Zadatak 641 (4A, 4B, TUPŠ)

Vrijednost razlomka $\frac{10^{13}-10^{9}}{10^{11}-10^{9}}$ jednaka je:

$A.\ 10^{9}$ $\quad B.\ 99$ $\quad C.\ 100$ $\quad D.\ 101$

Rješenje 641

Ponovimo!

$$a^{n}:a^{m}=a^{n-m} \quad , \quad \left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{2}-b^{2}=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

$$\frac{10^{13}-10^{9}}{10^{11}-10^{9}}=\frac{10^{9}\cdot\left(10^{4}-1\right)}{10^{9}\cdot\left(10^{2}-1\right)}=\frac{10^{9}\cdot\left(10^{4}-1\right)}{10^{9}\cdot\left(10^{2}-1\right)}=\frac{10^{4}-1}{10^{2}-1}=\frac{\left(10^{2}\right)^{2}-1}{10^{2}-1}=$$

$$=\frac{\left(10^{2}-1\right)\cdot\left(10^{2}+1\right)}{10^{2}-1}=\frac{\left(10^{2}-1\right)\cdot\left(10^{2}+1\right)}{10^{2}-1}=10^{2}+1=100+1=101.$$

Odgovor je pod D.

Vježba 641

Vrijednost razlomka $\frac{10^{12}-10^{8}}{10^{10}-10^{8}}$ jednaka je:

$A.\ 10^{9}$ $\quad B.\ 99$ $\quad C.\ 100$ $\quad D.\ 101$

Rezultat: D.

Zadatak 642 (4A, 4B, TUPŠ)

Racionaliziraj nazivnik u razlomku $\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}$.

Rješenje 642

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad (a-b)\cdot(a+b)=a^{2}-b^{2} \quad , \quad \left(\sqrt[4]{a}\right)^{2}=\sqrt{a} \quad , \quad \left(\sqrt{a}\right)^{2}=a.$$

$$\frac{n}{1}=n.$$

$$\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}\cdot\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt[4]{3}+\sqrt[4]{2}}=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\left(\sqrt[4]{3}-\sqrt[4]{2}\right)\cdot\left(\sqrt[4]{3}+\sqrt[4]{2}\right)}=$$

$$=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\left(\sqrt[4]{3}\right)^{2}-\left(\sqrt[4]{2}\right)^{2}}=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt{3}-\sqrt{2}}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=$$

$$=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2}=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{3-2}=$$

$$=\frac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)}{1}=\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right).$$

Vježba 642

Racionaliziraj nazivnik u razlomku $\frac{1}{\sqrt[4]{3}+\sqrt[4]{2}}$.

Rezultat: $\left(\sqrt[4]{3}-\sqrt[4]{2}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right).$

Zadatak 643 (4A, 4B, TUPŠ)

Čemu je jednako M ako je $v=\sqrt{\frac{T}{3\cdot M}}$?

Rješenje 643

Ponovimo!

$$\left(\sqrt{a}\right)^2=a.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\quad,\quad n\neq 0\quad,\quad n\neq 1.$$

$$v=\sqrt{\frac{T}{3\cdot M}}\ \Rightarrow\ v=\sqrt{\frac{T}{3\cdot M}}\ /^2\ \Rightarrow\ v^2=\left(\sqrt{\frac{T}{3\cdot M}}\right)^2\ \Rightarrow\ v^2=\frac{T}{3\cdot M}\ \Rightarrow$$

$$\Rightarrow\ v^2=\frac{T}{3\cdot M}\ /\cdot\frac{M}{v^2}\ \Rightarrow\ M=\frac{T}{3\cdot v^2}.$$

Vježba 643

Čemu je jednako T ako je $v=\sqrt{\frac{T}{3\cdot M}}$?

Rezultat: $T=3\cdot M\cdot v^2.$

Zadatak 644 (Vinko, srednja škola)

Pojednostavni $\frac{\left(\sqrt[3]{x}+4\right)^2-1}{x+125}\cdot\frac{1}{\sqrt[3]{x^2}-9}$.

Rješenje 644

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\left(\sqrt[n]{a}\right)^p=\sqrt[n]{a^p}\quad,\quad\left(\sqrt[n]{a}\right)^n=a.$$

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right)\quad,\quad a^1=a\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

$$\frac{\left(\sqrt[3]{x}+4\right)^2 - 1}{x+125} \cdot \frac{1}{\sqrt[3]{x^2}-9} = \frac{\left(\left(\sqrt[3]{x}+4\right)-1\right) \cdot \left(\left(\sqrt[3]{x}+4\right)+1\right)}{\left(\sqrt[3]{x}\right)^3 + 5^3} \cdot \frac{1}{\left(\sqrt[3]{x}\right)^2 - 3^2} =$$

$$= \frac{\left(\sqrt[3]{x}+4-1\right) \cdot \left(\sqrt[3]{x}+4+1\right)}{\left(\sqrt[3]{x}\right)^3 + 5^3} \cdot \frac{1}{\left(\sqrt[3]{x}\right)^2 - 3^2} =$$

$$= \frac{\left(\sqrt[3]{x}+3\right) \cdot \left(\sqrt[3]{x}+5\right)}{\left(\sqrt[3]{x}+5\right)\left(\left(\sqrt[3]{x}\right)^2 - 5 \cdot \sqrt[3]{x} + 5^2\right)} \cdot \frac{1}{\left(\sqrt[3]{x}-3\right) \cdot \left(\sqrt[3]{x}+3\right)} =$$

$$= \frac{\left(\sqrt[3]{x}+3\right) \cdot \left(\sqrt[3]{x}+5\right)}{\left(\sqrt[3]{x}+5\right)\left(\sqrt[3]{x^2} - 5 \cdot \sqrt[3]{x} + 25\right)} \cdot \frac{1}{\left(\sqrt[3]{x}-3\right) \cdot \left(\sqrt[3]{x}+3\right)} =$$

$$= \frac{\left(\sqrt[3]{x}+3\right) \cdot \left(\sqrt[3]{x}+5\right)}{\left(\sqrt[3]{x}+5\right)\left(\sqrt[3]{x^2} - 5 \cdot \sqrt[3]{x} + 25\right)} \cdot \frac{1}{\left(\sqrt[3]{x}-3\right) \cdot \left(\sqrt[3]{x}+3\right)} = \frac{1}{\sqrt[3]{x^2} - 5 \cdot \sqrt[3]{x} + 25} \cdot \frac{1}{\sqrt[3]{x}-3} =$$

$$= \frac{1}{\left(\sqrt[3]{x}-3\right) \cdot \left(\sqrt[3]{x^2} - 5 \cdot \sqrt[3]{x} + 25\right)} = \frac{1}{\left(\sqrt[3]{x}-3\right) \cdot \left(\left(\sqrt[3]{x}\right)^2 - 5 \cdot \sqrt[3]{x} + 25\right)} =$$

$$= \frac{1}{\left(\sqrt[3]{x}\right)^3 - 5 \cdot \left(\sqrt[3]{x}\right)^2 + 25 \cdot \sqrt[3]{x} - 3 \cdot \left(\sqrt[3]{x}\right)^2 + 15 \cdot \sqrt[3]{x} - 75} =$$

$$= \frac{1}{\left(\sqrt[3]{x}\right)^3 - 8 \cdot \left(\sqrt[3]{x}\right)^2 + 40 \cdot \sqrt[3]{x} - 75} = \frac{1}{x - 8 \cdot \sqrt[3]{x^2} + 40 \cdot \sqrt[3]{x} - 75}.$$

Vježba 644

Pojednostavni $\frac{\left(\sqrt[3]{x}+4\right)^2 - 1}{\sqrt[3]{x^2}-9} \cdot \frac{1}{x+125}.$

Rezultat: $\frac{1}{x - 8 \cdot \sqrt[3]{x^2} + 40 \cdot \sqrt[3]{x} - 75}.$

Zadatak 645 (Branimir, srednja škola)

Dokažite jednakost $\frac{4}{\sqrt{6}+\sqrt{2}} + \frac{3}{\sqrt{5}-\sqrt{2}} = \frac{1}{\sqrt{6}-\sqrt{5}}.$

Rješenje 645

Ponovimo!

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \quad , \quad n\neq 0 \quad , \quad n\neq 1.$$

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \frac{n}{1}=n.$$

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right) \quad , \quad a^1=a \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \quad , \quad n\neq 0 \quad , \quad n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$a-b=0 \quad \Rightarrow \quad a=b.$$

Ako su dva izraza jednaka njihova je razlika jednaka nuli. Zato preoblikujemo zadanu jednakost.

$$\frac{4}{\sqrt{6}+\sqrt{2}}+\frac{3}{\sqrt{5}-\sqrt{2}}=\frac{1}{\sqrt{6}-\sqrt{5}} \quad \Rightarrow \quad \frac{4}{\sqrt{6}+\sqrt{2}}+\frac{3}{\sqrt{5}-\sqrt{2}}-\frac{1}{\sqrt{6}-\sqrt{5}}=0.$$

Dalje slijedi:

$$\frac{4}{\sqrt{6}+\sqrt{2}}+\frac{3}{\sqrt{5}-\sqrt{2}}-\frac{1}{\sqrt{6}-\sqrt{5}}=$$

$$=\frac{4}{\sqrt{6}+\sqrt{2}}\cdot\frac{\sqrt{6}-\sqrt{2}}{\sqrt{6}-\sqrt{2}}+\frac{3}{\sqrt{5}-\sqrt{2}}\cdot\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}}-\frac{1}{\sqrt{6}-\sqrt{5}}\cdot\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}+\sqrt{5}}=$$

$$=\frac{4\cdot\left(\sqrt{6}-\sqrt{2}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{2}\right)^2}+\frac{3\cdot\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}-\frac{\sqrt{6}+\sqrt{5}}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}=$$

$$=\frac{4\cdot\left(\sqrt{6}-\sqrt{2}\right)}{6-2}+\frac{3\cdot\left(\sqrt{5}+\sqrt{2}\right)}{5-2}-\frac{\sqrt{6}+\sqrt{5}}{6-5}=\frac{4\cdot\left(\sqrt{6}-\sqrt{2}\right)}{4}+\frac{3\cdot\left(\sqrt{5}+\sqrt{2}\right)}{3}-\frac{\sqrt{6}+\sqrt{5}}{1}=$$

$$=\frac{4\cdot\left(\sqrt{6}-\sqrt{2}\right)}{4}+\frac{3\cdot\left(\sqrt{5}+\sqrt{2}\right)}{3}-\frac{\sqrt{6}+\sqrt{5}}{1}=\sqrt{6}-\sqrt{2}+\sqrt{5}+\sqrt{2}-\left(\sqrt{6}+\sqrt{5}\right)=$$

$$=\sqrt{6}-\sqrt{2}+\sqrt{5}+\sqrt{2}-\sqrt{6}-\sqrt{5}=\sqrt{6}-\sqrt{2}+\sqrt{5}+\sqrt{2}-\sqrt{6}-\sqrt{5}=0.$$

Vježba 645

Dokažite jednakost $\frac{4}{\sqrt{6}+\sqrt{2}}-\frac{3}{\sqrt{2}-\sqrt{5}}=\frac{1}{\sqrt{6}-\sqrt{5}}.$

Rezultat: Dokaz analogan.

Zadatak 646 (Nessy, FPMOZ)

Racionaliziraj nazivnik u razlomku $\frac{a}{\sqrt{a^2 \cdot \sqrt[3]{a}}}$.

Rješenje 646

Ponovimo!

$$a^n \cdot \sqrt[m]{b} = \sqrt[m]{a^{n \cdot m} \cdot b} \quad , \quad \left(\sqrt[n]{a}\right)^n = a \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a} \quad , \quad \sqrt[n]{a^n} = a \; , \; a \ge 0 \quad , \quad \sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \ne 0 \; , \; n \ne 1.$$

1.inačica

$$\frac{a}{\sqrt{a^2 \cdot \sqrt[3]{a}}} = \frac{a}{\sqrt{\sqrt[3]{a^6} \cdot a}} = \frac{a}{\sqrt[6]{a^6 \cdot a}} = \frac{a}{\sqrt[6]{a^6} \cdot \sqrt[6]{a}} = \frac{a}{a \cdot \sqrt[6]{a}} = \frac{a}{a \cdot \sqrt[6]{a}} = \frac{1}{\sqrt[6]{a}} =$$

$$= \begin{bmatrix} \text{racionalizacija} \\ \text{nazivnika} \end{bmatrix} = \frac{1}{\sqrt[6]{a}} \cdot \frac{\sqrt[6]{a^5}}{\sqrt[6]{a^5}} = \frac{\sqrt[6]{a^5}}{\sqrt[6]{a \cdot a^5}} = \frac{\sqrt[6]{a^5}}{\sqrt[6]{a^6}} = \frac{\sqrt[6]{a^5}}{a}.$$

2.inačica

$$\frac{a}{\sqrt{a^2 \cdot \sqrt[3]{a}}} = \frac{a}{\sqrt{a^2} \cdot \sqrt{\sqrt[3]{a}}} = \frac{a}{a \cdot \sqrt{\sqrt[3]{a}}} = \frac{a}{a \cdot \sqrt{\sqrt[3]{a}}} = \frac{1}{\sqrt{\sqrt[3]{a}}} = \frac{1}{\sqrt[6]{a}} =$$

$$= \begin{bmatrix} \text{racionalizacija} \\ \text{nazivnika} \end{bmatrix} = \frac{1}{\sqrt[6]{a}} \cdot \frac{\sqrt[6]{a^5}}{\sqrt[6]{a^5}} = \frac{\sqrt[6]{a^5}}{\sqrt[6]{a \cdot a^5}} = \frac{\sqrt[6]{a^5}}{\sqrt[6]{a^6}} = \frac{\sqrt[6]{a^5}}{a}.$$

Vježba 646

Racionaliziraj nazivnik u razlomku $\frac{a}{\sqrt[3]{a^3 \cdot \sqrt{a}}}$.

Rezultat: $\frac{\sqrt[6]{a^5}}{a}$.

Zadatak 647 (Željko, srednja škola)

Izračunajte $\frac{\left(a^2 + a \cdot b\right)^2}{a^2 - b^2} : \frac{(a+b)^2}{\left(a \cdot b - b^2\right)^2} : \left(b^2 \cdot a - b^3\right)$.

Rješenje 647

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$(a \cdot b)^n = a^n \cdot b^n \quad , \quad \frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c} \quad , \quad n = \frac{n}{1} \quad , \quad a^1 = a \quad , \quad a^n : a^m = a^{n-m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{\left(a^2+a\cdot b\right)^2}{a^2-b^2}:\frac{(a+b)^2}{\left(a\cdot b-b^2\right)^2}:\left(b^2\cdot a-b^3\right)=\frac{\left(a^2+a\cdot b\right)^2}{a^2-b^2}:\frac{(a+b)^2}{\left(a\cdot b-b^2\right)^2}:\frac{b^2\cdot a-b^3}{1}=$$

$$=\frac{\left(a^2+a\cdot b\right)^2}{a^2-b^2}\cdot\frac{\left(a\cdot b-b^2\right)^2}{(a+b)^2}\cdot\frac{1}{b^2\cdot a-b^3}=\frac{\left(a\cdot(a+b)\right)^2}{(a-b)\cdot(a+b)}\cdot\frac{\left(b\cdot(a-b)\right)^2}{(a+b)^2}\cdot\frac{1}{b^2\cdot(a-b)}=$$

$$=\frac{a^2\cdot(a+b)^2}{(a-b)\cdot(a+b)}\cdot\frac{b^2\cdot(a-b)^2}{(a+b)^2}\cdot\frac{1}{b^2\cdot(a-b)}=\frac{a^2\cdot(a+b)^2}{(a-b)\cdot(a+b)}\cdot\frac{b^2\cdot(a-b)^2}{(a+b)^2}\cdot\frac{1}{b^2\cdot(a-b)}=$$

$$=\frac{a^2}{(a-b)\cdot(a+b)}\cdot\frac{b^2\cdot(a-b)^2}{1}\cdot\frac{1}{b^2\cdot(a-b)}=\frac{a^2}{(a-b)\cdot(a+b)}\cdot\frac{b^2\cdot(a-b)^2}{1}\cdot\frac{1}{b^2\cdot(a-b)}=$$

$$=\frac{a^2}{a+b}\cdot\frac{1}{1}\cdot\frac{1}{1}=\frac{a^2}{a+b}.$$

Vježba 647

Izračunajte $\frac{\left(a^2+a\cdot b\right)^2}{(a+b)^2}:\frac{a^2-b^2}{\left(a\cdot b-b^2\right)^2}:\left(b^2\cdot a-b^3\right).$

Rezultat: $\frac{a^2}{a+b}.$

Zadatak 648 (Darko, srednja škola)

Skrati razlomak $\frac{x^2-1}{\sqrt{x^3-x+\sqrt{x}-1}}.$

Rješenje 648

Ponovimo!

$$a^1=a\quad ,\quad \sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}\quad ,\quad \sqrt{a^2}=a\ ,\ a\geq 0\quad ,\quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\left(\sqrt{a}\right)^2=a\quad ,\quad a^n\cdot a^m=a^{n+m}\quad ,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad ,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{x^2-1}{\sqrt{x^3}-x+\sqrt{x}-1}=\frac{(x-1)\cdot(x+1)}{\sqrt{x^2\cdot x}-x+\sqrt{x}-1}=\frac{(x-1)\cdot(x+1)}{\sqrt{x^2}\cdot\sqrt{x}-x+\sqrt{x}-1}=\frac{(x-1)\cdot(x+1)}{x\cdot\sqrt{x}-x+\sqrt{x}-1}=$$

$$=\frac{(x-1)\cdot(x+1)}{\left(x\cdot\sqrt{x}-x\right)+\left(\sqrt{x}-1\right)}=\frac{(x-1)\cdot(x+1)}{\left(x\cdot\sqrt{x}-x\right)+\left(\sqrt{x}-1\right)}=\frac{(x-1)\cdot(x+1)}{x\cdot\left(\sqrt{x}-1\right)+\left(\sqrt{x}-1\right)}=$$

$$=\frac{(x-1)\cdot(x+1)}{x\cdot\left(\sqrt{x}-1\right)+\left(\sqrt{x}-1\right)}=\frac{(x-1)\cdot(x+1)}{\left(\sqrt{x}-1\right)\cdot(x+1)}=\frac{(x-1)\cdot(x+1)}{\left(\sqrt{x}-1\right)\cdot(x+1)}=\frac{x-1}{\sqrt{x}-1}=\frac{\left(\sqrt{x}\right)^2-1}{\sqrt{x}-1}=$$

$$=\frac{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}{\sqrt{x}-1}=\frac{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}{\sqrt{x}-1}=\sqrt{x}+1.$$

2.inačica

$$\frac{x^2-1}{\sqrt{x^3}-x+\sqrt{x}-1}=\frac{(x-1)\cdot(x+1)}{\sqrt{x^2\cdot x}-x+\sqrt{x}-1}=\frac{(x-1)\cdot(x+1)}{\sqrt{x^2}\cdot\sqrt{x}-x+\sqrt{x}-1}=\frac{(x-1)\cdot(x+1)}{x\cdot\sqrt{x}-x+\sqrt{x}-1}=$$

$$=\frac{(x-1)\cdot(x+1)}{\left(x\cdot\sqrt{x}+\sqrt{x}\right)+(-x-1)}=\frac{(x-1)\cdot(x+1)}{\left(x\cdot\sqrt{x}+\sqrt{x}\right)-(x+1)}=\frac{(x-1)\cdot(x+1)}{\sqrt{x}\cdot(x+1)-(x+1)}=$$

$$=\frac{(x-1)\cdot(x+1)}{\sqrt{x}\cdot(x+1)-(x+1)}=\frac{(x-1)\cdot(x+1)}{(x+1)\cdot\left(\sqrt{x}-1\right)}=\frac{(x-1)\cdot(x+1)}{(x+1)\cdot\left(\sqrt{x}-1\right)}=\frac{x-1}{\sqrt{x}-1}=$$

$$=\frac{x-1}{\sqrt{x}-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}+1}=\frac{(x-1)\cdot\left(\sqrt{x}+1\right)}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}=\frac{(x-1)\cdot\left(\sqrt{x}+1\right)}{\left(\sqrt{x}\right)^2-1}=\frac{(x-1)\cdot\left(\sqrt{x}+1\right)}{x-1}=$$

$$=\frac{(x-1)\cdot\left(\sqrt{x}+1\right)}{x-1}=\sqrt{x}+1.$$

Vježba 648

Skrati razlomak $\dfrac{1-x^2}{1-\sqrt{x}+x-\sqrt{x^3}}$.

Rezultat: $1+\sqrt{x}.$

Zadatak 649 (Antonija, srednja škola)

Izračunaj: $\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(a^{-1}+b^{-1}\right)+\dfrac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\dfrac{1}{\sqrt{a}}+\dfrac{1}{\sqrt{b}}\right).$

Rješenje 649

Ponovimo!

$$a^1=a \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}.$$

$$a^n:a^m=a^{n-m} \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(a^{-1}+b^{-1}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^{3}}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)=$$

$$=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^{2}}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^{3}}\cdot\frac{\sqrt{b}+\sqrt{a}}{\sqrt{a}\cdot\sqrt{b}}=$$

$$=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^{2}}\cdot\frac{b+a}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^{3}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a\cdot b}}=$$

$$=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^{2}}\cdot\frac{a+b}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^{3}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a\cdot b}}=\frac{a+b}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^{2}}\cdot\frac{1}{\sqrt{a\cdot b}}=$$

$$=\frac{a+b}{\left(\sqrt{a\cdot b}\right)^{2}\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}+\frac{2}{\sqrt{a\cdot b}\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=\frac{a+b+2\cdot\sqrt{a\cdot b}}{\left(\sqrt{a\cdot b}\right)^{2}\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=$$

$$=\frac{a+2\cdot\sqrt{a\cdot b}+b}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=\frac{\left(\sqrt{a}\right)^{2}+2\cdot\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^{2}}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=\frac{\left(\sqrt{a}+\sqrt{b}\right)^{2}}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=\frac{1}{a\cdot b}.$$

Vježba 649

Izračunaj: $\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(a^{-1}+b^{-1}\right)+\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^{2}}\cdot\frac{2}{\sqrt{a\cdot b}}.$

Rezultat: $\frac{1}{a\cdot b}.$

Zadatak 650 (Vesna, gimnazija)

Skrati razlomak $\frac{x^{2}-y^{2}+2\cdot y-1}{x\cdot\left(x-y+1\right)+y\cdot\left(x-y+1\right)}.$

Rješenje 650

Ponovimo!

$$a^{2}-2\cdot a\cdot b+b^{2}=\left(a-b\right)^{2}\quad ,\quad a^{2}-b^{2}=\left(a-b\right)\cdot\left(a+b\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c\quad ,\quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{x^2-y^2+2\cdot y-1}{x\cdot(x-y+1)+y\cdot(x-y+1)}=\frac{x^2-\left(y^2-2\cdot y+1\right)}{x\cdot(x-y+1)+y\cdot(x-y+1)}=\frac{x^2-(y-1)^2}{(x-y+1)\cdot(x+y)}=$$

$$=\frac{(x-(y-1))\cdot(x+(y-1))}{(x-y+1)\cdot(x+y)}=\frac{(x-y+1)\cdot(x+y-1)}{(x-y+1)\cdot(x+y)}=\frac{(x-y+1)\cdot(x+y-1)}{(x-y+1)\cdot(x+y)}=\frac{x+y-1}{x+y}.$$

Vježba 650

Skrati razlomak $\frac{x\cdot(x-y+1)+y\cdot(x-y+1)}{x^2-y^2+2\cdot y-1}$.

Rezultat: $\frac{x+y}{x+y-1}$.

Zadatak 651 (Vesna, gimnazija)

Skrati razlomak $\frac{x^2-y\cdot z+x\cdot z-y^2}{x^2+y\cdot z-x\cdot z-y^2}$.

Rješenje 651

Ponovimo!

$$a^1=a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}, \quad n\neq 0 \quad , \quad n\neq 1.$$

$$\frac{x^2-y\cdot z+x\cdot z-y^2}{x^2+y\cdot z-x\cdot z-y^2}=\begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}=\frac{\left(x^2-y^2\right)+(-y\cdot z+x\cdot z)}{\left(x^2-y^2\right)+(y\cdot z-x\cdot z)}=$$

$$=\frac{(x-y)\cdot(x+y)+z\cdot(x-y)}{(x-y)\cdot(x+y)-z\cdot(x-y)}=\frac{(x-y)\cdot(x+y)+z\cdot(x-y)}{(x-y)\cdot(x+y)-z\cdot(x-y)}=\frac{(x-y)\cdot(x+y+z)}{(x-y)\cdot(x+y-z)}=$$

$$=\frac{(x-y)\cdot(x+y+z)}{(x-y)\cdot(x+y-z)}=\frac{x+y+z}{x+y-z}.$$

Vježba 651

Skrati razlomak $\frac{x^2+y\cdot z-x\cdot z-y^2}{x^2-y\cdot z+x\cdot z-y^2}$.

Rezultat: $\frac{x+y-z}{x+y+z}$.

Zadatak 652 (Marin, gimnazija)

Napiši u obliku umnoška algebarski izraz $x^3-7\cdot x-6$.

Rješenje 652

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad a^3 + b^3 = (a+b)\cdot\left(a^2 - a\cdot b + b^2\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

$$x^3 - 7\cdot x - 6 = x^3 - 7\cdot x - 6 = x^3 - 7\cdot x - 7 + 1 = \begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix} = \left(x^3 + 1\right) + (-7\cdot x - 7) =$$

$$= \left(x^3 + 1^3\right) - 7\cdot(x+1) = (x+1)\cdot\left(x^2 - x + 1\right) - 7\cdot(x+1) = (x+1)\cdot\left(x^2 - x + 1\right) - 7\cdot(x+1) =$$

$$= (x+1)\cdot\left(\left(x^2 - x + 1\right) - 7\right) = (x+1)\cdot\left(x^2 - x + 1 - 7\right) = (x+1)\cdot\left(x^2 - x - 6\right) = (x+1)\cdot\left(x^2 - x - 6\right) =$$

$$= (x+1)\cdot\left(x^2 - 3\cdot x + 2\cdot x - 6\right) = \begin{bmatrix}\text{u drugoj zagradi}\\ \text{metoda grupiranja}\end{bmatrix} = (x+1)\cdot\left(\left(x^2 - 3\cdot x\right) + (2\cdot x - 6)\right) =$$

$$= (x+1)\cdot(x\cdot(x-3) + 2\cdot(x-3)) = (x+1)\cdot(x\cdot(x-3) + 2\cdot(x-3)) = (x+1)\cdot(x-3)\cdot(x+2).$$

Vježba 652

Napiši u obliku umnoška algebarski izraz $x^3 - 7\cdot x + 6$.

Rezultat: $(x-1)\cdot(x+3)\cdot(x-2)$.

Zadatak 653 (Marin, gimnazija)

Izračunaj: $\left(\frac{a+b}{a^2\cdot b - a\cdot b^2} - \frac{a-b}{a^2\cdot b + a\cdot b^2}\right)\cdot\frac{a^4 - b^4}{4}$.

Rješenje 653

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad \left(a^n\right)^m = a^{n\cdot m} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a\cdot d - b\cdot c}{b\cdot d}.$$

$$a^2 - b^2 = (a-b)\cdot(a+b) \quad , \quad (a+b)^2 = a^2 + 2\cdot a\cdot b + b^2 \quad , \quad (a-b)^2 = a^2 - 2\cdot a\cdot b + b^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\left(\frac{a+b}{a^2\cdot b - a\cdot b^2} - \frac{a-b}{a^2\cdot b + a\cdot b^2}\right)\cdot\frac{a^4 - b^4}{4} = \left(\frac{a+b}{a\cdot b\cdot(a-b)} - \frac{a-b}{a\cdot b\cdot(a+b)}\right)\cdot\frac{\left(a^2\right)^2 - \left(b^2\right)^2}{4} =$$

$$= \frac{(a+b)^2 - (a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}\cdot\frac{\left(a^2 - b^2\right)\cdot\left(a^2 + b^2\right)}{4} =$$

$$= \frac{(a+b)^2 - (a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}\cdot\frac{(a-b)\cdot(a+b)\cdot\left(a^2 + b^2\right)}{4} =$$

$$=\frac{(a+b)^2-(a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}\cdot\frac{(a-b)\cdot(a+b)\cdot\left(a^2+b^2\right)}{4}=\frac{(a+b)^2-(a-b)^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}.$$

Dalje možemo na dva načina.

1.inačica

$$\left(\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}\right)\cdot\frac{a^4-b^4}{4}=\ldots=\frac{(a+b)^2-(a-b)^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{a^2+2\cdot a\cdot b+b^2-\left(a^2-2\cdot a\cdot b+b^2\right)}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{2\cdot a\cdot b+2\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=a^2+b^2.$$

2.inačica

$$\left(\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}\right)\cdot\frac{a^4-b^4}{4}=\ldots=\frac{(a+b)^2-(a-b)^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{((a+b)-(a-b))\cdot((a+b)+(a-b))}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{(a+b-a+b)\cdot(a+b+a-b)}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{(a+b-a+b)\cdot(a+b+a-b)}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{2\cdot b\cdot 2\cdot a}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=a^2+b^2.$$

Vježba 653

Izračunaj: $\left(\frac{a+b}{a^2\cdot b-a\cdot b^2}+\frac{b-a}{a^2\cdot b+a\cdot b^2}\right)\cdot\frac{a^4-b^4}{4}.$

Rezultat: $a^2+b^2.$

Zadatak 654 (4A, TUPŠ)

Ako je $t=\frac{1}{r}-\frac{m}{h}$, čemu je jednako m?

$A.\ m=h\cdot\left(\frac{1}{r}-t\right)$ $B.\ m=h\cdot\left(\frac{1}{r}+t\right)$ $C.\ m=\frac{1-r\cdot t}{r\cdot h}$ $D.\ m=\frac{1+r\cdot t}{r\cdot h}$

Rješenje 654

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

Najprije se u jednadžbi riješimo razlomaka tako cijelu jednadžbu pomnožimo zajedničkim nazivnikom.

$$t = \frac{1}{r} - \frac{m}{h} \Rightarrow t = \frac{1}{r} - \frac{m}{h} \ / \cdot r \cdot h \Rightarrow t \cdot r \cdot h = h - m \cdot r \Rightarrow m \cdot r = h - t \cdot r \cdot h \Rightarrow$$

$$\Rightarrow m \cdot r = h - t \cdot r \cdot h \ / : r \Rightarrow m = \frac{h}{r} - \frac{t \cdot r \cdot h}{r} \Rightarrow m = \frac{h}{r} - \frac{t \cdot r \cdot h}{r} \Rightarrow m = \frac{h}{r} - t \cdot h \Rightarrow$$

$$\Rightarrow m = h \cdot \left(\frac{1}{r} - t \right).$$

Odgovor je pod A.

2.inačica

$$t = \frac{1}{r} - \frac{m}{h} \Rightarrow \frac{m}{h} = \frac{1}{r} - t \Rightarrow \frac{m}{h} = \frac{1}{r} - t \ / \cdot h \Rightarrow m = h \cdot \left(\frac{1}{r} - t \right).$$

Odgovor je pod A.

Vježba 654

Ako je $t = \frac{1}{r} - \frac{m}{h}$, čemu je jednako r?

$A. \ r = \frac{h}{m + t \cdot h}$ $\qquad B. \ r = \frac{m + t \cdot h}{h}$ $\qquad C. \ r = \frac{m - t \cdot h}{h}$ $\qquad D. \ r = \frac{h}{m - t \cdot h}$

Rezultat: A.

Zadatak 655 (4A, TUPŠ)

Čemu je jednak pojednostavljen i do kraja skraćen algebarski izraz

$\left(3 \cdot a - \frac{6 \cdot a - 1}{3 \cdot a} \right) \cdot \frac{1}{3 \cdot a - 1}$, za $a \neq 0, \ a \neq -\frac{1}{3}$?

Rješenje 655

Ponovimo!

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

$$(a \cdot b)^n = a^n \cdot b^n \quad , \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2 \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\left(3\cdot a-\frac{6\cdot a-1}{3\cdot a}\right)\cdot\frac{1}{3\cdot a-1}=\left(\frac{3\cdot a}{1}-\frac{6\cdot a-1}{3\cdot a}\right)\cdot\frac{1}{3\cdot a-1}=\frac{(3\cdot a)^2-(6\cdot a-1)}{3\cdot a}\cdot\frac{1}{3\cdot a-1}=$$

$$=\frac{9\cdot a^2-6\cdot a+1}{3\cdot a}\cdot\frac{1}{3\cdot a-1}=\frac{(3\cdot a-1)^2}{3\cdot a}\cdot\frac{1}{3\cdot a-1}=\frac{(3\cdot a-1)^2}{3\cdot a}\cdot\frac{1}{3\cdot a-1}=\frac{3\cdot a-1}{3\cdot a}.$$

Vježba 655

Čemu je jednak pojednostavljen i do kraja skraćen algebarski izraz

$$\left(3\cdot a+\frac{1-6\cdot a}{3\cdot a}\right)\cdot\frac{1}{3\cdot a-1},\ \text{za}\ a\neq 0,\ a\neq-\frac{1}{3}?$$

Rezultat: $\frac{3\cdot a-1}{3\cdot a}$.

Zadatak 656 (4A, TUPŠ)

Napiši algebarski izraz $\left(x^{1.5}\cdot\sqrt[4]{x}\right)^{\frac{1}{2}}$ u obliku potencije s bazom x.

Rješenje 656

Ponovimo!

$$\sqrt[n]{a^m}=a^{\frac{m}{n}}\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\left(a^n\right)^m=a^{n\cdot m}.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad(a\cdot b)^n=a^n\cdot b^n.$$

Decimalni broj piše se u obliku decimalnog razlomka tako da se u brojnik napiše zadani decimalni broj bez decimalne točke, a u nazivnik se napiše dekadska jedinica (10, 100, 1000, 10000, 100000, …) koja ima toliko nula koliko decimalni broj ima decimala (znamenaka na decimalnom mjestu, tj. iza decimalne točke ili decimalnog zareza).
Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice.

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\left(x^{1.5}\cdot\sqrt[4]{x}\right)^{\frac{1}{2}}=\left(x^{\frac{15}{10}}\cdot x^{\frac{1}{4}}\right)^{\frac{1}{2}}=\left(x^{\frac{15}{10}}\cdot x^{\frac{1}{4}}\right)^{\frac{1}{2}}=\left(x^{\frac{3}{2}}\cdot x^{\frac{1}{4}}\right)^{\frac{1}{2}}=\left(x^{\frac{3}{2}+\frac{1}{4}}\right)^{\frac{1}{2}}=\left(x^{\frac{6+1}{4}}\right)^{\frac{1}{2}}=$$

$$=\left(x^{\frac{7}{4}}\right)^{\frac{1}{2}}=x^{\frac{7}{8}}=x^{0.875}.$$

2.inačica

$$\left(x^{1.5}\cdot\sqrt[4]{x}\right)^{\frac{1}{2}}=\left(x^{\frac{15}{10}}\cdot x^{\frac{1}{4}}\right)^{\frac{1}{2}}=\left(x^{\frac{15}{10}}\cdot x^{\frac{1}{4}}\right)^{\frac{1}{2}}=\left(x^{\frac{3}{2}}\cdot x^{\frac{1}{4}}\right)^{\frac{1}{2}}=\left(x^{\frac{3}{2}}\right)^{\frac{1}{2}}\cdot\left(x^{\frac{1}{4}}\right)^{\frac{1}{2}}=$$

$$=x^{\frac{3}{4}}\cdot x^{\frac{1}{8}}=x^{\frac{3}{4}+\frac{1}{8}}=x^{\frac{6+1}{8}}=x^{\frac{7}{8}}=x^{0.875}.$$

Vježba 656

Napiši algebarski izraz $\left(x^{1.5}\cdot\sqrt[4]{x}\right)^{0.5}$ u obliku potencije s bazom x.

Rezultat: $x^{\frac{7}{8}}=x^{0.875}.$

Zadatak 657 (Kate ☺, gimnazija)

Ako je $\frac{1}{a\cdot(b+1)}+\frac{1}{b\cdot(a+1)}=\frac{1}{(a+1)\cdot(b+1)}$, koliko je $\frac{1}{a}+\frac{1}{b}$?

Rješenje 657

Ponovimo!

$$\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice.

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Preoblikujemo zadanu jednakost.

$$\frac{1}{a\cdot(b+1)}+\frac{1}{b\cdot(a+1)}=\frac{1}{(a+1)\cdot(b+1)}\Rightarrow$$

$$\Rightarrow\frac{1}{a\cdot(b+1)}+\frac{1}{b\cdot(a+1)}=\frac{1}{(a+1)\cdot(b+1)}\ /\cdot(a+1)\cdot(b+1)\Rightarrow$$

$$\Rightarrow\frac{a+1}{a}+\frac{b+1}{b}=1\Rightarrow\frac{a}{a}+\frac{1}{a}+\frac{b}{b}+\frac{1}{b}=1\Rightarrow\frac{a}{a}+\frac{1}{a}+\frac{b}{b}+\frac{1}{b}=1\Rightarrow$$

$$\Rightarrow 1+\frac{1}{a}+1+\frac{1}{b}=1\Rightarrow 1+\frac{1}{a}+1+\frac{1}{b}=1\Rightarrow\frac{1}{a}+1+\frac{1}{b}=0\Rightarrow\frac{1}{a}+\frac{1}{b}=-1.$$

Vježba 657

Ako je $\frac{b}{b+1}+\frac{a}{a+1}=\frac{a\cdot b}{(a+1)\cdot(b+1)}$, koliko je $\frac{1}{a}+\frac{1}{b}$?

Rezultat: $\frac{1}{a}+\frac{1}{b}=-1.$

Zadatak 658 (Dada, gimnazija)

Pomnoži i zapiši rezultat u obliku potencije $\left(a^2\cdot b\right)^3\cdot\left(a^2\cdot b\right)^4$.

Rješenje 658

Ponovimo!

$$a^1=a\quad,\quad(a\cdot b)^n=a^n\cdot b^n\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^n\cdot a^m=a^{n+m}.$$

1.inačica

$$\left(a^2\cdot b\right)^3\cdot\left(a^2\cdot b\right)^4=\left(a^2\cdot b^1\right)^3\cdot\left(a^2\cdot b^1\right)^4=\left(a^2\right)^3\cdot\left(b^1\right)^3\cdot\left(a^2\right)^4\cdot\left(b^1\right)^4=$$

$$=a^6\cdot b^3\cdot a^8\cdot b^4=a^{6+8}\cdot b^{3+4}=a^{14}\cdot b^7.$$

2.inačica

$$\left(a^{2}\cdot b\right)^{3}\cdot\left(a^{2}\cdot b\right)^{4}=\left(a^{2}\cdot b\right)^{3+4}=\left(a^{2}\cdot b\right)^{7}=\left(a^{2}\cdot b^{1}\right)^{7}=\left(a^{2}\right)^{7}\cdot\left(b^{1}\right)^{7}=a^{14}\cdot b^{7}.$$

Vježba 658

Pomnoži i zapiši rezultat u obliku potencije $\left(a\cdot b^{2}\right)^{3}\cdot\left(a\cdot b^{2}\right)^{4}$.

Rezultat: $a^{7}\cdot b^{14}$.

Zadatak 659 (Dada, gimnazija)

Zapiši u obliku potencije s bazom 3 izraz $\left(27^{2}\cdot 81\cdot 9^{3}\right)^{4}$.

Rješenje 659

Ponovimo!

$$a^{1}=a\quad,\quad(a\cdot b)^{n}=a^{n}\cdot b^{n}\quad,\quad\left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad a^{n}\cdot a^{m}=a^{n+m}.$$

1.inačica

$$\left(27^{2}\cdot 81\cdot 9^{3}\right)^{4}=\left(27^{2}\cdot 81^{1}\cdot 9^{3}\right)^{4}=\left(27^{2}\right)^{4}\cdot\left(81^{1}\right)^{4}\cdot\left(9^{3}\right)^{4}=27^{8}\cdot 81^{4}\cdot 9^{12}=$$

$$=\left(3^{3}\right)^{8}\cdot\left(3^{4}\right)^{4}\cdot\left(3^{2}\right)^{12}=3^{24}\cdot 3^{16}\cdot 3^{24}=3^{24+16+24}=3^{64}.$$

2.inačica

$$\left(27^{2}\cdot 81\cdot 9^{3}\right)^{4}=\left(\left(3^{3}\right)^{2}\cdot 3^{4}\cdot\left(3^{2}\right)^{3}\right)^{4}=\left(3^{6}\cdot 3^{4}\cdot 3^{6}\right)^{4}=\left(3^{6+4+6}\right)^{4}=\left(3^{16}\right)^{4}=3^{64}.$$

3.inačica

$$\left(27^{2}\cdot 81\cdot 9^{3}\right)^{4}=\left(\left(3^{3}\right)^{2}\cdot 3^{4}\cdot\left(3^{2}\right)^{3}\right)^{4}=\left(3^{6}\cdot 3^{4}\cdot 3^{6}\right)^{4}=$$

$$=\left(3^{6}\right)^{4}\cdot\left(3^{4}\right)^{4}\cdot\left(3^{6}\right)^{4}=3^{24}\cdot 3^{16}\cdot 3^{24}=3^{24+16+24}=3^{64}.$$

Vježba 659

Zapiši u obliku potencije s bazom 3 izraz $\left(27^{2}\cdot 9^{2}\cdot 9^{3}\right)^{4}$.

Rezultat: 3^{64}.

Zadatak 660 (Lucy, gimnazija)

Napiši u obliku umnoška algebarski izraz $x^{2}\cdot y-x^{2}-y^{2}+1$.

Rješenje 660

Ponovimo!

$$a^{2}-b^{2}=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$x^{2}\cdot y-x^{2}-y^{2}+1=\left(x^{2}\cdot y-x^{2}\right)+\left(-y^{2}+1\right)=\left(x^{2}\cdot y-x^{2}\right)-\left(y^{2}-1\right)=$$

$$= x^2 \cdot (y-1) - (y-1) \cdot (y+1) = x^2 \cdot (y-1) - (y-1) \cdot (y+1) = (y-1) \cdot \left(x^2 - (y+1)\right) =$$

$$= (y-1) \cdot \left(x^2 - y - 1\right).$$

Vježba 660

Napiši u obliku umnoška algebarski izraz $x^2 \cdot y + 1 - x^2 - y^2$.

Rezultat: $(y-1) \cdot \left(x^2 - y - 1\right).$