Zadatak 721 (Tonka, ekonomska škola)

Izračunajte: $\frac{a^2-b^2}{a^3+b^3}-\frac{a-b}{a^2-a\cdot b+b^2}$.

Rješenje 721

Ponovimo!

$$x^2-y^2=(x-y)\cdot(x+y) \quad , \quad x^3+y^3=(x+y)\cdot\left(x^2-x\cdot y+y^2\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$\frac{a^2-b^2}{a^3+b^3}-\frac{a-b}{a^2-a\cdot b+b^2}=\frac{(a-b)\cdot(a+b)}{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}-\frac{a-b}{a^2-a\cdot b+b^2}=$$

$$=\frac{(a-b)\cdot(a+b)}{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}-\frac{a-b}{a^2-a\cdot b+b^2}=\frac{a-b}{a^2-a\cdot b+b^2}-\frac{a-b}{a^2-a\cdot b+b^2}=$$

$$=\frac{a-b}{a^2-a\cdot b+b^2}-\frac{a-b}{a^2-a\cdot b+b^2}=0.$$

Vježba 721

Izračunajte: $\frac{a^2-b^2}{a^3+b^3}+\frac{b-a}{a^2-a\cdot b+b^2}$.

Rezultat: 0.

Zadatak 722 (Franjo, srednja škola)

Dokaži: $(a-x)\cdot(b+y)-(b+x)\cdot(a-y)=(a+b)\cdot(y-x)$.

Rješenje 722

Ponovimo!

$$x^2-y^2=(x-y)\cdot(x+y) \quad , \quad x^3+y^3=(x+y)\cdot\left(x^2-x\cdot y+y^2\right).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$(a-x)\cdot(b+y)-(b+x)\cdot(a-y)=a\cdot b+a\cdot y-x\cdot b-x\cdot y-(b\cdot a-b\cdot y+x\cdot a-x\cdot y)=$$

$$=a\cdot b+a\cdot y-x\cdot b-x\cdot y-b\cdot a+b\cdot y-x\cdot a+x\cdot y=$$

$$=a\cdot b+a\cdot y-x\cdot b-x\cdot y-b\cdot a+b\cdot y-x\cdot a+x\cdot y=a\cdot y-x\cdot b+b\cdot y-x\cdot a=$$

$$=a\cdot y-a\cdot x+b\cdot y-b\cdot x=(a\cdot y-a\cdot x)+(b\cdot y-b\cdot x)=$$

$$=a\cdot(y-x)+b\cdot(y-x)=(y-x)\cdot(a+b)=(a+b)\cdot(y-x).$$

2.inačica

$$(a-x)\cdot(b+y)-(b+x)\cdot(a-y)=(a+b)\cdot(y-x) \Rightarrow$$

$$\Rightarrow (a-x)\cdot(b+y)-(b+x)\cdot(a-y)-(a+b)\cdot(y-x)=0 \Rightarrow$$

$$\Rightarrow a\cdot b+a\cdot y-x\cdot b-x\cdot y-(b\cdot a-b\cdot y+x\cdot a-x\cdot y)-(a\cdot y-a\cdot x+b\cdot y-b\cdot x)=0 \Rightarrow$$

$$\Rightarrow a\cdot b+a\cdot y-x\cdot b-x\cdot y-b\cdot a+b\cdot y-x\cdot a+x\cdot y-a\cdot y+a\cdot x-b\cdot y+b\cdot x=0 \Rightarrow$$

$$\Rightarrow a\cdot b+a\cdot y-x\cdot b-x\cdot y-b\cdot a+b\cdot y-x\cdot a+x\cdot y-a\cdot y+a\cdot x-b\cdot y+b\cdot x=0 \Rightarrow 0=0.$$

Vježba 722

Dokaži: $(a-x)\cdot(b+y)+(b+x)\cdot(y-a)=(a+b)\cdot(y-x)$.

Rezultat: Dokaz analogan.

Zadatak 723 (Franjo, srednja škola)

Izračunaj: $\left(a-\frac{a^2+9}{6}\right):\left(\frac{3}{a}-\frac{a}{3}\right)$.

Rješenje 723

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a^2-2\cdot a\cdot b+b^2=(a-b)^2.$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c} \quad , \quad a^1=a.$$

$$a^n:a^m=a^{n-m} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \quad , \quad n\neq 0 \quad , \quad n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(a-\frac{a^2+9}{6}\right):\left(\frac{3}{a}-\frac{a}{3}\right)=\left(\frac{a}{1}-\frac{a^2+9}{6}\right):\left(\frac{3}{a}-\frac{a}{3}\right)=\frac{6\cdot a-\left(a^2+9\right)}{6}:\frac{9-a^2}{3\cdot a}=$$

$$=\frac{6\cdot a-a^2-9}{6}\cdot\frac{3\cdot a}{9-a^2}=\frac{-a^2+6\cdot a-9}{6}\cdot\frac{3\cdot a}{-a^2+9}=\frac{-\left(a^2-6\cdot a+9\right)}{6}\cdot\frac{3\cdot a}{-\left(a^2-9\right)}=$$

$$=\frac{a^2-6\cdot a+9}{6}\cdot\frac{3\cdot a}{a^2-9}=\frac{(a-3)^2}{6}\cdot\frac{3\cdot a}{(a-3)\cdot(a+3)}=\frac{(a-3)^2}{6}\cdot\frac{3\cdot a}{(a-3)\cdot(a+3)}=$$

$$=\frac{a-3}{2}\cdot\frac{a}{a+3}=\frac{a\cdot(a-3)}{2\cdot(a+3)}.$$

Vježba 723

Izračunaj: $\left(\frac{a^2+9}{6}-a\right):\left(\frac{a}{3}-\frac{3}{a}\right).$

Rezultat: $\frac{a\cdot(a-3)}{2\cdot(a+3)}.$

Zadatak 724 (Iva, gimnazija)

Izračunaj: $\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}.$

Rješenje 724

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^n:a^m=a^{n-m}.$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$a^n:a^m=a^{n-m} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}=\frac{a+b}{a\cdot b\cdot(a-b)}-\frac{a-b}{a\cdot b\cdot(a+b)}=\frac{(a+b)^2-(a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}=$$

$$=\frac{((a+b)-(a-b))\cdot((a+b)+(a-b))}{a\cdot b\cdot(a-b)\cdot(a+b)}=\frac{(a+b-a+b)\cdot(a+b+a-b)}{a\cdot b\cdot(a-b)\cdot(a+b)}=$$

$$=\frac{(a+b-a+b)\cdot(a+b+a-b)}{a\cdot b\cdot(a-b)\cdot(a+b)}=\frac{2\cdot b\cdot 2\cdot a}{a\cdot b\cdot(a-b)\cdot(a+b)}=\frac{4\cdot a\cdot b}{a\cdot b\cdot(a-b)\cdot(a+b)}=$$

$$=\frac{4\cdot a\cdot b}{a\cdot b\cdot(a-b)\cdot(a+b)}=\frac{4}{(a-b)\cdot(a+b)}=\frac{4}{a^2-b^2}.$$

2.inačica

$$\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}=\frac{a+b}{a\cdot b\cdot(a-b)}-\frac{a-b}{a\cdot b\cdot(a+b)}=\frac{(a+b)^2-(a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}=$$

$$=\frac{a^2+2\cdot a\cdot b+b^2-\left(a^2-2\cdot a\cdot b+b^2\right)}{a\cdot b\cdot(a-b)\cdot(a+b)}=\frac{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}{a\cdot b\cdot(a-b)\cdot(a+b)}=$$

$$=\frac{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}{a\cdot b\cdot(a-b)\cdot(a+b)}=\frac{2\cdot a\cdot b+2\cdot a\cdot b}{a\cdot b\cdot(a-b)\cdot(a+b)}=\frac{4\cdot a\cdot b}{a\cdot b\cdot(a-b)\cdot(a+b)}=$$

$$=\frac{4\cdot a\cdot b}{a\cdot b\cdot(a-b)\cdot(a+b)}=\frac{4}{(a-b)\cdot(a+b)}=\frac{4}{a^2-b^2}.$$

Vježba 724

Izračunaj: $\frac{a+b}{a^2\cdot b-a\cdot b^2}+\frac{b-a}{a^2\cdot b+a\cdot b^2}.$

Rezultat: $\frac{4}{a^2-b^2}.$

Zadatak 725 (Petra, gimnazija)

Pojednostavni izraz: $\frac{x^4+1-2\cdot x^2}{1-x-x^2+x^3}.$

Rješenje 725

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^2-2\cdot a\cdot b+b^2=(a-b)^2\quad,\quad a^1=a\quad,\quad a^n:a^m=a^{n-m}.$$

$$a^n\cdot a^m=a^{n+m}\quad,\quad (a\cdot b)^n=a^n\cdot b^n,\quad (a-b)^2=(b-a)^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b},\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{x^4+1-2\cdot x^2}{1-x-x^2+x^3}=\frac{x^4-2\cdot x^2+1}{1-x-x^2\cdot(1-x)}=\frac{\left(x^2\right)^2-2\cdot x^2+1}{(1-x)-x^2\cdot(1-x)}=\frac{\left(x^2-1\right)^2}{(1-x)-x^2\cdot(1-x)}=$$

$$=\frac{((x-1)\cdot(x+1))^2}{(1-x)\cdot\left(1-x^2\right)}=\frac{(x-1)^2\cdot(x+1)^2}{(1-x)\cdot(1-x)\cdot(1+x)}=\frac{(x-1)^2\cdot(x+1)^2}{(1-x)^2\cdot(1+x)}=\frac{(x-1)^2\cdot(x+1)^2}{(x-1)^2\cdot(1+x)}=$$

$$=\frac{(x-1)^2\cdot(x+1)^2}{(x-1)^2\cdot(x+1)}=x+1.$$

Vježba 725

Pojednostavni izraz: $\frac{2\cdot x^2-1-x^4}{x+x^2-1-x^3}.$

Rezultat: $x+1.$

Zadatak 726 (Zvonimir, srednja škola)

Pojednostavni izraz: $\left(\frac{a}{a-b}-\frac{b}{a+b}+\frac{2\cdot a\cdot b}{a^2-b^2}\right)\cdot\left(\frac{a}{a-b}-\frac{b}{a+b}-\frac{2\cdot a\cdot b}{a^2-b^2}\right).$

Rješenje 726

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad a^1=a.$$

$$a^n\cdot a^m=a^{n+m}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

$$a^2-2\cdot a\cdot b+b^2=(a-b)^2\quad,\quad a^n\cdot b^n=(a\cdot b)^n\quad,\quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$\left(a^n\right)^m=a^{n\cdot m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left(\frac{a}{a-b}-\frac{b}{a+b}+\frac{2\cdot a\cdot b}{a^2-b^2}\right)\cdot\left(\frac{a}{a-b}-\frac{b}{a+b}-\frac{2\cdot a\cdot b}{a^2-b^2}\right)=$$

$$=\left(\frac{a}{a-b}-\frac{b}{a+b}+\frac{2\cdot a\cdot b}{(a-b)\cdot(a+b)}\right)\cdot\left(\frac{a}{a-b}-\frac{b}{a+b}-\frac{2\cdot a\cdot b}{(a-b)\cdot(a+b)}\right)=$$

$$=\frac{a\cdot(a+b)-b\cdot(a-b)+2\cdot a\cdot b}{(a-b)\cdot(a+b)}\cdot\frac{a\cdot(a+b)-b\cdot(a-b)-2\cdot a\cdot b}{(a-b)\cdot(a+b)}=$$

$$=\frac{a^2+a\cdot b-a\cdot b+b^2+2\cdot a\cdot b}{a^2-b^2}\cdot\frac{a^2+a\cdot b-a\cdot b+b^2-2\cdot a\cdot b}{a^2-b^2}=$$

$$=\frac{a^2+a\cdot b-a\cdot b+b^2+2\cdot a\cdot b}{a^2-b^2}\cdot\frac{a^2+a\cdot b-a\cdot b+b^2-2\cdot a\cdot b}{a^2-b^2}=$$

$$=\frac{a^2+b^2+2\cdot a\cdot b}{a^2-b^2}\cdot\frac{a^2+b^2-2\cdot a\cdot b}{a^2-b^2}=\frac{a^2+2\cdot a\cdot b+b^2}{a^2-b^2}\cdot\frac{a^2-2\cdot a\cdot b+b^2}{a^2-b^2}=$$

$$=\frac{(a+b)^2}{a^2-b^2}\cdot\frac{(a-b)^2}{a^2-b^2}=\frac{(a+b)^2\cdot(a-b)^2}{\left(a^2-b^2\right)^2}=\frac{\left((a+b)\cdot(a-b)\right)^2}{\left(a^2-b^2\right)^2}=$$

$$=\frac{\left(a^2-b^2\right)^2}{\left(a^2-b^2\right)^2}=\frac{\left(a^2-b^2\right)^2}{\left(a^2-b^2\right)^2}=1.$$

2.inačica

$$\left(\frac{a}{a-b}-\frac{b}{a+b}+\frac{2\cdot a\cdot b}{a^2-b^2}\right)\cdot\left(\frac{a}{a-b}-\frac{b}{a+b}-\frac{2\cdot a\cdot b}{a^2-b^2}\right)=$$

$$=\left(\left(\frac{a}{a-b}-\frac{b}{a+b}\right)+\frac{2\cdot a\cdot b}{a^2-b^2}\right)\cdot\left(\left(\frac{a}{a-b}-\frac{b}{a+b}\right)-\frac{2\cdot a\cdot b}{a^2-b^2}\right)=$$

$$=\left(\frac{a}{a-b}-\frac{b}{a+b}\right)^2-\left(\frac{2\cdot a\cdot b}{a^2-b^2}\right)^2=\left(\frac{a\cdot(a+b)-b\cdot(a-b)}{(a-b)\cdot(a+b)}\right)^2-\frac{(2\cdot a\cdot b)^2}{\left(a^2-b^2\right)^2}=$$

$$=\left(\frac{a^2+a\cdot b-a\cdot b+b^2}{a^2-b^2}\right)^2-\frac{4\cdot a^2\cdot b^2}{\left(a^2-b^2\right)^2}=\left(\frac{a^2+a\cdot b-a\cdot b+b^2}{a^2-b^2}\right)^2-\frac{4\cdot a^2\cdot b^2}{\left(a^2-b^2\right)^2}=$$

$$=\left(\frac{a^2+b^2}{a^2-b^2}\right)^2-\frac{4\cdot a^2\cdot b^2}{\left(a^2-b^2\right)^2}=\frac{\left(a^2+b^2\right)^2}{\left(a^2-b^2\right)^2}-\frac{4\cdot a^2\cdot b^2}{\left(a^2-b^2\right)^2}=$$

$$=\frac{\left(a^2+b^2\right)^2-4\cdot a^2\cdot b^2}{\left(a^2-b^2\right)^2}=\frac{\left(a^2\right)^2+2\cdot a^2\cdot b^2+\left(b^2\right)^2-4\cdot a^2\cdot b^2}{\left(a^2-b^2\right)^2}=$$

$$=\frac{\left(a^2\right)^2-2\cdot a^2\cdot b^2+\left(b^2\right)^2}{\left(a^2-b^2\right)^2}=\frac{\left(a^2-b^2\right)^2}{\left(a^2-b^2\right)^2}=\frac{\left(a^2-b^2\right)^2}{\left(a^2-b^2\right)^2}=1.$$

Vježba 726

Pojednostavni izraz: $\left(\frac{a}{a-b}-\frac{b}{a+b}+\frac{2\cdot a\cdot b}{a^2-b^2}\right)\cdot\left(\frac{a}{a-b}-\frac{b}{a+b}+\frac{2\cdot a\cdot b}{b^2-a^2}\right).$

Rezultat: 1.

Zadatak 727 (Zvonimir, srednja škola)

Pojednostavni izraz: $\left(\frac{(a+b)^3}{3\cdot a\cdot b}-a-b\right)^n:\left(\frac{(a-b)^2}{a\cdot b}+1\right)^n.$

Rješenje 727

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad(a+b)^3=a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3.$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad a^n:b^n=(a:b)^n \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left(\frac{(a+b)^3}{3\cdot a\cdot b}-a-b\right)^n:\left(\frac{(a-b)^2}{a\cdot b}+1\right)^n=\left(\frac{(a+b)^3}{3\cdot a\cdot b}-\frac{a}{1}-\frac{b}{1}\right)^n:\left(\frac{(a-b)^2}{a\cdot b}+\frac{1}{1}\right)^n=$$

$$=\left(\frac{(a+b)^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2}{3\cdot a\cdot b}\right)^n:\left(\frac{(a-b)^2+a\cdot b}{a\cdot b}\right)^n=$$

$$=\left(\frac{a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2}{3\cdot a\cdot b}\right)^n:\left(\frac{a^2-2\cdot a\cdot b+b^2+a\cdot b}{a\cdot b}\right)^n=$$

$$=\left(\frac{a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2}{3\cdot a\cdot b}\right)^n:\left(\frac{a^2-a\cdot b+b^2}{a\cdot b}\right)^n=$$

$$=\left(\frac{a^3+b^3}{3\cdot a\cdot b}\right)^n:\left(\frac{a^2-a\cdot b+b^2}{a\cdot b}\right)^n=\left(\frac{a^3+b^3}{3\cdot a\cdot b}:\frac{a^2-a\cdot b+b^2}{a\cdot b}\right)^n=$$

$$=\left(\frac{a^3+b^3}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-a\cdot b+b^2}\right)^n=\left(\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-a\cdot b+b^2}\right)^n=$$

$$=\left(\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{3\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-a\cdot b+b^2}\right)^n=\left(\frac{a+b}{3}\cdot\frac{1}{1}\right)^n=\left(\frac{a+b}{3}\right)^n.$$

Vježba 727

Pojednostavni izraz: $\left(\frac{(a+b)^3}{3\cdot a\cdot b}-a-b\right)^n:\left(\frac{(b-a)^2}{a\cdot b}+1\right)^n.$

Rezultat: $\left(\frac{a+b}{3}\right)^n.$

Zadatak 728 (4B, TUPŠ)

Rastavite na faktore izraz: $x^5+5\cdot x^3+9\cdot x.$

Rješenje 728

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad \left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2 .$$

$$a^2 - b^2 = (a-b) \cdot (a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$x^5 + 5 \cdot x^3 + 9 \cdot x = x \cdot \left(x^4 + 5 \cdot x^2 + 9\right) = x \cdot \left(x^4 + 6 \cdot x^2 - x^2 + 9\right) =$$

$$= x \cdot \left(x^4 + 6 \cdot x^2 + 9 - x^2\right) = x \cdot \left(\left(x^2\right)^2 + 2 \cdot 3 \cdot x^2 + 3^2 - x^2\right) = x \cdot \left(\left(x^2 + 3\right)^2 - x^2\right) =$$

$$= x \cdot \left(\left(x^2 + 3\right) - x\right) \cdot \left(\left(x^2 + 3\right) + x\right) = x \cdot \left(x^2 + 3 - x\right) \cdot \left(x^2 + 3 + x\right) =$$

$$= x \cdot \left(x^2 - x + 3\right) \cdot \left(x^2 + x + 3\right).$$

Vježba 728

Rastavite na faktore izraz: $x^6 + 5 \cdot x^4 + 9 \cdot x^2$.

Rezultat: $x^2 \cdot \left(x^2 - x + 3\right) \cdot \left(x^2 + x + 3\right).$

Zadatak 729 (Ivan, gimnazija)

Izračunaj: $\left(4 - \frac{4 + a^2}{a}\right) : \left(\frac{1}{2} - \frac{1}{a}\right).$

Rješenje 729

Ponovimo!

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad \frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c} \quad , \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2 .$$

$$a^1 = a \quad , \quad \frac{a^n}{a^m} = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\left(4 - \frac{4 + a^2}{a}\right) : \left(\frac{1}{2} - \frac{1}{a}\right) = \left(\frac{4}{1} - \frac{4 + a^2}{a}\right) : \left(\frac{1}{2} - \frac{1}{a}\right) = \frac{4 \cdot a - \left(4 + a^2\right)}{a} : \frac{a - 2}{2 \cdot a} =$$

$$= \frac{4 \cdot a - 4 - a^2}{a} \cdot \frac{2 \cdot a}{a - 2} = \frac{-a^2 + 4 \cdot a - 4}{a} \cdot \frac{2 \cdot a}{a - 2} = \frac{-\left(a^2 - 4 \cdot a + 4\right)}{a} \cdot \frac{2 \cdot a}{a - 2} =$$

$$= \frac{-(a-2)^2}{a} \cdot \frac{2 \cdot a}{a - 2} = \frac{-(a-2)^2}{a} \cdot \frac{2 \cdot a}{a - 2} = \frac{-(a-2)}{1} \cdot \frac{2}{1} = -2 \cdot (a - 2) = 2 \cdot (2 - a).$$

Vježba 729

Izračunaj: $\left(\frac{4+a^2}{a}-4\right):\left(\frac{1}{a}-\frac{1}{2}\right)$.

Rezultat: $2\cdot(2-a)$.

Zadatak 730 (Ivan, gimnazija)

Izračunaj: $\left(\frac{b}{a}-\frac{a}{b}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)$.

Rješenje 730

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad (a-b)^2=(b-a)^2.$$

$$\left(\frac{b}{a}-\frac{a}{b}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)=$$

$$=\left(\frac{b^2-a^2}{a\cdot b}\right)^2+a^2-\left(\frac{a^2-b^2}{a\cdot b}\right)^2=\frac{\left(b^2-a^2\right)^2}{(a\cdot b)^2}+a^2-\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}=$$

$$=\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}+a^2-\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}=\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}+a^2-\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}=a^2.$$

Vježba 730

Izračunaj: $\left(\frac{a}{b}-\frac{b}{a}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)$.

Rezultat: a^2.

Zadatak 731 (Domagoj, srednja škola)

Prikaži u obliku potencije s bazom 10 sljedeći brojevni izraz: $2^6\cdot 5^4+6\cdot 10^4$.

Rješenje 731

Ponovimo!

$$a^n\cdot a^m=a^{n+m} \quad , \quad a^n\cdot b^n=(a\cdot b)^n \quad , \quad a^1=a.$$

$$2^6\cdot 5^4+6\cdot 10^4=2^2\cdot 2^4\cdot 5^4+6\cdot 10^4=2^2\cdot\left(2^4\cdot 5^4\right)+6\cdot 10^4=$$

$$=4\cdot(2\cdot 5)^4+6\cdot 10^4=4\cdot 10^4+6\cdot 10^4=10\cdot 10^4=10^1\cdot 10^4=10^5.$$

Vježba 731

Prikaži u obliku potencije s bazom 10 sljedeći brojevni izraz: $2^6\cdot 5^4+2\cdot 3\cdot 10^4$.

Rezultat: 10^5.

Zadatak 732 (Domagoj, srednja škola)

Dokaži: $(a-x)\cdot(b+y)-(b+x)\cdot(a-y)=(a+b)\cdot(y-x).$

Rješenje 732

Ponovimo!

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$(a-x)\cdot(b+y)-(b+x)\cdot(a-y)=a\cdot b+a\cdot y-x\cdot b-x\cdot y-(b\cdot a-b\cdot y+x\cdot a-x\cdot y)=$$

$$=a\cdot b+a\cdot y-x\cdot b-x\cdot y-b\cdot a+b\cdot y-x\cdot a+x\cdot y=$$

$$=a\cdot b+a\cdot y-x\cdot b-x\cdot y-b\cdot a+b\cdot y-x\cdot a+x\cdot y=a\cdot y-x\cdot b+b\cdot y-x\cdot a=$$

$$=(a\cdot y-x\cdot a)+(-x\cdot b+b\cdot y)=a\cdot(y-x)+b\cdot(-x+y)=$$

$$=a\cdot(y-x)+b\cdot(y-x)=(y-x)\cdot(a+b)=(a+b)\cdot(y-x).$$

2.inačica

$$(a-x)\cdot(b+y)-(b+x)\cdot(a-y)=a\cdot b+a\cdot y-x\cdot b-x\cdot y-(b\cdot a-b\cdot y+x\cdot a-x\cdot y)=$$

$$=a\cdot b+a\cdot y-x\cdot b-x\cdot y-b\cdot a+b\cdot y-x\cdot a+x\cdot y=$$

$$=a\cdot b+a\cdot y-x\cdot b-x\cdot y-b\cdot a+b\cdot y-x\cdot a+x\cdot y=a\cdot y-x\cdot b+b\cdot y-x\cdot a=$$

$$=(a\cdot y+b\cdot y)+(-x\cdot b-x\cdot a)=y\cdot(a+b)-x\cdot(b+a)=$$

$$=y\cdot(a+b)-x\cdot(a+b)=(a+b)\cdot(y-x).$$

Vježba 732

Dokaži: $(a-x)\cdot(b+y)+(b+x)\cdot(y-a)=(a+b)\cdot(y-x).$

Rezultat: Dokaz analogan.

Zadatak 733 (Domagoj, srednja škola)

Napiši u obliku umnoška sljedeći višečlani algebarski izraz:

$$a^2\cdot b^3-a\cdot b\cdot c^2\cdot d-a\cdot b^2\cdot c\cdot d+c^3\cdot d^2.$$

Rješenje 733

Ponovimo!

$$a^1=a \quad , \quad a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$a^2\cdot b^3-a\cdot b\cdot c^2\cdot d-a\cdot b^2\cdot c\cdot d+c^3\cdot d^2=$$

$$=\left(a^2\cdot b^3-a\cdot b\cdot c^2\cdot d\right)+\left(-a\cdot b^2\cdot c\cdot d+c^3\cdot d^2\right)=$$

$$= a\cdot b\cdot\left(a\cdot b^2-c^2\cdot d\right)-c\cdot d\cdot\left(a\cdot b^2-c^2\cdot d\right)=$$

$$=\left(a\cdot b^2-c^2\cdot d\right)\cdot\left(a\cdot b-c\cdot d\right)=\left(a\cdot b-c\cdot d\right)\cdot\left(a\cdot b^2-c^2\cdot d\right).$$

2.inačica

$$a^2\cdot b^3-a\cdot b\cdot c^2\cdot d-a\cdot b^2\cdot c\cdot d+c^3\cdot d^2=$$

$$=\left(a^2\cdot b^3-a\cdot b^2\cdot c\cdot d\right)+\left(-a\cdot b\cdot c^2\cdot d+c^3\cdot d^2\right)=$$

$$=a\cdot b^2\cdot\left(a\cdot b-c\cdot d\right)-c^2\cdot d\cdot\left(a\cdot b-c\cdot d\right)=\left(a\cdot b-c\cdot d\right)\cdot\left(a\cdot b^2-c^2\cdot d\right).$$

Vježba 733

Napiši u obliku umnoška sljedeći višečlani algebarski izraz:

$$a^2\cdot b^3-a\cdot b^2\cdot c\cdot d-a\cdot b\cdot c^2\cdot d+c^3\cdot d^2.$$

Rezultat: $\left(a\cdot b-c\cdot d\right)\cdot\left(a\cdot b^2-c^2\cdot d\right).$

Zadatak 734 (Sofija ☺, gimnazija)

Dokaži da iz $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}=0$ slijedi $\left(a+b+c\right)^3=27\cdot a\cdot b\cdot c$ (a, b, c su realni brojevi).

Rješenje 734

Ponovimo!

$$\left(\sqrt[3]{a}\right)^3=a \quad , \quad \left(-a\right)^3=-a^3 \quad , \quad \sqrt[3]{a^3}=a \quad , \quad \left(a+b\right)^3=a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3.$$

$$a^1=a \quad , \quad a^n:a^m=a^{n-m} \quad , \quad \sqrt[3]{a}\cdot\sqrt[3]{b}=\sqrt[3]{a\cdot b} \quad , \quad \left(a\cdot b\right)^3=a^3\cdot b^3.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

1.inačica

Preoblikujemo zadanu jednadžbu.

$$\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}=0 \Rightarrow \sqrt[3]{a}+\sqrt[3]{b}=-\sqrt[3]{c} \Rightarrow \sqrt[3]{a}+\sqrt[3]{b}=-\sqrt[3]{c} \;/^3 \Rightarrow$$

$$\Rightarrow \left(\sqrt[3]{a}+\sqrt[3]{b}\right)^3=\left(-\sqrt[3]{c}\right)^3 \Rightarrow$$

$$\Rightarrow \left(\sqrt[3]{a}\right)^3+3\cdot\left(\sqrt[3]{a}\right)^2\cdot\sqrt[3]{b}+3\cdot\sqrt[3]{a}\cdot\left(\sqrt[3]{b}\right)^2+\left(\sqrt[3]{b}\right)^3=-\left(\sqrt[3]{c}\right)^3 \Rightarrow$$

$$\Rightarrow a+3\cdot\sqrt[3]{a}\cdot\sqrt[3]{b}\cdot\left(\sqrt[3]{a}+\sqrt[3]{b}\right)+b=-c \Rightarrow \left[\sqrt[3]{a}+\sqrt[3]{b}=-\sqrt[3]{c}\right] \Rightarrow$$

$$\Rightarrow a+3\cdot\sqrt[3]{a}\cdot\sqrt[3]{b}\cdot\left(-\sqrt[3]{c}\right)+b=-c \Rightarrow a-3\cdot\sqrt[3]{a}\cdot\sqrt[3]{b}\cdot\sqrt[3]{c}+b=-c \Rightarrow$$

$$\Rightarrow a-3\cdot\sqrt[3]{a\cdot b\cdot c}+b=-c \Rightarrow a+b+c=3\cdot\sqrt[3]{a\cdot b\cdot c} \Rightarrow a+b+c=3\cdot\sqrt[3]{a\cdot b\cdot c} \;/^3 \Rightarrow$$

$$\Rightarrow \left(a+b+c\right)^3=\left(3\cdot\sqrt[3]{a\cdot b\cdot c}\right)^3 \Rightarrow \left(a+b+c\right)^3=3^3\cdot\left(\sqrt[3]{a\cdot b\cdot c}\right)^3 \Rightarrow \left(a+b+c\right)^3=27\cdot a\cdot b\cdot c.$$

2.inačica

Uvedemo zamjene radi lakšeg računanja.

$$\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}=0 \Rightarrow \begin{bmatrix} a=m^3 \\ b=n^3 \\ c=t^3 \end{bmatrix} \Rightarrow \sqrt[3]{m^3}+\sqrt[3]{n^3}+\sqrt[3]{t^3}=0 \Rightarrow m+n+t=0 \Rightarrow$$

$$\Rightarrow m+n=-t \Rightarrow m+n=-t \;/^3 \Rightarrow (m+n)^3=(-t)^3 \Rightarrow$$

$$\Rightarrow m^3+3\cdot m^2\cdot n+3\cdot m\cdot n^2+n^3=-t^3 \Rightarrow m^3+3\cdot m\cdot n\cdot(m+n)+n^3=-t^3 \Rightarrow$$

$$\Rightarrow [m+n=-t] \Rightarrow m^3+3\cdot m\cdot n\cdot(-t)+n^3=-t^3 \Rightarrow m^3-3\cdot m\cdot n\cdot t+n^3=-t^3 \Rightarrow$$

$$\Rightarrow m^3+n^3+t^3=3\cdot m\cdot n\cdot t \Rightarrow \begin{bmatrix} a=m^3 \Rightarrow m=\sqrt[3]{a} \\ b=n^3 \Rightarrow n=\sqrt[3]{b} \\ c=t^3 \Rightarrow t=\sqrt[3]{c} \end{bmatrix} \Rightarrow a+b+c=3\cdot\sqrt[3]{a}\cdot\sqrt[3]{b}\cdot\sqrt[3]{c} \Rightarrow$$

$$\Rightarrow a+b+c=3\cdot\sqrt[3]{a\cdot b\cdot c} \Rightarrow a+b+c=3\cdot\sqrt[3]{a\cdot b\cdot c} \;/^3 \Rightarrow$$

$$\Rightarrow (a+b+c)^3=\left(3\cdot\sqrt[3]{a\cdot b\cdot c}\right)^3 \Rightarrow (a+b+c)^3=3^3\cdot\left(\sqrt[3]{a\cdot b\cdot c}\right)^3 \Rightarrow (a+b+c)^3=27\cdot a\cdot b\cdot c.$$

Vježba 734

Dokaži da iz $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}=0$ slijedi $(a+b+c)=3\cdot\sqrt[3]{a\cdot b\cdot c}$ (a, b, c su realni brojevi).

Rezultat: Dokaz analogan.

Zadatak 735 (Tomislav, gimnazija)

Zadan je izraz $\left(\frac{a+1}{5\cdot a-a^2}+\frac{2\cdot a+2}{a^2-25}\right):\frac{a+1}{a^2+10\cdot a+25}$ za sve vrijednosti realnog broja a za koje je definiran. Odredite **brojnik** do kraja skraćenoga razlomka nakon provedenih računskih operacija u zadanome izrazu.

$A.\ a \qquad B.\ a+5 \qquad C.\ a+9 \qquad D.\ 6$

Rješenje 735

Ponovimo!

$$a^1=a \quad , \quad a^n:a^m=a^{n-m} \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c} \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$a^2+2\cdot a\cdot b+b^2=(a+b)^2 \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left(\frac{a+1}{5\cdot a-a^2}+\frac{2\cdot a+2}{a^2-25}\right):\frac{a+1}{a^2+10\cdot a+25}=\left(\frac{a+1}{5\cdot a-a^2}-\frac{2\cdot a+2}{25-a^2}\right)\cdot\frac{a^2+10\cdot a+25}{a+1}=$$

$$=\left(\frac{a+1}{a\cdot(5-a)}-\frac{2\cdot(a+1)}{(5-a)\cdot(5+a)}\right)\cdot\frac{(a+5)^2}{a+1}=\frac{(a+1)\cdot(5+a)-2\cdot(a+1)\cdot a}{a\cdot(5-a)\cdot(5+a)}\cdot\frac{(a+5)^2}{a+1}=$$

$$=\frac{(a+1)\cdot(5+a-2\cdot a)}{a\cdot(5-a)\cdot(5+a)}\cdot\frac{(a+5)^2}{a+1}=\frac{(a+1)\cdot(5-a)}{a\cdot(5-a)\cdot(5+a)}\cdot\frac{(a+5)^2}{a+1}=\frac{(a+1)\cdot(5-a)}{a\cdot(5-a)\cdot(5+a)}\cdot\frac{(a+5)^2}{a+1}=$$

$$=\frac{1}{a}\cdot\frac{5+a}{1}=\frac{5+a}{a}=\frac{a+5}{a}.$$

Brojnik skraćenoga razlomka je a + 5.
Odgovor je pod B.

Vježba 735

Zadan je izraz $\left(\frac{a+1}{5\cdot a-a^2}-\frac{2\cdot a+2}{25-a^2}\right):\frac{a+1}{a^2+10\cdot a+25}$ za sve vrijednosti realnog broja a za koje je definiran. Odredite **brojnik** do kraja skraćenoga razlomka nakon provedenih računskih operacija u zadanome izrazu.

A. a *B.* $a+5$ *C.* $a+9$ *D.* 6

Rezultat: B.

Zadatak 736 (4B, TUPŠ)

Izračunaj $\frac{a^{-2}-b^{-2}}{a^{-1}+b^{-1}}$.

A. $\frac{b-a}{a\cdot b}$ *B.* $\frac{a\cdot b}{b-a}$ *C.* $a-b$ *D.* $\frac{a+b}{a-b}$

Rješenje 736

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad a^1=a\quad,\quad a^{-n}=\frac{1}{a^n}.$$

$$\frac{n}{1}=n\quad,\quad\frac{a^n}{a^m}=a^{n-m}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq0\ ,\ n\neq1.$$

1.inačica

$$\frac{a^{-2}-b^{-2}}{a^{-1}+b^{-1}}=\frac{\left(a^{-1}\right)^2-\left(b^{-1}\right)^2}{a^{-1}+b^{-1}}=\frac{\left(a^{-1}-b^{-1}\right)\cdot\left(a^{-1}+b^{-1}\right)}{a^{-1}+b^{-1}}=\frac{\left(a^{-1}-b^{-1}\right)\cdot\left(a^{-1}+b^{-1}\right)}{a^{-1}+b^{-1}}=$$

$$=\frac{a^{-1}-b^{-1}}{1}=a^{-1}-b^{-1}=\frac{1}{a}-\frac{1}{b}=\frac{b-a}{a\cdot b}.$$

Odgovor je pod A.

2.inačica

$$\frac{a^{-2}-b^{-2}}{a^{-1}+b^{-1}}=\frac{\frac{1}{a^2}-\frac{1}{b^2}}{\frac{1}{a}+\frac{1}{b}}=\frac{\frac{b^2-a^2}{a^2\cdot b^2}}{\frac{b+a}{a\cdot b}}=\frac{a\cdot b\cdot\left(b^2-a^2\right)}{a^2\cdot b^2\cdot(b+a)}=\frac{a\cdot b\cdot(b-a)\cdot(b+a)}{a^2\cdot b^2\cdot(b+a)}=$$

$$=\frac{a\cdot b\cdot(b-a)\cdot(b+a)}{a^2\cdot b^2\cdot(b+a)}=\frac{b-a}{a\cdot b}.$$

Odgovor je pod A.

Vježba 736

Izračunaj $\frac{a^{-1}+b^{-1}}{a^{-2}-b^{-2}}$.

$A.\ \frac{b-a}{a\cdot b}$ $\qquad B.\ \frac{a\cdot b}{b-a}$ $\qquad C.\ a-b$ $\qquad D.\ \frac{a+b}{a-b}$

Rezultat: B.

Zadatak 737 (Mirta, srednja škola)

Ako je $\frac{a}{b}=\frac{b}{c}$, onda je $\frac{a^2+b^2}{b^2+c^2}$ jednako:

$A.\ \frac{b}{c}$ $\qquad B.\ \frac{c}{a}$ $\qquad C.\ \frac{a}{c}$ $\qquad D.\ \frac{c}{b}$

Rješenje 737

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^n:a^m=a^{n-m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{a}{b}=\frac{b}{c}\ \Rightarrow\ \frac{a}{b}=\frac{b}{c}\ /\cdot b\cdot c\ \Rightarrow\ a\cdot c=b^2\ \Rightarrow\ b^2=a\cdot c.$$

Sada je:

$$\frac{a^2+b^2}{b^2+c^2}=\left[b^2=a\cdot c\right]=\frac{a^2+a\cdot c}{a\cdot c+c^2}=\frac{a\cdot(a+c)}{c\cdot(a+c)}=\frac{a\cdot(a+c)}{c\cdot(a+c)}=\frac{a}{c}.$$

Odgovor je pod C.

Vježba 737

Ako je $\frac{a}{b}=\frac{b}{c}$, onda je $\frac{b^2+c^2}{a^2+b^2}$ jednako:

$$A.\ \frac{b}{c} \qquad B.\ \frac{c}{a} \qquad C.\ \frac{a}{c} \qquad D.\ \frac{c}{b}$$

Rezultat: B.

Zadatak 738 (Mario, gimnazija)

Jednostavniji zapis algebarskog izraza $\left(1-\frac{a^2-b^2-c^2}{2\cdot b\cdot c}\right):\left(\frac{1}{a\cdot b}+\frac{1}{b\cdot c}+\frac{1}{a\cdot c}\right)$ jest:

$$A.\ \frac{c\cdot(a+b-c)}{2} \qquad B.\ \frac{b\cdot(a+c-b)}{2} \qquad C.\ \frac{a\cdot(b+c-a)}{2} \qquad D.\ \frac{1}{a\cdot b\cdot c}$$

Rješenje 738

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}, \ n\neq 0 \ , \ n\neq 1.$$

$$\left(1-\frac{a^2-b^2-c^2}{2\cdot b\cdot c}\right):\left(\frac{1}{a\cdot b}+\frac{1}{b\cdot c}+\frac{1}{a\cdot c}\right)=\left(\frac{1}{1}-\frac{a^2-b^2-c^2}{2\cdot b\cdot c}\right):\left(\frac{1}{a\cdot b}+\frac{1}{b\cdot c}+\frac{1}{a\cdot c}\right)=$$

$$=\frac{2\cdot b\cdot c-\left(a^2-b^2-c^2\right)}{2\cdot b\cdot c}:\frac{c+a+b}{a\cdot b\cdot c}=\frac{2\cdot b\cdot c-a^2+b^2+c^2}{2\cdot b\cdot c}:\frac{a+b+c}{a\cdot b\cdot c}=$$

$$=\frac{b^2+2\cdot b\cdot c+c^2-a^2}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=\frac{(b+c)^2-a^2}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=$$

$$=\frac{(b+c-a)\cdot(b+c+a)}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=\frac{(b+c-a)\cdot(a+b+c)}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=$$

$$=\frac{(b+c-a)\cdot(a+b+c)}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a+b+c}=\frac{b+c-a}{2}\cdot\frac{a}{1}=\frac{a\cdot(b+c-a)}{2}.$$

Odgovor je pod C.

Vježba 738

Jednostavniji zapis algebarskog izraza $\left(1+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}\right):\left(\frac{1}{a\cdot b}+\frac{1}{b\cdot c}+\frac{1}{a\cdot c}\right)$ jest:

$$A. \frac{c\cdot(a+b-c)}{2} \qquad B. \frac{b\cdot(a+c-b)}{2} \qquad C. \frac{a\cdot(b+c-a)}{2} \qquad D. \frac{1}{a\cdot b\cdot c}$$

Rezultat: C.

Zadatak 739 (Antonio, gimnazija)

Ako je $x\cdot y\cdot z=1$, onda je $\frac{x}{x\cdot y+x+1}+\frac{y}{y\cdot z+y+1}+\frac{z}{x\cdot z+z+1}=1$. Dokazati!

Rješenje 739

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \;,\; n\neq 0 \;,\; n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{x}{x\cdot y+x+1}+\frac{y}{y\cdot z+y+1}+\frac{z}{x\cdot z+z+1}=\frac{x}{x\cdot y+x+1}\cdot\frac{z}{z}+\frac{y}{y\cdot z+y+1}\cdot\frac{x\cdot z}{x\cdot z}+\frac{z}{x\cdot z+z+1}=$$

$$=\frac{x\cdot z}{z\cdot(x\cdot y+x+1)}+\frac{x\cdot y\cdot z}{x\cdot z\cdot(y\cdot z+y+1)}+\frac{z}{x\cdot z+z+1}=$$

$$=\frac{x\cdot z}{x\cdot y\cdot z+x\cdot z+z}+\frac{x\cdot y\cdot z}{x\cdot y\cdot z\cdot z+x\cdot y\cdot z+x\cdot z}+\frac{z}{x\cdot z+z+1}=\left[\begin{array}{c}\text{uvjet}\\ x\cdot y\cdot z=1\end{array}\right]=$$

$$=\frac{x\cdot z}{1+x\cdot z+z}+\frac{1}{1\cdot z+1+x\cdot z}+\frac{z}{x\cdot z+z+1}=\frac{x\cdot z}{x\cdot z+z+1}+\frac{1}{z+1+x\cdot z}+\frac{z}{x\cdot z+z+1}=$$

$$=\frac{x\cdot z}{x\cdot z+z+1}+\frac{1}{x\cdot z+z+1}+\frac{z}{x\cdot z+z+1}=\frac{x\cdot z+1+z}{x\cdot z+z+1}=\frac{x\cdot z+z+1}{x\cdot z+z+1}=\frac{x\cdot z+z+1}{x\cdot z+z+1}=1.$$

Vježba 739

Ako je $x\cdot y\cdot z=1$, onda je $\frac{x}{x\cdot y+x+1}+\frac{y}{y\cdot z+y+1}+\frac{z}{x\cdot z+z+1}-1=0$. Dokazati!

Rezultat: Dokaz analogan.

Zadatak 740 (Marija, srednja škola)

Napiši u obliku umnoška: $-2\cdot x^4\cdot y+12\cdot x^3\cdot y-18\cdot x^2\cdot y$.

Rješenje 740

Ponovimo!

$$a^1=a \quad , \quad a^n:a^m=a^{n-m} \quad , \quad a^2-2\cdot a\cdot b+b^2=(a-b)^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$-2 \cdot x^4 \cdot y + 12 \cdot x^3 \cdot y - 18 \cdot x^2 \cdot y = \left[\text{ izlučimo } -2 \cdot x^2 \cdot y \right] =$$

$$= -2 \cdot x^2 \cdot y \cdot \left(x^2 - 6 \cdot x + 9 \right) = -2 \cdot x^2 \cdot y \cdot \left(x^2 - 2 \cdot 3 \cdot x + 3^2 \right) = -2 \cdot x^2 \cdot y \cdot (x-3)^2 .$$

Vježba 740

Napiši u obliku umnoška: $2 \cdot x^4 \cdot y + 12 \cdot x^3 \cdot y + 18 \cdot x^2 \cdot y.$

Rezultat: $2 \cdot x^2 \cdot y \cdot (x+3)^2 .$