Zadatak 761 (Ivana, srednja škola)

Izračunaj: $\dfrac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}+\dfrac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}.$

Rješenje 761

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad (a-b)\cdot(a+b)=a^2-b^2\quad,\quad \left(\sqrt{a}\right)^2=a\quad,\quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad (a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

$$a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}+\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=$$

$$\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}\cdot\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}+\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}\cdot\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}=$$

$$=\frac{\left(\sqrt{x+y}+\sqrt{x-y}\right)^2}{\left(\sqrt{x+y}\right)^2-\left(\sqrt{x-y}\right)^2}+\frac{\left(\sqrt{x+y}-\sqrt{x-y}\right)^2}{\left(\sqrt{x+y}\right)^2-\left(\sqrt{x-y}\right)^2}=$$

$$=\frac{\left(\sqrt{x+y}\right)^2+2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2}{x+y-(x-y)}+\frac{\left(\sqrt{x+y}\right)^2-2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2}{x+y-(x-y)}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}+\frac{x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}+\frac{x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}=$$

$$=\frac{x+2\cdot\sqrt{(x+y)\cdot(x-y)}+x}{y+y}+\frac{x-2\cdot\sqrt{(x+y)\cdot(x-y)}+x}{y+y}=$$

$$=\frac{2\cdot x+2\cdot\sqrt{(x+y)\cdot(x-y)}}{2\cdot y}+\frac{2\cdot x-2\cdot\sqrt{(x+y)\cdot(x-y)}}{2\cdot y}=$$

$$=\frac{2\cdot x+2\cdot\sqrt{(x+y)\cdot(x-y)}+2\cdot x-2\cdot\sqrt{(x+y)\cdot(x-y)}}{2\cdot y}=$$

$$=\frac{2\cdot x+2\cdot\sqrt{(x+y)\cdot(x-y)}+2\cdot x-2\cdot\sqrt{(x+y)\cdot(x-y)}}{2\cdot y}=\frac{2\cdot x+2\cdot x}{2\cdot y}=\frac{4\cdot x}{2\cdot y}=\frac{4\cdot x}{2\cdot y}=\frac{2\cdot x}{y}.$$

2.inačica

$$\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}+\frac{\sqrt{x+y}-\sqrt{x-y}}{\sqrt{x+y}+\sqrt{x-y}}=$$

$$=\frac{\left(\sqrt{x+y}+\sqrt{x-y}\right)^2+\left(\sqrt{x+y}-\sqrt{x-y}\right)^2}{\left(\sqrt{x+y}-\sqrt{x-y}\right)\cdot\left(\sqrt{x+y}+\sqrt{x-y}\right)}=$$

$$=\frac{\left(\sqrt{x+y}\right)^2+2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2+\left(\sqrt{x+y}\right)^2-2\cdot\sqrt{x+y}\cdot\sqrt{x-y}+\left(\sqrt{x-y}\right)^2}{\left(\sqrt{x+y}\right)^2-\left(\sqrt{x-y}\right)^2}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y+x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-(x-y)}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y+x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}=$$

$$=\frac{x+y+2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y+x+y-2\cdot\sqrt{(x+y)\cdot(x-y)}+x-y}{x+y-x+y}=\frac{x+x+x+x}{y+y}=$$

$$=\frac{4\cdot x}{2\cdot y}=\frac{4\cdot x}{2\cdot y}=\frac{2\cdot x}{y}.$$

Vježba 761

Izračunaj: $\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}-\frac{\sqrt{x-y}-\sqrt{x+y}}{\sqrt{x+y}+\sqrt{x-y}}.$

Rezultat: $\frac{2\cdot x}{y}.$

Zadatak 762 (Tihomir, srednja škola)

Izračunaj: $(a+b+c)^2-(b+c-a)^2+(a+b-c)^2-(a+c-b)^2.$

$A.\ 8\cdot a\cdot b$ $B.\ 6\cdot a\cdot b$ $C.\ 8\cdot a\cdot c$ $D.\ 8\cdot b\cdot c$

Rješenje 762

Ponovimo!

$$(x+y+z)^2=x^2+y^2+z^2+2\cdot x\cdot y+2\cdot x\cdot z+2\cdot y\cdot z \quad , \quad x^2-y^2=(x-y)\cdot(x+y).$$

$$(-a)^2=a^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$(a+b+c)^2-(b+c-a)^2+(a+b-c)^2-(a+c-b)^2=$$

$$=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c-\left(b^2+c^2+(-a)^2+2\cdot b\cdot c+2\cdot b\cdot(-a)+2\cdot c\cdot(-a)\right)+$$

$$+a^2+b^2+(-c)^2+2\cdot a\cdot b+2\cdot a\cdot(-c)+2\cdot b\cdot(-c)-$$

$$-\left(a^2+c^2+(-b)^2+2\cdot a\cdot c+2\cdot a\cdot(-b)+2\cdot c\cdot(-b)\right)=$$

$$=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c-\left(b^2+c^2+a^2+2\cdot b\cdot c-2\cdot b\cdot a-2\cdot c\cdot a\right)+$$

$$+a^2+b^2+c^2+2\cdot a\cdot b-2\cdot a\cdot c-2\cdot b\cdot c-$$

$$-\left(a^2+c^2+b^2+2\cdot a\cdot c-2\cdot a\cdot b-2\cdot c\cdot b\right)=$$

$$=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c-b^2-c^2-a^2-2\cdot b\cdot c+2\cdot b\cdot a+2\cdot c\cdot a+$$

$$+a^2+b^2+c^2+2\cdot a\cdot b-2\cdot a\cdot c-2\cdot b\cdot c-$$

$$-a^2-c^2-b^2-2\cdot a\cdot c+2\cdot a\cdot b+2\cdot c\cdot b=$$

$$=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c-b^2-c^2-a^2-2\cdot b\cdot c+2\cdot b\cdot a+2\cdot c\cdot a+$$

$$+a^2+b^2+c^2+2\cdot a\cdot b-2\cdot a\cdot c-2\cdot b\cdot c-$$

$$-a^2-c^2-b^2-2\cdot a\cdot c+2\cdot a\cdot b+2\cdot c\cdot b=$$

$$=2\cdot a\cdot b+2\cdot a\cdot b+2\cdot a\cdot b+2\cdot a\cdot b=8\cdot a\cdot b.$$

Odgovor je pod A.

2.inačica

$$(a+b+c)^2-(b+c-a)^2+(a+b-c)^2-(a+c-b)^2=$$

$$=\left((a+b+c)^2-(b+c-a)^2\right)+\left((a+b-c)^2-(a+c-b)^2\right)=$$

$$=\left((a+b+c)-(b+c-a)\right)\cdot\left((a+b+c)+(b+c-a)\right)+\left((a+b-c)-(a+c-b)\right)\cdot\left((a+b-c)+(a+c-b)\right)=$$

$$=(a+b+c-b-c+a)\cdot(a+b+c+b+c-a)+(a+b-c-a-c+b)\cdot(a+b-c+a+c-b)=$$

$$=(a+b+c-b-c+a)\cdot(a+b+c+b+c-a)+(a+b-c-a-c+b)\cdot(a+b-c+a+c-b)=$$

$$=(a+a)\cdot(b+c+b+c)+(b-c-c+b)\cdot(a+a)=2\cdot a\cdot(2\cdot b+2\cdot c)+(2\cdot b-2\cdot c)\cdot 2\cdot a=$$

$$=2\cdot a\cdot(2\cdot b+2\cdot c+2\cdot b-2\cdot c)=2\cdot a\cdot(2\cdot b+2\cdot c+2\cdot b-2\cdot c)=2\cdot a\cdot(2\cdot b+2\cdot b)=$$

$$=2\cdot a\cdot 4\cdot b=8\cdot a\cdot b.$$

Odgovor je pod A.

Vježba 762

Izračunaj: $(a+b+c)^2-(a-b-c)^2+(c-a-b)^2-(b-a-c)^2.$

$A.\ 8\cdot a\cdot b \qquad B.\ 6\cdot a\cdot b \qquad C.\ 8\cdot a\cdot c \qquad D.\ 8\cdot b\cdot c$

Rezultat: A.

Zadatak 763 (Ivan, gimnazija)

Dokaži da iz $(a-b)^2+(c-a)^2+(b-c)^2=(a+b-2\cdot c)^2+(a-2\cdot b+c)^2+(b+c-2\cdot a)^2$ slijedi a = b = c.

Rješenje 763

Ponovimo!

$$(x-y)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad (x+y+z)^2=x^2+y^2+z^2+2\cdot x\cdot y+2\cdot x\cdot z+2\cdot y\cdot z.$$

$$(-a)^2=a^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n.$$

$$(a-b)^2+(c-a)^2+(b-c)^2=(a+b-2\cdot c)^2+(a-2\cdot b+c)^2+(b+c-2\cdot a)^2 \Rightarrow$$

$$\Rightarrow a^2-2\cdot a\cdot b+b^2+c^2-2\cdot c\cdot a+a^2+b^2-2\cdot b\cdot c+c^2=$$

$$=a^2+b^2+(-2\cdot c)^2+2\cdot a\cdot b+2\cdot a\cdot(-2\cdot c)+2\cdot b\cdot(-2\cdot c)+$$

$$+a^2+(-2\cdot b)^2+c^2+2\cdot a\cdot(-2\cdot b)+2\cdot a\cdot c+2\cdot(-2\cdot b)\cdot c+$$

$$+b^2+c^2+(-2\cdot a)^2+2\cdot b\cdot c+2\cdot b\cdot(-2\cdot a)+2\cdot c\cdot(-2\cdot a) \Rightarrow$$

$$\Rightarrow a^2-2\cdot a\cdot b+b^2+c^2-2\cdot c\cdot a+a^2+b^2-2\cdot b\cdot c+c^2=$$

$$=a^2+b^2+4\cdot c^2+2\cdot a\cdot b-4\cdot a\cdot c-4\cdot b\cdot c+a^2+4\cdot b^2+c^2-4\cdot a\cdot b+2\cdot a\cdot c-4\cdot b\cdot c+$$

$$+b^2+c^2+4\cdot a^2+2\cdot b\cdot c-4\cdot b\cdot a-4\cdot c\cdot a \Rightarrow$$

$$\Rightarrow a^2-2\cdot a\cdot b+b^2+c^2-2\cdot c\cdot a+a^2+b^2-2\cdot b\cdot c+c^2=$$

$$=a^2+b^2+4\cdot c^2+2\cdot a\cdot b-4\cdot a\cdot c-4\cdot b\cdot c+a^2+4\cdot b^2+c^2-4\cdot a\cdot b+2\cdot a\cdot c-4\cdot b\cdot c+$$

$$+b^2+c^2+4\cdot a^2+2\cdot b\cdot c-4\cdot b\cdot a-4\cdot c\cdot a \Rightarrow$$

$$\Rightarrow 0=4\cdot c^2+4\cdot b^2-4\cdot b\cdot c+4\cdot a^2-4\cdot b\cdot a-4\cdot c\cdot a \Rightarrow$$

$$\Rightarrow 4\cdot c^2+4\cdot b^2-4\cdot b\cdot c+4\cdot a^2-4\cdot b\cdot a-4\cdot c\cdot a=0 \Rightarrow$$

$$\Rightarrow 4\cdot c^2+4\cdot b^2-4\cdot b\cdot c+4\cdot a^2-4\cdot b\cdot a-4\cdot c\cdot a=0 \ /:2 \Rightarrow$$

$$\Rightarrow 2\cdot c^2+2\cdot b^2-2\cdot b\cdot c+2\cdot a^2-2\cdot b\cdot a-2\cdot c\cdot a=0 \Rightarrow$$

$$\Rightarrow a^2-2\cdot a\cdot b+b^2+a^2-2\cdot a\cdot c+c^2+b^2-2\cdot b\cdot c+c^2=0 \Rightarrow$$

$$\Rightarrow \left(a^2-2\cdot a\cdot b+b^2\right)+\left(a^2-2\cdot a\cdot c+c^2\right)+\left(b^2-2\cdot b\cdot c+c^2\right)=0 \Rightarrow$$

$$\Rightarrow\ (a-b)^2+(a-c)^2+(b-c)^2=0\ \Rightarrow\ \left.\begin{matrix} a-b=0 \\ a-c=0 \\ b-c=0 \end{matrix}\right\}\ \Rightarrow\ \left.\begin{matrix} a=b \\ a=c \\ b=c \end{matrix}\right\}\ \Rightarrow\ a=b=c.$$

Vježba 763

Dokaži da iz $(b-a)^2+(a-c)^2+(c-b)^2=(a+b-2\cdot c)^2+(a-2\cdot b+c)^2+(b+c-2\cdot a)^2$ slijedi a = b = c.

Rezultat: Dokaz analogan.

Zadatak 764 (Pax, gimnazija)

Dokaži da za realne brojeve $a,\ b>0$ vrijedi $\frac{a}{b}+\frac{b}{a}\geq 2.$

Rješenje 764

Ponovimo!

$$a\geq b\ ,\ c>0\ \Rightarrow\ a\cdot c\geq b\cdot c\quad ,\quad a^2\geq 0\ ,\ a\in R\quad ,\quad (a-b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$\left(\sqrt{a}\right)^2=a\quad ,\quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}\quad ,\quad a^1=a\quad ,\quad a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

Polazimo od očigledne nejednakosti.

$$\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)^2\geq 0\ \Rightarrow\ \left(\sqrt{\frac{a}{b}}\right)^2-2\cdot\sqrt{\frac{a}{b}}\cdot\sqrt{\frac{b}{a}}+\left(\sqrt{\frac{b}{a}}\right)^2\geq 0\ \Rightarrow\ \frac{a}{b}-2\cdot\sqrt{\frac{a}{b}\cdot\frac{b}{a}}+\frac{b}{a}\geq 0\ \Rightarrow$$

$$\Rightarrow\ \frac{a}{b}-2\cdot\sqrt{\frac{a}{b}\cdot\frac{b}{a}}+\frac{b}{a}\geq 0\ \Rightarrow\ \frac{a}{b}-2\cdot\sqrt{1}+\frac{b}{a}\geq 0\ \Rightarrow\ \frac{a}{b}-2\cdot 1+\frac{b}{a}\geq 0\ \Rightarrow\ \frac{a}{b}-2+\frac{b}{a}\geq 0\ \Rightarrow$$

$$\Rightarrow\ \frac{a}{b}+\frac{b}{a}\geq 2.$$

2.inačica

Polazimo od zadane nejednakosti.

$$\frac{a}{b}+\frac{b}{a}\geq 2\ \Rightarrow\ \frac{a}{b}+\frac{b}{a}\geq 2\ /\cdot\ a\cdot b\ \Rightarrow\ a^2+b^2\geq 2\cdot a\cdot b\ \Rightarrow\ a^2-2\cdot a\cdot b+b^2\geq 0\ \Rightarrow\ (a-b)^2\geq 0.$$

Dobili smo točnu nejednakost, znači da je početna tvrdnja istinita.

Vježba 764

Dokaži da za realne brojeve $a,\ b>0$ vrijedi $\frac{1}{2}\cdot\left(\frac{a}{b}+\frac{b}{a}\right)\geq 1.$

Rezultat: Dokaz analogan.

Zadatak 765 (Pax, gimnazija)

Rastavi na faktore: $(a\cdot b+a\cdot c+b\cdot c)\cdot(a+b+c)-a\cdot b\cdot c.$

Rješenje 765

Ponovimo!

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

$$(a \cdot b + a \cdot c + b \cdot c) \cdot (a+b+c) - a \cdot b \cdot c =$$

$$= a^2 \cdot b + a \cdot b^2 + a \cdot b \cdot c + a^2 \cdot c + a \cdot b \cdot c + a \cdot c^2 + a \cdot b \cdot c + b^2 \cdot c + b \cdot c^2 - a \cdot b \cdot c =$$

$$= a^2 \cdot b + a \cdot b^2 + a \cdot b \cdot c + a^2 \cdot c + a \cdot b \cdot c + a \cdot c^2 + a \cdot b \cdot c + b^2 \cdot c + b \cdot c^2 - a \cdot b \cdot c =$$

$$= a^2 \cdot b + a \cdot b^2 + a^2 \cdot c + a \cdot b \cdot c + a \cdot c^2 + a \cdot b \cdot c + b^2 \cdot c + b \cdot c^2 = \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] =$$

$$= \left(a^2 \cdot b + a^2 \cdot c \right) + \left(a \cdot b^2 + a \cdot b \cdot c \right) + \left(a \cdot b \cdot c + a \cdot c^2 \right) + \left(b^2 \cdot c + b \cdot c^2 \right) =$$

$$= a^2 \cdot (b+c) + a \cdot b \cdot (b+c) + a \cdot c \cdot (b+c) + b \cdot c \cdot (b+c) = (b+c) \cdot \left(a^2 + a \cdot b + a \cdot c + b \cdot c \right) =$$

$$= \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] = (b+c) \cdot \left(\left(a^2 + a \cdot b \right) + (a \cdot c + b \cdot c) \right) = (b+c) \cdot \left(a \cdot (a+b) + c \cdot (a+b) \right) =$$

$$= (b+c) \cdot (a+b) \cdot (a+c).$$

Vježba 765

Rastavi na faktore: $(a+b+c) \cdot (a \cdot b + a \cdot c + b \cdot c) - a \cdot b \cdot c.$

Rezultat: $(b+c) \cdot (a+b) \cdot (a+c).$

Zadatak 766 (Pax, gimnazija)

Rastavi na faktore: $a^2 \cdot b + a \cdot b^2 + a^2 \cdot c + a \cdot c^2 + b^2 \cdot c + b \cdot c^2 + 3 \cdot a \cdot b \cdot c.$

Rješenje 766

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad a^n \cdot a^m = a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$a^2 \cdot b + a \cdot b^2 + a^2 \cdot c + a \cdot c^2 + b^2 \cdot c + b \cdot c^2 + 3 \cdot a \cdot b \cdot c =$$

$$= a^2 \cdot b + a \cdot b^2 + a \cdot b \cdot c + a^2 \cdot c + a \cdot c^2 + a \cdot b \cdot c + b^2 \cdot c + b \cdot c^2 + a \cdot b \cdot c = \left[\begin{array}{l} \text{metoda} \\ \text{grupiranja} \end{array} \right] =$$

$$= \left(a^2 \cdot b + a \cdot b^2 + a \cdot b \cdot c \right) + \left(a^2 \cdot c + a \cdot c^2 + a \cdot b \cdot c \right) + \left(b^2 \cdot c + b \cdot c^2 + a \cdot b \cdot c \right) =$$

$$= a \cdot b \cdot (a+b+c) + a \cdot c \cdot (a+c+b) + b \cdot c \cdot (b+c+a) =$$

$$= a \cdot b \cdot (a+b+c) + a \cdot c \cdot (a+b+c) + b \cdot c \cdot (a+b+c) = (a+b+c) \cdot (a \cdot b + a \cdot c + b \cdot c).$$

Vježba 766

Rastavi na faktore: $a \cdot b \cdot (a+b) + a \cdot c \cdot (a+c) + b \cdot c \cdot (b+c) + 3 \cdot a \cdot b \cdot c.$

Rezultat: $(a+b+c) \cdot (a \cdot b + a \cdot c + b \cdot c).$

Zadatak 767 (Pax, gimnazija)

Rastavi na faktore: $a^4 + b^4 + 2 \cdot a^3 \cdot b + 2 \cdot a^2 \cdot b^2 + 2 \cdot a \cdot b^3.$

Rješenje 767

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$a^4 + b^4 + 2 \cdot a^3 \cdot b + 2 \cdot a^2 \cdot b^2 + 2 \cdot a \cdot b^3 =$$

$$= a^4 + a^2 \cdot b^2 + 2 \cdot a^3 \cdot b + 2 \cdot a \cdot b^3 + b^4 + a^2 \cdot b^2 = \left[\begin{array}{l}\text{metoda} \\ \text{grupiranja}\end{array}\right] =$$

$$= \left(a^4 + a^2 \cdot b^2\right) + \left(2 \cdot a^3 \cdot b + 2 \cdot a \cdot b^3\right) + \left(b^4 + a^2 \cdot b^2\right) =$$

$$= a^2 \cdot \left(a^2 + b^2\right) + 2 \cdot a \cdot b \cdot \left(a^2 + b^2\right) + b^2 \cdot \left(b^2 + a^2\right) =$$

$$= a^2 \cdot \left(a^2 + b^2\right) + 2 \cdot a \cdot b \cdot \left(a^2 + b^2\right) + b^2 \cdot \left(a^2 + b^2\right) =$$

$$= \left(a^2 + b^2\right) \cdot \left(a^2 + 2 \cdot a \cdot b + b^2\right) = \left(a^2 + b^2\right) \cdot (a+b)^2.$$

Vježba 767

Rastavi na faktore: $a^4 + b^4 + 2 \cdot \left(a^3 \cdot b + a^2 \cdot b^2 + a \cdot b^3\right).$

Rezultat: $\left(a^2 + b^2\right) \cdot (a+b)^2.$

Zadatak 768 (Tonka, gimnazija)

Provedite računsku operaciju $\frac{x^2}{2-x} + x + 2$ te napišite rezultat u obliku do kraja skraćenoga razlomka.

Rješenje 768

Ponovimo!

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$\frac{x^2}{2-x} + x + 2 = \frac{x^2}{2-x} + \frac{x+2}{1} = \frac{x^2 + (x+2) \cdot (2-x)}{2-x} = \frac{x^2 + (2+x) \cdot (2-x)}{2-x} =$$

$$= \frac{x^2 + 2^2 - x^2}{2-x} = \frac{x^2 + 4 - x^2}{2-x} = \frac{x^2 + 4 - x^2}{2-x} = \frac{4}{2-x}.$$

Vježba 768

Provedite računsku operaciju $x+2-\frac{x^2}{x-2}$ te napišite rezultat u obliku do kraja skraćenoga razlomka.

Rezultat: $\frac{4}{2-x}.$

Zadatak 769 (Trnoružica, gimnazija)

Pomnožite izraze a + 3 i a – 2 i od umnoška oduzmite kvadrat broja a. Dobiveni izraz pojednostavnite do kraja.

Rješenje 769

Ponovimo!

$$a^1=a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zadani tekst preoblikujemo u matematički izraz.

Pomnožite izraze a + 3 i a – 2 …

$$(a+3)\cdot(a-2)$$

… i od umnoška oduzmite kvadrat broja a.

$$(a+3)\cdot(a-2)-a^2=a^2-2\cdot a+3\cdot a-6-a^2=a^2-2\cdot a+3\cdot a-6-a^2=$$
$$=-2\cdot a+3\cdot a-6=a-6.$$

Vježba 769

Pomnožite izraze a + 2 i a – 3 i od umnoška oduzmite kvadrat broja a. Dobiveni izraz pojednostavnite do kraja.

Rezultat: $-a-6.$

Zadatak 770 (4B, TUPŠ)

Izračunajmo: $\frac{7}{2\cdot a-4}-\frac{3}{a+2}-\frac{12}{a^2-4}.$

Rješenje 770

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{7}{2\cdot a-4}-\frac{3}{a+2}-\frac{12}{a^2-4}=\left[\begin{array}{l}\text{nazivnike rastavimo}\\ \text{na faktore}\end{array}\right]=$$

$$=\frac{7}{2\cdot(a-2)}-\frac{3}{a+2}-\frac{12}{(a-2)\cdot(a+2)}=\left[\begin{array}{l}\text{nazivnike svedemo na}\\ \text{zajednički nazivnik}\end{array}\right]=$$

$$=\frac{7\cdot(a+2)-3\cdot 2\cdot(a-2)-12\cdot 2}{2\cdot(a-2)\cdot(a+2)}=\left[\begin{array}{l}\text{izvršimo naznačene}\\ \text{operacije u brojniku}\end{array}\right]=$$

$$=\frac{7\cdot a+14-6\cdot(a-2)-24}{2\cdot(a-2)\cdot(a+2)}=\frac{7\cdot a+14-6\cdot a+12-24}{2\cdot(a-2)\cdot(a+2)}=\frac{a+2}{2\cdot(a-2)\cdot(a+2)}=$$

$$=\frac{a+2}{2\cdot(a-2)\cdot(a+2)}=\frac{1}{2\cdot(a-2)}.$$

Vježba 770

Izračunajmo: $\frac{7}{2\cdot a-4}-\frac{3}{a+2}+\frac{12}{4-a^2}$.

Rezultat: $\frac{1}{2\cdot(a-2)}$.

Zadatak 771 (4B, TUPŠ)

Izračunajmo: $\frac{1}{x+2}-\frac{1}{2-x}-\frac{8}{x^3-4\cdot x}$.

Rješenje 771

Ponovimo!

$$a^1=a \quad , \quad a^n:a^m=a^{n-m} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{1}{x+2}-\frac{1}{2-x}-\frac{8}{x^3-4\cdot x}=\left[\begin{array}{l}\text{nazivnike rastavimo}\\ \text{na faktore}\end{array}\right]=\frac{1}{x+2}-\frac{1}{-(x-2)}-\frac{8}{x^3-4\cdot x}=$$

$$=\frac{1}{x+2}+\frac{1}{x-2}-\frac{8}{x^3-4\cdot x}=\frac{1}{x+2}+\frac{1}{x-2}-\frac{8}{x\cdot\left(x^2-4\right)}=\frac{1}{x+2}+\frac{1}{x-2}-\frac{8}{x\cdot(x-2)\cdot(x+2)}=$$

$$=\left[\begin{array}{l}\text{nazivnike svedemo na}\\ \text{zajednički nazivnik}\end{array}\right]=\frac{x\cdot(x-2)+x\cdot(x+2)-8}{x\cdot(x-2)\cdot(x+2)}=\left[\begin{array}{l}\text{izvršimo naznačene}\\ \text{operacije u brojniku}\end{array}\right]=$$

$$=\frac{x^2-2\cdot x+x^2+2\cdot x-8}{x\cdot(x-2)\cdot(x+2)}=\frac{x^2-2\cdot x+x^2+2\cdot x-8}{x\cdot(x-2)\cdot(x+2)}=\frac{x^2+x^2-8}{x\cdot(x-2)\cdot(x+2)}=$$

$$=\frac{2\cdot x^2-8}{x\cdot(x-2)\cdot(x+2)}=\frac{2\cdot\left(x^2-4\right)}{x\cdot(x-2)\cdot(x+2)}=\frac{2\cdot(x-2)\cdot(x+2)}{x\cdot(x-2)\cdot(x+2)}=\frac{2\cdot(x-2)\cdot(x+2)}{x\cdot(x-2)\cdot(x+2)}=\frac{2}{x}.$$

Vježba 771

Izračunajmo: $\frac{1}{x+2}+\frac{1}{x-2}-\frac{8}{x^{3}-4\cdot x}$.

Rezultat: $\frac{2}{x}$.

Zadatak 772 (Goran, srednja škola)

Izračunajmo: $\left[\frac{(a+b)^{3}}{3\cdot a\cdot b}-a-b\right]^{n}:\left[\frac{(a-b)^{2}}{a\cdot b}+1\right]^{n}$.

Rješenje 772

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad a^{1}=a\quad,\quad a^{n}\cdot a^{m}=a^{n+m}.$$

$$(a+b)^{3}=a^{3}+3\cdot a^{2}\cdot b+3\cdot a\cdot b^{2}+b^{3}\quad,\quad(a-b)^{2}=a^{2}-2\cdot a\cdot b+b^{2}\quad,\quad a^{n}:b^{n}=(a:b)^{n}.$$

$$a^{3}+b^{3}=(a+b)\cdot\left(a^{2}-a\cdot b+b^{2}\right)\quad,\quad\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left[\frac{(a+b)^{3}}{3\cdot a\cdot b}-a-b\right]^{n}:\left[\frac{(a-b)^{2}}{a\cdot b}+1\right]^{n}=$$

$$=\left[\frac{(a+b)^{3}}{3\cdot a\cdot b}-(a+b)\right]^{n}:\left[\frac{(a-b)^{2}}{a\cdot b}+1\right]^{n}=\left[\frac{(a+b)^{3}}{3\cdot a\cdot b}-\frac{a+b}{1}\right]^{n}:\left[\frac{(a-b)^{2}}{a\cdot b}+\frac{1}{1}\right]^{n}=$$

$$=\left[\frac{(a+b)^{3}-3\cdot a\cdot b\cdot(a+b)}{3\cdot a\cdot b}\right]^{n}:\left[\frac{(a-b)^{2}+a\cdot b}{a\cdot b}\right]^{n}=$$

$$=\left[\frac{a^{3}+3\cdot a^{2}\cdot b+3\cdot a\cdot b^{2}+b^{3}-3\cdot a^{2}\cdot b-3\cdot a\cdot b^{2}}{3\cdot a\cdot b}\right]^{n}:\left[\frac{a^{2}-2\cdot a\cdot b+b^{2}+a\cdot b}{a\cdot b}\right]^{n}=$$

$$=\left[\frac{a^{3}+3\cdot a^{2}\cdot b+3\cdot a\cdot b^{2}+b^{3}-3\cdot a^{2}\cdot b-3\cdot a\cdot b^{2}}{3\cdot a\cdot b}\right]^{n}:\left[\frac{a^{2}-2\cdot a\cdot b+b^{2}+a\cdot b}{a\cdot b}\right]^{n}=$$

$$=\left[\frac{a^{3}+b^{3}}{3\cdot a\cdot b}\right]^{n}:\left[\frac{a^{2}-a\cdot b+b^{2}}{a\cdot b}\right]^{n}=\left[\frac{a^{3}+b^{3}}{3\cdot a\cdot b}:\frac{a^{2}-a\cdot b+b^{2}}{a\cdot b}\right]^{n}=$$

$$=\left[\frac{a^3+b^3}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-a\cdot b+b^2}\right]^n=\left[\frac{a^3+b^3}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-a\cdot b+b^2}\right]^n=$$

$$=\left[\frac{a^3+b^3}{3}\cdot\frac{1}{a^2-a\cdot b+b^2}\right]^n=\left[\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{3}\cdot\frac{1}{a^2-a\cdot b+b^2}\right]^n=$$

$$=\left[\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{3}\cdot\frac{1}{a^2-a\cdot b+b^2}\right]^n=\left[\frac{a+b}{3}\cdot\frac{1}{1}\right]^n=\left[\frac{a+b}{3}\right]^n.$$

Vježba 772

Izračunajmo: $\left[\frac{(a-b)^2}{a\cdot b}+1\right]^n:\left[\frac{(a+b)^3}{3\cdot a\cdot b}-a-b\right]^n.$

Rezultat: $\left[\frac{3}{a+b}\right]^n.$

Zadatak 773 (Leon, srednja škola)

Izvedi operacije: $\left[\frac{m^2}{m^2-n^2}-\frac{m^2\cdot n}{m^2+n^2}\cdot\left(\frac{m}{m\cdot n+n^2}+\frac{n}{m^2+m\cdot n}\right)\right]:\frac{n}{m-n}.$

Rješenje 773

Ponovimo!

$$a^1=a\quad,\quad a^n:a^m=a^{n-m}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad a^n\cdot a^m=a^{n+m}.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left[\frac{m^2}{m^2-n^2}-\frac{m^2\cdot n}{m^2+n^2}\cdot\left(\frac{m}{m\cdot n+n^2}+\frac{n}{m^2+m\cdot n}\right)\right]:\frac{n}{m-n}=$$

$$\left[\frac{m^2}{m^2-n^2}-\frac{m^2\cdot n}{m^2+n^2}\cdot\left(\frac{m}{n\cdot(m+n)}+\frac{n}{m\cdot(m+n)}\right)\right]:\frac{n}{m-n}=$$

$$=\left[\frac{m^2}{m^2-n^2}-\frac{m^2\cdot n}{m^2+n^2}\cdot\frac{m^2+n^2}{n\cdot m\cdot(m+n)}\right]:\frac{n}{m-n}=\left[\frac{m^2}{m^2-n^2}-\frac{m^2\cdot n}{m^2+n^2}\cdot\frac{m^2+n^2}{n\cdot m\cdot(m+n)}\right]:\frac{n}{m-n}=$$

$$=\left[\frac{m^2}{m^2-n^2}-\frac{m}{1}\cdot\frac{1}{m+n}\right]:\frac{n}{m-n}=\left[\frac{m^2}{m^2-n^2}-\frac{m}{m+n}\right]:\frac{n}{m-n}=$$

$$=\left[\frac{m^2}{(m-n)\cdot(m+n)}-\frac{m}{m+n}\right]:\frac{n}{m-n}=\frac{m^2-m\cdot(m-n)}{(m-n)\cdot(m+n)}:\frac{n}{m-n}=$$

$$=\frac{m^2-m^2+m\cdot n}{(m-n)\cdot(m+n)}:\frac{n}{m-n}=\frac{m^2-m^2+m\cdot n}{(m-n)\cdot(m+n)}:\frac{n}{m-n}=\frac{m\cdot n}{(m-n)\cdot(m+n)}:\frac{n}{m-n}=$$

$$=\frac{m\cdot n}{(m-n)\cdot(m+n)}\cdot\frac{m-n}{n}=\frac{m\cdot n}{(m-n)\cdot(m+n)}\cdot\frac{m-n}{n}=\frac{m}{m+n}\cdot\frac{1}{1}=\frac{m}{m+n}.$$

Vježba 773

Izvedi operacije: $\frac{n}{m-n}:\left[\frac{m^2}{m^2-n^2}-\frac{m^2\cdot n}{m^2+n^2}\cdot\left(\frac{m}{m\cdot n+n^2}+\frac{n}{m^2+m\cdot n}\right)\right].$

Rezultat: $\frac{m+n}{m}.$

Zadatak 774 (4B, TUPŠ + Tonka ☺. gimnazija)

Pojednostavnite izraz: $\frac{\left(a^2-b^2\right)^2}{(a+b)^2}.$

Rješenje 774

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad(a\cdot b)^n=a^n\cdot a^m\quad,\quad\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n.$$

$$\frac{n}{1}=n.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{\left(a^2-b^2\right)^2}{(a+b)^2}=\frac{((a-b)\cdot(a+b))^2}{(a+b)^2}=\frac{(a-b)^2\cdot(a+b)^2}{(a+b)^2}=\frac{(a-b)^2\cdot(a+b)^2}{(a+b)^2}=\frac{(a-b)^2}{1}=(a-b)^2.$$

2.inačica

$$\frac{\left(a^2-b^2\right)^2}{(a+b)^2}=\left(\frac{a^2-b^2}{a+b}\right)^2=\left(\frac{(a-b)\cdot(a+b)}{a+b}\right)^2=\left(\frac{(a-b)\cdot(a+b)}{a+b}\right)^2=$$

$$=\left(\frac{a-b}{1}\right)^2=(a-b)^2.$$

Vježba 774

Pojednostavnite izraz: $\frac{(a+b)^2}{\left(a^2-b^2\right)^2}$.

Rezultat: $\frac{1}{(a-b)^2}$.

Zadatak 775 (4B, TUPŠ + Tonka ☺. gimnazija)

Za x ≠ – 1 izraz $\frac{x+2+\frac{1}{x}}{\frac{1}{x}+1}$ jednak je:

$A.\ -x \qquad B.\ \frac{1}{x+1} \qquad C.\ x+1 \qquad D.\ 1$

Rješenje 775

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \frac{\frac{a}{n}}{\frac{b}{n}}=\frac{a}{b}.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \frac{1}{a}\cdot a=1 \quad , \quad \frac{a^n}{a^m}=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \ , \ n\neq 0 \ , \ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

1.inačica

$$\frac{x+2+\frac{1}{x}}{\frac{1}{x}+1}=\frac{\frac{x+2}{1}+\frac{1}{x}}{\frac{1}{x}+\frac{1}{1}}=\frac{\frac{x\cdot(x+2)+1}{x}}{\frac{1+x}{x}}=\frac{\frac{x^2+2\cdot x+1}{x}}{\frac{1+x}{x}}=\frac{\frac{x^2+2\cdot x+1}{x}}{\frac{1+x}{x}}=\frac{x^2+2\cdot x+1}{1+x}=$$

$$=\frac{(x+1)^2}{x+1}=\frac{(x+1)^2}{x+1}=\frac{x+1}{1}=x+1.$$

Odgovor je pod C.

2.inačica

$$\frac{x+2+\frac{1}{x}}{\frac{1}{x}+1}=\left[\begin{array}{l}\text{proširimo}\\ \text{razlomak sa } x\end{array}\right]=\frac{\left(x+2+\frac{1}{x}\right)\cdot x}{\left(\frac{1}{x}+1\right)\cdot x}=\frac{x^2+2\cdot x+1}{1+x}=\frac{(x+1)^2}{x+1}=$$

$$=\frac{(x+1)^2}{x+1}=\frac{x+1}{1}=x+1.$$

Odgovor je pod C.

Vježba 775

Za x ≠ – 1 izraz $\dfrac{x+\frac{1}{x}+2}{1+\frac{1}{x}}$ jednak je:

$$A.\ -x \qquad B.\ \frac{1}{x+1} \qquad C.\ x+1 \qquad D.\ 1$$

Rezultat: C.

Zadatak 776 (4B, TUPŠ + Tonka ☺. gimnazija)

Reducirani oblik izraza $\dfrac{x^2-1}{x^2-y^2}:\dfrac{x-1}{x-y}$ glasi:

$$A.\ 1 \qquad B.\ \frac{x-1}{x-y} \qquad C.\ \frac{x-y}{x-1} \qquad D.\ \frac{x+1}{x+y}$$

Rješenje 776

Ponovimo!

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{x^2-1}{x^2-y^2}:\frac{x-1}{x-y}=\frac{x^2-1}{x^2-y^2}\cdot\frac{x-y}{x-1}=\frac{(x-1)\cdot(x+1)}{(x-y)\cdot(x+y)}\cdot\frac{x-y}{x-1}=\frac{(x-1)\cdot(x+1)}{(x-y)\cdot(x+y)}\cdot\frac{x-y}{x-1}=$$

$$=\frac{x+1}{x+y}\cdot\frac{1}{1}=\frac{x+1}{x+y}.$$

Odgovor je pod D.

Vježba 776

Reducirani oblik izraza $\dfrac{x-1}{x-y}:\dfrac{x^2-1}{x^2-y^2}$ glasi:

$$A.\ \frac{x-y}{x-1} \qquad B.\ \frac{x+y}{x+1} \qquad C.\ \frac{x}{x-1} \qquad D.\ \frac{y+1}{x+y}$$

Rezultat: B.

Zadatak 777 (4B, TUPŠ + Tonka ☺. gimnazija)

$$\text{Za } a \neq -\frac{1}{2} \text{ izraz } \frac{1-\frac{1}{1+2\cdot a}+2\cdot a}{1+\frac{1}{1+2\cdot a}} \text{ jednak je:}$$

$$A.\ 2\cdot a+1 \qquad B.\ 2\cdot a+2 \qquad C.\ 2\cdot a \qquad D.\ a$$

Rješenje 777

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad a^{1}=a.$$

$$a^{n}\cdot a^{m}=a^{n+m} \quad , \quad a^{n}:a^{m}=a^{n-m} \quad , \quad \frac{\frac{a}{n}}{\frac{b}{n}}=\frac{a}{b} \quad , \quad a\cdot\frac{1}{a}=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \ , \ n\neq 0 \ , \ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

1.inačica

$$\frac{1-\frac{1}{1+2\cdot a}+2\cdot a}{1+\frac{1}{1+2\cdot a}}=\frac{\frac{1}{1}-\frac{1}{1+2\cdot a}+\frac{2\cdot a}{1}}{\frac{1}{1}+\frac{1}{1+2\cdot a}}=\frac{\frac{1+2\cdot a-1+2\cdot a\cdot(1+2\cdot a)}{1+2\cdot a}}{\frac{1+2\cdot a+1}{1+2\cdot a}}=$$

$$=\frac{\frac{1+2\cdot a-1+2\cdot a+4\cdot a^{2}}{1+2\cdot a}}{\frac{2+2\cdot a}{1+2\cdot a}}=\frac{\frac{1+2\cdot a-1+2\cdot a+4\cdot a^{2}}{1+2\cdot a}}{\frac{2+2\cdot a}{1+2\cdot a}}=\frac{\frac{2\cdot a+2\cdot a+4\cdot a^{2}}{1+2\cdot a}}{\frac{2+2\cdot a}{1+2\cdot a}}=$$

$$=\frac{\frac{4\cdot a+4\cdot a^{2}}{1+2\cdot a}}{\frac{2+2\cdot a}{1+2\cdot a}}=\frac{\frac{4\cdot a+4\cdot a^{2}}{1+2\cdot a}}{\frac{2+2\cdot a}{1+2\cdot a}}=\frac{4\cdot a+4\cdot a^{2}}{2+2\cdot a}=\frac{4\cdot a\cdot(1+a)}{2\cdot(1+a)}=\frac{4\cdot a\cdot(1+a)}{2\cdot(1+a)}=\frac{2\cdot a}{1}=2\cdot a.$$

Odgovor je pod C.

2.inačica

$$\frac{1-\frac{1}{1+2\cdot a}+2\cdot a}{1+\frac{1}{1+2\cdot a}}=\left[\begin{array}{l}\text{razlomak proširimo}\\ \text{izrazom } 1+2\cdot a\end{array}\right]=\frac{\left(1-\frac{1}{1+2\cdot a}+2\cdot a\right)\cdot(1+2\cdot a)}{\left(1+\frac{1}{1+2\cdot a}\right)\cdot(1+2\cdot a)}=$$

$$=\frac{1+2\cdot a-1+2\cdot a\cdot(1+2\cdot a)}{1+2\cdot a+1}=\frac{1+2\cdot a-1+2\cdot a+4\cdot a^{2}}{1+2\cdot a+1}=\frac{1+2\cdot a-1+2\cdot a+4\cdot a^{2}}{1+2\cdot a+1}=$$

$$=\frac{2\cdot a+2\cdot a+4\cdot a^{2}}{2+2\cdot a}=\frac{4\cdot a+4\cdot a^{2}}{2+2\cdot a}=\frac{4\cdot a\cdot(1+a)}{2\cdot(1+a)}=\frac{4\cdot a\cdot(1+a)}{2\cdot(1+a)}=\frac{2\cdot a}{1}=2\cdot a.$$

Odgovor je pod C.

Vježba 777

Za $a\neq-\frac{1}{2}$ izraz $\frac{1+2\cdot a-\frac{1}{1+2\cdot a}}{1+\frac{1}{1+2\cdot a}}$ jednak je :

A. $2\cdot a+1$ *B.* $2\cdot a+2$ *C.* $2\cdot a$ *D.* a

Rezultat: C.