Zadatak 621 (Ana, gimnazija)

Pomnoži $\frac{x^2+y^2}{x^2-y^2}\cdot\frac{x^2+2\cdot x\cdot y+y^2}{x^4-y^4}\cdot\frac{x-y}{x+y}$.

Rješenje 621

Ponovimo!

$$a^2+2\cdot a\cdot b+b^2=(a+b)^2 \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{x^2+y^2}{x^2-y^2}\cdot\frac{x^2+2\cdot x\cdot y+y^2}{x^4-y^4}\cdot\frac{x-y}{x+y}=\frac{x^2+y^2}{(x-y)\cdot(x+y)}\cdot\frac{(x+y)^2}{\left(x^2\right)^2-\left(y^2\right)^2}\cdot\frac{x-y}{x+y}=$$

$$=\frac{x^2+y^2}{(x-y)\cdot(x+y)}\cdot\frac{(x+y)^2}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)}\cdot\frac{x-y}{x+y}=$$

$$=\frac{x^2+y^2}{(x-y)\cdot(x+y)}\cdot\frac{(x+y)^2}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)}\cdot\frac{x-y}{x+y}=$$

$$=\frac{1}{(x-y)\cdot(x+y)}\cdot\frac{(x+y)^2}{x^2-y^2}\cdot\frac{x-y}{x+y}=\frac{1}{(x-y)\cdot(x+y)}\cdot\frac{(x+y)^2}{x^2-y^2}\cdot\frac{x-y}{x+y}=$$

$$=\frac{1}{x+y}\cdot\frac{(x+y)^2}{x^2-y^2}\cdot\frac{1}{x+y}=\frac{1}{x+y}\cdot\frac{(x+y)^2}{x^2-y^2}\cdot\frac{1}{x+y}=\frac{1}{x^2-y^2}.$$

Vježba 621

Pomnoži $\frac{x^2+y^2}{x+y}\cdot\frac{x^2+2\cdot x\cdot y+y^2}{x^4-y^4}\cdot\frac{x-y}{x^2-y^2}$.

Rezultat: $\frac{1}{x^2-y^2}$.

Zadatak 622 (Ana, gimnazija)

Izračunajte: $\frac{2}{a^2-2\cdot a+1}:\left(\frac{\sqrt{a}+1}{\sqrt{a}-1}-\frac{\sqrt{a}-1}{\sqrt{a}+1}\right)$.

Rješenje 622

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left(\sqrt{a}\right)^2=a.$$

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \;,\; n\neq 0 \;,\; n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$\frac{2}{a^2-2\cdot a+1}:\left(\frac{\sqrt{a}+1}{\sqrt{a}-1}-\frac{\sqrt{a}-1}{\sqrt{a}+1}\right)=\frac{2}{(a-1)^2}:\frac{\left(\sqrt{a}+1\right)^2-\left(\sqrt{a}-1\right)^2}{\left(\sqrt{a}-1\right)\cdot\left(\sqrt{a}+1\right)}=$$

$$=\frac{2}{(a-1)^2}:\frac{\left(\left(\sqrt{a}+1\right)-\left(\sqrt{a}-1\right)\right)\cdot\left(\left(\sqrt{a}+1\right)+\left(\sqrt{a}-1\right)\right)}{\left(\sqrt{a}\right)^2-1^2}=$$

$$=\frac{2}{(a-1)^2}:\frac{\left(\sqrt{a}+1-\sqrt{a}+1\right)\cdot\left(\sqrt{a}+1+\sqrt{a}-1\right)}{a-1}=$$

$$=\frac{2}{(a-1)^2}:\frac{\left(\sqrt{a}+1-\sqrt{a}+1\right)\cdot\left(\sqrt{a}+1+\sqrt{a}-1\right)}{a-1}=\frac{2}{(a-1)^2}:\frac{2\cdot 2\cdot\sqrt{a}}{a-1}=$$

$$=\frac{2}{(a-1)^2}:\frac{4\cdot\sqrt{a}}{a-1}=\frac{2}{(a-1)^2}\cdot\frac{a-1}{4\cdot\sqrt{a}}=\frac{2}{(a-1)^2}\cdot\frac{a-1}{4\cdot\sqrt{a}}=\frac{1}{a-1}\cdot\frac{1}{2\cdot\sqrt{a}}=\frac{1}{2\cdot(a-1)\cdot\sqrt{a}}=$$

$$=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{1}{2\cdot(a-1)\cdot\sqrt{a}}\cdot\frac{\sqrt{a}}{\sqrt{a}}=\frac{\sqrt{a}}{2\cdot(a-1)\cdot\left(\sqrt{a}\right)^2}=\frac{\sqrt{a}}{2\cdot a\cdot(a-1)}.$$

Vježba 622

Izračunajte: $\frac{2}{a^2-2\cdot a+1}:\left(\frac{\sqrt{a}+1}{\sqrt{a}-1}+\frac{1-\sqrt{a}}{\sqrt{a}+1}\right)$.

Rezultat: $\frac{\sqrt{a}}{2\cdot a\cdot(a-1)}$.

Zadatak 623 (Martin, srednja škola)

Dokaži da za sve realne brojeve a, b i c vrijedi nejednakost $a^2+b^2+c^2\geq a\cdot b+b\cdot c+c\cdot a$.

Rješenje 623

Ponovimo!

$$a^2\geq 0 \;,\; a\in R \quad , \quad (a-b)^2\geq 0 \;\Rightarrow\; a^2+b^2\geq 2\cdot a\cdot b \quad , \quad a^2-2\cdot a\cdot b+b^2=(a-b)^2.$$

$$a \geq b \;,\; c > 0 \;\Rightarrow\; \frac{a}{c} \geq \frac{b}{c} \quad , \quad a \geq b \;,\; c > 0 \;\Rightarrow\; a \cdot c \geq b \cdot c \quad , \quad \left.\begin{array}{l} a \geq b \\ c \geq d \end{array}\right\} \Rightarrow a + c \geq b + d.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

Postavimo sustav nejednakosti koje su očito istinite.

$$\left.\begin{array}{l} a^2 + b^2 \geq 2 \cdot a \cdot b \\ b^2 + c^2 \geq 2 \cdot b \cdot c \\ c^2 + a^2 \geq 2 \cdot c \cdot a \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{nejednakosti} \end{array}\right] \Rightarrow$$

$$\Rightarrow a^2 + b^2 + b^2 + c^2 + c^2 + a^2 \geq 2 \cdot a \cdot b + 2 \cdot b \cdot c + 2 \cdot c \cdot a \Rightarrow$$

$$\Rightarrow 2 \cdot a^2 + 2 \cdot b^2 + 2 \cdot c^2 \geq 2 \cdot a \cdot b + 2 \cdot b \cdot c + 2 \cdot c \cdot a \Rightarrow$$

$$\Rightarrow 2 \cdot a^2 + 2 \cdot b^2 + 2 \cdot c^2 \geq 2 \cdot a \cdot b + 2 \cdot b \cdot c + 2 \cdot c \cdot a \;/:2 \Rightarrow$$

$$\Rightarrow a^2 + b^2 + c^2 \geq a \cdot b + b \cdot c + c \cdot a. \quad \text{Dokaz gotov.}$$

2.inačica

Preoblikujemo zadanu nejednakost kako bismo dobili nejednakost za koju smo sigurni da je točna.

$$a^2 + b^2 + c^2 \geq a \cdot b + b \cdot c + c \cdot a \Rightarrow a^2 + b^2 + c^2 \geq a \cdot b + b \cdot c + c \cdot a \;/\cdot 2 \Rightarrow$$

$$\Rightarrow 2 \cdot a^2 + 2 \cdot b^2 + 2 \cdot c^2 \geq 2 \cdot a \cdot b + 2 \cdot b \cdot c + 2 \cdot c \cdot a \Rightarrow$$

$$\Rightarrow 2 \cdot a^2 + 2 \cdot b^2 + 2 \cdot c^2 - 2 \cdot a \cdot b - 2 \cdot b \cdot c - 2 \cdot c \cdot a \geq 0 \Rightarrow$$

$$\Rightarrow a^2 - 2 \cdot a \cdot b + b^2 + b^2 - 2 \cdot b \cdot c + c^2 + c^2 - 2 \cdot c \cdot a - a^2 \geq 0 \Rightarrow$$

$$\Rightarrow \left(a^2 - 2 \cdot a \cdot b + b^2\right) + \left(b^2 - 2 \cdot b \cdot c + c^2\right) + \left(c^2 - 2 \cdot c \cdot a + a^2\right) \geq 0 \Rightarrow$$

$$\Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 \geq 0.$$

Posljednja nejednakost je očito istinita pa je i zadana nejednakost točna.

Vježba 623

Dokaži da za sve realne brojeve a, b i c vrijedi nejednakost

$a \cdot (a - b) + b \cdot (b - c) + c \cdot (c - a) \geq 0.$

Rezultat: Dokaz analogan.

Zadatak 624 (Leon, srednja škola)

Rastavi na faktore $(a - b)^3 - a + b.$

Rješenje 624

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad a^2 - b^2 = (a - b) \cdot (a + b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$(a-b)^3-a+b=(a-b)^3-(a-b)=(a-b)^3-(a-b)=\begin{bmatrix}\text{izlučimo}\\ a-b\end{bmatrix}=(a-b)\cdot\left((a-b)^2-1\right)=$$

$$=(a-b)\cdot(a-b-1)\cdot(a-b+1).$$

Vježba 624

Rastavi na faktore $(a-b)^2-a+b.$

Rezultat: $(a-b)\cdot(a-b-1).$

Zadatak 625 (Leon, srednja škola)

Rastavi na faktore $16\cdot x^4-16\cdot x^3+4\cdot x-1.$

Rješenje 625

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad a^1=a.$$

$$a^2-2\cdot a\cdot b+b^2=(a-b)^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$16\cdot x^4-16\cdot x^3+4\cdot x-1=\begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}=16\cdot x^4-1-16\cdot x^3+4\cdot x=$$

$$=\left(16\cdot x^4-1\right)+\left(-16\cdot x^3+4\cdot x\right)=\left(\left(4\cdot x^2\right)^2-1^2\right)-4\cdot x\cdot\left(4\cdot x^2-1\right)=$$

$$=\left(4\cdot x^2-1\right)\cdot\left(4\cdot x^2+1\right)-4\cdot x\cdot\left(4\cdot x^2-1\right)=\left(4\cdot x^2-1\right)\cdot\left(4\cdot x^2+1\right)-4\cdot x\cdot\left(4\cdot x^2-1\right)=$$

$$=\begin{bmatrix}\text{izlučimo}\\ 4\cdot x^2-1\end{bmatrix}=\left(4\cdot x^2-1\right)\cdot\left(4\cdot x^2+1-4\cdot x\right)=\left(4\cdot x^2-1\right)\cdot\left(4\cdot x^2-4\cdot x+1\right)=$$

$$=\left((2\cdot x)^2-1^2\right)\cdot(2\cdot x-1)^2=(2\cdot x-1)\cdot(2\cdot x+1)\cdot(2\cdot x-1)^2=$$

$$=(2\cdot x-1)^3\cdot(2\cdot x+1).$$

Vježba 625

Rastavi na faktore $x^4-4\cdot x^3+16\cdot x-16.$

Rezultat: $(x-2)^3\cdot(x+2).$

Zadatak 626 (Tibor, srednja škola)

Ako je $a + b = x + y$ i $a^2 + b^2 = x^2 + y^2$, dokazati da je $a^3 + b^3 = x^3 + y^3$.

Rješenje 626

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad(a+b)^3=a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Kvadriramo prvu jednakost.

$$a+b=x+y \Rightarrow a+b=x+y \;/^2 \Rightarrow (a+b)^2=(x+y)^2 \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b+b^2=x^2+2\cdot x\cdot y+y^2 \Rightarrow \left[\begin{matrix}\text{druga jednakost}\\ a^2+b^2=x^2+y^2\end{matrix}\right] \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b+b^2=a^2+2\cdot x\cdot y+b^2 \Rightarrow a^2+2\cdot a\cdot b+b^2=a^2+2\cdot x\cdot y+b^2 \Rightarrow$$

$$\Rightarrow 2\cdot a\cdot b=2\cdot x\cdot y \Rightarrow 2\cdot a\cdot b=2\cdot x\cdot y \;/:2 \Rightarrow a\cdot b=x\cdot y.$$

Sada prvu jednakost kubiramo.

$$a+b=x+y \Rightarrow a+b=x+y \;/^3 \Rightarrow (a+b)^3=(x+y)^3 \Rightarrow$$

$$\Rightarrow a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3=x^3+3\cdot x^2\cdot y+3\cdot x\cdot y^2+y^3 \Rightarrow$$

$$\Rightarrow a^3+3\cdot a\cdot b\cdot(a+b)+b^3=x^3+3\cdot x\cdot y\cdot(x+y)+y^3 \Rightarrow \left[\begin{matrix}\text{uvjeti}\\ a\cdot b=x\cdot y\\ a+b=x+y\end{matrix}\right] \Rightarrow$$

$$\Rightarrow a^3+3\cdot a\cdot b\cdot(a+b)+b^3=x^3+3\cdot a\cdot b\cdot(a+b)+y^3 \Rightarrow$$

$$\Rightarrow a^3+3\cdot a\cdot b\cdot(a+b)+b^3=x^3+3\cdot a\cdot b\cdot(a+b)+y^3 \Rightarrow a^3+b^3=x^3+y^3.$$ Dokaz gotov.

Vježba 626

Ako je $a - x = y - b$ i $a^2 - x^2 = y^2 - b^2$, dokazati da je $a^3 + b^3 = x^3 + y^3$.

Rezultat: Dokaz analogan.

Zadatak 627 (Marin, srednja škola)

Dokaži, ako je $a > 0$, $b > 0$, $c > 0$, tada je $\frac{3}{a+b+c}<\frac{1}{a+b}+\frac{1}{a+c}+\frac{1}{b+c}$.

Rješenje 627

Ponovimo!

$$a>b \Rightarrow \frac{1}{a}<\frac{1}{b}\,,\; a\neq 0,\; b\neq 0 \quad,\quad \left.\begin{matrix}a<b\\ c<d\end{matrix}\right\} \Rightarrow a+c<b+d \quad,\quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

Budući da je $a > 0$, $b > 0$, $c > 0$, vrijedi:

$$\left.\begin{matrix}a+b+c>a+b\\ a+b+c>a+c\\ a+b+c>b+c\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\frac{1}{a+b+c}<\frac{1}{a+b}\\ \frac{1}{a+b+c}<\frac{1}{a+c}\\ \frac{1}{a+b+c}<\frac{1}{b+c}\end{matrix}\right\} \Rightarrow \left[\begin{matrix}\text{zbrojimo}\\ \text{nejednakosti}\end{matrix}\right] \Rightarrow$$

$$\Rightarrow \frac{1}{a+b+c}+\frac{1}{a+b+c}+\frac{1}{a+b+c}<\frac{1}{a+b}+\frac{1}{a+c}+\frac{1}{b+c} \Rightarrow$$

$$\Rightarrow \frac{3}{a+b+c}<\frac{1}{a+b}+\frac{1}{a+c}+\frac{1}{b+c}.$$ Dokaz gotov.

Vježba 627

Dokaži, ako je a > 0, b > 0, c > 0, tada je $\frac{1}{a+b+c}<\frac{1}{3\cdot(a+b)}+\frac{1}{3\cdot(a+c)}+\frac{1}{3\cdot(b+c)}$.

Rezultat: Dokaz analogan.

Zadatak 628 (Tom, gimnazija)

Neka su x, y i z realni brojevi takvi da je x + y + z = x · y · z. Dokažite da vrijedi identitet
$x\cdot(1-y^2)\cdot(1-z^2)+y\cdot(1-z^2)\cdot(1-x^2)+z\cdot(1-x^2)\cdot(1-y^2)=4\cdot x\cdot y\cdot z.$

Rješenje 628

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Lijevu stranu jednakosti preoblikujemo na sljedeći način:

$$x\cdot(1-y^2)\cdot(1-z^2)+y\cdot(1-z^2)\cdot(1-x^2)+z\cdot(1-x^2)\cdot(1-y^2)=$$

$$=x\cdot(1-y^2-z^2+z^2\cdot y^2)+y\cdot(1-z^2-x^2+x^2\cdot z^2)+z\cdot(1-x^2-y^2+x^2\cdot y^2)=$$

$$=x-x\cdot y^2-x\cdot z^2+x\cdot z^2\cdot y^2+y-y\cdot z^2-y\cdot x^2+y\cdot x^2\cdot z^2+z-z\cdot x^2-z\cdot y^2+z\cdot x^2\cdot y^2=$$

$$=\begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}=$$

$$=(x+y+z)+(z\cdot x^2\cdot y^2-y\cdot x^2-x\cdot y^2)+(x\cdot z^2\cdot y^2-y\cdot z^2-z\cdot y^2)+(y\cdot x^2\cdot z^2-x\cdot z^2-z\cdot x^2)=$$

$$=(x+y+z)+x\cdot y\cdot(x\cdot y\cdot z-x-y)+y\cdot z\cdot(x\cdot y\cdot z-z-y)+x\cdot z\cdot(x\cdot y\cdot z-z-x)=$$

$$=\begin{bmatrix}x+y+z=x\cdot y\cdot z\\ x\cdot y\cdot z=x+y+z\end{bmatrix}=$$

$$=x\cdot y\cdot z+x\cdot y\cdot(x+y+z-x-y)+y\cdot z\cdot(x+y+z-z-y)+x\cdot z\cdot(x+y+z-z-x)=$$

$$=x\cdot y\cdot z+x\cdot y\cdot(x+y+z-x-y)+y\cdot z\cdot(x+y+z-z-y)+x\cdot z\cdot(x+y+z-z-x)=$$

$$=x\cdot y\cdot z+x\cdot y\cdot z+x\cdot y\cdot z+x\cdot y\cdot z=4\cdot x\cdot y\cdot z.$$

Vježba 628

Neka su x, y i z realni brojevi takvi da je x · (1 – y · z) + y + z = 0. Dokažite da vrijedi identitet
$x\cdot(1-y^2)\cdot(1-z^2)+y\cdot(1-z^2)\cdot(1-x^2)+z\cdot(1-x^2)\cdot(1-y^2)=4\cdot x\cdot y\cdot z.$

Rezultat: Dokaz analogan.

Zadatak 629 (Ante, srednja škola)

Pojednostavnite: $\sqrt{\frac{a+2\cdot\sqrt{a\cdot b}+9\cdot b}{\sqrt{a}-2\cdot\sqrt[4]{a\cdot b}+3\cdot\sqrt{b}}-2\cdot\sqrt{b}}.$

A. $\sqrt[4]{a}-\sqrt[4]{b}$ B. $\sqrt[4]{a}+\sqrt[4]{b}$ C. $\sqrt[4]{a}$ D. $\sqrt[4]{b}$

Rješenje 629

Ponovimo!

$$a^1=a \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^2+2\cdot a\cdot b+b^2=(a+b)^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n.$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \sqrt{a^2}=a \; , \; a\geq 0 \quad , \quad \sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

Ako označimo

$$\sqrt[4]{a}=u>0 \quad , \quad \sqrt[4]{b}=v>0,$$

tada je

$$\sqrt{a}=u^2 \;\Rightarrow\; a=u^4 \quad , \quad \sqrt{b}=v^2 \;\Rightarrow\; b=v^4.$$

Dalje slijedi:

$$\sqrt{\frac{a+2\cdot\sqrt{a\cdot b}+9\cdot b}{\sqrt{a}-2\cdot\sqrt[4]{a\cdot b}+3\cdot\sqrt{b}}-2\cdot\sqrt{b}}=\sqrt{\frac{a+2\cdot\sqrt{a}\cdot\sqrt{b}+9\cdot b}{\sqrt{a}-2\cdot\sqrt[4]{a}\cdot\sqrt[4]{b}+3\cdot\sqrt{b}}-2\cdot\sqrt{b}}=$$

$$=\begin{bmatrix}\sqrt[4]{a}=u \; , \; \sqrt[4]{b}=v\\ \sqrt{a}=u^2 \; , \; \sqrt{b}=v^2\\ a=u^4 \; , \; b=v^4\end{bmatrix}=\sqrt{\frac{u^4+2\cdot u^2\cdot v^2+9\cdot v^4}{u^2-2\cdot u\cdot v+3\cdot v^2}-2\cdot v^2}=$$

$$=\sqrt{\frac{u^4+6\cdot u^2\cdot v^2+9\cdot v^4-4\cdot u^2\cdot v^2}{u^2-2\cdot u\cdot v+3\cdot v^2}-2\cdot v^2}=$$

$$=\sqrt{\frac{\left(u^4+6\cdot u^2\cdot v^2+9\cdot v^4\right)-4\cdot u^2\cdot v^2}{u^2-2\cdot u\cdot v+3\cdot v^2}-2\cdot v^2}=\sqrt{\frac{\left(u^2+3\cdot v^2\right)^2-(2\cdot u\cdot v)^2}{u^2-2\cdot u\cdot v+3\cdot v^2}-2\cdot v^2}=$$

$$=\sqrt{\frac{\left(u^2+3\cdot v^2-2\cdot u\cdot v\right)\cdot\left(u^2+3\cdot v^2+2\cdot u\cdot v\right)}{u^2-2\cdot u\cdot v+3\cdot v^2}-2\cdot v^2}=$$

$$=\sqrt{\frac{\left(u^2-2\cdot u\cdot v+3\cdot v^2\right)\cdot\left(u^2+2\cdot u\cdot v+3\cdot v^2\right)}{u^2-2\cdot u\cdot v+3\cdot v^2}-2\cdot v^2}=$$

$$=\sqrt{u^2+2\cdot u\cdot v+3\cdot v^2-2\cdot v^2}=\sqrt{u^2+2\cdot u\cdot v+v^2}=\sqrt{(u+v)^2}=u+v=\begin{bmatrix}\sqrt[4]{a}=u\\ \sqrt[4]{b}=v\end{bmatrix}=\sqrt[4]{a}+\sqrt[4]{b}.$$

Odgovor je pod B.

Vježba 629

Pojednostavnite: $\sqrt{\dfrac{2\cdot a+4\cdot\sqrt{a\cdot b}+18\cdot b}{2\cdot\sqrt{a}-4\cdot\sqrt[4]{a\cdot b}+6\cdot\sqrt{b}}-2\cdot\sqrt{b}}.$

$A.\ \sqrt[4]{a}-\sqrt[4]{b}$ $\quad B.\ \sqrt[4]{a}+\sqrt[4]{b}$ $\quad C.\ \sqrt[4]{a}$ $\quad D.\ \sqrt[4]{b}$

Rezultat: B.

Zadatak 630 (Tomislav, srednja škola)

Pojednostavnite: $\dfrac{9-4\cdot a^2}{27-8\cdot a^3}\cdot\dfrac{9\cdot a+6\cdot a^2+4\cdot a^3}{9+6\cdot a}.$

Rješenje 630

Ponovimo!

$$(a\cdot b)^n=a^n\cdot b^n \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right).$$

$$a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{9-4\cdot a^2}{27-8\cdot a^3}\cdot\frac{9\cdot a+6\cdot a^2+4\cdot a^3}{9+6\cdot a}=\frac{3^2-(2\cdot a)^2}{3^3-(2\cdot a)^3}\cdot\frac{a\cdot\left(9+6\cdot a+4\cdot a^2\right)}{3\cdot(3+2\cdot a)}=$$

$$=\frac{(3-2\cdot a)\cdot(3+2\cdot a)}{(3-2\cdot a)\cdot\left(3^2+3\cdot 2\cdot a+(2\cdot a)^2\right)}\cdot\frac{a\cdot\left(9+6\cdot a+4\cdot a^2\right)}{3\cdot(3+2\cdot a)}=$$

$$=\frac{(3-2\cdot a)\cdot(3+2\cdot a)}{(3-2\cdot a)\cdot\left(9+6\cdot a+4\cdot a^2\right)}\cdot\frac{a\cdot\left(9+6\cdot a+4\cdot a^2\right)}{3\cdot(3+2\cdot a)}=$$

$$=\frac{(3-2\cdot a)\cdot(3+2\cdot a)}{(3-2\cdot a)\cdot\left(9+6\cdot a+4\cdot a^2\right)}\cdot\frac{a\cdot\left(9+6\cdot a+4\cdot a^2\right)}{3\cdot(3+2\cdot a)}=\frac{a}{3}.$$

Vježba 630

Pojednostavnite: $\dfrac{4\cdot a^2-9}{8\cdot a^3-27}\cdot\dfrac{9\cdot a+6\cdot a^2+4\cdot a^3}{9+6\cdot a}.$

Rezultat: $\dfrac{a}{3}.$

Zadatak 631 (Tomislav, srednja škola)

Pojednostavnite: $\left(\frac{1}{x-1}-\frac{1}{x+1}\right)\cdot\frac{x^2-2\cdot x-3}{2\cdot x-6}$.

Rješenje 631

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$\left(\frac{1}{x-1}-\frac{1}{x+1}\right)\cdot\frac{x^2-2\cdot x-3}{2\cdot x-6}=\frac{x+1-(x-1)}{(x-1)\cdot(x+1)}\cdot\frac{x^2-3\cdot x+x-3}{2\cdot(x-3)}=\frac{x+1-x+1}{(x-1)\cdot(x+1)}\cdot\frac{\left(x^2-3\cdot x\right)+(x-3)}{2\cdot(x-3)}=$$

$$=\frac{x+1-x+1}{(x-1)\cdot(x+1)}\cdot\frac{x\cdot(x-3)+(x-3)}{2\cdot(x-3)}=\frac{2}{(x-1)\cdot(x+1)}\cdot\frac{x\cdot(x-3)+(x-3)}{2\cdot(x-3)}=$$

$$=\frac{2}{(x-1)\cdot(x+1)}\cdot\frac{(x-3)\cdot(x+1)}{2\cdot(x-3)}=\frac{2}{(x-1)\cdot(x+1)}\cdot\frac{(x-3)\cdot(x+1)}{2\cdot(x-3)}=\frac{1}{x-1}.$$

Vježba 631

Pojednostavnite: $\left(\frac{1}{x-1}-\frac{1}{x+1}\right)\cdot\frac{2\cdot x^2-4\cdot x-6}{4\cdot x-12}$.

Rezultat: $\frac{1}{x-1}$.

Zadatak 632 (Martin, tehnička škola)

Racionaliziraj nazivnik $\frac{\sqrt{a\cdot b}}{a\cdot\sqrt{b}-b\cdot\sqrt{a}}$.

Rješenje 632

Ponovimo!

$$(x-y)\cdot(x+y)=x^2-y^2 \quad , \quad \sqrt{x}\cdot\sqrt{y}=\sqrt{x\cdot y} \quad , \quad \left(\sqrt{x}\right)^2=x \quad , \quad \left(x\cdot\sqrt{y}\right)^2=x^2\cdot y.$$

$$x^1=x \quad , \quad x^n\cdot x^m=x^{n+m} \quad , \quad \sqrt{x^2}=x \;,\; x\geq 0 \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$\frac{\sqrt{a\cdot b}}{a\cdot\sqrt{b}-b\cdot\sqrt{a}}=\frac{\sqrt{a\cdot b}}{a\cdot\sqrt{b}-b\cdot\sqrt{a}}\cdot\frac{a\cdot\sqrt{b}+b\cdot\sqrt{a}}{a\cdot\sqrt{b}+b\cdot\sqrt{a}}=\frac{\sqrt{a\cdot b}\cdot\left(a\cdot\sqrt{b}+b\cdot\sqrt{a}\right)}{\left(a\cdot\sqrt{b}-b\cdot\sqrt{a}\right)\cdot\left(a\cdot\sqrt{b}+b\cdot\sqrt{a}\right)}=$$

$$=\frac{\sqrt{a\cdot b}\cdot\left(a\cdot\sqrt{b}+b\cdot\sqrt{a}\right)}{\left(a\cdot\sqrt{b}\right)^2-\left(b\cdot\sqrt{a}\right)^2}=\frac{a\cdot\sqrt{a\cdot b\cdot b}+b\cdot\sqrt{a\cdot b\cdot a}}{a^2\cdot b-b^2\cdot a}=\frac{a\cdot\sqrt{a\cdot b^2}+b\cdot\sqrt{a^2\cdot b}}{a^2\cdot b-b^2\cdot a}=$$

$$=\frac{a\cdot\sqrt{a}\cdot\sqrt{b^2}+b\cdot\sqrt{a^2}\cdot\sqrt{b}}{a^2\cdot b-b^2\cdot a}=\frac{a\cdot\sqrt{a}\cdot b+b\cdot a\cdot\sqrt{b}}{a^2\cdot b-b^2\cdot a}=\frac{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)}{a\cdot b\cdot(a-b)}=$$

$$=\frac{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)}{a\cdot b\cdot(a-b)}=\frac{\sqrt{a}+\sqrt{b}}{a-b}.$$

Vježba 632

Racionaliziraj nazivnik $\frac{\sqrt{a\cdot b}}{a\cdot\sqrt{b}+b\cdot\sqrt{a}}$.

Rezultat: $\frac{\sqrt{a}-\sqrt{b}}{a-b}$.

Zadatak 633 (4A, 4B, TUPŠ)

Pojednostavni izraz $\dfrac{\frac{10\cdot x^3}{x^2-1}}{\frac{5\cdot x}{x+1}}$.

Rješenje 633

Ponovimo!

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad a^1=a\quad,\quad\frac{a^n}{a^m}=a^{n-m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{\frac{10\cdot x^3}{x^2-1}}{\frac{5\cdot x}{x+1}}=\frac{\frac{10\cdot x^3}{x^2-1}}{\frac{5\cdot x}{x+1}}=\frac{\frac{2\cdot x^2}{x^2-1}}{\frac{1}{x+1}}=\frac{2\cdot x^2\cdot(x+1)}{x^2-1}=\frac{2\cdot x^2\cdot(x+1)}{(x-1)\cdot(x+1)}=\frac{2\cdot x^2\cdot(x+1)}{(x-1)\cdot(x+1)}=\frac{2\cdot x^2}{x-1}.$$

Vježba 633

Pojednostavni izraz $\dfrac{\frac{10\cdot x^3}{x^2-1}}{\frac{5\cdot x}{x-1}}$.

Rezultat: $\frac{2\cdot x^2}{x+1}.$

Zadatak 634 (4A, 4B, TUPŠ)

Pojednostavni izraz $\frac{4\cdot\left(x\cdot\sqrt{z}\right)^2\cdot y\cdot z}{\left(2\cdot y\cdot z\right)^2}.$

Rješenje 634

Ponovimo!

$$\left(a\cdot b\right)^n=a^n\cdot b^n \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \frac{a^n}{a^m}=a^{n-m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{4\cdot\left(x\cdot\sqrt{z}\right)^2\cdot y\cdot z}{\left(2\cdot y\cdot z\right)^2}=\frac{4\cdot x^2\cdot\left(\sqrt{z}\right)^2\cdot y\cdot z}{4\cdot y^2\cdot z^2}=\frac{4\cdot x^2\cdot z\cdot y\cdot z}{4\cdot y^2\cdot z^2}=\frac{4\cdot x^2\cdot y\cdot z^2}{4\cdot y^2\cdot z^2}=\frac{4\cdot x^2\cdot y\cdot z^2}{4\cdot y^2\cdot z^2}=\frac{x^2}{y}.$$

Vježba 634

Pojednostavni izraz $\frac{4\cdot\left(x\cdot\sqrt{z}\right)^2\cdot y^2\cdot z}{\left(2\cdot y\cdot z\right)^2}.$

Rezultat: $x^2.$

Zadatak 635 (4A, 4B, TUPŠ)

Rastavi na faktore $y^4+8\cdot y^3+16\cdot y^2-\left(y^2+8\cdot y+16\right).$

Rješenje 635

Ponovimo!

$$\left(a+b\right)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad a^2-b^2=\left(a-b\right)\cdot\left(a+b\right) \quad , \quad a^1=a.$$

$$a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

1.inačica

$$y^4+8\cdot y^3+16\cdot y^2-\left(y^2+8\cdot y+16\right)=\left[\begin{matrix}\text{iz prva tri člana}\\ \text{izlučimo } y^2\end{matrix}\right]=$$

$$=y^2\cdot\left(y^2+8\cdot y+16\right)-\left(y^2+8\cdot y+16\right)=y^2\cdot\left(y^2+8\cdot y+16\right)-\left(y^2+8\cdot y+16\right)=$$

$$=\left(y^2+8\cdot y+16\right)\cdot\left(y^2-1\right)=\left(y+4\right)^4\cdot\left(y-1\right)\cdot\left(y+1\right).$$

2.inačica

$$y^4+8\cdot y^3+16\cdot y^2-\left(y^2+8\cdot y+16\right)=y^4+8\cdot y^3+16\cdot y^2-y^2-8\cdot y-16=$$

$$=y^4-y^2+8\cdot y^3-8\cdot y+16\cdot y^2-16=\begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}=$$

$$=\left(y^4-y^2\right)+\left(8\cdot y^3-8\cdot y\right)+\left(16\cdot y^2-16\right)=y^2\cdot\left(y^2-1\right)+8\cdot y\cdot\left(y^2-1\right)+16\cdot\left(y^2-1\right)=$$

$$=y^2\cdot\left(y^2-1\right)+8\cdot y\cdot\left(y^2-1\right)+16\cdot\left(y^2-1\right)=\left(y^2-1\right)\cdot\left(y^2+8\cdot y+16\right)=$$

$$=(y-1)\cdot(y+1)\cdot(y+4)^2.$$

Vježba 635

Rastavi na faktore $y^4+10\cdot y^3+25\cdot y^2-\left(y^2+10\cdot y+25\right)$.

Rezultat: $(y-1)\cdot(y+1)\cdot(y+5)^2.$

Zadatak 636 (Lussy, gimnazija)

Pojednostavnite $\frac{2\cdot\sqrt{5}}{\sqrt{10}}$.

Rješenje 636

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{2\cdot\sqrt{5}}{\sqrt{10}}=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=\frac{2\cdot\sqrt{5}}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}}=\frac{2\cdot\sqrt{5}\cdot\sqrt{10}}{\left(\sqrt{10}\right)^2}=\frac{2\cdot\sqrt{5\cdot 10}}{10}=\frac{2\cdot\sqrt{50}}{10}=$$

$$=\begin{bmatrix}\text{djelomično}\\ \text{korjenovanje}\end{bmatrix}=\frac{2\cdot\sqrt{25\cdot 2}}{10}=\frac{2\cdot\sqrt{25}\cdot\sqrt{2}}{10}=\frac{2\cdot 5\cdot\sqrt{2}}{10}=\frac{10\cdot\sqrt{2}}{10}=\frac{10\cdot\sqrt{2}}{10}=\sqrt{2}.$$

2.inačica

$$\frac{2\cdot\sqrt{5}}{\sqrt{10}}=2\cdot\sqrt{\frac{5}{10}}=2\cdot\sqrt{\frac{5}{10}}=2\cdot\sqrt{\frac{1}{2}}=2\cdot\frac{\sqrt{1}}{\sqrt{2}}=2\cdot\frac{1}{\sqrt{2}}=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=$$

$$=2\cdot\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=2\cdot\frac{\sqrt{2}}{\left(\sqrt{2}\right)^2}=2\cdot\frac{\sqrt{2}}{2}=2\cdot\frac{\sqrt{2}}{2}=\sqrt{2}.$$

Vježba 636

Pojednostavnite $\frac{2\cdot\sqrt{3}}{\sqrt{6}}$.

Rezultat: $\sqrt{2}.$

Zadatak 637 (Ivan, gimnazija)

Pojednostavnite $\left(\frac{1}{\sqrt{x}+1}+\frac{1}{x-1}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}.$

Rješenje 637

Ponovimo!

$$\left(\sqrt{a}\right)^2=a \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left(\frac{1}{\sqrt{x}+1}+\frac{1}{x-1}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\left(\frac{1}{\sqrt{x}+1}+\frac{1}{\left(\sqrt{x}\right)^2-1}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\left(\frac{1}{\sqrt{x}+1}+\frac{1}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\frac{\sqrt{x}-1+1}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{\sqrt{x}-1+1}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\frac{\sqrt{x}}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{\sqrt{x}}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{1}{\sqrt{x}-1}.$$

2.inačica

$$\left(\frac{1}{\sqrt{x}+1}+\frac{1}{x-1}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{1}{\sqrt{x}+1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}+\frac{1}{x-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\frac{1}{\sqrt{x}+1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}+\frac{1}{\left(\sqrt{x}\right)^2-1}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{1}{\sqrt{x}}+\frac{1}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=$$

$$=\frac{1}{\sqrt{x}}+\frac{1}{\left(\sqrt{x}-1\right)\cdot\left(\sqrt{x}+1\right)}\cdot\frac{\sqrt{x}+1}{\sqrt{x}}=\frac{1}{\sqrt{x}}+\frac{1}{\left(\sqrt{x}-1\right)\cdot\sqrt{x}}=\frac{\sqrt{x}-1+1}{\left(\sqrt{x}-1\right)\cdot\sqrt{x}}=$$

$$=\frac{\sqrt{x}-1+1}{\left(\sqrt{x}-1\right)\cdot\sqrt{x}}=\frac{\sqrt{x}}{\left(\sqrt{x}-1\right)\cdot\sqrt{x}}=\frac{\sqrt{x}}{\left(\sqrt{x}-1\right)\cdot\sqrt{x}}=\frac{1}{\sqrt{x}-1}.$$

Vježba 637

Pojednostavnite $\left(\frac{1}{\sqrt{x}+1}-\frac{1}{1-x}\right)\cdot\frac{\sqrt{x}+1}{\sqrt{x}}.$

Rezultat: $\frac{1}{\sqrt{x}-1}.$

Zadatak 638 (Paula, maturantica)

Rastavite na faktore $4\cdot x^2-12\cdot x\cdot y+9\cdot y^2+2\cdot x-3\cdot y.$

Rješenje 638

Ponovimo!

$$a^1=a \quad , \quad a^n:a^m=a^{n-m} \quad , \quad a^2-2\cdot a\cdot b+b^2=(a-b)^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$4\cdot x^2-12\cdot x\cdot y+9\cdot y^2+2\cdot x-3\cdot y=4\cdot x^2-6\cdot x\cdot y-6\cdot x\cdot y+9\cdot y^2+2\cdot x-3\cdot y=$$

$$=\left(4\cdot x^2-6\cdot x\cdot y+2\cdot x\right)+\left(-6\cdot x\cdot y+9\cdot y^2-3\cdot y\right)=$$

$$=2\cdot x\cdot(2\cdot x-3\cdot y+1)-3\cdot y\cdot(2\cdot x-3\cdot y+1)=2\cdot x\cdot(2\cdot x-3\cdot y+1)-3\cdot y\cdot(2\cdot x-3\cdot y+1)=$$

$$=(2\cdot x-3\cdot y+1)\cdot(2\cdot x-3\cdot y)=(2\cdot x-3\cdot y)\cdot(2\cdot x-3\cdot y+1).$$

2.inačica (Paula Manjkas)

$$4\cdot x^2-12\cdot x\cdot y+9\cdot y^2+2\cdot x-3\cdot y=\left(4\cdot x^2-12\cdot x\cdot y+9\cdot y^2\right)+(2\cdot x-3\cdot y)=$$

$$=(2\cdot x-3\cdot y)^2+(2\cdot x-3\cdot y)=(2\cdot x-3\cdot y)\cdot(2\cdot x-3\cdot y+1).$$

Vježba 638

Rastavite na faktore $x^2-4\cdot x\cdot y+4\cdot y^2+x-2\cdot y.$

Rezultat: $(x-2\cdot y)\cdot(x-2\cdot y+1).$

Zadatak 639 (Jele, gimnazija)

Ako je $\frac{3\cdot a-b}{a+3\cdot b}=2,$ koliko je $\frac{a-3\cdot b}{3\cdot a+b}$?

$A.\ \frac{1}{2}$ $\quad B.\ \frac{2}{11}$ $\quad C.\ \frac{3}{4}$ $\quad D.\ \frac{1}{4}$

Rješenje 639

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Preoblikujemo zadanu jednadžbu.

$$\frac{3\cdot a-b}{a+3\cdot b}=2 \Rightarrow \frac{3\cdot a-b}{a+3\cdot b}=2\ /\cdot\left(a+3\cdot b\right) \Rightarrow 3\cdot a-b=2\cdot\left(a+3\cdot b\right) \Rightarrow 3\cdot a-b=2\cdot a+6\cdot b \Rightarrow$$

$$\Rightarrow 3\cdot a-2\cdot a=6\cdot b+b \Rightarrow a=7\cdot b.$$

Sada je:

$$\frac{a-3\cdot b}{3\cdot a+b}=\left[a=7\cdot b\right]=\frac{7\cdot b-3\cdot b}{3\cdot 7\cdot b+b}=\frac{4\cdot b}{21\cdot b+b}=\frac{4\cdot b}{22\cdot b}=\frac{4\cdot b}{22\cdot b}=\frac{2}{11}.$$

Odgovor je pod B.

Vježba 639

Ako je $\frac{a+3\cdot b}{3\cdot a-b}=\frac{1}{2}$, koliko je $\frac{a-3\cdot b}{3\cdot a+b}$?

$A.\ \frac{1}{2}$ $\quad B.\ \frac{2}{11}$ $\quad C.\ \frac{3}{4}$ $\quad D.\ \frac{1}{4}$

Rezultat: B.

Zadatak 640 (4A, 4B, TUPŠ)

Ako je a · (a – b) = 11 i b · (a – b) = 13, tada je:

$A.\ a^2-b^2=143$ $\quad B.\ a^2-b^2=2$ $\quad C.\ a^2-b^2=24$ $\quad D.\ a^2-b^2=48$

Rješenje 640

Ponovimo!

$$\left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a+c=b+d \quad , \quad \left(a-b\right)\cdot\left(a+b\right)=a^2-b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

$$\left.\begin{array}{l} a\cdot\left(a-b\right)=11 \\ b\cdot\left(a-b\right)=13 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow a\cdot\left(a-b\right)+b\cdot\left(a-b\right)=11+13 \Rightarrow$$

$$\Rightarrow a\cdot\left(a-b\right)+b\cdot\left(a-b\right)=24 \Rightarrow a\cdot\left(a-b\right)+b\cdot\left(a-b\right)=24 \Rightarrow \left(a-b\right)\cdot\left(a+b\right)=24 \Rightarrow$$

$$\Rightarrow a^2-b^2=24.$$

Odgovor je pod C.

Vježba 640

Ako je a · (a – b) = 30 i b · (a – b) = 18, tada je:

$A.\ a^2-b^2=143$ $\quad B.\ a^2-b^2=2$ $\quad C.\ a^2-b^2=24$ $\quad D.\ a^2-b^2=48$

Rezultat: D.