Zadatak 481 (Dragana, komercijalna škola)

Rastavi brojnik i nazivnik na faktore: $\frac{x \cdot y - y \cdot z}{z^3 + 3 \cdot z}$.

Rješenje 481

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$\frac{x \cdot y - y \cdot z}{z^3 + 3 \cdot z} = \frac{y \cdot (x - z)}{z \cdot \left(z^2 + 3\right)}.$$

Vježba 481

Rastavi brojnik i nazivnik na faktore: $\frac{z^3 + 3 \cdot z}{x \cdot y - y \cdot z}$.

Rezultat: $\frac{z \cdot \left(z^2 + 3\right)}{y \cdot (x - z)}$.

Zadatak 482 (Dragana, komercijalna škola)

Skrati razlomak $\frac{16 \cdot x^3 - 36 \cdot x \cdot y^2}{6 \cdot x \cdot y - 9 \cdot y^2}$.

Rješenje 482

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad a^2 - b^2 = (a - b) \cdot (a + b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \neq 0 \; , \; n \neq 1.$$

$$\frac{16 \cdot x^3 - 36 \cdot x \cdot y^2}{6 \cdot x \cdot y - 9 \cdot y^2} = \begin{bmatrix} \text{u brojniku izlučimo } 4 \cdot x \\ \text{u nazivniku izlučimo } 3 \cdot y \end{bmatrix} = \frac{4 \cdot x \cdot \left(4 \cdot x^2 - 9 \cdot y^2\right)}{3 \cdot y \cdot (2 \cdot x - 3 \cdot y)} =$$

$$= \frac{4 \cdot x \cdot (2 \cdot x - 3 \cdot y) \cdot (2 \cdot x + 3 \cdot y)}{3 \cdot y \cdot (2 \cdot x - 3 \cdot y)} = \frac{4 \cdot x \cdot (2 \cdot x - 3 \cdot y) \cdot (2 \cdot x + 3 \cdot y)}{3 \cdot y \cdot (2 \cdot x - 3 \cdot y)} = \frac{4 \cdot x \cdot (2 \cdot x + 3 \cdot y)}{3 \cdot y}.$$

Vježba 482

Skrati razlomak $\frac{6 \cdot x \cdot y - 9 \cdot y^2}{16 \cdot x^3 - 36 \cdot x \cdot y^2}$.

Rezultat: $\frac{3 \cdot y}{4 \cdot x \cdot (2 \cdot x + 3 \cdot y)}$.

Zadatak 483 (BB, gimnazija)

Provjeri je li istinita sljedeća tvrdnja: ako je a > b, onda je (a + 1) · b < a · (b + 1).

Rješenje 483

Ponovimo!

$$x < y \Rightarrow y > x \quad , \quad x + z < y + z \Rightarrow x < y.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$(a + 1) \cdot b < a \cdot (b + 1) \Rightarrow a \cdot b + b < a \cdot b + a \Rightarrow a \cdot b + b < a \cdot b + a \Rightarrow b < a \Rightarrow a > b.$$

Tvrdnja je točna.

Vježba 483

Provjeri je li istinita sljedeća tvrdnja: ako je a > b, onda je (a + 2) · b < a · (b + 2).

Rezultat: Dokaz analogan.

Zadatak 484 (BB, gimnazija)

Ako su a i b pozitivni realni brojevi, pokaži da je $\frac{a \cdot b}{(a + b)^2} \le \frac{1}{4}$.

Rješenje 484

Ponovimo!

$$(x - y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad x \ge y \Rightarrow x + z \ge y + z \, , \, z \in R \quad , \quad (x + y)^2 = x^2 + 2 \cdot x \cdot y + y^2.$$

$$x \ge y \, , \, z > 0 \Rightarrow x \cdot z \ge y \cdot z \quad , \quad x \ge y \Rightarrow y \le x \quad , \quad x^2 \ge 0 \text{ za } x \in R.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \, , \, n \ne 0 \, , \, n \ne 1.$$

Budući da je kvadrat realnog broja nenegativan broj (veći ili jednak 0), slijedi:

$$(a - b)^2 \ge 0 \Rightarrow a^2 - 2 \cdot a \cdot b + b^2 \ge 0 \Rightarrow a^2 - 2 \cdot a \cdot b + b^2 + 4 \cdot a \cdot b \ge 4 \cdot a \cdot b \Rightarrow$$

$$\Rightarrow a^2 + 2 \cdot a \cdot b + b^2 \ge 4 \cdot a \cdot b \Rightarrow (a + b)^2 \ge 4 \cdot a \cdot b \Rightarrow 4 \cdot a \cdot b \le (a + b)^2 \Rightarrow$$

$$\Rightarrow 4 \cdot a \cdot b \le (a + b)^2 \; / \cdot \frac{1}{4 \cdot (a + b)^2} \Rightarrow \frac{4 \cdot a \cdot b}{4 \cdot (a + b)^2} \le \frac{(a + b)^2}{4 \cdot (a + b)^2} \Rightarrow \frac{4 \cdot a \cdot b}{4 \cdot (a + b)^2} \le \frac{(a + b)^2}{4 \cdot (a + b)^2} \Rightarrow$$

$$\Rightarrow \frac{a \cdot b}{(a + b)^2} \le \frac{1}{4}.$$

Vježba 484

Ako su a i b pozitivni realni brojevi, pokaži da je $\frac{4 \cdot a \cdot b}{(a + b)^2} \le 1$.

Rezultat: Dokaz analogan.

Zadatak 485 (Ivan, gimnazija)

Rastavi na faktore $a^2 - b^2 - a \cdot x - b \cdot x$.

Rješenje 485

Ponovimo!

$$x^2 - y^2 = (x - y) \cdot (x + y).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$a^2 - b^2 - a \cdot x - b \cdot x = (a - b) \cdot (a + b) - x \cdot (a + b) = (a - b) \cdot (a + b) - x \cdot (a + b) =$$

$$= (a + b) \cdot ((a - b) - x) = (a + b) \cdot (a - b - x).$$

Vježba 485

Rastavi na faktore $a^2 - b^2 + a \cdot x - b \cdot x$.

Rezultat: $(a - b) \cdot (a + b + x)$.

Zadatak 486 (Josip, gimnazija)

Skrati razlomak $\dfrac{a^2 - a \cdot b + b \cdot c - c^2}{b^2 - a^2 - 2 \cdot a \cdot c - c^2}$.

Rješenje 486

Ponovimo!

$$x^2 - y^2 = (x - y) \cdot (x + y) \quad , \quad (x + y)^2 = x^2 + 2 \cdot x \cdot y + y^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \;,\; n \neq 0 \;,\; n \neq 1.$$

$$\frac{a^2 - a \cdot b + b \cdot c - c^2}{b^2 - a^2 - 2 \cdot a \cdot c - c^2} = \frac{a^2 - c^2 - a \cdot b + b \cdot c}{b^2 - \left(a^2 + 2 \cdot a \cdot c + c^2\right)} = \frac{(a - c) \cdot (a + c) - b \cdot (a - c)}{b^2 - (a + c)^2} =$$

$$= \frac{(a - c) \cdot (a + c) - b \cdot (a - c)}{b^2 - (a + c)^2} = \frac{(a - c) \cdot (a + c) - b \cdot (a - c)}{(b - (a + c)) \cdot (b + (a + c))} = \frac{(a - c) \cdot ((a + c) - b)}{(b - a - c) \cdot (b + a + c)} =$$

$$= \frac{(a - c) \cdot (a + c - b)}{(b - a - c) \cdot (b + a + c)} = \frac{(a - c) \cdot (a - b + c)}{(-a + b - c) \cdot (a + b + c)} = \frac{(a - c) \cdot (a - b + c)}{-(a - b + c) \cdot (a + b + c)} = \frac{(a - c) \cdot (a - b + c)}{-(a - b + c) \cdot (a + b + c)} =$$

$$= -\frac{a - c}{a + b + c} = \frac{c - a}{a + b + c}.$$

Vježba 486

Skrati razlomak $\dfrac{b^2 - a^2 - 2 \cdot a \cdot c - c^2}{a^2 - a \cdot b + b \cdot c - c^2}$.

Rezultat: $\dfrac{a + b + c}{c - a}$.

Zadatak 487 (Josip, gimnazija)

Skrati razlomak $\frac{a^2+6\cdot a+9-b^2}{a^2+2\cdot a\cdot b+b^2-9}$.

Rješenje 487

Ponovimo!

$$(x+y)^2=x^2+2\cdot x\cdot y+y^2 \quad , \quad x^2-y^2=(x-y)\cdot(x+y).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{a^2+6\cdot a+9-b^2}{a^2+2\cdot a\cdot b+b^2-9}=\frac{(a+3)^2-b^2}{(a+b)^2-9}=\frac{((a+3)-b)\cdot((a+3)+b)}{((a+b)-3)\cdot((a+b)+3)}=\frac{(a+3-b)\cdot(a+3+b)}{(a+b-3)\cdot(a+b+3)}=$$

$$=\frac{(a+3-b)\cdot(a+3+b)}{(a+b-3)\cdot(a+b+3)}=\frac{a+3-b}{a+b-3}=\frac{a-b+3}{a+b-3}.$$

Vježba 487

Skrati razlomak $\frac{a^2+2\cdot a\cdot b+b^2-9}{a^2+6\cdot a+9-b^2}$.

Rezultat: $\frac{a+b-3}{a-b+3}$.

Zadatak 488 (Ivan, gimnazija)

Izraz $\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}$ za $a\neq b\neq 0$ identičan je razlomku:

$$A.\ \frac{b^2}{b-a} \qquad B.\ \frac{a^2}{a-b} \qquad C.\ \frac{1}{a-b^2} \qquad D.\ \frac{1}{a\cdot b}$$

Rješenje 488

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left(\frac{b}{1-\frac{a}{b}}+\frac{a}{1-\frac{b}{a}}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}=\left(\frac{b}{\frac{1}{1}-\frac{a}{b}}+\frac{a}{\frac{1}{1}-\frac{b}{a}}\right)\cdot\frac{1}{\frac{1}{1}-\frac{a^2}{b^2}}=\left(\frac{b}{\frac{b-a}{b}}+\frac{a}{\frac{a-b}{a}}\right)\cdot\frac{1}{\frac{b^2-a^2}{b^2}}=$$

$$=\left(\frac{\frac{b}{1}}{\frac{b-a}{b}}+\frac{\frac{a}{1}}{\frac{a-b}{a}}\right)\cdot\frac{\frac{1}{1}}{\frac{b^2-a^2}{b^2}}=\left(\frac{b^2}{b-a}+\frac{a^2}{a-b}\right)\cdot\frac{b^2}{b^2-a^2}=\left(\frac{b^2}{b-a}+\frac{a^2}{-(b-a)}\right)\cdot\frac{b^2}{b^2-a^2}=$$

$$=\left(\frac{b^2}{b-a}-\frac{a^2}{b-a}\right)\cdot\frac{b^2}{b^2-a^2}=\frac{b^2-a^2}{b-a}\cdot\frac{b^2}{b^2-a^2}=\frac{b^2-a^2}{b-a}\cdot\frac{b^2}{b^2-a^2}=\frac{b^2}{b-a}.$$

Odgovor je pod A.

Vježba 488

Izraz $\left(\frac{b}{1-\frac{a}{b}}-\frac{a}{\frac{b}{a}-1}\right)\cdot\frac{1}{1-\frac{a^2}{b^2}}$ za $a\neq b\neq 0$ identičan je razlomku:

$A.\ \frac{b^2}{b-a}$ $\quad B.\ \frac{a^2}{a-b}$ $\quad C.\ \frac{1}{a-b^2}$ $\quad D.\ \frac{1}{a\cdot b}$

Rezultat: A.

Zadatak 489 (Mario, tehnička škola)

Vrijednost razlomka $\frac{(m-n)^2+4\cdot m\cdot n}{(m+n)^2-4\cdot m\cdot n}$ za m = 111111, n = 777777 jednaka je:

$A.\ \frac{1}{49}$ $\quad B.\ \frac{16}{9}$ $\quad C.\ \frac{3}{4}$ $\quad D.\ \frac{1}{6}$

Rješenje 489

Ponovimo!

$$(x-y)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad (x+y)^2=x^2+2\cdot x\cdot y+y^2 \quad , \quad \left(\frac{x}{y}\right)^n=\frac{x^n}{y^n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{(m-n)^2+4\cdot m\cdot n}{(m+n)^2-4\cdot m\cdot n}=\frac{m^2-2\cdot m\cdot n+n^2+4\cdot m\cdot n}{m^2+2\cdot m\cdot n+n^2-4\cdot m\cdot n}=\frac{m^2+2\cdot m\cdot n+n^2}{m^2-2\cdot m\cdot n+n^2}=\frac{(m+n)^2}{(m-n)^2}=$$

$$=\left(\frac{m+n}{m-n}\right)^2=\left(\frac{111111+777777}{111111-777777}\right)^2=\left(\frac{111111\cdot(1+7)}{111111\cdot(1-7)}\right)^2=\left(\frac{111111\cdot 8}{111111\cdot(-6)}\right)^2=\left(\frac{111111\cdot 8}{111111\cdot(-6)}\right)^2=$$

$$=\left(-\frac{8}{6}\right)^2=\left(-\frac{8}{6}\right)^2=\left(-\frac{4}{3}\right)^2=\frac{16}{9}.$$

Odgovor je pod B.

Vježba 489

Vrijednost razlomka $\dfrac{(m-n)^2+4\cdot m\cdot n}{(m+n)^2-4\cdot m\cdot n}$ za m = 111, n = 777 jednaka je:

$A.\ \frac{1}{49}$ $B.\ \frac{16}{9}$ $C.\ \frac{3}{4}$ $D.\ \frac{1}{6}$

Rezultat: B.

Zadatak 490 (Gabrijela, ekonomska škola)

Pojednostavnite: $\left(\dfrac{\sqrt[4]{a^3\cdot b}-\sqrt[4]{a\cdot b^3}}{a\cdot\sqrt{b}-b\cdot\sqrt{a}}\right)^{-4}$.

Rješenje 490

Ponovimo!

$$x^1=x\quad,\quad x^n\cdot x^m=x^{n+m}\quad,\quad \sqrt{x\cdot y}=\sqrt{x}\cdot\sqrt{y}\quad,\quad \left(\sqrt[n]{x}\right)^n=n.$$

$$\sqrt[n\cdot p]{x^{m\cdot p}}=\sqrt[n]{x^m}\quad,\quad \left(\frac{x}{y}\right)^{-n}=\left(\frac{y}{x}\right)^n\quad,\quad \left(\sqrt[n]{x}\right)^p=\sqrt[n]{x^p}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left(\frac{\sqrt[4]{a^3\cdot b}-\sqrt[4]{a\cdot b^3}}{a\cdot\sqrt{b}-b\cdot\sqrt{a}}\right)^{-4}=\left(\frac{a\cdot\sqrt{b}-b\cdot\sqrt{a}}{\sqrt[4]{a^3\cdot b}-\sqrt[4]{a\cdot b^3}}\right)^4=\left(\frac{\left(\sqrt{a}\right)^2\cdot\sqrt{b}-\left(\sqrt{b}\right)^2\cdot\sqrt{a}}{\sqrt[4]{a\cdot a^2\cdot b}-\sqrt[4]{a\cdot b\cdot b^2}}\right)^4=$$

$$=\left(\frac{\sqrt{a}\cdot\sqrt{b}\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\sqrt[4]{a\cdot b\cdot a^2}-\sqrt[4]{a\cdot b\cdot b^2}}\right)^4=\left(\frac{\sqrt{a\cdot b}\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\sqrt[4]{a\cdot b}\cdot\sqrt[4]{a^2}-\sqrt[4]{a\cdot b}\cdot\sqrt[4]{b^2}}\right)^4=$$

$$=\left(\frac{\sqrt{a\cdot b}\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\sqrt[4]{a\cdot b}\cdot\left(\sqrt[4]{a^2}-\sqrt[4]{b^2}\right)}\right)^4=\left(\frac{\sqrt{a\cdot b}\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\sqrt[4]{a\cdot b}\cdot\left(\sqrt[4]{a^2}-\sqrt[4]{b^2}\right)}\right)^4=\left(\frac{\sqrt{a\cdot b}\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\sqrt[4]{a\cdot b}\cdot\left(\sqrt{a}-\sqrt{b}\right)}\right)^4=$$

$$=\left(\frac{\sqrt{a\cdot b}\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\sqrt[4]{a\cdot b}\cdot\left(\sqrt{a}-\sqrt{b}\right)}\right)^4=\left(\frac{\sqrt{a\cdot b}}{\sqrt[4]{a\cdot b}}\right)^4=\frac{\left(\sqrt{a\cdot b}\right)^4}{\left(\sqrt[4]{a\cdot b}\right)^4}=\frac{\sqrt{a^4\cdot b^4}}{\left(\sqrt[4]{a\cdot b}\right)^4}=\frac{a^2\cdot b^2}{a\cdot b}=\frac{a^2\cdot b^2}{a\cdot b}=a\cdot b.$$

Vježba 490

Pojednostavnite: $\left(\dfrac{a\cdot\sqrt{b}-b\cdot\sqrt{a}}{\sqrt[4]{a^3}\cdot b-\sqrt[4]{a\cdot b^3}}\right)^{-4}$.

Rezultat: $\dfrac{1}{a\cdot b}$.

Zadatak 491 (Gabrijela, ekonomska škola)

Pojednostavnite: $\left(\sqrt[3]{\dfrac{a^2}{b^2}}+\sqrt[3]{\dfrac{b^2}{a^2}}+1\right)\cdot\left(\sqrt[3]{\dfrac{a}{b}}-\sqrt[3]{\dfrac{b}{a}}\right)$.

Rješenje 491

Ponovimo!

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right)\quad,\quad\left(\sqrt[n]{a}\right)^n=a\quad,\quad\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot c}.$$

$$\left(\sqrt[3]{\frac{a^2}{b^2}}+\sqrt[3]{\frac{b^2}{a^2}}+1\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right)=\left(\sqrt[3]{\frac{a^2}{b^2}}+1+\sqrt[3]{\frac{b^2}{a^2}}\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right)=$$

$$=\left(\sqrt[3]{\left(\frac{a}{b}\right)^2}+1+\sqrt[3]{\left(\frac{b}{a}\right)^2}\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right)=\left(\sqrt[3]{\frac{a}{b}}\right)^3-\left(\sqrt[3]{\frac{b}{a}}\right)^3=\frac{a}{b}-\frac{b}{a}=\frac{a^2-b^2}{a\cdot b}.$$

Vježba 491

Pojednostavnite: $\left(\sqrt[3]{\dfrac{a^2}{b^2}}+\sqrt[3]{\dfrac{b^2}{a^2}}-1\right)\cdot\left(\sqrt[3]{\dfrac{a}{b}}+\sqrt[3]{\dfrac{b}{a}}\right)$.

Rezultat: $\dfrac{a^2+b^2}{a\cdot b}$.

Zadatak 492 (4A, TUPŠ)

Rastavite na faktore: $x^2\cdot y-2\cdot x^3+2\cdot x\cdot y-4\cdot x^2+y-2\cdot x$.

$A.\ (y-2\cdot x)^2\cdot(x+1)$ $\qquad B.\ (y-2\cdot x)\cdot(x+1)^2$

$C.\ (y-2\cdot x)\cdot\left(x^2-1\right)$ $\qquad D.\ (y-2\cdot x)\cdot(x+1)$

Rješenje 492

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad a^1=a\quad,\quad a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$x^2\cdot y-2\cdot x^3+2\cdot x\cdot y-4\cdot x^2+y-2\cdot x=\begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}=\left(x^2\cdot y-2\cdot x^3\right)+\left(2\cdot x\cdot y-4\cdot x^2\right)+(y-2\cdot x)=$$

$$= x^2 \cdot (y - 2 \cdot x) + 2 \cdot x \cdot (y - 2 \cdot x) + (y - 2 \cdot x) = x^2 \cdot (y - 2 \cdot x) + 2 \cdot x \cdot (y - 2 \cdot x) + (y - 2 \cdot x) =$$

$$= (y - 2 \cdot x) \cdot \left(x^2 + 2 \cdot x + 1\right) = (y - 2 \cdot x) \cdot (x + 1)^2.$$

Odgovor je pod B.

Vježba 492

Rastavite na faktore: $x^2 \cdot y - x^3 + 2 \cdot x \cdot y - 2 \cdot x^2 + y - x.$

$$A.\ (y - x)^2 \cdot (x + 1) \qquad B.\ (y - x) \cdot (x + 1)^2$$
$$C.\ (y - x) \cdot (x + 1) \qquad D.\ (y - x) \cdot (x - 1)$$

Rezultat: B.

Zadatak 493 (Lajlica, gimnazija)

Riješi izraz: $\left(\dfrac{a^4 - a^2 + 2 \cdot a - 1}{\left(a^2 + 1\right)^2 - a^2} - \dfrac{\left(a^2 - 1\right)^2 - a^2}{a^4 + 2 \cdot a^3 + a^2 - 1} \right) : \dfrac{a^2 - 2 \cdot a}{a^3 - 1}.$

Rješenje 493

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$(a - b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \quad , \quad a^2 - b^2 = (a - b) \cdot (a + b) \quad , \quad \left(a^n\right)^m = a^{n \cdot m}.$$

$$a^n \cdot b^n = (a \cdot b)^n \quad , \quad \frac{a}{n} - \frac{b}{n} = \frac{a - b}{n} \quad , \quad \frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}.$$

$$(a + b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad a^3 - b^3 = (a - b) \cdot \left(a^2 + a \cdot b + b^2\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\left(\frac{a^4 - a^2 + 2 \cdot a - 1}{\left(a^2 + 1\right)^2 - a^2} - \frac{\left(a^2 - 1\right)^2 - a^2}{a^4 + 2 \cdot a^3 + a^2 - 1} \right) : \frac{a^2 - 2 \cdot a}{a^3 - 1} =$$

$$= \left(\frac{a^4 - \left(a^2 - 2 \cdot a + 1\right)}{\left(a^2 + 1\right)^2 - a^2} - \frac{\left(a^2 - 1\right)^2 - a^2}{a^2 \cdot \left(a^2 + 2 \cdot a + 1\right) - 1} \right) : \frac{a^2 - 2 \cdot a}{a^3 - 1} =$$

$$= \left(\frac{a^4 - (a - 1)^2}{\left(a^2 + 1\right)^2 - a^2} - \frac{\left(a^2 - 1\right)^2 - a^2}{a^2 \cdot (a + 1)^2 - 1} \right) : \frac{a^2 - 2 \cdot a}{a^3 - 1} =$$

$$=\left(\frac{\left(a^2\right)^2-(a-1)^2}{\left(a^2+1\right)^2-a^2}-\frac{\left(a^2-1\right)^2-a^2}{(a\cdot(a+1))^2-1}\right):\frac{a^2-2\cdot a}{a^3-1}=$$

$$=\left(\frac{\left(a^2-(a-1)\right)\cdot\left(a^2+(a-1)\right)}{\left(a^2+1-a\right)\cdot\left(a^2+1+a\right)}-\frac{\left(a^2-1-a\right)\cdot\left(a^2-1+a\right)}{(a\cdot(a+1)-1)\cdot(a\cdot(a+1)+1)}\right):\frac{a^2-2\cdot a}{a^3-1}=$$

$$=\left(\frac{\left(a^2-a+1\right)\cdot\left(a^2+a-1\right)}{\left(a^2-a+1\right)\cdot\left(a^2+a+1\right)}-\frac{\left(a^2-a-1\right)\cdot\left(a^2+a-1\right)}{\left(a^2+a-1\right)\cdot\left(a^2+a+1\right)}\right):\frac{a^2-2\cdot a}{a^3-1}=$$

$$=\left(\frac{\left(a^2-a+1\right)\cdot\left(a^2+a-1\right)}{\left(a^2-a+1\right)\cdot\left(a^2+a+1\right)}-\frac{\left(a^2-a-1\right)\cdot\left(a^2+a-1\right)}{\left(a^2+a-1\right)\cdot\left(a^2+a+1\right)}\right):\frac{a^2-2\cdot a}{a^3-1}=$$

$$=\left(\frac{a^2+a-1}{a^2+a+1}-\frac{a^2-a-1}{a^2+a+1}\right):\frac{a^2-2\cdot a}{a^3-1}=\frac{a^2+a-1-\left(a^2-a-1\right)}{a^2+a+1}:\frac{a^2-2\cdot a}{a^3-1}=$$

$$=\frac{a^2+a-1-a^2+a+1}{a^2+a+1}:\frac{a^2-2\cdot a}{a^3-1}=\frac{a^2+a-1-a^2+a+1}{a^2+a+1}:\frac{a^2-2\cdot a}{a^3-1}=\frac{a+a}{a^2+a+1}:\frac{a^2-2\cdot a}{a^3-1}=$$

$$=\frac{2\cdot a}{a^2+a+1}:\frac{a^2-2\cdot a}{a^3-1}=\frac{2\cdot a}{a^2+a+1}\cdot\frac{a^3-1}{a^2-2\cdot a}=\frac{2\cdot a}{a^2+a+1}\cdot\frac{(a-1)\cdot\left(a^2+a+1\right)}{a\cdot(a-2)}=$$

$$=\frac{2\cdot a}{a^2+a+1}\cdot\frac{(a-1)\cdot\left(a^2+a+1\right)}{a\cdot(a-2)}=\frac{2\cdot(a-1)}{a-2}.$$

Vježba 493

Riješi izraz: $\left(\frac{a^4-a^2+2\cdot a-1}{\left(a^2+1\right)^2-a^2}+\frac{a^2-\left(a^2-1\right)^2}{a^4+2\cdot a^3+a^2-1}\right):\frac{2\cdot a-a^2}{1-a^3}.$

Rezultat: $\frac{2\cdot(a-1)}{a-2}.$

Zadatak 494 (IT, gimnazija)

Ako je $x^2\cdot y^3=80$, a $x^3\cdot y^4=50$, izračunaj $x\cdot y^2$.

Rješenje 494

Ponovimo!

$$\frac{a^n}{a^m}=a^{n-m}\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c}\quad,\quad \left.\begin{matrix}a=b\\c=d\end{matrix}\right\}\Rightarrow\frac{a}{c}=\frac{b}{d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$(a\cdot b)^n=a^n\cdot b^n\quad ,\quad \left(a^n\right)^m=a^{n\cdot m}.$$

1.inačica

$$\left.\begin{array}{l} x^2\cdot y^3=80 \\ x^3\cdot y^4=50 \end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{podijelimo}\\ \text{jednakosti}\end{array}\right]\Rightarrow\frac{x^3\cdot y^4}{x^2\cdot y^3}=\frac{50}{80}\Rightarrow\frac{x^3\cdot y^4}{x^2\cdot y^3}=\frac{50}{80}\Rightarrow x\cdot y=\frac{5}{8}.$$

Sada prvu jednakost preoblikujemo na ovaj način:

$$\left.\begin{array}{l} x^2\cdot y^3=80 \\ x\cdot y=\frac{5}{8} \end{array}\right\}\Rightarrow\left.\begin{array}{l} x\cdot y\cdot x\cdot y^2=80 \\ x\cdot y=\frac{5}{8} \end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{metoda}\\ \text{supstitucije}\end{array}\right]\Rightarrow\frac{5}{8}\cdot x\cdot y^2=80\Rightarrow$$

$$\Rightarrow\frac{5}{8}\cdot x\cdot y^2=80\ /\cdot\frac{8}{5}\Rightarrow x\cdot y^2=80\cdot\frac{8}{5}\Rightarrow x\cdot y^2=80\cdot\frac{8}{5}\Rightarrow x\cdot y^2=16\cdot 8\Rightarrow x\cdot y^2=128.$$

2.inačica

Računamo x.

$$\left.\begin{array}{l} x^2\cdot y^3=80 \\ x^3\cdot y^4=50 \end{array}\right\}\Rightarrow\left.\begin{array}{l} x^2\cdot y^3=80\ /\ 4 \\ x^3\cdot y^4=50\ /\ 3 \end{array}\right\}\Rightarrow\left.\begin{array}{l} \left(x^2\cdot y^3\right)^4=80^4 \\ \left(x^3\cdot y^4\right)^3=50^3 \end{array}\right\}\Rightarrow\left.\begin{array}{l} \left(x^2\right)^4\cdot\left(y^3\right)^4=80^4 \\ \left(x^3\right)^3\cdot\left(y^4\right)^3=50^3 \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l} x^8\cdot y^{12}=80^4 \\ x^9\cdot y^{12}=50^3 \end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{podijelimo}\\ \text{jednakosti}\end{array}\right]\Rightarrow\frac{x^9\cdot y^{12}}{x^8\cdot y^{12}}=\frac{50^3}{80^4}\Rightarrow$$

$$\Rightarrow\frac{x^9\cdot y^{12}}{x^8\cdot y^{12}}=\frac{50^3}{80^4}\Rightarrow x=\frac{50^3}{80^4}.$$

Računamo y.

$$\left.\begin{array}{l} x^2\cdot y^3=80 \\ x^3\cdot y^4=50 \end{array}\right\}\Rightarrow\left.\begin{array}{l} x^2\cdot y^3=80\ /\ 3 \\ x^3\cdot y^4=50\ /\ 2 \end{array}\right\}\Rightarrow\left.\begin{array}{l} \left(x^2\cdot y^3\right)^3=80^3 \\ \left(x^3\cdot y^4\right)^2=50^2 \end{array}\right\}\Rightarrow\left.\begin{array}{l} \left(x^2\right)^3\cdot\left(y^3\right)^3=80^3 \\ \left(x^3\right)^2\cdot\left(y^4\right)^2=50^2 \end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l} x^6\cdot y^9=80^3 \\ x^6\cdot y^8=50^2 \end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{podijelimo}\\ \text{jednakosti}\end{array}\right]\Rightarrow\frac{x^6\cdot y^9}{x^6\cdot y^8}=\frac{80^3}{50^2}\Rightarrow\frac{x^6\cdot y^9}{x^6\cdot y^8}=\frac{80^3}{50^2}\Rightarrow y=\frac{80^3}{50^2}.$$

Sada je:

$$x\cdot y^2=\frac{50^3}{80^4}\cdot\left(\frac{80^3}{50^2}\right)^2\Rightarrow x\cdot y^2=\frac{50^3}{80^4}\cdot\frac{\left(80^3\right)^2}{\left(50^2\right)^2}\Rightarrow x\cdot y^2=\frac{50^3}{80^4}\cdot\frac{80^6}{50^4}\Rightarrow$$

$$\Rightarrow x\cdot y^2=\frac{50^3}{80^4}\cdot\frac{80^6}{50^4}\Rightarrow x\cdot y^2=\frac{80^2}{50}\Rightarrow x\cdot y^2=\frac{6\,400}{50}\Rightarrow x\cdot y^2=128.$$

Vježba 494

Ako je $\frac{x^2 \cdot y^3}{2} = 40,$ a $\frac{x^3 \cdot y^4}{5} = 10,$ izračunaj $x \cdot y^2$.

Rezultat: $x \cdot y^2 = 128.$

Zadatak 495 (Marija, gimnazija)

Kvadrat zbroja bilo kojega realnog broja i njegove recipročne vrijednosti veći je od kvadrata razlike tog broja i njegove recipročne vrijednosti za:

$A.\ 2 \qquad B.\ 4 \qquad C.\ 1 \qquad D.\ 8$

Rješenje 495

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \quad , \quad a \cdot \frac{1}{a} = 1.$$

$$a^2 - b^2 = (a-b) \cdot (a+b) \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad \frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Neka je x bilo koji realan broj različit od nule.
Zapišimo u obliku izraza rečenicu "Kvadrat zbroja bilo kojega realnog broja i njegove recipročne vrijednosti":

$$\left(x + \frac{1}{x}\right)^2.$$

Zapišimo u obliku izraza rečenicu " Kvadrat razlike bilo kojega realnog broja i njegove recipročne vrijednosti":

$$\left(x - \frac{1}{x}\right)^2.$$

Računamo za koliko je $\left(x + \frac{1}{x}\right)^2$ veći od $\left(x - \frac{1}{x}\right)^2$.

1.inačica

$$\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2 = x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} - \left(x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2}\right) =$$

$$= x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} - x^2 + 2 \cdot x \cdot \frac{1}{x} - \frac{1}{x^2} = x^2 + 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} - x^2 + 2 \cdot x \cdot \frac{1}{x} - \frac{1}{x^2} = 2 + 2 = 4.$$

Odgovor je pod B.

2.inačica

$$\left(x + \frac{1}{x}\right)^2 - \left(x - \frac{1}{x}\right)^2 = \left(\left(x + \frac{1}{x}\right) - \left(x - \frac{1}{x}\right)\right) \cdot \left(\left(x + \frac{1}{x}\right) + \left(x - \frac{1}{x}\right)\right) =$$

$$= \left(x + \frac{1}{x} - x + \frac{1}{x}\right) \cdot \left(x + \frac{1}{x} + x - \frac{1}{x}\right) = \left(x + \frac{1}{x} - x + \frac{1}{x}\right) \cdot \left(x + \frac{1}{x} + x - \frac{1}{x}\right) =$$

$$= \left(\frac{1}{x}+\frac{1}{x}\right)\cdot(x+x)=\frac{2}{x}\cdot 2\cdot x=\frac{2}{x}\cdot 2\cdot x=2\cdot 2=4.$$

Odgovor je pod B.

Vježba 495

Kvadrat razlike bilo kojega realnog broja i njegove recipročne vrijednosti manji je od kvadrata zbroja tog broja i njegove recipročne vrijednosti za:

A. 2 *B.* 4 *C.* 1 *D.* 8

Rezultat: B.

Zadatak 496 (Tina, gimnazija)

Dokaži ako je a + b + c = 0, tada vrijedi a · (a + c) = b · (b + c).

Rješenje 496

Ponovimo!

$$x\cdot(-y)=-x\cdot y.$$

$$\left.\begin{array}{l} a+b+c=0 \\ a+b+c=0 \\ a\cdot(a+c)=b\cdot(b+c)\end{array}\right\} \Rightarrow \left.\begin{array}{l} a+c=-b \\ b+c=-a \\ a\cdot(a+c)=b\cdot(b+c)\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda} \\ \text{supstitucije}\end{array}\right] \Rightarrow$$

$$\Rightarrow a\cdot(-b)=b\cdot(-a) \Rightarrow -a\cdot b=-a\cdot b \Rightarrow -a\cdot b=-a\cdot b \;/\cdot(-1) \Rightarrow a\cdot b=a\cdot b.$$

Budući da su lijeva i desna strana jednake, vrijedi dana jednakost.

Vježba 496

Dokaži ako je a + b + c = 0, tada vrijedi $a^2 - b^2 = c\cdot(b-a)$.

Rezultat: Dokaz analogan.

Zadatak 497 (Tina, gimnazija)

Dokaži ako je a + b + c = 0, tada vrijedi $a^3 + b^3 + c^3 = 3\cdot a\cdot b\cdot c$.

Rješenje 497

Ponovimo!

$$(-a-b)^3=-(a+b)^3 \quad , \quad (a+b)^3=a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left.\begin{array}{l} a+b+c=0 \\ a^3+b^3+c^3=3\cdot a\cdot b\cdot c\end{array}\right\} \Rightarrow \left.\begin{array}{l} c=-a-b \\ a^3+b^3+c^3=3\cdot a\cdot b\cdot c\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda} \\ \text{supstitucije}\end{array}\right] \Rightarrow$$

$$\Rightarrow a^3+b^3+(-a-b)^3=3\cdot a\cdot b\cdot(-a-b) \Rightarrow a^3+b^3-(a+b)^3=-3\cdot a\cdot b\cdot(a+b) \Rightarrow$$

$$\Rightarrow a^3+b^3-\left(a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3\right)=-3\cdot a\cdot b\cdot(a+b) \Rightarrow$$

$$\Rightarrow a^3+b^3-a^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2-b^3=-3\cdot a\cdot b\cdot(a+b) \Rightarrow$$

$$\Rightarrow a^3+b^3-a^3-3\cdot a^2\cdot b-3\cdot a\cdot b^2-b^3=-3\cdot a\cdot b\cdot(a+b) \Rightarrow$$

$$\Rightarrow -3\cdot a^2\cdot b-3\cdot a\cdot b^2=-3\cdot a\cdot b\cdot(a+b) \Rightarrow -3\cdot a\cdot b\cdot(a+b)=-3\cdot a\cdot b\cdot(a+b).$$

Budući da su lijeva i desna strana jednake, vrijedi dana jednakost.

Vježba 497

Dokaži ako je a + b + c = 0, tada vrijedi $a^3 + b^3 = c \cdot (3 \cdot a \cdot b - c^2)$.

Rezultat: Dokaz analogan.

Zadatak 498 (Antonio, tehnička škola)

Ako je $a \cdot x + b \cdot y = 0$, dokaži da je $\frac{a^2}{a^2 + b^2} + \frac{x^2}{x^2 + y^2} = 1.$

Rješenje 498

Ponovimo!

$$(a \cdot b)^n = a^n \cdot b^n \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \quad , \quad \frac{a}{n} + \frac{b}{n} = \frac{a + b}{n}.$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad \frac{a}{b} = \frac{\frac{a}{n}}{\frac{b}{n}}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

Iz jednakosti

$$a \cdot x + b \cdot y = 0$$

izračunamo, na primjer, y:

$$a \cdot x + b \cdot y = 0 \ \Rightarrow \ b \cdot y = -a \cdot x \ \Rightarrow \ b \cdot y = -a \cdot x \ / \cdot \frac{1}{b} \ \Rightarrow \ y = -\frac{a}{b} \cdot x.$$

Tada je:

$$\frac{a^2}{a^2 + b^2} + \frac{x^2}{x^2 + y^2} = \left[\begin{array}{c} \text{zamjena} \\ y = -\frac{a}{b} \cdot x \end{array} \right] = \frac{a^2}{a^2 + b^2} + \frac{x^2}{x^2 + \left(-\frac{a}{b} \cdot x\right)^2} = \frac{a^2}{a^2 + b^2} + \frac{x^2}{x^2 + \frac{a^2}{b^2} \cdot x^2} =$$

$$= \frac{a^2}{a^2 + b^2} + \frac{x^2}{x^2 \cdot \left(1 + \frac{a^2}{b^2}\right)} = \frac{a^2}{a^2 + b^2} + \frac{x^2}{x^2 \cdot \left(1 + \frac{a^2}{b^2}\right)} = \frac{a^2}{a^2 + b^2} + \frac{1}{1 + \frac{a^2}{b^2}} =$$

$$= \frac{a^2}{a^2 + b^2} + \frac{1}{\frac{1}{1} + \frac{a^2}{b^2}} = \frac{a^2}{a^2 + b^2} + \frac{1}{\frac{b^2 + a^2}{b^2}} = \frac{a^2}{a^2 + b^2} + \frac{\frac{1}{1}}{\frac{b^2 + a^2}{b^2}} = \frac{a^2}{a^2 + b^2} + \frac{b^2}{b^2 + a^2} =$$

$$=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}=\frac{a^2+b^2}{a^2+b^2}=\frac{a^2+b^2}{a^2+b^2}=1.$$

2.inačica

Preoblikujemo jednakost.

$$a\cdot x+b\cdot y=0 \Rightarrow a\cdot x=-b\cdot y \Rightarrow a\cdot x=-b\cdot y \;/\cdot\frac{1}{b\cdot x} \Rightarrow \frac{a}{b}=-\frac{y}{x} \Rightarrow \frac{a}{b}=-\frac{y}{x} \;/^2 \Rightarrow$$

$$\Rightarrow \left(\frac{a}{b}\right)^2=\left(-\frac{y}{x}\right)^2 \Rightarrow \frac{a^2}{b^2}=\frac{y^2}{x^2}.$$

Tada je:

$$\frac{a^2}{a^2+b^2}+\frac{x^2}{x^2+y^2}=\frac{\frac{a^2}{b^2}}{\frac{a^2+b^2}{b^2}}+\frac{x^2}{x^2+y^2}=\frac{\frac{a^2}{b^2}}{\frac{a^2+b^2}{b^2}}+\frac{x^2}{x^2+y^2}=\frac{\frac{a^2}{b^2}}{\frac{a^2}{b^2}+\frac{b^2}{b^2}}+\frac{x^2}{x^2+y^2}=$$

$$=\frac{\frac{a^2}{b^2}}{\frac{a^2}{b^2}+\frac{b^2}{b^2}}+\frac{x^2}{x^2+y^2}=\frac{\frac{a^2}{b^2}}{\frac{a^2}{b^2}+1}+\frac{x^2}{x^2+y^2}=\left[\begin{array}{c}\text{zamjena}\\ \frac{a^2}{b^2}=\frac{y^2}{x^2}\end{array}\right]=\frac{\frac{y^2}{x^2}}{\frac{y^2}{x^2}+1}+\frac{x^2}{x^2+y^2}$$

$$=\frac{\frac{y^2}{x^2}}{\frac{y^2}{x^2}+\frac{1}{1}}+\frac{x^2}{x^2+y^2}=\frac{\frac{y^2}{x^2}}{\frac{y^2+x^2}{x^2}}+\frac{x^2}{x^2+y^2}=\frac{\frac{y^2}{x^2}}{\frac{y^2+x^2}{x^2}}+\frac{x^2}{x^2+y^2}=\frac{\frac{y^2}{1}}{\frac{y^2+x^2}{1}}+\frac{x^2}{x^2+y^2}=$$

$$=\frac{y^2}{y^2+x^2}+\frac{x^2}{x^2+y^2}=\frac{y^2}{x^2+y^2}+\frac{x^2}{x^2+y^2}=\frac{y^2+x^2}{x^2+y^2}=\frac{x^2+y^2}{x^2+y^2}=\frac{x^2+y^2}{x^2+y^2}=1.$$

Vježba 498

Ako je $\frac{a}{b}+\frac{y}{x}=0$, dokaži da je $\frac{a^2}{a^2+b^2}+\frac{x^2}{x^2+y^2}=1.$

Rezultat: Dokaz analogan.

Zadatak 499 (Dario, gimnazija)

Što je rezultat sređivanja izraza $\frac{x^3-y^3}{x^3+x^2\cdot y+x\cdot y^2}+\frac{2\cdot y^2-x\cdot y}{x\cdot y}$ za sve x, y za koje je izraz definiran?

Rješenje 499

Ponovimo!

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right) \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\frac{x^3 - y^3}{x^3 + x^2 \cdot y + x \cdot y^2} + \frac{2 \cdot y^2 - x \cdot y}{x \cdot y} = \frac{(x-y) \cdot \left(x^2 + x \cdot y + y^2\right)}{x \cdot \left(x^2 + x \cdot y + y^2\right)} + \frac{y \cdot (2 \cdot y - x)}{x \cdot y} =$$

$$= \frac{(x-y) \cdot \left(x^2 + x \cdot y + y^2\right)}{x \cdot \left(x^2 + x \cdot y + y^2\right)} + \frac{y \cdot (2 \cdot y - x)}{x \cdot y} = \frac{x-y}{x} + \frac{2 \cdot y - x}{x} = \frac{x - y + 2 \cdot y - x}{x} =$$

$$= \frac{x - y + 2 \cdot y - x}{x} = \frac{-y + 2 \cdot y}{x} = \frac{y}{x}.$$

Vježba 499

Što je rezultat sređivanja izraza $\frac{x^3 - y^3}{x^3 + x^2 \cdot y + x \cdot y^2} + \frac{2 \cdot y^2 - x \cdot y}{x \cdot y} - \frac{y}{x}$ za sve x, y za koje je izraz definiran?

Rezultat: 0.

Zadatak 500 (Dario, gimnazija)

Izraz $a^2 - 2 \cdot a \cdot b - 3 \cdot b^2$ napišite kao umnožak dvaju binoma.

Rješenje 500

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad x^2 - y^2 = (x-y) \cdot (x+y) \quad , \quad (a \cdot b)^n = a^n \cdot b^n.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$a^2 - 2 \cdot a \cdot b - 3 \cdot b^2 = a^2 - 2 \cdot a \cdot b + b^2 - 4 \cdot b^2 = a^2 - 2 \cdot a \cdot b + b^2 - 4 \cdot b^2 =$$

$$= \left(a^2 - 2 \cdot a \cdot b + b^2\right) - 4 \cdot b^2 = (a-b)^2 - (2 \cdot b)^2 = (a - b - 2 \cdot b) \cdot (a - b + 2 \cdot b) = (a - 3 \cdot b) \cdot (a + b).$$

2.inačica

$$a^2 - 2 \cdot a \cdot b - 3 \cdot b^2 = a^2 - 3 \cdot a \cdot b + a \cdot b - 3 \cdot b^2 = a^2 - 3 \cdot a \cdot b + a \cdot b - 3 \cdot b^2 =$$

$$= \left(a^2 - 3 \cdot a \cdot b\right) + \left(a \cdot b - 3 \cdot b^2\right) = a \cdot (a - 3 \cdot b) + b \cdot (a - 3 \cdot b) = a \cdot (a - 3 \cdot b) + b \cdot (a - 3 \cdot b) =$$

$$= (a - 3 \cdot b) \cdot (a + b).$$

Vježba 500

Izraz $a^2 - 3 \cdot a \cdot b - 4 \cdot b^2$ napišite kao umnožak dvaju binoma.

Rezultat: $(a - 4 \cdot b) \cdot (a + b)$.