

Zadatak 761 (Ivana, srednja škola)

Izračunaj: $\frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} + \frac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} + \sqrt{x-y}}$.

Rješenje 761

Ponovimo!

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \quad (a-b) \cdot (a+b) = a^2 - b^2, \quad (\sqrt{a})^2 = a, \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}.$$

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2, \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2, \quad \frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

$$a^1 = a, \quad a^n \cdot a^m = a^{n+m}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n}, \quad n \neq 0, \quad n \neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

1. inačica

$$\begin{aligned} & \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} + \frac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} + \sqrt{x-y}} = \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \\ & \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \cdot \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} + \sqrt{x-y}} + \frac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} + \sqrt{x-y}} \cdot \frac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} = \\ & = \frac{(\sqrt{x+y} + \sqrt{x-y})^2}{(\sqrt{x+y})^2 - (\sqrt{x-y})^2} + \frac{(\sqrt{x+y} - \sqrt{x-y})^2}{(\sqrt{x+y})^2 - (\sqrt{x-y})^2} = \\ & = \frac{(\sqrt{x+y})^2 + 2 \cdot \sqrt{x+y} \cdot \sqrt{x-y} + (\sqrt{x-y})^2}{x+y - (x-y)} + \frac{(\sqrt{x+y})^2 - 2 \cdot \sqrt{x+y} \cdot \sqrt{x-y} + (\sqrt{x-y})^2}{x+y - (x-y)} = \\ & = \frac{x+y + 2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y}{x+y - x+y} + \frac{x+y - 2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y}{x+y - x+y} = \\ & = \frac{x+y + 2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y}{x+y - x+y} + \frac{x+y - 2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y}{x+y - x+y} = \\ & = \frac{x + 2 \cdot \sqrt{(x+y) \cdot (x-y)} + x}{y+y} + \frac{x - 2 \cdot \sqrt{(x+y) \cdot (x-y)} + x}{y+y} = \end{aligned}$$

$$\begin{aligned}
&= \frac{2 \cdot x + 2 \cdot \sqrt{(x+y) \cdot (x-y)}}{2 \cdot y} + \frac{2 \cdot x - 2 \cdot \sqrt{(x+y) \cdot (x-y)}}{2 \cdot y} = \\
&= \frac{2 \cdot x + 2 \cdot \sqrt{(x+y) \cdot (x-y)} + 2 \cdot x - 2 \cdot \sqrt{(x+y) \cdot (x-y)}}{2 \cdot y} = \\
&= \frac{2 \cdot x + 2 \cdot \sqrt{(x+y) \cdot (x-y)} + 2 \cdot x - 2 \cdot \sqrt{(x+y) \cdot (x-y)}}{2 \cdot y} = \frac{2 \cdot x + 2 \cdot x}{2 \cdot y} = \frac{4 \cdot x}{2 \cdot y} = \frac{4 \cdot x}{2 \cdot y} = \frac{2 \cdot x}{y}.
\end{aligned}$$

2. inačica

$$\begin{aligned}
&\frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} + \frac{\sqrt{x+y} - \sqrt{x-y}}{\sqrt{x+y} + \sqrt{x-y}} = \\
&= \frac{(\sqrt{x+y} + \sqrt{x-y})^2 + (\sqrt{x+y} - \sqrt{x-y})^2}{(\sqrt{x+y} - \sqrt{x-y}) \cdot (\sqrt{x+y} + \sqrt{x-y})} = \\
&= \frac{(\sqrt{x+y})^2 + 2 \cdot \sqrt{x+y} \cdot \sqrt{x-y} + (\sqrt{x-y})^2 + (\sqrt{x+y})^2 - 2 \cdot \sqrt{x+y} \cdot \sqrt{x-y} + (\sqrt{x-y})^2}{(\sqrt{x+y})^2 - (\sqrt{x-y})^2} = \\
&= \frac{x+y+2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y+x+y-2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y}{x+y-(x-y)} = \\
&= \frac{x+y+2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y+x+y-2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y}{x+y-x+y} = \\
&= \frac{x+y+2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y+x+y-2 \cdot \sqrt{(x+y) \cdot (x-y)} + x-y}{x+y-x+y} = \frac{x+x+x+x}{y+y} = \\
&= \frac{4 \cdot x}{2 \cdot y} = \frac{4 \cdot x}{2 \cdot y} = \frac{2 \cdot x}{y}.
\end{aligned}$$

Vježba 761

Izračunaj: $\frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} - \frac{\sqrt{x-y} - \sqrt{x+y}}{\sqrt{x+y} + \sqrt{x-y}}$.

Rezultat: $\frac{2 \cdot x}{y}$.

Zadatak 762 (Tihomir, srednja škola)

Izračunaj: $(a+b+c)^2 - (b+c-a)^2 + (a+b-c)^2 - (a+c-b)^2$.

A. $8 \cdot a \cdot b$ B. $6 \cdot a \cdot b$ C. $8 \cdot a \cdot c$ D. $8 \cdot b \cdot c$

Rješenje 762

Ponovimo!

$$(x+y+z)^2 = x^2 + y^2 + z^2 + 2 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y \cdot z, \quad x^2 - y^2 = (x-y) \cdot (x+y).$$

$$(-a)^2 = a^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1. inačica

$$\begin{aligned} & (a+b+c)^2 - (b+c-a)^2 + (a+b-c)^2 - (a+c-b)^2 = \\ & = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c - \left(b^2 + c^2 + (-a)^2 + 2 \cdot b \cdot c + 2 \cdot b \cdot (-a) + 2 \cdot c \cdot (-a) \right) + \\ & \quad + a^2 + b^2 + (-c)^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot (-c) + 2 \cdot b \cdot (-c) - \\ & \quad - \left(a^2 + c^2 + (-b)^2 + 2 \cdot a \cdot c + 2 \cdot a \cdot (-b) + 2 \cdot c \cdot (-b) \right) = \\ & = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c - \left(b^2 + c^2 + a^2 + 2 \cdot b \cdot c - 2 \cdot b \cdot a - 2 \cdot c \cdot a \right) + \\ & \quad + a^2 + b^2 + c^2 + 2 \cdot a \cdot b - 2 \cdot a \cdot c - 2 \cdot b \cdot c - \\ & \quad - \left(a^2 + c^2 + b^2 + 2 \cdot a \cdot c - 2 \cdot a \cdot b - 2 \cdot c \cdot b \right) = \\ & = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c - b^2 - c^2 - a^2 - 2 \cdot b \cdot c + 2 \cdot b \cdot a + 2 \cdot c \cdot a + \\ & \quad + a^2 + b^2 + c^2 + 2 \cdot a \cdot b - 2 \cdot a \cdot c - 2 \cdot b \cdot c - \\ & \quad - a^2 - c^2 - b^2 - 2 \cdot a \cdot c + 2 \cdot a \cdot b + 2 \cdot c \cdot b = \\ & = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c - b^2 - c^2 - a^2 - 2 \cdot b \cdot c + 2 \cdot b \cdot a + 2 \cdot c \cdot a + \\ & \quad + a^2 + b^2 + c^2 + 2 \cdot a \cdot b - 2 \cdot a \cdot c - 2 \cdot b \cdot c - \\ & \quad - a^2 - c^2 - b^2 - 2 \cdot a \cdot c + 2 \cdot a \cdot b + 2 \cdot c \cdot b = \\ & = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c - b^2 - c^2 - a^2 - 2 \cdot b \cdot c + 2 \cdot b \cdot a + 2 \cdot c \cdot a + \\ & \quad + a^2 + b^2 + c^2 + 2 \cdot a \cdot b - 2 \cdot a \cdot c - 2 \cdot b \cdot c - \\ & \quad - a^2 - c^2 - b^2 - 2 \cdot a \cdot c + 2 \cdot a \cdot b + 2 \cdot c \cdot b = \\ & = 2 \cdot a \cdot b + 2 \cdot a \cdot b + 2 \cdot a \cdot b + 2 \cdot a \cdot b = 8 \cdot a \cdot b. \end{aligned}$$

Odgovor je pod A.

2. inačica

$$\begin{aligned} & (a+b+c)^2 - (b+c-a)^2 + (a+b-c)^2 - (a+c-b)^2 = \\ & = \left((a+b+c)^2 - (b+c-a)^2 \right) + \left((a+b-c)^2 - (a+c-b)^2 \right) = \\ & = \left((a+b+c) - (b+c-a) \right) \cdot \left((a+b+c) + (b+c-a) \right) + \left((a+b-c) - (a+c-b) \right) \cdot \left((a+b-c) + (a+c-b) \right) = \\ & = (a+b+c-b-c+a) \cdot (a+b+c+b+c-a) + (a+b-c-a-c+b) \cdot (a+b-c+a+c-b) = \\ & = (a+b+c-b-c+a) \cdot (a+b+c+b+c-a) + (a+b-c-a-c+b) \cdot (a+b-c+a+c-b) = \\ & = (a+a) \cdot (b+c+b+c) + (b-c-c+b) \cdot (a+a) = 2 \cdot a \cdot (2 \cdot b + 2 \cdot c) + (2 \cdot b - 2 \cdot c) \cdot 2 \cdot a = \\ & = 2 \cdot a \cdot (2 \cdot b + 2 \cdot c + 2 \cdot b - 2 \cdot c) = 2 \cdot a \cdot (2 \cdot b + 2 \cdot c + 2 \cdot b - 2 \cdot c) = 2 \cdot a \cdot (2 \cdot b + 2 \cdot b) = \\ & = 2 \cdot a \cdot 4 \cdot b = 8 \cdot a \cdot b. \end{aligned}$$

Odgovor je pod A.

Vježba 762

Izračunaj: $(a+b+c)^2 - (a-b-c)^2 + (c-a-b)^2 - (b-a-c)^2$.

A. $8 \cdot a \cdot b$ B. $6 \cdot a \cdot b$ C. $8 \cdot a \cdot c$ D. $8 \cdot b \cdot c$

Rezultat: A.

Zadatak 763 (Ivan, gimnazija)

Dokaži da iz $(a-b)^2 + (c-a)^2 + (b-c)^2 = (a+b-2 \cdot c)^2 + (a-2 \cdot b+c)^2 + (b+c-2 \cdot a)^2$ slijedi $a = b = c$.

Rješenje 763

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2, \quad (x+y+z)^2 = x^2 + y^2 + z^2 + 2 \cdot x \cdot y + 2 \cdot x \cdot z + 2 \cdot y \cdot z.$$

$$(-a)^2 = a^2, \quad (a \cdot b)^n = a^n \cdot b^n.$$

$$\begin{aligned} (a-b)^2 + (c-a)^2 + (b-c)^2 &= (a+b-2 \cdot c)^2 + (a-2 \cdot b+c)^2 + (b+c-2 \cdot a)^2 \Rightarrow \\ &\Rightarrow a^2 - 2 \cdot a \cdot b + b^2 + c^2 - 2 \cdot c \cdot a + a^2 + b^2 - 2 \cdot b \cdot c + c^2 = \\ &= a^2 + b^2 + (-2 \cdot c)^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot (-2 \cdot c) + 2 \cdot b \cdot (-2 \cdot c) + \\ &+ a^2 + (-2 \cdot b)^2 + c^2 + 2 \cdot a \cdot (-2 \cdot b) + 2 \cdot a \cdot c + 2 \cdot (-2 \cdot b) \cdot c + \\ &+ b^2 + c^2 + (-2 \cdot a)^2 + 2 \cdot b \cdot c + 2 \cdot b \cdot (-2 \cdot a) + 2 \cdot c \cdot (-2 \cdot a) \Rightarrow \\ &\Rightarrow a^2 - 2 \cdot a \cdot b + b^2 + c^2 - 2 \cdot c \cdot a + a^2 + b^2 - 2 \cdot b \cdot c + c^2 = \\ &= a^2 + b^2 + 4 \cdot c^2 + 2 \cdot a \cdot b - 4 \cdot a \cdot c - 4 \cdot b \cdot c + a^2 + 4 \cdot b^2 + c^2 - 4 \cdot a \cdot b + 2 \cdot a \cdot c - 4 \cdot b \cdot c + \\ &+ b^2 + c^2 + 4 \cdot a^2 + 2 \cdot b \cdot c - 4 \cdot b \cdot a - 4 \cdot c \cdot a \Rightarrow \\ &\Rightarrow a^2 - 2 \cdot a \cdot b + b^2 + c^2 - 2 \cdot c \cdot a + a^2 + b^2 - 2 \cdot b \cdot c + c^2 = \\ &= a^2 + b^2 + 4 \cdot c^2 + 2 \cdot a \cdot b - 4 \cdot a \cdot c - 4 \cdot b \cdot c + a^2 + 4 \cdot b^2 + c^2 - 4 \cdot a \cdot b + 2 \cdot a \cdot c - 4 \cdot b \cdot c + \\ &+ b^2 + c^2 + 4 \cdot a^2 + 2 \cdot b \cdot c - 4 \cdot b \cdot a - 4 \cdot c \cdot a \Rightarrow \\ &\Rightarrow 0 = 4 \cdot c^2 + 4 \cdot b^2 - 4 \cdot b \cdot c + 4 \cdot a^2 - 4 \cdot b \cdot a - 4 \cdot c \cdot a \Rightarrow \\ &\Rightarrow 4 \cdot c^2 + 4 \cdot b^2 - 4 \cdot b \cdot c + 4 \cdot a^2 - 4 \cdot b \cdot a - 4 \cdot c \cdot a = 0 \Rightarrow \\ &\Rightarrow 4 \cdot c^2 + 4 \cdot b^2 - 4 \cdot b \cdot c + 4 \cdot a^2 - 4 \cdot b \cdot a - 4 \cdot c \cdot a = 0 \quad /: 2 \Rightarrow \\ &\Rightarrow 2 \cdot c^2 + 2 \cdot b^2 - 2 \cdot b \cdot c + 2 \cdot a^2 - 2 \cdot b \cdot a - 2 \cdot c \cdot a = 0 \Rightarrow \\ &\Rightarrow a^2 - 2 \cdot a \cdot b + b^2 + a^2 - 2 \cdot a \cdot c + c^2 + b^2 - 2 \cdot b \cdot c + c^2 = 0 \Rightarrow \\ &\Rightarrow (a^2 - 2 \cdot a \cdot b + b^2) + (a^2 - 2 \cdot a \cdot c + c^2) + (b^2 - 2 \cdot b \cdot c + c^2) = 0 \Rightarrow \end{aligned}$$

$$\Rightarrow (a-b)^2 + (a-c)^2 + (b-c)^2 = 0 \Rightarrow \left. \begin{array}{l} a-b=0 \\ a-c=0 \\ b-c=0 \end{array} \right\} \Rightarrow \left. \begin{array}{l} a=b \\ a=c \\ b=c \end{array} \right\} \Rightarrow a=b=c.$$

Vježba 763

Dokaži da iz $(b-a)^2 + (a-c)^2 + (c-b)^2 = (a+b-2\cdot c)^2 + (a-2\cdot b+c)^2 + (b+c-2\cdot a)^2$ slijedi $a = b = c$.

Rezultat: Dokaz analogan.

Zadatak 764 (Pax, gimnazija)

Dokaži da za realne brojeve $a, b > 0$ vrijedi $\frac{a}{b} + \frac{b}{a} \geq 2$.

Rješenje 764

Ponovimo!

$$a \geq b, c > 0 \Rightarrow a \cdot c \geq b \cdot c, \quad a^2 \geq 0, \quad a \in \mathbb{R}, \quad (a-b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

$$(\sqrt{a})^2 = a, \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}, \quad a^1 = a, \quad a^n \cdot a^m = a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

1. inačica

Polazimo od očigledne nejednakosti.

$$\left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}} \right)^2 \geq 0 \Rightarrow \left(\sqrt{\frac{a}{b}} \right)^2 - 2 \cdot \sqrt{\frac{a}{b}} \cdot \sqrt{\frac{b}{a}} + \left(\sqrt{\frac{b}{a}} \right)^2 \geq 0 \Rightarrow \frac{a}{b} - 2 \cdot \sqrt{\frac{a}{b} \cdot \frac{b}{a}} + \frac{b}{a} \geq 0 \Rightarrow$$

$$\Rightarrow \frac{a}{b} - 2 \cdot \sqrt{\frac{a}{b} \cdot \frac{b}{a}} + \frac{b}{a} \geq 0 \Rightarrow \frac{a}{b} - 2 \cdot \sqrt{1} + \frac{b}{a} \geq 0 \Rightarrow \frac{a}{b} - 2 \cdot 1 + \frac{b}{a} \geq 0 \Rightarrow \frac{a}{b} - 2 + \frac{b}{a} \geq 0 \Rightarrow$$

$$\Rightarrow \frac{a}{b} + \frac{b}{a} \geq 2.$$

2. inačica

Polazimo od zadane nejednakosti.

$$\frac{a}{b} + \frac{b}{a} \geq 2 \Rightarrow \frac{a}{b} + \frac{b}{a} \geq 2 \cdot \frac{a \cdot b}{a \cdot b} \Rightarrow a^2 + b^2 \geq 2 \cdot a \cdot b \Rightarrow a^2 - 2 \cdot a \cdot b + b^2 \geq 0 \Rightarrow (a-b)^2 \geq 0.$$

Dobili smo točnu nejednakost, znači da je početna tvrdnja istinita.

Vježba 764

Dokaži da za realne brojeve $a, b > 0$ vrijedi $\frac{1}{2} \cdot \left(\frac{a}{b} + \frac{b}{a} \right) \geq 1$.

Rezultat: Dokaz analogan.

Zadatak 765 (Pax, gimnazija)

Rastavi na faktore: $(a \cdot b + a \cdot c + b \cdot c) \cdot (a + b + c) - a \cdot b \cdot c$.

Rješenje 765

Ponovimo!

$$a^1 = a, \quad a^n \cdot a^m = a^{n+m}, \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Množenje zagrada

$$\begin{aligned} (a+b) \cdot (c+d) &= a \cdot c + a \cdot d + b \cdot c + b \cdot d. \\ (a \cdot b + a \cdot c + b \cdot c) \cdot (a+b+c) - a \cdot b \cdot c &= \\ &= a^2 \cdot b + a \cdot b^2 + a \cdot b \cdot c + a^2 \cdot c + a \cdot b \cdot c + a \cdot c^2 + a \cdot b \cdot c + b^2 \cdot c + b \cdot c^2 - a \cdot b \cdot c = \\ &= a^2 \cdot b + a \cdot b^2 + a \cdot b \cdot c + a^2 \cdot c + a \cdot b \cdot c + a \cdot c^2 + a \cdot b \cdot c + b^2 \cdot c + b \cdot c^2 - a \cdot b \cdot c = \\ &= a^2 \cdot b + a \cdot b^2 + a^2 \cdot c + a \cdot b \cdot c + a \cdot c^2 + a \cdot b \cdot c + b^2 \cdot c + b \cdot c^2 = \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] = \\ &= (a^2 \cdot b + a^2 \cdot c) + (a \cdot b^2 + a \cdot b \cdot c) + (a \cdot b \cdot c + a \cdot c^2) + (b^2 \cdot c + b \cdot c^2) = \\ &= a^2 \cdot (b+c) + a \cdot b \cdot (b+c) + a \cdot c \cdot (b+c) + b \cdot c \cdot (b+c) = (b+c) \cdot (a^2 + a \cdot b + a \cdot c + b \cdot c) = \\ &= \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] = (b+c) \cdot \left((a^2 + a \cdot b) + (a \cdot c + b \cdot c) \right) = (b+c) \cdot (a \cdot (a+b) + c \cdot (a+b)) = \\ &= (b+c) \cdot (a+b) \cdot (a+c). \end{aligned}$$

Vježba 765

Rastavi na faktore: $(a+b+c) \cdot (a \cdot b + a \cdot c + b \cdot c) - a \cdot b \cdot c$.

Rezultat: $(b+c) \cdot (a+b) \cdot (a+c)$.

Zadatak 766 (Pax, gimnazija)

Rastavi na faktore: $a^2 \cdot b + a \cdot b^2 + a^2 \cdot c + a \cdot c^2 + b^2 \cdot c + b \cdot c^2 + 3 \cdot a \cdot b \cdot c$.

Rješenje 766

Ponovimo!

$$a^1 = a, \quad a^n : a^m = a^{n-m}, \quad a^n \cdot a^m = a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$\begin{aligned} a \cdot (b+c) &= a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c). \\ a^2 \cdot b + a \cdot b^2 + a^2 \cdot c + a \cdot c^2 + b^2 \cdot c + b \cdot c^2 + 3 \cdot a \cdot b \cdot c &= \\ &= a^2 \cdot b + a \cdot b^2 + a \cdot b \cdot c + a^2 \cdot c + a \cdot c^2 + a \cdot b \cdot c + b^2 \cdot c + b \cdot c^2 + a \cdot b \cdot c = \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] = \\ &= (a^2 \cdot b + a \cdot b^2 + a \cdot b \cdot c) + (a^2 \cdot c + a \cdot c^2 + a \cdot b \cdot c) + (b^2 \cdot c + b \cdot c^2 + a \cdot b \cdot c) = \\ &= a \cdot b \cdot (a+b+c) + a \cdot c \cdot (a+c+b) + b \cdot c \cdot (b+c+a) = \\ &= a \cdot b \cdot (a+b+c) + a \cdot c \cdot (a+b+c) + b \cdot c \cdot (a+b+c) = (a+b+c) \cdot (a \cdot b + a \cdot c + b \cdot c). \end{aligned}$$

Vježba 766

Rastavi na faktore: $a \cdot b \cdot (a+b) + a \cdot c \cdot (a+c) + b \cdot c \cdot (b+c) + 3 \cdot a \cdot b \cdot c$.

Rezultat: $(a+b+c) \cdot (a \cdot b + a \cdot c + b \cdot c)$.

Zadatak 767 (Pax, gimnazija)

Rastavi na faktore: $a^4 + b^4 + 2 \cdot a^3 \cdot b + 2 \cdot a^2 \cdot b^2 + 2 \cdot a \cdot b^3$.

Rješenje 767

Ponovimo!

$$a^1 = a, \quad a^n : a^m = a^{n-m}, \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$\begin{aligned} a \cdot (b+c) &= a \cdot b + a \cdot c, & a \cdot b + a \cdot c &= a \cdot (b+c). \\ a^4 + b^4 + 2 \cdot a^3 \cdot b + 2 \cdot a^2 \cdot b^2 + 2 \cdot a \cdot b^3 &= \\ = a^4 + a^2 \cdot b^2 + 2 \cdot a^3 \cdot b + 2 \cdot a \cdot b^3 + b^4 + a^2 \cdot b^2 &= \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] = \\ = (a^4 + a^2 \cdot b^2) + (2 \cdot a^3 \cdot b + 2 \cdot a \cdot b^3) + (b^4 + a^2 \cdot b^2) &= \\ = a^2 \cdot (a^2 + b^2) + 2 \cdot a \cdot b \cdot (a^2 + b^2) + b^2 \cdot (b^2 + a^2) &= \\ = a^2 \cdot (a^2 + b^2) + 2 \cdot a \cdot b \cdot (a^2 + b^2) + b^2 \cdot (a^2 + b^2) &= \\ = (a^2 + b^2) \cdot (a^2 + 2 \cdot a \cdot b + b^2) = (a^2 + b^2) \cdot (a+b)^2. \end{aligned}$$

Vježba 767

Rastavi na faktore: $a^4 + b^4 + 2 \cdot (a^3 \cdot b + a^2 \cdot b^2 + a \cdot b^3)$.

Rezultat: $(a^2 + b^2) \cdot (a+b)^2$.

Zadatak 768 (Tonka, gimnazija)

Provedite računsku operaciju $\frac{x^2}{2-x} + x + 2$ te napišite rezultat u obliku do kraja skraćenoga razlomka.

Rješenje 768

Ponovimo!

$$n = \frac{n}{1}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$\begin{aligned} \frac{x^2}{2-x} + x + 2 &= \frac{x^2}{2-x} + \frac{x+2}{1} = \frac{x^2 + (x+2) \cdot (2-x)}{2-x} = \frac{x^2 + (2+x) \cdot (2-x)}{2-x} = \\ &= \frac{x^2 + 2^2 - x^2}{2-x} = \frac{x^2 + 4 - x^2}{2-x} = \frac{x^2 + 4 - x^2}{2-x} = \frac{4}{2-x}. \end{aligned}$$

Vježba 768

Provedite računsku operaciju $x + 2 - \frac{x^2}{x-2}$ te napišite rezultat u obliku do kraja skraćenoga razlomka.

Rezultat: $\frac{4}{2-x}$.

Zadatak 769 (Trnoružica, gimnazija)

Pomnožite izraze $a + 3$ i $a - 2$ i od umnoška oduzmite kvadrat broja a . Dobiveni izraz pojednostavnite do kraja.

Rješenje 769

Ponovimo!

$$a^1 = a, \quad a^n \cdot a^m = a^{n+m}.$$

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

Zadani tekst preoblikujemo u matematički izraz.

Pomnožite izraze $a + 3$ i $a - 2$...

$$(a+3) \cdot (a-2)$$

... i od umnoška oduzmite kvadrat broja a .

$$\begin{aligned} (a+3) \cdot (a-2) - a^2 &= a^2 - 2 \cdot a + 3 \cdot a - 6 - a^2 = a^2 - 2 \cdot a + 3 \cdot a - 6 - a^2 = \\ &= -2 \cdot a + 3 \cdot a - 6 = a - 6. \end{aligned}$$

Vježba 769

Pomnožite izraze $a + 2$ i $a - 3$ i od umnoška oduzmite kvadrat broja a . Dobiveni izraz pojednostavnite do kraja.

Rezultat: $-a - 6$.

Zadatak 770 (4B, TUPŠ)

Izračunajmo: $\frac{7}{2 \cdot a - 4} - \frac{3}{a + 2} - \frac{12}{a^2 - 4}$.

Rješenje 770

Ponovimo!

$$a^2 - b^2 = (a-b) \cdot (a+b), \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\frac{7}{2 \cdot a - 4} - \frac{3}{a + 2} - \frac{12}{a^2 - 4} = \left[\begin{array}{l} \text{nazivnike rastavimo} \\ \text{na faktore} \end{array} \right] =$$

$$\begin{aligned}
&= \frac{7}{2 \cdot (a-2)} - \frac{3}{a+2} - \frac{12}{(a-2) \cdot (a+2)} = \left[\begin{array}{l} \text{nazivnike svedemo na} \\ \text{zajednički nazivnik} \end{array} \right] = \\
&= \frac{7 \cdot (a+2) - 3 \cdot 2 \cdot (a-2) - 12 \cdot 2}{2 \cdot (a-2) \cdot (a+2)} = \left[\begin{array}{l} \text{izvršimo naznačene} \\ \text{operacije u brojniku} \end{array} \right] = \\
&= \frac{7 \cdot a + 14 - 6 \cdot (a-2) - 24}{2 \cdot (a-2) \cdot (a+2)} = \frac{7 \cdot a + 14 - 6 \cdot a + 12 - 24}{2 \cdot (a-2) \cdot (a+2)} = \frac{a+2}{2 \cdot (a-2) \cdot (a+2)} = \\
&= \frac{a+2}{2 \cdot (a-2) \cdot (a+2)} = \frac{1}{2 \cdot (a-2)}.
\end{aligned}$$

Vježba 770

Izračunajmo: $\frac{7}{2 \cdot a - 4} - \frac{3}{a+2} + \frac{12}{4-a^2}$.

Rezultat: $\frac{1}{2 \cdot (a-2)}$.

Zadatak 771 (4B, TUPŠ)

Izračunajmo: $\frac{1}{x+2} - \frac{1}{2-x} - \frac{8}{x^3-4 \cdot x}$.

Rješenje 771

Ponovimo!

$$a^1 = a, \quad a^n : a^m = a^{n-m}, \quad a^2 - b^2 = (a-b) \cdot (a+b), \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned}
&\frac{1}{x+2} - \frac{1}{2-x} - \frac{8}{x^3-4 \cdot x} = \left[\begin{array}{l} \text{nazivnike rastavimo} \\ \text{na faktore} \end{array} \right] = \frac{1}{x+2} - \frac{1}{-(x-2)} - \frac{8}{x^3-4 \cdot x} = \\
&= \frac{1}{x+2} + \frac{1}{x-2} - \frac{8}{x^3-4 \cdot x} = \frac{1}{x+2} + \frac{1}{x-2} - \frac{8}{x \cdot (x^2-4)} = \frac{1}{x+2} + \frac{1}{x-2} - \frac{8}{x \cdot (x-2) \cdot (x+2)} = \\
&= \left[\begin{array}{l} \text{nazivnike svedemo na} \\ \text{zajednički nazivnik} \end{array} \right] = \frac{x \cdot (x-2) + x \cdot (x+2) - 8}{x \cdot (x-2) \cdot (x+2)} = \left[\begin{array}{l} \text{izvršimo naznačene} \\ \text{operacije u brojniku} \end{array} \right] = \\
&= \frac{x^2 - 2 \cdot x + x^2 + 2 \cdot x - 8}{x \cdot (x-2) \cdot (x+2)} = \frac{x^2 - 2 \cdot x + x^2 + 2 \cdot x - 8}{x \cdot (x-2) \cdot (x+2)} = \frac{x^2 + x^2 - 8}{x \cdot (x-2) \cdot (x+2)} = \\
&= \frac{2 \cdot x^2 - 8}{x \cdot (x-2) \cdot (x+2)} = \frac{2 \cdot (x^2 - 4)}{x \cdot (x-2) \cdot (x+2)} = \frac{2 \cdot (x-2) \cdot (x+2)}{x \cdot (x-2) \cdot (x+2)} = \frac{2 \cdot (x-2) \cdot (x+2)}{x \cdot (x-2) \cdot (x+2)} = \frac{2}{x}.
\end{aligned}$$

Vježba 771

Izračunajmo: $\frac{1}{x+2} + \frac{1}{x-2} - \frac{8}{x^3 - 4 \cdot x}$.

Rezultat: $\frac{2}{x}$.

Zadatak 772 (Goran, srednja škola)

Izračunajmo: $\left[\frac{(a+b)^3}{3 \cdot a \cdot b} - a - b \right]^n : \left[\frac{(a-b)^2}{a \cdot b} + 1 \right]^n$.

Rješenje 772

Ponovimo!

$$n = \frac{n}{1}, \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad a^1 = a, \quad a^n \cdot a^m = a^{n+m}.$$

$$(a+b)^3 = a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3, \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2, \quad a^n : b^n = (a:b)^n.$$

$$a^3 + b^3 = (a+b) \cdot (a^2 - a \cdot b + b^2), \quad \frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned} & \left[\frac{(a+b)^3}{3 \cdot a \cdot b} - a - b \right]^n : \left[\frac{(a-b)^2}{a \cdot b} + 1 \right]^n = \\ & = \left[\frac{(a+b)^3}{3 \cdot a \cdot b} - (a+b) \right]^n : \left[\frac{(a-b)^2}{a \cdot b} + 1 \right]^n = \left[\frac{(a+b)^3}{3 \cdot a \cdot b} - \frac{a+b}{1} \right]^n : \left[\frac{(a-b)^2}{a \cdot b} + \frac{1}{1} \right]^n = \\ & = \left[\frac{(a+b)^3 - 3 \cdot a \cdot b \cdot (a+b)}{3 \cdot a \cdot b} \right]^n : \left[\frac{(a-b)^2 + a \cdot b}{a \cdot b} \right]^n = \\ & = \left[\frac{a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3 - 3 \cdot a^2 \cdot b - 3 \cdot a \cdot b^2}{3 \cdot a \cdot b} \right]^n : \left[\frac{a^2 - 2 \cdot a \cdot b + b^2 + a \cdot b}{a \cdot b} \right]^n = \\ & = \left[\frac{a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3 - 3 \cdot a^2 \cdot b - 3 \cdot a \cdot b^2}{3 \cdot a \cdot b} \right]^n : \left[\frac{a^2 - 2 \cdot a \cdot b + b^2 + a \cdot b}{a \cdot b} \right]^n = \\ & = \left[\frac{a^3 + b^3}{3 \cdot a \cdot b} \right]^n : \left[\frac{a^2 - a \cdot b + b^2}{a \cdot b} \right]^n = \left[\frac{a^3 + b^3}{3 \cdot a \cdot b} : \frac{a^2 - a \cdot b + b^2}{a \cdot b} \right]^n = \end{aligned}$$

$$\begin{aligned}
&= \left[\frac{a^3 + b^3}{3 \cdot a \cdot b} \cdot \frac{a \cdot b}{a^2 - a \cdot b + b^2} \right]^n = \left[\frac{a^3 + b^3}{3 \cdot a \cdot b} \cdot \frac{a \cdot b}{a^2 - a \cdot b + b^2} \right]^n = \\
&= \left[\frac{a^3 + b^3}{3} \cdot \frac{1}{a^2 - a \cdot b + b^2} \right]^n = \left[\frac{(a+b) \cdot (a^2 - a \cdot b + b^2)}{3} \cdot \frac{1}{a^2 - a \cdot b + b^2} \right]^n = \\
&= \left[\frac{(a+b) \cdot (a^2 - a \cdot b + b^2)}{3} \cdot \frac{1}{a^2 - a \cdot b + b^2} \right]^n = \left[\frac{a+b}{3} \cdot \frac{1}{1} \right]^n = \left[\frac{a+b}{3} \right]^n.
\end{aligned}$$

Vježba 772

Izračunajmo: $\left[\frac{(a-b)^2}{a \cdot b} + 1 \right]^n : \left[\frac{(a+b)^3}{3 \cdot a \cdot b} - a - b \right]^n$.

Rezultat: $\left[\frac{3}{a+b} \right]^n$.

Zadatak 773 (Leon, srednja škola)

Izvedi operacije: $\left[\frac{m^2}{m^2 - n^2} - \frac{m^2 \cdot n}{m^2 + n^2} \cdot \left(\frac{m}{m \cdot n + n^2} + \frac{n}{m^2 + m \cdot n} \right) \right] : \frac{n}{m-n}$.

Rješenje 773

Ponovimo!

$$a^1 = a, \quad a^n : a^m = a^{n-m}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad a^n \cdot a^m = a^{n+m}.$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \quad a^2 - b^2 = (a-b) \cdot (a+b), \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

$$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\left[\frac{m^2}{m^2 - n^2} - \frac{m^2 \cdot n}{m^2 + n^2} \cdot \left(\frac{m}{m \cdot n + n^2} + \frac{n}{m^2 + m \cdot n} \right) \right] : \frac{n}{m-n} =$$

$$\left[\frac{m^2}{m^2 - n^2} - \frac{m^2 \cdot n}{m^2 + n^2} \cdot \left(\frac{m}{n \cdot (m+n)} + \frac{n}{m \cdot (m+n)} \right) \right] : \frac{n}{m-n} =$$

$$\begin{aligned}
&= \left[\frac{m^2}{m^2 - n^2} - \frac{m^2 \cdot n}{m^2 + n^2} \cdot \frac{m^2 + n^2}{n \cdot m \cdot (m+n)} \right] : \frac{n}{m-n} = \left[\frac{m^2}{m^2 - n^2} - \frac{m^2 \cdot n}{m^2 + n^2} \cdot \frac{m^2 + n^2}{n \cdot m \cdot (m+n)} \right] : \frac{n}{m-n} = \\
&= \left[\frac{m^2}{m^2 - n^2} - \frac{m}{1} \cdot \frac{1}{m+n} \right] : \frac{n}{m-n} = \left[\frac{m^2}{m^2 - n^2} - \frac{m}{m+n} \right] : \frac{n}{m-n} = \\
&= \left[\frac{m^2}{(m-n) \cdot (m+n)} - \frac{m}{m+n} \right] : \frac{n}{m-n} = \frac{m^2 - m \cdot (m-n)}{(m-n) \cdot (m+n)} : \frac{n}{m-n} = \\
&= \frac{m^2 - m^2 + m \cdot n}{(m-n) \cdot (m+n)} : \frac{n}{m-n} = \frac{m^2 - m^2 + m \cdot n}{(m-n) \cdot (m+n)} : \frac{n}{m-n} = \frac{m \cdot n}{(m-n) \cdot (m+n)} : \frac{n}{m-n} = \\
&= \frac{m \cdot n}{(m-n) \cdot (m+n)} \cdot \frac{m-n}{n} = \frac{m \cdot n}{(m-n) \cdot (m+n)} \cdot \frac{m-n}{n} = \frac{m}{m+n} \cdot \frac{1}{1} = \frac{m}{m+n}.
\end{aligned}$$

Vježba 773

Izvedi operacije: $\frac{n}{m-n} : \left[\frac{m^2}{m^2 - n^2} - \frac{m^2 \cdot n}{m^2 + n^2} \cdot \left(\frac{m}{m \cdot n + n^2} + \frac{n}{m^2 + m \cdot n} \right) \right]$.

Rezultat: $\frac{m+n}{m}$.

Zadatak 774 (4B, TUPŠ + Tonka ☺. gimnazija)

Pojednostavnite izraz: $\frac{(a^2 - b^2)^2}{(a+b)^2}$.

Rješenje 774

Ponovimo!

$$a^2 - b^2 = (a-b) \cdot (a+b) \quad , \quad (a \cdot b)^n = a^n \cdot a^m \quad , \quad \frac{a^n}{b^n} = \left(\frac{a}{b} \right)^n .$$

$$\frac{n}{1} = n.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

1. inačica

$$\frac{(a^2 - b^2)^2}{(a+b)^2} = \frac{((a-b) \cdot (a+b))^2}{(a+b)^2} = \frac{(a-b)^2 \cdot (a+b)^2}{(a+b)^2} = \frac{(a-b)^2 \cdot (a+b)^2}{(a+b)^2} = \frac{(a-b)^2}{1} = (a-b)^2.$$

2. inačica

$$\frac{(a^2 - b^2)^2}{(a+b)^2} = \left(\frac{a^2 - b^2}{a+b}\right)^2 = \left(\frac{(a-b) \cdot (a+b)}{a+b}\right)^2 = \left(\frac{(a-b) \cdot (a+b)}{a+b}\right)^2 =$$

$$= \left(\frac{a-b}{1}\right)^2 = (a-b)^2.$$

Vježba 774

Pojednostavnite izraz: $\frac{(a+b)^2}{(a^2 - b^2)^2}$.

Rezultat: $\frac{1}{(a-b)^2}$.

Zadatak 775 (4B, TUPŠ + Tonka ☺. gimnazija)

Za $x \neq -1$ izraz $\frac{x+2+\frac{1}{x}}{\frac{1}{x}+1}$ jednak je:

- A. $-x$ B. $\frac{1}{x+1}$ C. $x+1$ D. 1

Rješenje 775

Ponovimo!

$$n = \frac{n}{1}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad a^1 = a, \quad a^n \cdot a^m = a^{n+m}, \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c}.$$

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2, \quad \frac{1}{a} \cdot a = 1, \quad \frac{a^n}{a^m} = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n}, \quad n \neq 0, \quad n \neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

1. inačica

$$\frac{x+2+\frac{1}{x}}{\frac{1}{x}+1} = \frac{\frac{x+2}{x} + \frac{1}{x}}{\frac{1}{x} + \frac{1}{1}} = \frac{\frac{x \cdot (x+2) + 1}{x}}{\frac{1+x}{x}} = \frac{x^2 + 2 \cdot x + 1}{x} = \frac{x^2 + 2 \cdot x + 1}{\frac{1+x}{x}} = \frac{x^2 + 2 \cdot x + 1}{1+x} =$$

$$= \frac{(x+1)^2}{x+1} = \frac{(x+1)^2}{x+1} = \frac{x+1}{1} = x+1.$$

Odgovor je pod C.

2. inačica

$$\frac{x+2+\frac{1}{x}}{\frac{1}{x}+1} = \left[\begin{array}{l} \text{proširimo} \\ \text{razlomak sa } x \end{array} \right] = \frac{\left(x+2+\frac{1}{x}\right) \cdot x}{\left(\frac{1}{x}+1\right) \cdot x} = \frac{x^2+2 \cdot x+1}{1+x} = \frac{(x+1)^2}{x+1} =$$

$$= \frac{(x+1)^2}{x+1} = \frac{x+1}{1} = x+1.$$

Odgovor je pod C.

Vježba 775

Za $x \neq -1$ izraz $\frac{x+\frac{1}{x}+2}{1+\frac{1}{x}}$ jednak je:

- A. $-x$ B. $\frac{1}{x+1}$ C. $x+1$ D. 1

Rezultat: C.

Zadatak 776 (4B, TUPŠ + Tonka ☺. gimnazija)

Reducirani oblik izraza $\frac{x^2-1}{x^2-y^2} : \frac{x-1}{x-y}$ glasi:

- A. 1 B. $\frac{x-1}{x-y}$ C. $\frac{x-y}{x-1}$ D. $\frac{x+1}{x+y}$

Rješenje 776

Ponovimo!

$$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}, \quad a^2 - b^2 = (a-b) \cdot (a+b), \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\frac{x^2-1}{x^2-y^2} : \frac{x-1}{x-y} = \frac{x^2-1}{x^2-y^2} \cdot \frac{x-y}{x-1} = \frac{(x-1) \cdot (x+1)}{(x-y) \cdot (x+y)} \cdot \frac{x-y}{x-1} = \frac{(x-1) \cdot (x+1)}{(x-y) \cdot (x+y)} \cdot \frac{x-y}{x-1} =$$

$$= \frac{x+1}{x+y} \cdot \frac{1}{1} = \frac{x+1}{x+y}.$$

Odgovor je pod D.

Vježba 776

Reducirani oblik izraza $\frac{x-1}{x-y} : \frac{x^2-1}{x^2-y^2}$ glasi:

$$A. \frac{x-y}{x-1} \quad B. \frac{x+y}{x+1} \quad C. \frac{x}{x-1} \quad D. \frac{y+1}{x+y}$$

Rezultat: B.

Zadatak 777 (4B, TUPŠ + Tonka ☺. gimnazija)

Za $a \neq -\frac{1}{2}$ izraz $\frac{1 - \frac{1}{1+2 \cdot a} + 2 \cdot a}{1 + \frac{1}{1+2 \cdot a}}$ jednak je :

$$A. 2 \cdot a + 1 \quad B. 2 \cdot a + 2 \quad C. 2 \cdot a \quad D. a$$

Rješenje 777

Ponovimo!

$$n = \frac{n}{1}, \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad a^1 = a.$$

$$a^n \cdot a^m = a^{n+m}, \quad a^n : a^m = a^{n-m}, \quad \frac{\frac{a}{b}}{\frac{1}{n}} = \frac{a}{b} \cdot n, \quad a \cdot \frac{1}{a} = 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n}, \quad n \neq 0, \quad n \neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

1. inačica

$$\begin{aligned} \frac{1 - \frac{1}{1+2 \cdot a} + 2 \cdot a}{1 + \frac{1}{1+2 \cdot a}} &= \frac{1 - \frac{1}{1+2 \cdot a} + \frac{2 \cdot a}{1}}{1 + \frac{1}{1+2 \cdot a}} = \frac{1+2 \cdot a - 1 + 2 \cdot a \cdot (1+2 \cdot a)}{1+2 \cdot a + 1} = \\ &= \frac{1+2 \cdot a - 1 + 2 \cdot a + 4 \cdot a^2}{1+2 \cdot a} = \frac{1+2 \cdot a - 1 + 2 \cdot a + 4 \cdot a^2}{1+2 \cdot a} = \frac{2 \cdot a + 2 \cdot a + 4 \cdot a^2}{1+2 \cdot a} = \\ &= \frac{4 \cdot a + 4 \cdot a^2}{1+2 \cdot a} = \frac{4 \cdot a + 4 \cdot a^2}{1+2 \cdot a} = \frac{4 \cdot a + 4 \cdot a^2}{2+2 \cdot a} = \frac{4 \cdot a \cdot (1+a)}{2 \cdot (1+a)} = \frac{4 \cdot a \cdot (1+a)}{2 \cdot (1+a)} = \frac{2 \cdot a}{1} = 2 \cdot a. \end{aligned}$$

Odgovor je pod C.

2. inačica

$$\frac{1 - \frac{1}{1+2 \cdot a} + 2 \cdot a}{1 + \frac{1}{1+2 \cdot a}} = \left[\begin{array}{l} \text{razlomak proširimo} \\ \text{izrazom } 1+2 \cdot a \end{array} \right] = \frac{\left(1 - \frac{1}{1+2 \cdot a} + 2 \cdot a\right) \cdot (1+2 \cdot a)}{\left(1 + \frac{1}{1+2 \cdot a}\right) \cdot (1+2 \cdot a)} =$$

$$= \frac{1+2 \cdot a - 1 + 2 \cdot a \cdot (1+2 \cdot a)}{1+2 \cdot a + 1} = \frac{1+2 \cdot a - 1 + 2 \cdot a + 4 \cdot a^2}{1+2 \cdot a + 1} = \frac{1+2 \cdot a - 1 + 2 \cdot a + 4 \cdot a^2}{1+2 \cdot a + 1} =$$

$$= \frac{2 \cdot a + 2 \cdot a + 4 \cdot a^2}{2+2 \cdot a} = \frac{4 \cdot a + 4 \cdot a^2}{2+2 \cdot a} = \frac{4 \cdot a \cdot (1+a)}{2 \cdot (1+a)} = \frac{4 \cdot a \cdot (1+a)}{2 \cdot (1+a)} = \frac{2 \cdot a}{1} = 2 \cdot a.$$

Odgovor je pod C.

Vježba 777

Za $a \neq -\frac{1}{2}$ izraz $\frac{1+2 \cdot a - \frac{1}{1+2 \cdot a}}{1 + \frac{1}{1+2 \cdot a}}$ jednak je :

- A. $2 \cdot a + 1$ B. $2 \cdot a + 2$ C. $2 \cdot a$ D. a

Rezultat: C.

Zadatak 778 (4B, TUPŠ + Tonka 😊. gimnazija)

Skrati razlomak: $\frac{x^2 - 6 \cdot x + 9}{5 \cdot x - 15}$.

- A. $\frac{x+3}{5}$ B. $\frac{x-2}{3}$ C. $\frac{x-3}{5}$ D. $\frac{x+6}{5}$

Rješenje 778

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$\frac{x^2 - 6 \cdot x + 9}{5 \cdot x - 15} = \frac{(x-3)^2}{5 \cdot (x-3)} = \frac{(x-3)^2}{5 \cdot (x-3)} = \frac{x-3}{5}.$$

Odgovor je pod C.

Vježba 778

Skrati razlomak: $\frac{x^2 + 6 \cdot x + 9}{5 \cdot x + 15}$.

- A. $\frac{x+3}{5}$ B. $\frac{x-2}{3}$ C. $\frac{x-3}{5}$ D. $\frac{x+6}{5}$

Rezultat: A.

Zadatak 779 (4B, TUPŠ + Tonka ☺. gimnazija)Skrati razlomak: $\frac{a^2 + 4 \cdot a + 4}{a^2 + 3 \cdot a + 2}$.

$$A. \frac{a+1}{a+2} \quad B. \frac{a+2}{a+1} \quad C. \frac{2 \cdot a+1}{a+1} \quad D. \frac{a+2}{2 \cdot a+1}$$

Rješenje 779

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2, \quad a^1 = a, \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned} \frac{a^2 + 4 \cdot a + 4}{a^2 + 3 \cdot a + 2} &= \left[\begin{array}{l} \text{u nazivniku} \\ \text{matoda grupiranja} \end{array} \right] = \frac{(a+2)^2}{a^2 + a + 2 \cdot a + 2} = \frac{(a+2)^2}{(a^2 + a) + (2 \cdot a + 2)} = \\ &= \frac{(a+2)^2}{a \cdot (a+1) + 2 \cdot (a+1)} = \frac{(a+2)^2}{(a+1) \cdot (a+2)} = \frac{(a+2)^2}{(a+1) \cdot (a+2)} = \frac{a+2}{a+1}. \end{aligned}$$

Odgovor je pod B.

Vježba 779Skrati razlomak: $\frac{a^2 + 3 \cdot a + 2}{a^2 + 4 \cdot a + 4}$.

$$A. \frac{a+1}{a+2} \quad B. \frac{a+2}{a+1} \quad C. \frac{2 \cdot a+1}{a+1} \quad D. \frac{a+2}{2 \cdot a+1}$$

Rezultat: A.**Zadatak 780 (4B, TUPŠ + Tonka ☺. gimnazija)**Skrati razlomak: $\frac{3 \cdot x^3 \cdot y + 24 \cdot y}{x^2 - 4}$.**Rješenje 780**

Ponovimo!

$$a^3 + b^3 = (a+b) \cdot (a^2 - a \cdot b + b^2), \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned} \frac{3 \cdot x^3 \cdot y + 24 \cdot y}{x^2 - 4} &= \frac{3 \cdot y \cdot (x^3 + 8)}{x^2 - 2^2} = \frac{3 \cdot y \cdot (x^3 + 2^3)}{(x-2) \cdot (x+2)} = \frac{3 \cdot y \cdot (x+2) \cdot (x^2 - 2 \cdot x + 2^2)}{(x-2) \cdot (x+2)} = \\ &= \frac{3 \cdot y \cdot (x+2) \cdot (x^2 - 2 \cdot x + 4)}{(x-2) \cdot (x+2)} = \frac{3 \cdot y \cdot (x+2) \cdot (x^2 - 2 \cdot x + 4)}{(x-2) \cdot (x+2)} = \frac{3 \cdot y \cdot (x^2 - 2 \cdot x + 4)}{x-2}. \end{aligned}$$

Vježba 780

Skrati razlomak: $\frac{x^2 - 4}{3 \cdot x^3 \cdot y + 24 \cdot y}$.

Rezultat: $\frac{x-2}{3 \cdot y \cdot (x^2 - 2 \cdot x + 4)}$.

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