin{array}{l} \text{u zadnjem nazivniku primijenimo} \\ \text{metodu grupiranja} \end{array} \right] = \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{(x^2-1)+(x-1)} = \\ &= \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{(x-1) \cdot (x+1) + (x-1)} = \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{(x-1) \cdot (x+1+1)} = \\ &= \frac{1}{x-1} + \frac{1}{x+2} - \frac{1}{(x-1) \cdot (x+2)} = \left[\begin{array}{l} \text{nađemo najmanji} \\ \text{zajednički nazivnik} \end{array} \right] = \frac{x+2-x-1-1}{(x+2) \cdot (x-1)} = \frac{x+2-x-1-1}{(x+2) \cdot (x-1)} = \\ &= \frac{2 \cdot x}{(x+2) \cdot (x-1)}. \end{aligned}$$

Vježba 798

Izračunaj: $\frac{1}{x+2} - \frac{1}{1-x} + \frac{1}{x+2} - \frac{1}{x^2+x-2}$.

Rezultat: $\frac{2 \cdot x}{(x+2) \cdot (x-1)}$.

Zadatak 799 (Dora, gimnazija)

Rastavi izraz $\frac{a}{b} + \frac{b}{a}$ na umnožak dva faktora, ako je jedan od njih $\frac{a-b}{b-a}$, $a \neq b$, $a \neq 0$, $b \neq 0$.

Rješenje 799

Ponovimo!

$$\frac{a-c}{b-d} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad \frac{a+c}{b+d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad a^1 = a, \quad a^n \cdot a^m = a^{n+m},$$

$$\frac{a}{b} \cdot \frac{b}{a} = 1, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Neka je x traženi faktor. Tada vrijedi:

$$\left(\frac{a-b}{b-a}\right) \cdot x = \frac{a}{b} + \frac{b}{a} \Rightarrow \frac{a^2-b^2}{a \cdot b} \cdot x = \frac{a^2+b^2}{a \cdot b} \Rightarrow \frac{a^2-b^2}{a \cdot b} \cdot x = \frac{a^2+b^2}{a \cdot b} / \frac{a \cdot b}{a^2-b^2} \Rightarrow$$

$$\Rightarrow x = \frac{a^2+b^2}{a \cdot b} \cdot \frac{a \cdot b}{a^2-b^2} \Rightarrow x = \frac{a^2+b^2}{a \cdot b} \cdot \frac{a \cdot b}{a^2-b^2} \Rightarrow$$

$$\Rightarrow x = \frac{a^2+b^2}{1} \cdot \frac{1}{a^2-b^2} \Rightarrow x = \frac{a^2+b^2}{a^2-b^2}.$$

Vježba 799

Rastavi izraz $\frac{a}{b} - \frac{b}{a}$ na umnožak dva faktora, ako je jedan od njih $\frac{a+b}{b+a}$, $a \neq b$, $a \neq 0$, $b \neq 0$.

Rezultat: $\frac{a^2-b^2}{a^2+b^2}$.

Zadatak 800 (Dora, gimnazija)

Pojednostavni izraz $\frac{a^n - a^{-n}}{a^n + a^{-n} - 2}$.

A. $\frac{a^n+1}{a^n-1}$ B. $\frac{a^n-1}{a^n+1}$ C. $\frac{a^n}{a^n+1}$ D. $\frac{a^n-1}{a^n}$

Rješenje 800

Ponovimo!

$$a^{-n} = \frac{1}{a^n}, \quad n = \frac{n}{1}, \quad \frac{a-c}{b-d} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad a^1 = a, \quad a^n \cdot a^m = a^{n+m},$$

$$a^2 - b^2 = (a-b) \cdot (a+b), \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}, \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2,$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i

jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n}, \quad n \neq 0, \quad n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1. inačica

$$\begin{aligned} \frac{a^n - a^{-n}}{a^n + a^{-n} - 2} &= \frac{a^n - \frac{1}{a^n}}{a^n + \frac{1}{a^n} - 2} = \frac{\frac{a^n}{1} - \frac{1}{a^n}}{1 + \frac{1}{a^n} - \frac{2}{1}} = \frac{\frac{(a^n)^2 - 1}{a^n}}{\frac{(a^n)^2 + 1 - 2 \cdot a^n}{a^n}} = \frac{\frac{(a^n)^2 - 1}{a^n}}{\frac{(a^n)^2 - 2 \cdot a^n + 1}{a^n}} = \\ &= \frac{\frac{(a^n)^2 - 1}{a^n}}{\frac{(a^n)^2 - 2 \cdot a^n + 1}{a^n}} = \frac{(a^n)^2 - 1}{(a^n)^2 - 2 \cdot a^n + 1} = \frac{(a^n)^2 - 1}{(a^n)^2 - 2 \cdot a^n + 1} = \frac{(a^n - 1) \cdot (a^n + 1)}{(a^n - 1)^2} = \\ &= \frac{(a^n - 1) \cdot (a^n + 1)}{(a^n - 1)^2} = \frac{a^n + 1}{a^n - 1}. \end{aligned}$$

Odgovor je pod A.

2. inačica

$$\begin{aligned} \frac{a^n - a^{-n}}{a^n + a^{-n} - 2} &= \frac{a^n - \frac{1}{a^n}}{a^n + \frac{1}{a^n} - 2} = \left[\begin{array}{l} \text{proširimo} \\ \text{razlomak} \end{array} \right] = \frac{a^n - \frac{1}{a^n}}{a^n + \frac{1}{a^n} - 2} \cdot \frac{a^n}{a^n} = \\ &= \frac{a^n \cdot \left(a^n - \frac{1}{a^n} \right)}{a^n \cdot \left(a^n + \frac{1}{a^n} - 2 \right)} = \frac{(a^n)^2 - 1}{(a^n)^2 + 1 - 2 \cdot a^n} = \frac{(a^n)^2 - 1}{(a^n)^2 - 2 \cdot a^n + 1} = \frac{(a^n - 1) \cdot (a^n + 1)}{(a^n - 1)^2} = \\ &= \frac{(a^n - 1) \cdot (a^n + 1)}{(a^n - 1)^2} = \frac{a^n + 1}{a^n - 1}. \end{aligned}$$

Odgovor je pod A.

Vježba 800

Pojednostavni izraz $\frac{a^n + a^{-n} - 2}{a^n - a^{-n}}$.

A. $\frac{a^n + 1}{a^n - 1}$ B. $\frac{a^n - 1}{a^n + 1}$ C. $\frac{a^n}{a^n + 1}$ D. $\frac{a^n - 1}{a^n}$

Rezultat: B.

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