Zadatak 461 (Doris, srednja škola)

Napiši u obliku umnoška algebarski izraz $8 \cdot a^4 + 24 \cdot a^3 - 27 \cdot a - 81.$

Rješenje 461

Ponovimo!

$$(a \cdot b)^n = a^n \cdot b^n \quad , \quad a^3 - b^3 = (a-b) \cdot \left(a^2 + a \cdot b + b^2\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

$$8 \cdot a^4 + 24 \cdot a^3 - 27 \cdot a - 81 = \left[\begin{array}{l}\text{grupiramo prvi i drugi član,} \\ \text{grupiramo treći i posljednji član}\end{array}\right] = \left(8 \cdot a^4 + 24 \cdot a^3\right) + (-27 \cdot a - 81) =$$

$$= 8 \cdot a^3 \cdot (a+3) - 27 \cdot (a+3) = 8 \cdot a^3 \cdot (a+3) - 27 \cdot (a+3) = (a+3) \cdot \left(8 \cdot a^3 - 27\right) =$$

$$= (a+3) \cdot \left((2 \cdot a)^3 - 3^3\right) = (a+3) \cdot (2 \cdot a - 3) \cdot \left((2 \cdot a)^2 + 2 \cdot a \cdot 3 + 3^2\right) = (a+3) \cdot (2 \cdot a - 3) \cdot \left(4 \cdot a^2 + 6 \cdot a + 9\right).$$

2.inačica

$$8 \cdot a^4 + 24 \cdot a^3 - 27 \cdot a - 81 = \left[\begin{array}{l}\text{grupiramo prvi i treći član,} \\ \text{grupiramo drugi i posljednji član}\end{array}\right] = \left(8 \cdot a^4 - 27 \cdot a\right) + \left(24 \cdot a^3 - 81\right) =$$

$$= a \cdot \left(8 \cdot a^3 - 27\right) + 3 \cdot \left(8 \cdot a^3 - 27\right) = a \cdot \left(8 \cdot a^3 - 27\right) + 3 \cdot \left(8 \cdot a^3 - 27\right) = \left(8 \cdot a^3 - 27\right) \cdot (a+3) =$$

$$= \left((2 \cdot a)^3 - 3^3\right) \cdot (a+3) = (2 \cdot a - 3) \cdot \left((2 \cdot a)^2 + 2 \cdot a \cdot 3 + 3^2\right) \cdot (a+3) =$$

$$= (2 \cdot a - 3) \cdot \left(4 \cdot a^2 + 6 \cdot a + 9\right) \cdot (a+3) = (a+3) \cdot (2 \cdot a - 3) \cdot \left(4 \cdot a^2 + 6 \cdot a + 9\right).$$

Vježba 461

Napiši u obliku umnoška algebarski izraz $81 + 27 \cdot a - 8 \cdot a^4 - 24 \cdot a^3.$

Rezultat: $(3+a) \cdot (3 - 2 \cdot a) \cdot \left(4 \cdot a^2 + 6 \cdot a + 9\right).$

Zadatak 462 (Ivana, gimnazija)

Izračunaj $(a-b)^2$ ako je $a^3 + b^3 = 637$ i $a + b = 13.$

Rješenje 462

Ponovimo!

$$x^3 + y^3 = (x+y) \cdot \left(x^2 - x \cdot y + y^2\right) \quad , \quad (x+y)^2 = x^2 + 2 \cdot x \cdot y + y^2.$$

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Budući da je $a^3 + b^3 = 637,$ slijedi:

$$a^3 + b^3 = 637 \Rightarrow (a+b) \cdot \left(a^2 - a \cdot b + b^2\right) = 637 \Rightarrow [\, a + b = 13 \,] \Rightarrow 13 \cdot \left(a^2 - a \cdot b + b^2\right) = 637 \Rightarrow$$

$$\Rightarrow 13 \cdot \left(a^2 - a \cdot b + b^2\right) = 637 \; / : 13 \Rightarrow a^2 - a \cdot b + b^2 = 49.$$

Za a + b = 13 vrijedi:

$$(a+b) = 13 \Rightarrow \begin{bmatrix} \text{kvadriramo} \\ \text{jednakost} \end{bmatrix} \Rightarrow (a+b) = 13 \; /^2 \Rightarrow (a+b)^2 = 13^2 \Rightarrow$$

$$\Rightarrow a^2 + 2 \cdot a \cdot b + b^2 = 169.$$

Iz sustava jednakosti dobije se vrijednost za a · b.

$$\left.\begin{matrix} a^2 + 2 \cdot a \cdot b + b^2 = 169 \\ a^2 - a \cdot b + b^2 = 49 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{oduzmemo} \\ \text{jednakosti} \end{bmatrix} \Rightarrow \left(a^2 + 2 \cdot a \cdot b + b^2\right) - \left(a^2 - a \cdot b + b^2\right) = 169 - 49 \Rightarrow$$

$$\Rightarrow a^2 + 2 \cdot a \cdot b + b^2 - a^2 + a \cdot b - b^2 = 120 \Rightarrow a^2 + 2 \cdot a \cdot b + b^2 - a^2 + a \cdot b - b^2 = 120 \Rightarrow$$

$$\Rightarrow 2 \cdot a \cdot b + a \cdot b = 120 \Rightarrow 3 \cdot a \cdot b = 120 \Rightarrow 3 \cdot a \cdot b = 120 \; / : 3 \Rightarrow a \cdot b = 40.$$

Sada računamo $(a-b)^2$.

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2 = a^2 - 2 \cdot a \cdot b + b^2 = a^2 - 4 \cdot a \cdot b + 2 \cdot a \cdot b + b^2 = a^2 + 2 \cdot a \cdot b + b^2 - 4 \cdot a \cdot b =$$

$$= \left(a^2 + 2 \cdot a \cdot b + b^2\right) - 4 \cdot a \cdot b = \begin{bmatrix} a^2 + 2 \cdot a \cdot b + b^2 = 169 \\ a \cdot b = 40 \end{bmatrix} = 169 - 4 \cdot 40 = 169 - 160 = 9.$$

Vježba 462

Izračunaj $(a-b)^2$ ako je $a^3 + b^3 = 407$ i $a + b = 11$.

Rezultat: 9.

Zadatak 463 (Tin, gimnazija)

Ako je za realne pozitivne brojeve a i b, a < b, tada je vrijednost izraza

$$x = \left(\sqrt{a + b + 2 \cdot \sqrt{a \cdot b}} + \sqrt{a + b - 2 \cdot \sqrt{a \cdot b}}\right)^2 \text{ jednaka:}$$

$A.\ a+b \qquad B.\ a-b \qquad C.\ 4 \cdot a \qquad D.\ 4 \cdot a$

Rješenje 463

Ponovimo!

$$\left(\sqrt{x}\right)^2 = x \quad , \quad \sqrt{x} \cdot \sqrt{y} = \sqrt{x \cdot y} \quad , \quad (x+y)^2 = x^2 + 2 \cdot x \cdot y + y^2.$$

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad \sqrt{x^2} = x \; , \; x \ge 0 \quad , \quad (x \cdot y)^n = x^n \cdot y^n.$$

$$x^2 - y^2 = (x-y) \cdot (x+y).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, x ≥ 0, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, x < 0, je $|x| = -x$.

1.inačica

$$x=\left(\sqrt{a+b+2\cdot\sqrt{a\cdot b}}+\sqrt{a+b-2\cdot\sqrt{a\cdot b}}\right)^2 \Rightarrow x=\left(\sqrt{a+2\cdot\sqrt{a\cdot b}+b}+\sqrt{a-2\cdot\sqrt{a\cdot b}+b}\right)^2 \Rightarrow$$

$$\Rightarrow x=\left(\sqrt{\left(\sqrt{a}\right)^2+2\cdot\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2}+\sqrt{\left(\sqrt{a}\right)^2-2\cdot\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2}\right)^2 \Rightarrow$$

$$\Rightarrow x=\left(\sqrt{\left(\sqrt{a}+\sqrt{b}\right)^2}+\sqrt{\left(\sqrt{a}-\sqrt{b}\right)^2}\right)^2 \Rightarrow x=\left(\left|\sqrt{a}+\sqrt{b}\right|+\left|\sqrt{a}-\sqrt{b}\right|\right)^2 \Rightarrow \begin{bmatrix}\sqrt{a}+\sqrt{b}>0\\ \sqrt{a}-\sqrt{b}<0\end{bmatrix} \Rightarrow$$

$$\Rightarrow x=\left(\left|\underbrace{\sqrt{a}+\sqrt{b}}_{\text{pozitivno}}\right|+\left|\underbrace{\sqrt{a}-\sqrt{b}}_{\text{negativno}}\right|\right)^2 \Rightarrow x=\left(\sqrt{a}+\sqrt{b}-\left(\sqrt{a}-\sqrt{b}\right)\right)^2 \Rightarrow x=\left(\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}\right)^2 \Rightarrow$$

$$\Rightarrow x=\left(\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}\right)^2 \Rightarrow x=\left(\sqrt{b}+\sqrt{b}\right)^2 \Rightarrow x=\left(2\cdot\sqrt{b}\right)^2 \Rightarrow x=2^2\cdot\left(\sqrt{b}\right)^2 \Rightarrow x=4\cdot b.$$

Odgovor je pod D.

2.inačica

$$x=\left(\sqrt{a+b+2\cdot\sqrt{a\cdot b}}+\sqrt{a+b-2\cdot\sqrt{a\cdot b}}\right)^2 \Rightarrow$$

$$\Rightarrow x=\left(\sqrt{a+b+2\cdot\sqrt{a\cdot b}}\right)^2+2\cdot\sqrt{a+b+2\cdot\sqrt{a\cdot b}}\cdot\sqrt{a+b-2\cdot\sqrt{a\cdot b}}+\left(\sqrt{a+b-2\cdot\sqrt{a\cdot b}}\right)^2 \Rightarrow$$

$$\Rightarrow x=a+b+2\cdot\sqrt{a\cdot b}+2\cdot\sqrt{\left(a+b+2\cdot\sqrt{a\cdot b}\right)\cdot\left(a+b-2\cdot\sqrt{a\cdot b}\right)}+a+b-2\cdot\sqrt{a\cdot b} \Rightarrow$$

$$\Rightarrow x=a+b+2\cdot\sqrt{a\cdot b}+2\cdot\sqrt{\left(\left(a+b\right)+2\cdot\sqrt{a\cdot b}\right)\cdot\left(\left(a+b\right)-2\cdot\sqrt{a\cdot b}\right)}+a+b-2\cdot\sqrt{a\cdot b} \Rightarrow$$

$$\Rightarrow x=a+b+2\cdot\sqrt{\left(a+b\right)^2-\left(2\cdot\sqrt{a\cdot b}\right)^2}+a+b \Rightarrow$$

$$\Rightarrow x=2\cdot a+2\cdot b+2\cdot\sqrt{a^2+2\cdot a\cdot b+b^2-2^2\cdot\left(\sqrt{a\cdot b}\right)^2} \Rightarrow$$

$$\Rightarrow x=2\cdot a+2\cdot b+2\cdot\sqrt{a^2+2\cdot a\cdot b+b^2-4\cdot a\cdot b} \Rightarrow x=2\cdot a+2\cdot b+2\cdot\sqrt{a^2-2\cdot a\cdot b+b^2} \Rightarrow$$

$$\Rightarrow x=2\cdot a+2\cdot b+2\cdot\sqrt{\left(a-b\right)^2} \Rightarrow x=2\cdot a+2\cdot b+2\cdot\left|a-b\right| \Rightarrow \left[a-b<0\right] \Rightarrow$$

$$\Rightarrow x=2\cdot a+2\cdot b+2\cdot\left|\underbrace{a-b}_{\text{negativno}}\right| \Rightarrow x=2\cdot a+2\cdot b-2\cdot\left(a-b\right) \Rightarrow x=2\cdot a+2\cdot b-2\cdot a+2\cdot b \Rightarrow$$

$$\Rightarrow x=2\cdot a+2\cdot b-2\cdot a+2\cdot b \Rightarrow x=2\cdot b+2\cdot b \Rightarrow x=4\cdot b.$$

Odgovor je pod D.

Vježba 463

Ako je za realne pozitivne brojeve a i b, a < b, tada je vrijednost izraza

$x=\left(\sqrt{a+b+2\cdot\sqrt{a\cdot b}}-\sqrt{a+b-2\cdot\sqrt{a\cdot b}}\right)^2$ jednaka:

$A.\ a+b \qquad B.\ a-b \qquad C.\ 4\cdot a \qquad D.\ 4\cdot a$

Rezultat: C.

Zadatak 464 (Ivan, gimnazija)

Ako je $a+b>0$ i $a\cdot b\neq 0$, tada vrijedi $\frac{a}{b^2}+\frac{b}{a^2}\geq\frac{1}{a}+\frac{1}{b}$.

Rješenje 464

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right)\quad,\quad a\geq b\ ,\ c>0\ \Rightarrow\ a\cdot c\geq b\cdot c.$$

$$a^2\geq 0\text{ za svaki }a\in R\quad,\quad (a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad a\geq b\ ,\ c>0\ \Rightarrow\ \frac{a}{c}\geq\frac{b}{c}.$$

$$a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^n:a^m=a^{n-m}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

1.inačica

Pomoću niza transformacija postavljenu nejednakost napišemo u sljedećem obliku:

$$\frac{a}{b^2}+\frac{b}{a^2}\geq\frac{1}{a}+\frac{1}{b}\ \Rightarrow\ \frac{a^3+b^3}{a^2\cdot b^2}\geq\frac{b+a}{a\cdot b}\ \Rightarrow\ \frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{a^2\cdot b^2}\geq\frac{a+b}{a\cdot b}\ \Rightarrow$$

$$\Rightarrow\ \frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{a^2\cdot b^2}\geq\frac{a+b}{a\cdot b}\ /\cdot\frac{1}{a+b}\ \Rightarrow\ \frac{a^2-a\cdot b+b^2}{a^2\cdot b^2}\geq\frac{1}{a\cdot b}\ \Rightarrow$$

$$\Rightarrow\ \frac{a^2-a\cdot b+b^2}{a^2\cdot b^2}-\frac{1}{a\cdot b}\geq 0\ \Rightarrow\ \frac{a^2-a\cdot b+b^2-a\cdot b}{a^2\cdot b^2}\geq 0\ \Rightarrow$$

$$\Rightarrow\ \frac{a^2-2\cdot a\cdot b+b^2}{a^2\cdot b^2}\geq 0\ \Rightarrow\ \frac{(a-b)^2}{a^2\cdot b^2}\geq 0.$$

Budući da je kvadrat realnog broja nenegativan broj (broj jednak ili veći od nule) posljednja nejednakost očito vrijedi pa zato vrijedi i postavljena nejednakost. Znak jednakosti u dokazanoj nejednakosti vrijedi samo ako je a = b.

2.inačica

Pomoću niza transformacija postavljenu nejednakost napišemo u sljedećem obliku:

$$\frac{a}{b^2}+\frac{b}{a^2}\geq\frac{1}{a}+\frac{1}{b}\ \Rightarrow\ \frac{a^3+b^3}{a^2\cdot b^2}\geq\frac{b+a}{a\cdot b}\ \Rightarrow\ \frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{a^2\cdot b^2}\geq\frac{a+b}{a\cdot b}\ \Rightarrow$$

$$\Rightarrow\ \frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{a^2\cdot b^2}\geq\frac{a+b}{a\cdot b}\ /\cdot\frac{a^2\cdot b^2}{a+b}\ \Rightarrow\ a^2-a\cdot b+b^2\geq a\cdot b\ \Rightarrow$$

$$\Rightarrow\ a^2-a\cdot b+b^2-a\cdot b\geq 0\ \Rightarrow\ a^2-2\cdot a\cdot b+b^2\geq 0\ \Rightarrow\ (a-b)^2\geq 0.$$

Budući da je kvadrat realnog broja nenegativan broj (broj jednak ili veći od nule) posljednja nejednakost očito vrijedi pa zato vrijedi i postavljena nejednakost. Znak jednakosti u dokazanoj nejednakosti vrijedi samo ako je a = b.

Vježba 464

Ako je $a+b>0$ i $a\cdot b\neq 0$, tada vrijedi $\frac{1}{a}\cdot\left(\frac{b}{a}-1\right)+\frac{1}{b}\cdot\left(\frac{a}{b}-1\right)\geq 0$.

Rezultat: Dokaz analogan.

Zadatak 465 (Goran, gimnazija)

Ako je $\left(x+\sqrt{x^2+1}\right)\cdot\left(y+\sqrt{y^2+1}\right)=1,$ koliko je $x+y$?

Rješenje 465

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\left(\sqrt{a}\right)^2=a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Postavljenu jednakost transformiramo na dva načina.

$$\left.\begin{array}{l}\left(x+\sqrt{x^2+1}\right)\cdot\left(y+\sqrt{y^2+1}\right)=1\\ \left(x+\sqrt{x^2+1}\right)\cdot\left(y+\sqrt{y^2+1}\right)=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\left(x+\sqrt{x^2+1}\right)\cdot\left(y+\sqrt{y^2+1}\right)=1\ /\cdot\frac{1}{x+\sqrt{x^2+1}}\\ \left(x+\sqrt{x^2+1}\right)\cdot\left(y+\sqrt{y^2+1}\right)=1\ /\cdot\frac{1}{y+\sqrt{y^2+1}}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}y+\sqrt{y^2+1}=\frac{1}{x+\sqrt{x^2+1}}\\ x+\sqrt{x^2+1}=\frac{1}{y+\sqrt{y^2+1}}\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}y+\sqrt{y^2+1}=\frac{1}{x+\sqrt{x^2+1}}\cdot\frac{x-\sqrt{x^2+1}}{x-\sqrt{x^2+1}}\\ x+\sqrt{x^2+1}=\frac{1}{y+\sqrt{y^2+1}}\cdot\frac{y-\sqrt{y^2+1}}{y-\sqrt{y^2+1}}\end{array}\right\}\Rightarrow\left.\begin{array}{l}y+\sqrt{y^2+1}=\frac{x-\sqrt{x^2+1}}{\left(x+\sqrt{x^2+1}\right)\cdot\left(x-\sqrt{x^2+1}\right)}\\ x+\sqrt{x^2+1}=\frac{y-\sqrt{y^2+1}}{\left(y+\sqrt{y^2+1}\right)\cdot\left(y-\sqrt{y^2+1}\right)}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}y+\sqrt{y^2+1}=\frac{x-\sqrt{x^2+1}}{x^2-\left(\sqrt{x^2+1}\right)^2}\\ x+\sqrt{x^2+1}=\frac{y-\sqrt{y^2+1}}{y^2-\left(\sqrt{y^2+1}\right)^2}\end{array}\right\}\Rightarrow\left.\begin{array}{l}y+\sqrt{y^2+1}=\frac{x-\sqrt{x^2+1}}{x^2-\left(x^2+1\right)}\\ x+\sqrt{x^2+1}=\frac{y-\sqrt{y^2+1}}{y^2-\left(y^2+1\right)}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}y+\sqrt{y^2+1}=\frac{x-\sqrt{x^2+1}}{x^2-x^2-1}\\ x+\sqrt{x^2+1}=\frac{y-\sqrt{y^2+1}}{y^2-y^2-1}\end{array}\right\}\Rightarrow\left.\begin{array}{l}y+\sqrt{y^2+1}=\frac{x-\sqrt{x^2+1}}{x^2-x^2-1}\\ x+\sqrt{x^2+1}=\frac{y-\sqrt{y^2+1}}{y^2-y^2-1}\end{array}\right\}\Rightarrow\left.\begin{array}{l}y+\sqrt{y^2+1}=\frac{x-\sqrt{x^2+1}}{-1}\\ x+\sqrt{x^2+1}=\frac{y-\sqrt{y^2+1}}{-1}\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} y+\sqrt{y^2+1}=-\left(x-\sqrt{x^2+1}\right) \\ x+\sqrt{x^2+1}=-\left(y-\sqrt{y^2+1}\right) \end{array}\right\} \Rightarrow \left.\begin{array}{l} y+\sqrt{y^2+1}=-x+\sqrt{x^2+1} \\ x+\sqrt{x^2+1}=-y+\sqrt{y^2+1} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} y+x=\sqrt{x^2+1}-\sqrt{y^2+1} \\ x+y=\sqrt{y^2+1}-\sqrt{x^2+1} \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednakosti} \end{array}\right] \Rightarrow$$

$$\Rightarrow y+x+x+y=\sqrt{x^2+1}-\sqrt{y^2+1}+\sqrt{y^2+1}-\sqrt{x^2+1} \Rightarrow$$

$$\Rightarrow 2\cdot x+2\cdot y=\sqrt{x^2+1}-\sqrt{y^2+1}+\sqrt{y^2+1}-\sqrt{x^2+1} \Rightarrow 2\cdot x+2\cdot y=0 \Rightarrow$$

$$\Rightarrow 2\cdot x+2\cdot y=0 \;/:2 \Rightarrow x+y=0.$$

Vježba 465

Ako je $\left(x-\sqrt{x^2+1}\right)\cdot\left(y-\sqrt{y^2+1}\right)=1,$ koliko je $x+y$?

Rezultat: 0.

Zadatak 466 (Ana, gimnazija)

Ako je $3\cdot a-b+2\cdot c+5\cdot d=11$ te $a+5\cdot b+2\cdot c-d=9,$ koliko je $a+b+c+d$?

$A.\ 2 \qquad B.\ 3 \qquad C.\ 4 \qquad D.\ 5$

Rješenje 466

Ponovimo!

$$\left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a+c=b+d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Iz sustava jednadžbi dobije se:

$$\left.\begin{array}{l} 3\cdot a-b+2\cdot c+5\cdot d=11 \\ a+5\cdot b+2\cdot c-d=9 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow 3\cdot a-b+2\cdot c+5\cdot d+a+5\cdot b+2\cdot c-d=11+9 \Rightarrow$$

$$\Rightarrow 4\cdot a+4\cdot b+4\cdot c+4\cdot d=20 \Rightarrow 4\cdot(a+b+c+d)=20 \Rightarrow$$

$$\Rightarrow 4\cdot(a+b+c+d)=20 \;/:4 \Rightarrow a+b+c+d=5.$$

Odgovor je pod D.

Vježba 466

Ako je $3\cdot a-b+2\cdot c+5\cdot d=5$ te $a+5\cdot b+2\cdot c-d=3,$ koliko je $a+b+c+d$?

$A.\ 2 \qquad B.\ 3 \qquad C.\ 4 \qquad D.\ 5$

Rezultat: A.

Zadatak 467 (ABC, ekonomska škola)

Učenik je na džepnome računalu zbrojio brojeve A i B. Dobiveni rezultat podijelio je s C. Taj je rezultat pomnožio s D. Koji izraz opisuje taj račun?

$A.\ \dfrac{A+B}{C\cdot D} \qquad B.\ \dfrac{(A+B)\cdot D}{C} \qquad C.\ (A+B:C)\cdot D \qquad D.\ A+B:\dfrac{C}{D}$

Rješenje 467

Ponovimo!

$$a : b = \frac{a}{b} \quad , \quad \frac{a}{b} \cdot c = \frac{a \cdot c}{b}.$$

Prevedimo tekst zadatka u matematički simbolički zapis.
Učenik je na džepnome računalu zbrojio brojeve A i B.

$$A + B.$$

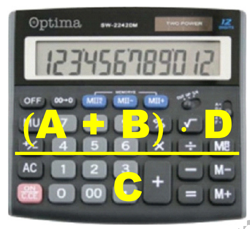
Dobiveni rezultat podijelio je s C.

$$(A + B) : C = \frac{A + B}{C}.$$

Taj je rezultat pomnožio s D.

$$\frac{A + B}{C} \cdot D = \frac{(A + B) \cdot D}{C}.$$

Odgovor je pod B.



Vježba 467

Učenik je na džepnome računalu zbrojio brojeve A i B. Dobiveni rezultat podijelio je s umnoškom brojeva C i D. Koji izraz opisuje taj račun?

$A.\ \frac{A+B}{C \cdot D} \qquad B.\ \frac{(A+B) \cdot D}{C} \qquad C.\ (A + B : C) \cdot D \qquad D.\ A + B : \frac{C}{D}$

Rezultat: A.

Zadatak 468 (4A, 4B, TUPŠ)

Iz izraza $s = v_0 \cdot t + \frac{a \cdot t^2}{2}$ izvedi a.

$A.\ \frac{v_0 - 2 \cdot s}{t^2} \qquad B.\ \frac{s - v_0 \cdot t}{2} \qquad C.\ \frac{v_0 \cdot t - s}{2} \qquad D.\ \frac{2 \cdot s}{t^2} - \frac{2 \cdot v_0}{t}$

Rješenje 468

Ponovimo!

$$a = b \;\Rightarrow\; b = a \quad , \quad \frac{a - b}{n} = \frac{a}{n} - \frac{b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$s = v_0 \cdot t + \frac{a \cdot t^2}{2} \;\Rightarrow\; s = v_0 \cdot t + \frac{a \cdot t^2}{2} \ / \cdot 2 \;\Rightarrow\; 2 \cdot s = 2 \cdot v_0 \cdot t + a \cdot t^2 \;\Rightarrow\; 2 \cdot v_0 \cdot t + a \cdot t^2 = 2 \cdot s \;\Rightarrow$$

$$\Rightarrow\; a \cdot t^2 = 2 \cdot s - 2 \cdot v_0 \cdot t \;\Rightarrow\; a \cdot t^2 = 2 \cdot s - 2 \cdot v_0 \cdot t \ / \cdot \frac{1}{t^2} \;\Rightarrow\; a = \frac{2 \cdot s - 2 \cdot v_0 \cdot t}{t^2} \;\Rightarrow$$

$$\Rightarrow\ a=\frac{2\cdot s}{t^2}-\frac{2\cdot v_0\cdot t}{t^2}\ \Rightarrow\ a=\frac{2\cdot s}{t^2}-\frac{2\cdot v_0\cdot t}{t^2}\ \Rightarrow\ a=\frac{2\cdot s}{t^2}-\frac{2\cdot v_0}{t}.$$

Odgovor je pod D.

Vježba 468

Iz izraza $s=v_0\cdot t-\frac{a\cdot t^2}{2}$ izvedi a.

$A.\ \frac{v_0+2\cdot s}{t^2}$ $\quad B.\ \frac{s+v_0\cdot t}{2}$ $\quad C.\ \frac{v_0\cdot t+s}{2}$ $\quad D.\ \frac{2\cdot v_0}{t}-\frac{2\cdot s}{t^2}$

Rezultat: D.

Zadatak 469 (4A, 4B, TUPŠ)

Koji izraz dobijemo nakon skraćivanja razlomka $\frac{2^{x+y}+2^x}{2^y+4^y}$?

Rješenje 469

Ponovimo!

$$a^n\cdot a^m=a^{n+m}\quad,\quad \left(a^n\right)^m=\left(a^m\right)^n\quad,\quad a^2=a\cdot a\quad,\quad \frac{a^n}{a^m}=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{2^{x+y}+2^x}{2^y+4^y}=\frac{2^x\cdot 2^y+2^x}{2^y+\left(2^2\right)^y}=\frac{2^x\cdot 2^y+2^x}{2^y+\left(2^y\right)^2}=\frac{2^x\cdot 2^y+2^x}{2^y+2^y\cdot 2^y}=\frac{2^x\cdot 2^y+2^x}{2^y+2^y\cdot 2^y}=$$

$$=\frac{2^x\cdot\left(2^y+1\right)}{2^y\cdot\left(1+2^y\right)}=\frac{2^x\cdot\left(2^y+1\right)}{2^y\cdot\left(1+2^y\right)}=\frac{2^x}{2^y}=2^{x-y}.$$

Vježba 469

Koji izraz dobijemo nakon skraćivanja razlomka $\frac{3^{x+y}+3^x}{3^y+9^y}$?

Rezultat: 3^{x-y}.

Zadatak 470 (4A, 4B, TUPŠ)

Dokaži da vrijednost izraza $\dfrac{2-x+4\cdot x^2+\dfrac{5\cdot x^2-6\cdot x+3}{x-1}}{2\cdot x+1+\dfrac{2\cdot x}{x-1}}:(2\cdot x-1)$ ne ovisi o x.

Rješenje 470

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Transformacijom postavljenog izraza dobije se izraz u kojem se ne pojavljuje x.

$$\frac{2-x+4\cdot x^2+\frac{5\cdot x^2-6\cdot x+3}{x-1}}{2\cdot x+1+\frac{2\cdot x}{x-1}}:(2\cdot x-1)=\frac{\frac{2-x+4\cdot x^2}{1}+\frac{5\cdot x^2-6\cdot x+3}{x-1}}{\frac{2\cdot x+1}{1}+\frac{2\cdot x}{x-1}}\cdot\frac{1}{2\cdot x-1}=$$

$$=\frac{\frac{(x-1)\cdot\left(2-x+4\cdot x^2\right)+5\cdot x^2-6\cdot x+3}{x-1}}{\frac{(x-1)\cdot(2\cdot x+1)+2\cdot x}{x-1}}\cdot\frac{1}{2\cdot x-1}=\frac{\frac{(x-1)\cdot\left(2-x+4\cdot x^2\right)+5\cdot x^2-6\cdot x+3}{x-1}}{\frac{(x-1)\cdot(2\cdot x+1)+2\cdot x}{x-1}}\cdot\frac{1}{2\cdot x-1}=$$

$$=\frac{\frac{(x-1)\cdot\left(2-x+4\cdot x^2\right)+5\cdot x^2-6\cdot x+3}{1}}{\frac{(x-1)\cdot(2\cdot x+1)+2\cdot x}{1}}\cdot\frac{1}{2\cdot x-1}=\frac{(x-1)\cdot\left(2-x+4\cdot x^2\right)+5\cdot x^2-6\cdot x+3}{(x-1)\cdot(2\cdot x+1)+2\cdot x}\cdot\frac{1}{2\cdot x-1}=$$

$$=\frac{2\cdot x-x^2+4\cdot x^3-2+x-4\cdot x^2+5\cdot x^2-6\cdot x+3}{2\cdot x^2+x-2\cdot x-1+2\cdot x}\cdot\frac{1}{2\cdot x-1}=$$

$$=\frac{2\cdot x-x^2+4\cdot x^3-2+x-4\cdot x^2+5\cdot x^2-6\cdot x+3}{2\cdot x^2+x-2\cdot x-1+2\cdot x}\cdot\frac{1}{2\cdot x-1}=\frac{2\cdot x+4\cdot x^3-2+x-6\cdot x+3}{2\cdot x^2+x-1}\cdot\frac{1}{2\cdot x-1}=$$

$$=\frac{4\cdot x^3-3\cdot x+1}{2\cdot x^2+x-1}\cdot\frac{1}{2\cdot x-1}=\frac{4\cdot x^3-3\cdot x+1}{\left(2\cdot x^2+x-1\right)\cdot(2\cdot x-1)}=$$

$$=\frac{4\cdot x^3-3\cdot x+1}{4\cdot x^3-2\cdot x^2+2\cdot x^2-x-2\cdot x+1}=\frac{4\cdot x^3-3\cdot x+1}{4\cdot x^3-2\cdot x^2+2\cdot x^2-x-2\cdot x+1}=$$

$$=\frac{4\cdot x^3-3\cdot x+1}{4\cdot x^3-x-2\cdot x+1}=\frac{4\cdot x^3-3\cdot x+1}{4\cdot x^3-3\cdot x+1}=\frac{4\cdot x^3-3\cdot x+1}{4\cdot x^3-3\cdot x+1}=1.$$

Vježba 470

Dokaži da vrijednost izraza $\dfrac{2\cdot x+1+\dfrac{2\cdot x}{x-1}}{2-x+4\cdot x^2+\dfrac{5\cdot x^2-6\cdot x+3}{x-1}}\cdot(2\cdot x-1)$ ne ovisi o x.

Rezultat: 1.

Zadatak 471 (Mahira, gimnazija)

Pojednostavnite: $\left(m-(1-m)^{-1}\right)^{-1}\cdot\left[\dfrac{m\cdot(m-2)+m^0}{\dfrac{1}{m^{-2}-m+1}}\right]^{-1}$.

Rješenje 471

Ponovimo!

$$a^{-n}=\frac{1}{a^n}\quad,\quad a^1=a\quad,\quad n=\frac{n}{1}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad a^0=1.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad (a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Transformacijom postavljenog izraza dobije se:

$$\left(m-(1-m)^{-1}\right)^{-1}\cdot\left[\frac{m\cdot(m-2)+m^0}{\frac{1}{m^{-2}-m+1}}\right]^{-1}=\left(m-\frac{1}{(1-m)^1}\right)^{-1}\cdot\left[\frac{m^2-2\cdot m+1}{\frac{1}{m^{-2}-m+1}}\right]^{-1}=$$

$$=\left(m-\frac{1}{1-m}\right)^{-1}\cdot\left[\frac{\frac{m^2-2\cdot m+1}{1}}{\frac{1}{m^{-2}-m+1}}\right]^{-1}=\left(\frac{m}{1}-\frac{1}{1-m}\right)^{-1}\cdot\left[\frac{\left(m^2-2\cdot m+1\right)\cdot\left(m^{-2}-m+1\right)}{1}\right]^{-1}=$$

$$=\left(\frac{m\cdot(1-m)-1}{1-m}\right)^{-1}\cdot\frac{1}{\left(m^2-2\cdot m+1\right)\cdot\left(m^{-2}-m+1\right)}=\frac{1-m}{m\cdot(1-m)-1}\cdot\frac{1}{\left(m^2-2\cdot m+1\right)\cdot\left(m^{-2}-m+1\right)}=$$

$$=\frac{1-m}{m-m^{2}-1}\cdot\frac{1}{(m-1)^{2}\cdot\left(\frac{1}{m^{2}}-m+1\right)}=\frac{-(m-1)}{m-m^{2}-1}\cdot\frac{1}{(m-1)^{2}\cdot\left(\frac{1}{m^{2}}-\frac{m}{1}+\frac{1}{1}\right)}=$$

$$=\frac{m-1}{-\left(m-m^{2}-1\right)}\cdot\frac{1}{(m-1)^{2}\cdot\frac{1-m^{3}+m^{2}}{m^{2}}}=\frac{m-1}{m^{2}-m+1}\cdot\frac{1}{(m-1)^{2}\cdot\frac{1-m^{3}+m^{2}}{m^{2}}}=$$

$$=\frac{m-1}{m^{2}-m+1}\cdot\frac{1}{(m-1)^{2}\cdot\frac{1-m^{3}+m^{2}}{m^{2}}}=\frac{1}{m^{2}-m+1}\cdot\frac{1}{(m-1)\cdot\frac{1-m^{3}+m^{2}}{m^{2}}}=$$

$$=\frac{1}{m^{2}-m+1}\cdot\frac{1}{\frac{m-1}{1}\cdot\frac{1-m^{3}+m^{2}}{m^{2}}}=\frac{1}{m^{2}-m+1}\cdot\frac{1}{\frac{(m-1)\cdot\left(1-m^{3}+m^{2}\right)}{m^{2}}}=$$

$$=\frac{1}{m^{2}-m+1}\cdot\frac{\frac{1}{1}}{\frac{(m-1)\cdot\left(1-m^{3}+m^{2}\right)}{m^{2}}}=\frac{1}{m^{2}-m+1}\cdot\frac{m^{2}}{(m-1)\cdot\left(1-m^{3}+m^{2}\right)}=$$

$$=\frac{1}{m^{2}-m+1}\cdot\frac{\frac{1}{1}}{\frac{(m-1)\cdot\left(1-m^{3}+m^{2}\right)}{m^{2}}}=\frac{m^{2}}{\left(m^{2}-m+1\right)\cdot(m-1)\cdot\left(1-m^{3}+m^{2}\right)}=$$

$$=\frac{m^{2}}{(m-1)\cdot\left(m^{2}-m+1\right)\cdot\left(1-m^{3}+m^{2}\right)}.$$

Vježba 471

Pojednostavnite: $\frac{1}{m-(1-m)^{-1}}\cdot\frac{\frac{1}{m^{-2}-m+1}}{m\cdot(m-2)+m^{0}}.$

Rezultat: $\frac{m^{2}}{(m-1)\cdot\left(m^{2}-m+1\right)\cdot\left(1-m^{3}+m^{2}\right)}.$

Zadatak 472 (Ante, tehnička škola)

Pojednostavnite: $\left(a^{5}+3\cdot a^{4}-7\cdot a^{3}\right):a^{3}.$

Rješenje 472

Ponovimo!

$$a^{n}:a^{m}=\frac{a^{n}}{a^{m}}=a^{n-m}\quad,\quad a^{1}=a\quad,\quad a^{0}=1\quad,\quad a:b=\frac{a}{b}.$$

$$(a+b):c=a:c+b:c\quad,\quad\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}\quad,\quad\frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}\quad,\quad a\cdot b:c=b\cdot a:c.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

$$\left(a^5 + 3 \cdot a^4 - 7 \cdot a^3\right) : a^3 = a^5 : a^3 + 3 \cdot a^4 : a^3 - 7 \cdot a^3 : a^3 = a^2 + 3 \cdot a^1 - 7 \cdot a^0 =$$

$$= a^2 + 3 \cdot a - 7 \cdot 1 = a^2 + 3 \cdot a - 7.$$

2.inačica

$$\left(a^5 + 3 \cdot a^4 - 7 \cdot a^3\right) : a^3 = \frac{a^5 + 3 \cdot a^4 - 7 \cdot a^3}{a^3} = \frac{a^5}{a^3} + 3 \cdot \frac{a^4}{a^3} - 7 \cdot \frac{a^3}{a^3} = \frac{a^5}{a^3} + 3 \cdot \frac{a^4}{a^3} - 7 \cdot \frac{a^3}{a^3} =$$

$$= a^2 + 3 \cdot a^1 - 7 \cdot 1 = a^2 + 3 \cdot a - 7.$$

3.inačica

$$\left(a^5 + 3 \cdot a^4 - 7 \cdot a^3\right) : a^3 = a^3 \cdot \left(a^2 + 3 \cdot a - 7\right) : a^3 = \left(a^2 + 3 \cdot a - 7\right) \cdot a^3 : a^3 =$$

$$= \left(a^2 + 3 \cdot a - 7\right) \cdot a^0 = \left(a^2 + 3 \cdot a - 7\right) \cdot 1 = a^2 + 3 \cdot a - 7.$$

Vježba 472

Pojednostavnite: $\left(a^6 + 3 \cdot a^5 - 7 \cdot a^4\right) : a^4.$

Rezultat: $a^2 + 3 \cdot a - 7.$

Zadatak 473 (Mateo, srednja škola)

Ako je b = 2 · a, c = 3 · b, d = 4 · c, e = 5 · d, izrazi pomoću a zbroj a + b + c + d + e.

Rješenje 473

Ponovimo!

$$(a \cdot b) \cdot c = a \cdot (b \cdot c).$$

$$\left.\begin{array}{l} b = 2 \cdot a \\ c = 3 \cdot b \\ d = 4 \cdot c \\ e = 5 \cdot d \end{array}\right\} \Rightarrow \left.\begin{array}{l} b = 2 \cdot a \\ c = 3 \cdot 2 \cdot a = 6 \cdot a \\ d = 4 \cdot 6 \cdot a = 24 \cdot a \\ e = 5 \cdot 24 \cdot a = 120 \cdot a \end{array}\right\} \Rightarrow a + b + c + d + e = a + 2 \cdot a + 6 \cdot a + 24 \cdot a + 120 \cdot a = 153 \cdot a.$$

Vježba 473

Ako je b = 4 · a, c = 3 · b, d = 2 · c, e = 5 · d, izrazi pomoću a zbroj a + b + c + d + e.

Rezultat: 161 · a.

Zadatak 474 (Kvarta, gimnazija)

Ako je a · b^2 = 5, a a^2 · b^5 = 15, izračunaj a i b.

Rješenje 474

Ponovimo!

$$a^1 = a \quad , \quad (a \cdot b)^n = a^n \cdot b^n \quad , \quad \left(a^n\right)^m = a^{n \cdot m} \quad , \quad (a \cdot b) \cdot c = a \cdot (b \cdot c).$$

$$\left.\begin{matrix} a = b \\ c = d \end{matrix}\right\} \Rightarrow \frac{a}{c} = \frac{b}{d} \quad , \quad \frac{a^n}{a^m} = a^{n-m} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad a^n \cdot a^m = a^{n+m}.$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

$$\left.\begin{matrix} a \cdot b^2 = 5 \\ a^2 \cdot b^5 = 15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a \cdot b^2 = 5 \ /^2 \\ a^2 \cdot b^5 = 15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \left(a \cdot b^2\right)^2 = 5^2 \\ a^2 \cdot b^5 = 15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a^2 \cdot b^4 = 25 \\ a^2 \cdot b^5 = 15 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{podijelimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow$$

$$\Rightarrow \frac{a^2 \cdot b^5}{a^2 \cdot b^4} = \frac{15}{25} \Rightarrow \frac{a^2 \cdot b^5}{a^2 \cdot b^4} = \frac{15}{25} \Rightarrow \frac{b^5}{b^4} = \frac{3}{5} \Rightarrow b^1 = \frac{3}{5} \Rightarrow b = \frac{3}{5}.$$

Računamo a.

$$\left.\begin{matrix} a \cdot b^2 = 5 \\ b = \frac{3}{5} \end{matrix}\right\} \Rightarrow a \cdot \left(\frac{3}{5}\right)^2 = 5 \Rightarrow a \cdot \frac{9}{25} = 5 \Rightarrow \frac{9}{25} \cdot a = 5 \Rightarrow \frac{9}{25} \cdot a = 5 \ / \cdot \frac{25}{9} \Rightarrow$$

$$\Rightarrow a = 5 \cdot \frac{25}{9} \Rightarrow a = \frac{5}{1} \cdot \frac{25}{9} \Rightarrow a = \frac{125}{9}.$$

2.inačica

$$\left.\begin{matrix} a \cdot b^2 = 5 \\ a^2 \cdot b^5 = 15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a \cdot b^2 = 5 \ /^2 \\ a^2 \cdot b^5 = 15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \left(a \cdot b^2\right)^2 = 5^2 \\ a^2 \cdot b^5 = 15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a^2 \cdot b^4 = 25 \\ a^2 \cdot b^5 = 15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a^2 \cdot b^4 = 25 \\ a^2 \cdot b^4 \cdot b^1 = 15 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a^2 \cdot b^4 = 25 \\ a^2 \cdot b^4 \cdot b = 15 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{supstitucije} \end{bmatrix} \Rightarrow \left.\begin{matrix} a^2 \cdot b^4 = 25 \\ \left(a^2 \cdot b^4\right) \cdot b = 15 \end{matrix}\right\} \Rightarrow 25 \cdot b = 15 \Rightarrow$$

$$\Rightarrow 25 \cdot b = 15 \ /: 25 \Rightarrow b = \frac{15}{25} \Rightarrow b = \frac{15}{25} \Rightarrow b = \frac{3}{5}.$$

Računamo a.

$$\left.\begin{matrix} a \cdot b^2 = 5 \\ b = \frac{3}{5} \end{matrix}\right\} \Rightarrow a \cdot \left(\frac{3}{5}\right)^2 = 5 \Rightarrow a \cdot \frac{9}{25} = 5 \Rightarrow \frac{9}{25} \cdot a = 5 \Rightarrow \frac{9}{25} \cdot a = 5 \ / \cdot \frac{25}{9} \Rightarrow$$

$$\Rightarrow a = 5 \cdot \frac{25}{9} \Rightarrow a = \frac{5}{1} \cdot \frac{25}{9} \Rightarrow a = \frac{125}{9}.$$

3.inačica

$$\left.\begin{array}{l} a\cdot b^2=5 \\ a^2\cdot b^5=15 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a\cdot b^2=5\ /\cdot\dfrac{1}{b^2} \\ a^2\cdot b^5=15 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=\dfrac{5}{b^2} \\ a^2\cdot b^5=15 \end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda}\\ \text{supstitucije}\end{array}\right] \Rightarrow \left(\frac{5}{b^2}\right)^2\cdot b^5=15 \Rightarrow$$

$$\Rightarrow \frac{5^2}{\left(b^2\right)^2}\cdot b^5=15 \Rightarrow \frac{25}{b^4}\cdot b^5=15 \Rightarrow \frac{25}{b^4}\cdot\frac{b^5}{1}=15 \Rightarrow 25\cdot b^1=15 \Rightarrow 25\cdot b=15 \Rightarrow$$

$$\Rightarrow 25\cdot b=15\ /:25 \Rightarrow b=\frac{15}{25} \Rightarrow b=\frac{15}{25} \Rightarrow b=\frac{3}{5}.$$

Računamo a.

$$\left.\begin{array}{l} a\cdot b^2=5 \\ b=\dfrac{3}{5} \end{array}\right\} \Rightarrow a\cdot\left(\frac{3}{5}\right)^2=5 \Rightarrow a\cdot\frac{9}{25}=5 \Rightarrow \frac{9}{25}\cdot a=5 \Rightarrow \frac{9}{25}\cdot a=5\ /\cdot\frac{25}{9} \Rightarrow$$

$$\Rightarrow a=5\cdot\frac{25}{9} \Rightarrow a=\frac{5}{1}\cdot\frac{25}{9} \Rightarrow a=\frac{125}{9}.$$

Vježba 474

Ako je $a^2 \cdot b = 5$, a $a^5 \cdot b^2 = 15$, izračunaj a i b.

Rezultat: $a=\frac{3}{5},\ b=\frac{125}{9}.$

Zadatak 475 (Kvarta, gimnazija)

Pojednostavnite: $\left(-2^3\right)^4+2\cdot\left(-2^4\right)^3+3\cdot\left(-2^2\right)^6.$

Rješenje 475

Ponovimo!

$$(-a)^{2\cdot n}=a^{2\cdot n} \quad , \quad (-a)^{2\cdot n-1}=-a^{2\cdot n-1} \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^1=a.$$

$$a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(-2^3\right)^4+2\cdot\left(-2^4\right)^3+3\cdot\left(-2^2\right)^6=\left(2^3\right)^4+2\cdot\left(-\left(2^4\right)^3\right)+3\cdot\left(2^2\right)^6=$$

$$=2^{12}+2\cdot\left(-2^{12}\right)+3\cdot 2^{12}=2^{12}-2\cdot 2^{12}+3\cdot 2^{12}=2^{12}-2\cdot 2^{12}+3\cdot 2^{12}=2^{12}\cdot(1-2+3)=$$

$$=2^{12}\cdot 2=2^{12}\cdot 2^1=2^{13}.$$

Vježba 475

Pojednostavnite: $\left(-2^3\right)^4+4\cdot\left(-2^4\right)^3+3\cdot\left(-2^2\right)^6.$

Rezultat: 0.

Zadatak 476 (4A, 4B, TUPŠ)

Učenik je na džepnome računalu zbrojio brojeve A i B. Dobiveni rezultat podijelio je s C. Taj je rezultat pomnožio s D. Koji izraz opisuje taj račun?

$$A.\ \frac{A+B}{C\cdot D} \qquad B.\ \frac{(A+B)\cdot D}{C} \qquad C.\ (A+B:C)\cdot D \qquad D.\ A+B:\frac{C}{D}$$

Rješenje 476

Ponovimo!

$$a\cdot\frac{b}{c}=\frac{a\cdot b}{c}.$$

Kako zapisati:

- zbroj brojeva x i y:

$$x+y.$$

- umnožak brojeva x i y:

$$x\cdot y.$$

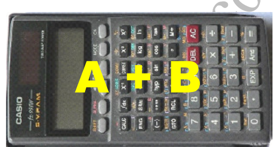
- količnik (kvocijent) brojeva x i y:

$$x:y \text{ ili } \frac{x}{y}.$$

Zakon distribucije množenja prema zbrajanju.

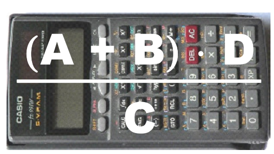
$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Prevodimo tekstualni materijal u matematički simbolički zapis.
Učenik je na džepnome računalu zbrojio brojeve A i B.



Dobiveni rezultat podijelio je s C.

Taj je rezultat pomnožio s D.



Odgovor je pod B.

Vježba 476

Učenik je na džepnome računalu zbrojio brojeve A i B. Dobiveni rezultat podijelio je s umnoškom brojeva C i D. Koji izraz opisuje taj račun?

$$A.\ \frac{A+B}{C\cdot D} \qquad B.\ \frac{(A+B)\cdot D}{C} \qquad C.\ (A+B:C)\cdot D \qquad D.\ A+B:\frac{C}{D}$$

Rezultat: A.

Zadatak 477 (4A, 4B, TUPŠ)

Čemu je jednako k ako je $m = \frac{k}{2} - 3 \cdot p$?

$A.\ k = m + 3 \cdot p \qquad B.\ k = m + 6 \cdot p \qquad C.\ k = 2 \cdot m + 3 \cdot p \qquad D.\ k = 2 \cdot m + 3 \cdot p$

Rješenje 477

Ponovimo!

$$a + b = b + a \quad , \quad a = b \Rightarrow b = a.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$m = \frac{k}{2} - 3 \cdot p \Rightarrow -\frac{k}{2} = -3 \cdot p - m \Rightarrow -\frac{k}{2} = -3 \cdot p - m \ / \cdot (-2) \Rightarrow k = 6 \cdot p + 2 \cdot m \Rightarrow k = 2 \cdot m + 6 \cdot p.$$

Odgovor je pod C.

2.inačica

$$m = \frac{k}{2} - 3 \cdot p \Rightarrow \frac{k}{2} - 3 \cdot p = m \Rightarrow \frac{k}{2} = m + 3 \cdot p \Rightarrow \frac{k}{2} = m + 3 \cdot p \ / \cdot 2 \Rightarrow k = 2 \cdot m + 6 \cdot p.$$

Odgovor je pod C.

3.inačica

$$m = \frac{k}{2} - 3 \cdot p \Rightarrow m = \frac{k}{2} - 3 \cdot p \ / \cdot 2 \Rightarrow 2 \cdot m = k - 6 \cdot p \Rightarrow -k = -6 \cdot p - 2 \cdot m \Rightarrow$$

$$\Rightarrow -k = -6 \cdot p - 2 \cdot m \ / \cdot (-1) \Rightarrow k = 6 \cdot p + 2 \cdot m \Rightarrow k = 2 \cdot m + 6 \cdot p.$$

Odgovor je pod C.

4.inačica

$$m = \frac{k}{2} - 3 \cdot p \Rightarrow m = \frac{k}{2} - 3 \cdot p \ / \cdot 2 \Rightarrow 2 \cdot m = k - 6 \cdot p \Rightarrow k - 6 \cdot p = 2 \cdot m \Rightarrow k = 2 \cdot m + 6 \cdot p.$$

Odgovor je pod C.

Vježba 477

Čemu je jednako k ako je $m = k - 3 \cdot p$?

$A.\ k = m + 3 \cdot p \qquad B.\ k = m + 6 \cdot p \qquad C.\ k = 2 \cdot m + 3 \cdot p \qquad D.\ k = 2 \cdot m + 3 \cdot p$

Rezultat: A.

Zadatak 478 (4A, 4B, TUPŠ)

Izrazite m iz formule $F = G \cdot \frac{m \cdot M}{r^2}$.

Rješenje 478

Ponovimo!

$$a = b \Rightarrow b = a.$$

1.inačica

$$F = G \cdot \frac{m \cdot M}{r^2} \Rightarrow F = G \cdot \frac{m \cdot M}{r^2} \ / \cdot r^2 \Rightarrow r^2 \cdot F = G \cdot m \cdot M \Rightarrow r^2 \cdot F = G \cdot m \cdot M \ / \cdot \frac{1}{G \cdot M} \Rightarrow$$

$$\Rightarrow \frac{r^2 \cdot F}{G \cdot M} = m \Rightarrow m = \frac{r^2 \cdot F}{G \cdot M}.$$

2.inačica

$$F = G \cdot \frac{m \cdot M}{r^2} \Rightarrow G \cdot \frac{m \cdot M}{r^2} = F \Rightarrow G \cdot \frac{m \cdot M}{r^2} = F \; / \cdot r^2 \Rightarrow G \cdot m \cdot M = r^2 \cdot F \Rightarrow$$

$$\Rightarrow G \cdot m \cdot M = r^2 \cdot F \; / \cdot \frac{1}{G \cdot M} \Rightarrow m = \frac{r^2 \cdot F}{G \cdot M}.$$

3.inačica

$$F = G \cdot \frac{m \cdot M}{r^2} \Rightarrow F = G \cdot \frac{m \cdot M}{r^2} \; / \cdot \frac{r^2}{G \cdot M} \Rightarrow \frac{F \cdot r^2}{G \cdot M} = m \Rightarrow m = \frac{r^2 \cdot F}{G \cdot M}.$$

Vježba 478

Izrazite M iz formule $F = G \cdot \frac{m \cdot M}{r^2}$.

Rezultat: $M = \frac{r^2 \cdot F}{G \cdot m}$.

Zadatak 479 (4A, 4B, TUPŠ)

Izraz $a^2 - 2 \cdot a \cdot b - 3 \cdot b^2$ napišite kao umnožak dvaju binoma.

Rješenje 479

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad (x \cdot y)^2 = x^2 \cdot y^2 \quad , \quad x^2 - y^2 = (x-y) \cdot (x+y).$$

Algebarski izrazi su izrazi u kojima se pojavljuju brojevi i slova te znakovi računskih operacija i zagrade.

Monomi su jednočlani algebarski izrazi. U njima nema zbrajanja niti oduzimanja. Na primjer,

$$6 \cdot x \quad , \quad 7 \cdot x \cdot y \cdot z^3 \quad , \quad \frac{1}{2} \cdot a^5 \cdot b^3.$$

Binomi su dvočlani algebarski izrazi. Oni su zbroj (razlika) dva monoma. Na primjer,

$$6 \cdot x + 7 \quad , \quad 7 \cdot x \cdot y \cdot z^3 - a \cdot b^2 \quad , \quad \frac{1}{2} \cdot a^5 \cdot b^3 + \frac{7}{9} \cdot c.$$

Postavljeni izraz transformiramo na sljedeći način:

$$a^2 - 2 \cdot a \cdot b - 3 \cdot b^2 = a^2 - 2 \cdot a \cdot b - 3 \cdot b^2 = a^2 - 2 \cdot a \cdot b + b^2 - 4 \cdot b^2 = \left(a^2 - 2 \cdot a \cdot b + b^2\right) - 4 \cdot b^2 =$$

$$= (a-b)^2 - (2 \cdot b)^2 = (a - b - 2 \cdot b) \cdot (a - b + 2 \cdot b) = (a - 3 \cdot b) \cdot (a + b).$$

Vježba 479

Izraz $a^2 - 4 \cdot a \cdot b + 3 \cdot b^2$ napišite kao umnožak dvaju binoma.

Rezultat: $(a - 3 \cdot b) \cdot (a - b)$.

Zadatak 480 (4A, 4B, TUPŠ)

Skrati razlomak: $\frac{x^3 \cdot y - x \cdot y^3}{x^3 \cdot y - 2 \cdot x^2 \cdot y^2 + x \cdot y^3}$.

Rješenje 480

Ponovimo!

$$a^2 - b^2 = (a-b) \cdot (a+b) \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$\frac{x^3\cdot y-x\cdot y^3}{x^3\cdot y-2\cdot x^2\cdot y^2+x\cdot y^3}=\frac{x\cdot y\cdot\left(x^2-y^2\right)}{x\cdot y\cdot\left(x^2-2\cdot x\cdot y+y^2\right)}=\frac{x\cdot y\cdot\left(x^2-y^2\right)}{x\cdot y\cdot\left(x^2-2\cdot x\cdot y+y^2\right)}=$$

$$=\frac{x^2-y^2}{x^2-2\cdot x\cdot y+y^2}=\frac{(x-y)\cdot(x+y)}{(x-y)^2}=\frac{(x-y)\cdot(x+y)}{(x-y)^2}=\frac{x+y}{x-y}.$$

Vježba 480

Skrati razlomak: $\dfrac{x^3\cdot y-2\cdot x^2\cdot y^2+x\cdot y^3}{x^3\cdot y-x\cdot y^3}.$

Rezultat: $\dfrac{x-y}{x+y}.$