Zadatak 421 (Ivana, gimnazija)

Skrati razlomak $\frac{2+\sqrt{2\cdot x}}{x+\sqrt{2\cdot x}}$.

Rješenje 421

Ponovimo!

$$\left(\sqrt{a}\right)^2=a \quad , \quad \sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b} \quad , \quad \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{2+\sqrt{2\cdot x}}{x+\sqrt{2\cdot x}}=\frac{\left(\sqrt{2}\right)^2+\sqrt{2}\cdot\sqrt{x}}{\left(\sqrt{x}\right)^2+\sqrt{2}\cdot\sqrt{x}}=\frac{\sqrt{2}\cdot\left(\sqrt{2}+\sqrt{x}\right)}{\sqrt{x}\cdot\left(\sqrt{x}+\sqrt{2}\right)}=\frac{\sqrt{2}\cdot\left(\sqrt{2}+\sqrt{x}\right)}{\sqrt{x}\cdot\left(\sqrt{x}+\sqrt{2}\right)}=\frac{\sqrt{2}}{\sqrt{x}}=\sqrt{\frac{2}{x}}.$$

Vježba 421

Skrati razlomak $\frac{3+\sqrt{3\cdot x}}{x+\sqrt{3\cdot x}}$.

Rezultat: $\sqrt{\frac{3}{x}}$.

Zadatak 422 (Ivana, gimnazija)

Skrati razlomak $\frac{a+b+2\cdot\sqrt{a\cdot b}}{a\cdot\sqrt{b}+b\cdot\sqrt{a}}$.

Rješenje 422

Ponovimo!

$$\left(\sqrt{a}\right)^2=a \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b} \quad , \quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}.$$

$$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{a+b+2\cdot\sqrt{a\cdot b}}{a\cdot\sqrt{b}+b\cdot\sqrt{a}}=\frac{a+2\cdot\sqrt{a\cdot b}+b}{a\cdot\sqrt{b}+b\cdot\sqrt{a}}=\frac{\left(\sqrt{a}\right)^2+2\cdot\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2}{\left(\sqrt{a}\right)^2\cdot\sqrt{b}+\left(\sqrt{b}\right)^2\cdot\sqrt{a}}=\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\sqrt{a}\cdot\sqrt{b}\cdot\left(\sqrt{a}+\sqrt{b}\right)}=$$

$$=\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{\sqrt{a}\cdot\sqrt{b}\cdot\left(\sqrt{a}+\sqrt{b}\right)}=\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}\cdot\sqrt{b}}=\frac{\sqrt{a}}{\sqrt{a}\cdot\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{a}\cdot\sqrt{b}}=\frac{\sqrt{a}}{\sqrt{a}\cdot\sqrt{b}}+\frac{\sqrt{b}}{\sqrt{a}\cdot\sqrt{b}}=\frac{1}{\sqrt{b}}+\frac{1}{\sqrt{a}}=\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}.$$

Vježba 422

Skrati razlomak $\frac{a\cdot\sqrt{b}+b\cdot\sqrt{a}}{a+b+2\cdot\sqrt{a\cdot b}}$.

Rezultat: $\frac{\sqrt{a}\cdot\sqrt{b}}{\sqrt{a}+\sqrt{b}}$.

Zadatak 423 (Docx, gimnazija)

Koristeći se zakonom distribucije množenja prema zbrajanju napiši u obliku produkta (rastavi na faktore) izraz: $(2\cdot a+4\cdot b)\cdot(x+y)+(2\cdot x+6\cdot y)\cdot(a+2\cdot b)$.

Rješenje 423

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

1.inačica

$$(2\cdot a+4\cdot b)\cdot(x+y)+(2\cdot x+6\cdot y)\cdot(a+2\cdot b)=2\cdot(a+2\cdot b)\cdot(x+y)+2\cdot(x+3\cdot y)\cdot(a+2\cdot b)=$$

$$=2\cdot(a+2\cdot b)\cdot(x+y)+2\cdot(x+3\cdot y)\cdot(a+2\cdot b)=2\cdot(a+2\cdot b)\cdot(x+y+x+3\cdot y)=$$

$$=2\cdot(a+2\cdot b)\cdot(2\cdot x+4\cdot y)=2\cdot(a+2\cdot b)\cdot 2\cdot(x+2\cdot y)=4\cdot(a+2\cdot b)\cdot(x+2\cdot y).$$

2.inačica

$$(2\cdot a+4\cdot b)\cdot(x+y)+(2\cdot x+6\cdot y)\cdot(a+2\cdot b)=$$

$$=2\cdot a\cdot x+2\cdot a\cdot y+4\cdot b\cdot x+4\cdot b\cdot y+2\cdot a\cdot x+4\cdot b\cdot x+6\cdot a\cdot y+12\cdot b\cdot y=$$

$$=4\cdot a\cdot x+8\cdot a\cdot y+8\cdot b\cdot x+16\cdot b\cdot y=4\cdot(a\cdot x+2\cdot a\cdot y+2\cdot b\cdot x+4\cdot b\cdot y)=$$

$$=4\cdot((a\cdot x+2\cdot b\cdot x)+(2\cdot a\cdot y+4\cdot b\cdot y))=4\cdot(x\cdot(a+2\cdot b)+2\cdot y\cdot(a+2\cdot b))=$$

$$=4\cdot(x\cdot(a+2\cdot b)+2\cdot y\cdot(a+2\cdot b))=4\cdot(a+2\cdot b)\cdot(x+2\cdot y).$$

Vježba 423

Koristeći se zakonom distribucije množenja prema zbrajanju napiši u obliku produkta (rastavi na faktore) izraz: $2\cdot a\cdot x+2\cdot a\cdot y+3\cdot x+3\cdot y$.

Rezultat: $(2\cdot a+3)\cdot(x+y)$.

Zadatak 424 (Docx, gimnazija)

Koristeći se zakonom distribucije množenja prema zbrajanju napiši u obliku produkta (rastavi na faktore) izraz: $a\cdot c+b\cdot c+a+b$.

Rješenje 424

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$a\cdot c+b\cdot c+a+b=\begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}=(a\cdot c+b\cdot c)+(a+b)=(a\cdot c+b\cdot c)+(a+b)=$$

$$=(a+b)\cdot c+(a+b)=(a+b)\cdot c+(a+b)=(a+b)\cdot(c+1).$$

2.inačica

$$a\cdot c+b\cdot c+a+b=\begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}=(a\cdot c+a)+(b\cdot c+b)=(a\cdot c+a)+(b\cdot c+b)=$$

$$=a\cdot(c+1)+b\cdot(c+1)=a\cdot(c+1)+b\cdot(c+1)=(a+b)\cdot(c+1).$$

Vježba 424

Koristeći se zakonom distribucije množenja prema zbrajanju napiši u obliku produkta (rastavi na faktore) izraz: $a\cdot c-b\cdot c+a-b.$

Rezultat: $(a-b)\cdot(c+1).$

Zadatak 425 (Petra, gimnazija)

Pojednostavnite: $\dfrac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\left(1+\dfrac{b^2+c^2-a^2}{2\cdot b\cdot c}\right):\dfrac{a-b-c}{a\cdot b\cdot c}.$

Rješenje 425

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad(a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad(a-b)^2=(b-a)^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\;,\;n\neq 0\;,\;n\neq 1.$$

1.inačica

$$\frac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\left(1+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}\right):\frac{a-b-c}{a\cdot b\cdot c}=\frac{\frac{b+c-a}{a\cdot(b+c)}}{\frac{b+c+a}{a\cdot(b+c)}}\cdot\left(\frac{1}{1}+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}\right):\frac{a-b-c}{a\cdot b\cdot c}=$$

$$=\frac{\frac{b+c-a}{a\cdot(b+c)}}{\frac{b+c+a}{a\cdot(b+c)}}\cdot\frac{2\cdot b\cdot c+b^2+c^2-a^2}{2\cdot b\cdot c}:\frac{a-b-c}{a\cdot b\cdot c}=\frac{\frac{b+c-a}{1}}{\frac{b+c+a}{1}}\cdot\frac{\left(b^2+2\cdot b\cdot c+c^2\right)-a^2}{2\cdot b\cdot c}:\frac{a-b-c}{a\cdot b\cdot c}=$$

$$=\frac{b+c-a}{b+c+a}\cdot\frac{(b+c)^2-a^2}{2\cdot b\cdot c}:\frac{a-b-c}{a\cdot b\cdot c}=\frac{b+c-a}{b+c+a}\cdot\frac{(b+c-a)\cdot(b+c+a)}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a-b-c}=$$

$$=\frac{b+c-a}{b+c+a}\cdot\frac{(b+c-a)\cdot(b+c+a)}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a-b-c}=\frac{b+c-a}{1}\cdot\frac{b+c-a}{2}\cdot\frac{a}{a-b-c}=$$

$$=\frac{-(a-b-c)}{1}\cdot\frac{-(a-b-c)}{2}\cdot\frac{a}{a-b-c}=\frac{a-b-c}{1}\cdot\frac{a-b-c}{2}\cdot\frac{a}{a-b-c}=\frac{a-b-c}{1}\cdot\frac{a-b-c}{2}\cdot\frac{a}{a-b-c}=$$

$$=\frac{1}{1}\cdot\frac{a-b-c}{2}\cdot\frac{a}{1}=\frac{a\cdot(a-b-c)}{2}=\frac{a}{2}\cdot(a-b-c).$$

2.inačica

$$\frac{\frac{1}{a}-\frac{1}{b+c}}{\frac{1}{a}+\frac{1}{b+c}}\cdot\left(1+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}\right):\frac{a-b-c}{a\cdot b\cdot c}=\frac{\frac{b+c-a}{a\cdot(b+c)}}{\frac{b+c+a}{a\cdot(b+c)}}\cdot\left(\frac{1}{1}+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}\right)\cdot\frac{a\cdot b\cdot c}{a-b-c}=$$

$$=\frac{\frac{b+c-a}{a\cdot(b+c)}}{\frac{b+c+a}{a\cdot(b+c)}}\cdot\frac{2\cdot b\cdot c+b^2+c^2-a^2}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a-b-c}=\frac{\frac{b+c-a}{1}}{\frac{b+c+a}{1}}\cdot\frac{2\cdot b\cdot c+b^2+c^2-a^2}{2\cdot b\cdot c}\cdot\frac{a\cdot b\cdot c}{a-b-c}=$$

$$=\frac{b+c-a}{b+c+a}\cdot\frac{2\cdot b\cdot c+b^2+c^2-a^2}{2}\cdot\frac{a}{a-b-c}=\frac{b+c-a}{b+c+a}\cdot\frac{\left(b^2+2\cdot b\cdot c+c^2\right)-a^2}{2}\cdot\frac{a}{a-b-c}=$$

$$=\frac{b+c-a}{b+c+a}\cdot\frac{(b+c)^2-a^2}{2}\cdot\frac{a}{a-b-c}=\frac{b+c-a}{b+c+a}\cdot\frac{(b+c-a)\cdot(b+c+a)}{2}\cdot\frac{a}{a-b-c}=$$

$$=\frac{b+c-a}{b+c+a}\cdot\frac{(b+c-a)\cdot(b+c+a)}{2}\cdot\frac{a}{a-b-c}=\frac{b+c-a}{1}\cdot\frac{b+c-a}{2}\cdot\frac{a}{a-b-c}=\frac{(b+c-a)^2}{2}\cdot\frac{a}{a-b-c}=$$

$$=\frac{(a-b-c)^2}{2}\cdot\frac{a}{a-b-c}=\frac{(a-b-c)^2}{2}\cdot\frac{a}{a-b-c}=\frac{a-b-c}{2}\cdot\frac{a}{1}=\frac{a}{2}\cdot(a-b-c).$$

Vježba 425

Pojednostavnite: $\frac{\frac{1}{b+c}-\frac{1}{a}}{\frac{1}{b+c}+\frac{1}{a}}\cdot\left(1+\frac{b^2+c^2-a^2}{2\cdot b\cdot c}\right):\frac{b+c-a}{a\cdot b\cdot c}.$

Rezultat: $\frac{a}{2}\cdot(a-b-c).$

Zadatak 426 (Mirjana, gimnazija)

Ako je $x\cdot y=a,\ x\cdot z=b,\ y\cdot z=c$ te $a\neq 0,\ b\neq 0,\ c\neq 0$, koliko je $x^2+y^2+z^2$?

Rješenje 426

Ponovimo!

$$a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad(a\cdot b)^n=a^n\cdot b^n\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\left.\begin{array}{l}a=b\\c=d\end{array}\right\}\Rightarrow a\cdot c=b\cdot d\quad,\quad\left.\begin{array}{l}a=b\\c=d\end{array}\right\}\Rightarrow\frac{a}{c}=\frac{b}{d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

Pomnožimo jednadžbe sustava.

$$\left.\begin{matrix} x\cdot y=a \\ x\cdot z=b \\ y\cdot z=c \end{matrix}\right\} \Rightarrow \begin{bmatrix}\text{pomnožimo} \\ \text{jednadžbe}\end{bmatrix} \Rightarrow x\cdot y\cdot x\cdot z\cdot y\cdot z=a\cdot b\cdot c \Rightarrow x^2\cdot y^2\cdot z^2=a\cdot b\cdot c.$$

Računamo x^2.

$$\left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ y\cdot z=c \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ y\cdot z=c \;/^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ (y\cdot z)^2=c^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ y^2\cdot z^2=c^2 \end{matrix}\right\}.$$

$$\Rightarrow \begin{bmatrix}\text{podijelimo} \\ \text{jednadžbe}\end{bmatrix} \Rightarrow \frac{x^2\cdot y^2\cdot z^2}{y^2\cdot z^2}=\frac{a\cdot b\cdot c}{c^2} \Rightarrow \frac{x^2\cdot y^2\cdot z^2}{y^2\cdot z^2}=\frac{a\cdot b\cdot c}{c^2} \Rightarrow x^2=\frac{a\cdot b}{c}.$$

Računamo y^2.

$$\left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ x\cdot z=b \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ x\cdot z=b \;/^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ (x\cdot z)^2=b^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ x^2\cdot z^2=b^2 \end{matrix}\right\}.$$

$$\Rightarrow \begin{bmatrix}\text{podijelimo} \\ \text{jednadžbe}\end{bmatrix} \Rightarrow \frac{x^2\cdot y^2\cdot z^2}{x^2\cdot z^2}=\frac{a\cdot b\cdot c}{b^2} \Rightarrow \frac{x^2\cdot y^2\cdot z^2}{x^2\cdot z^2}=\frac{a\cdot b\cdot c}{b^2} \Rightarrow y^2=\frac{a\cdot c}{b}.$$

Računamo z^2.

$$\left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ x\cdot y=a \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ x\cdot y=a \;/^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ (x\cdot y)^2=a^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ x^2\cdot y^2=a^2 \end{matrix}\right\}.$$

$$\Rightarrow \begin{bmatrix}\text{podijelimo} \\ \text{jednadžbe}\end{bmatrix} \Rightarrow \frac{x^2\cdot y^2\cdot z^2}{x^2\cdot y^2}=\frac{a\cdot b\cdot c}{a^2} \Rightarrow \frac{x^2\cdot y^2\cdot z^2}{x^2\cdot y^2}=\frac{a\cdot b\cdot c}{a^2} \Rightarrow z^2=\frac{b\cdot c}{a}.$$

Sada je:

$$x^2+y^2+z^2=\frac{a\cdot b}{c}+\frac{a\cdot c}{b}+\frac{b\cdot c}{a} \Rightarrow x^2+y^2+z^2=\frac{a^2\cdot b^2+a^2\cdot c^2+b^2\cdot c^2}{a\cdot b\cdot c}.$$

2.inačica

Pomnožimo jednadžbe sustava.

$$\left.\begin{matrix} x\cdot y=a \\ x\cdot z=b \\ y\cdot z=c \end{matrix}\right\} \Rightarrow \begin{bmatrix}\text{pomnožimo} \\ \text{jednadžbe}\end{bmatrix} \Rightarrow x\cdot y\cdot x\cdot z\cdot y\cdot z=a\cdot b\cdot c \Rightarrow x^2\cdot y^2\cdot z^2=a\cdot b\cdot c.$$

Računamo x^2.

$$\left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ y\cdot z=c \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ y\cdot z=c \;/^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ (y\cdot z)^2=c^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2\cdot y^2\cdot z^2=a\cdot b\cdot c \\ y^2\cdot z^2=c^2 \end{matrix}\right\}.$$

$$\Rightarrow \begin{bmatrix}\text{podijelimo} \\ \text{jednadžbe}\end{bmatrix} \Rightarrow \frac{x^2\cdot y^2\cdot z^2}{y^2\cdot z^2}=\frac{a\cdot b\cdot c}{c^2} \Rightarrow \frac{x^2\cdot y^2\cdot z^2}{y^2\cdot z^2}=\frac{a\cdot b\cdot c}{c^2} \Rightarrow x^2=\frac{a\cdot b}{c}.$$

Računamo y^2.

$$\left.\begin{matrix} x^2=\dfrac{a\cdot b}{c} \\ x\cdot y=a \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2=\dfrac{a\cdot b}{c} \\ x\cdot y=a \;/^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2=\dfrac{a\cdot b}{c} \\ (x\cdot y)^2=a^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2=\dfrac{a\cdot b}{c} \\ x^2\cdot y^2=a^2 \end{matrix}\right\} \Rightarrow \begin{bmatrix}\text{metoda} \\ \text{supstitucije}\end{bmatrix} \Rightarrow$$

$$\Rightarrow \frac{a \cdot b}{c} \cdot y^2 = a^2 \Rightarrow \frac{a \cdot b}{c} \cdot y^2 = a^2 \;/ \cdot \frac{c}{a \cdot b} \Rightarrow y^2 = \frac{a \cdot c}{b}.$$

Računamo z^2.

$$\left.\begin{aligned} x^2 &= \frac{a \cdot b}{c} \\ x \cdot z &= b \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x^2 &= \frac{a \cdot b}{c} \\ x \cdot z &= b \;/^2 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x^2 &= \frac{a \cdot b}{c} \\ (x \cdot z)^2 &= b^2 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} x^2 &= \frac{a \cdot b}{c} \\ x^2 \cdot z^2 &= b^2 \end{aligned}\right\} \Rightarrow \left[\begin{matrix}\text{metoda} \\ \text{supstitucije}\end{matrix}\right] \Rightarrow$$

$$\Rightarrow \frac{a \cdot b}{c} \cdot z^2 = b^2 \Rightarrow \frac{a \cdot b}{c} \cdot z^2 = b^2 \;/ \cdot \frac{c}{a \cdot b} \Rightarrow z^2 = \frac{b \cdot c}{a}.$$

Sada je:

$$x^2 + y^2 + z^2 = \frac{a \cdot b}{c} + \frac{a \cdot c}{b} + \frac{b \cdot c}{a} \Rightarrow x^2 + y^2 + z^2 = \frac{a^2 \cdot b^2 + a^2 \cdot c^2 + b^2 \cdot c^2}{a \cdot b \cdot c}.$$

Vježba 426

Ako je $x \cdot y = 2,\ x \cdot z = 3,\ y \cdot z = 6,$ koliko je $x^2 + y^2 + z^2$?

Rezultat: 14.

Zadatak 427 (Ana, srednja škola)

Prikažite u obliku potencije s bazom 2: $2^8 + 4^5 + 8^3 + 16^2$.

Rješenje 427

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad n = \frac{n}{1}.$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$2^8 + 4^5 + 8^3 + 16^2 = 2^8 + \left(2^2\right)^5 + \left(2^3\right)^3 + \left(2^4\right)^2 = 2^8 + 2^{10} + 2^9 + 2^8 =$$

$$= 2^8 + 2^8 \cdot 2^2 + 2^8 \cdot 2^1 + 2^8 = 2^8 + 2^8 \cdot 2^2 + 2^8 \cdot 2^1 + 2^8 = 2^8 \cdot \left(1 + 2^2 + 2^1 + 1\right) =$$

$$= 2^8 \cdot (1 + 4 + 2 + 1) = 2^8 \cdot 8 = 2^8 \cdot 2^3 = 2^{8+3} = 2^{11}.$$

2.inačica

$$2^8 + 4^5 + 8^3 + 16^2 = 2^8 + \left(2^2\right)^5 + \left(2^3\right)^3 + \left(2^4\right)^2 = 2^8 + 2^{10} + 2^9 + 2^8 =$$

$$= 2^{10} \cdot 2^{-2} + 2^{10} + 2^{10} \cdot 2^{-1} + 2^{10} \cdot 2^{-2} = 2^{10} \cdot 2^{-2} + 2^{10} + 2^{10} \cdot 2^{-1} + 2^{10} \cdot 2^{-2} =$$

$$= 2^{10} \cdot \left(2^{-2} + 1 + 2^{-1} + 2^{-2}\right) = 2^{10} \cdot \left(\frac{1}{2^2} + 1 + \frac{1}{2} + \frac{1}{2^2}\right) = 2^{10} \cdot \left(\frac{1}{4} + 1 + \frac{1}{2} + \frac{1}{4}\right) =$$

$$= 2^{10} \cdot \left(\frac{1}{4} + \frac{1}{1} + \frac{1}{2} + \frac{1}{4}\right) = 2^{10} \cdot \frac{1 + 4 + 2 + 1}{4} = 2^{10} \cdot \frac{8}{4} = 2^{10} \cdot \frac{8}{4} = 2^{10} \cdot 2 = 2^{10} \cdot 2^1 = 2^{11}.$$

Vježba 427

Prikažite u obliku potencije s bazom 2: $2^8 + 4^5 + 8^3 + 4^4$.

Rezultat: 2^{11}.

Zadatak 428 (Marina, srednja škola)

Ako je $x^2 - x + 2 = 0$, onda je $x^4 - 2 \cdot x^3 + x^2 + 7$ jednako:

$A.\ 11 \qquad B.\ 13 \qquad C.\ 15 \qquad D.\ 17$

Rješenje 428

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

Zadanu jednadžbu transformiramo na sljedeći način.

$$x^2 - x + 2 = 0 \Rightarrow x^2 - x = -2 \Rightarrow \begin{bmatrix} \text{kvadriramo} \\ \text{jednadžbu} \end{bmatrix} \Rightarrow x^2 - x = -2 \ /^2 \Rightarrow \left(x^2 - x\right)^2 = (-2)^2 \Rightarrow$$

$$\Rightarrow \left(x^2\right)^2 - 2 \cdot x^2 \cdot x + x^2 = 4 \Rightarrow x^4 - 2 \cdot x^3 + x^2 = 4.$$

Sada je

$$x^4 - 2 \cdot x^3 + x^2 + 7 = \underbrace{\left(x^4 - 2 \cdot x^3 + x^2\right)}_{=4} + 7 = 4 + 7 = 11.$$

Odgovor je pod A.

2.inačica

$$x^2 - x + 2 = 0 \Rightarrow \begin{bmatrix} \text{kvadriramo} \\ \text{jednadžbu} \end{bmatrix} \Rightarrow x^2 - x + 2 = 0 \ /^2 \Rightarrow \left(x^2 - x + 2\right)^2 = 0^2 \Rightarrow$$

$$\Rightarrow \left(x^2\right)^2 + (-x)^2 + 2^2 + 2 \cdot x^2 \cdot (-x) + 2 \cdot x^2 \cdot 2 + 2 \cdot (-x) \cdot 2 = 0 \Rightarrow$$

$$\Rightarrow x^4 + x^2 + 4 - 2 \cdot x^3 + 4 \cdot x^2 - 4 \cdot x = 0 \Rightarrow x^4 - 2 \cdot x^3 + x^2 + 4 \cdot x^2 - 4 \cdot x + 4 = 0 \Rightarrow$$

$$\Rightarrow x^4 - 2 \cdot x^3 + x^2 + 7 - 7 + 4 \cdot x^2 - 4 \cdot x + 4 = 0 \Rightarrow \left(x^4 - 2 \cdot x^3 + x^2 + 7\right) - 7 + 4 \cdot x^2 - 4 \cdot x + 4 = 0 \Rightarrow$$

$$\Rightarrow \left(x^4 - 2 \cdot x^3 + x^2 + 7\right) - 7 + 4 \cdot x^2 - 4 \cdot x + 4 = 0 \Rightarrow \left(x^4 - 2 \cdot x^3 + x^2 + 7\right) - 7 + 4 \cdot x^2 - 4 \cdot x + 8 - 4 = 0 \Rightarrow$$

$$\Rightarrow \left(x^4 - 2 \cdot x^3 + x^2 + 7\right) - 7 + \left(4 \cdot x^2 - 4 \cdot x + 8\right) - 4 = 0 \Rightarrow$$

$$\Rightarrow \left(x^4 - 2 \cdot x^3 + x^2 + 7\right) + \left(4 \cdot x^2 - 4 \cdot x + 8\right) = 7 + 4 \Rightarrow \left(x^4 - 2 \cdot x^3 + x^2 + 7\right) + \left(4 \cdot x^2 - 4 \cdot x + 8\right) = 11 \Rightarrow$$

$$\Rightarrow \left(x^4 - 2 \cdot x^3 + x^2 + 7\right) + 4 \cdot \left(x^2 - x + 2\right) = 11 \Rightarrow \left(x^4 - 2 \cdot x^3 + x^2 + 7\right) + 4 \cdot \underbrace{\left(x^2 - x + 2\right)}_{=0} = 11 \Rightarrow$$

$$\Rightarrow \left(x^4-2\cdot x^3+x^2+7\right)+4\cdot 0=11 \Rightarrow \left(x^4-2\cdot x^3+x^2+7\right)+0=11 \Rightarrow x^4-2\cdot x^3+x^2+7=11.$$

Odgovor je pod A.

Vježba 428

Ako je $x^2-x+2=0$, onda je $x^4-2\cdot x^3+x^2+9$ jednako:

$A.\ 11 \qquad B.\ 13 \qquad C.\ 15 \qquad D.\ 17$

Rezultat: B.

Zadatak 429 (Ivan, gimnazija)

Koliko je $\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1+\sqrt[16]{a}\right)\cdot\left(1-\sqrt[16]{a}\right)$, za $a=2$?

$A.\ -1 \qquad B.\ -2 \qquad C.\ -3 \qquad D.\ -4$

Rješenje 429

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad \left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m} \quad , \quad \sqrt[n\cdot p]{a^{m\cdot p}}=\sqrt[n]{a^m}.$$

$$\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1+\sqrt[16]{a}\right)\cdot\left(1-\sqrt[16]{a}\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\underbrace{\left(1+\sqrt[16]{a}\right)\cdot\left(1-\sqrt[16]{a}\right)}_{\text{razlika kvadrata}}=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1^2-\left(\sqrt[16]{a}\right)^2\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1-\sqrt[16]{a^2}\right)=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1-\sqrt[16]{a^2}\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1-\sqrt[8]{a}\right)=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\underbrace{\left(1+\sqrt[8]{a}\right)\cdot\left(1-\sqrt[8]{a}\right)}_{\text{razlika kvadrata}}=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1^2-\left(\sqrt[8]{a}\right)^2\right)=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1-\sqrt[8]{a^2}\right)=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1-\sqrt[8]{a^2}\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1-\sqrt[4]{a}\right)=\left(1+\sqrt{a}\right)\cdot\underbrace{\left(1+\sqrt[4]{a}\right)\cdot\left(1-\sqrt[4]{a}\right)}_{\text{razlika kvadrata}}=\left(1+\sqrt{a}\right)\cdot\left(1^2-\left(\sqrt[4]{a}\right)^2\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1-\sqrt[4]{a^2}\right)=\left(1+\sqrt{a}\right)\cdot\left(1-\sqrt[4]{a^2}\right)=\left(1+\sqrt{a}\right)\cdot\left(1-\sqrt{a}\right)=\begin{bmatrix}\text{razlika}\\ \text{kvadrata}\end{bmatrix}=$$

$$=1^2-\left(\sqrt{a}\right)^2=1-a=\begin{bmatrix}\text{uvjet}\\ a=2\end{bmatrix}=1-2=-1.$$

Odgovor je pod A.

Vježba 429

Koliko je $\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1+\sqrt[16]{a}\right)\cdot\left(1-\sqrt[16]{a}\right)$, za $a=3$?

$A.\ -1 \qquad B.\ -2 \qquad C.\ -3 \qquad D.\ -4$

Rezultat: B.

Zadatak 430 (Matija, strukovna škola)

Izračunaj: $8\cdot a^2\cdot b^4-7\cdot a^2\cdot b^4+2\cdot a^2\cdot b^4$.

Rješenje 430

Ponovimo!

Umnožak n jednakih faktora a napisan u obliku a^n zove se n – ta potencija broja a.

$$a^n=\underbrace{a\cdot a\cdot a\cdot\ldots\cdot a}_{\text{n faktora}}.$$

Izraz a^n zove se potencija. Broj a je baza, a broj n je eksponent potencije. Dvije potencije a^n i b^m jednake su, ako imaju jednake baze i eksponente.

$$a^n=b^m\ \Rightarrow\ \left.\begin{matrix}a=b\\ n=m\end{matrix}\right\}.$$

Ako nekim brojem pomnožimo potenciju taj se broj zove koeficijent potencije.

Izraz	Koeficijent potencije	Potencije
$5\cdot a^3$	5	a^3
$-7\cdot b^8$	-7	b^8
$\frac{3}{2}\cdot c^4$	$\frac{3}{2}$	c^4
$0.9\cdot x$	0.9	x
$13\cdot a^2\cdot b^3$	13	$a^2\cdot b^3$
$x^8\cdot y^2$	1	$x^8\cdot y^2$
$-a\cdot b^2\cdot c^7$	-1	$a\cdot b^2\cdot c^7$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Zbrajati i oduzimati mogu se samo jednake potencije. Zbroje se ili oduzmu koeficijenti i taj rezultat pomnoži sa zadanom potencijom.

$$\alpha\cdot a^n+\beta\cdot a^n=(\alpha+\beta)\cdot a^n$$
$$\alpha\cdot a^n-\beta\cdot a^n=(\alpha-\beta)\cdot a^n.$$

1.inačica

$$8\cdot a^2\cdot b^4-7\cdot a^2\cdot b^4+2\cdot a^2\cdot b^4=8\cdot a^2\cdot b^4-7\cdot a^2\cdot b^4+2\cdot a^2\cdot b^4=$$
$$=(8-7+2)\cdot a^2\cdot b^4=3\cdot a^2\cdot b^4.$$

2.inačica

$$8\cdot a^2\cdot b^4-7\cdot a^2\cdot b^4+2\cdot a^2\cdot b^4=8\cdot a^2\cdot b^4-7\cdot a^2\cdot b^4+2\cdot a^2\cdot b^4=$$
$$=1\cdot a^2\cdot b^4+2\cdot a^2\cdot b^4=1\cdot a^2\cdot b^4+2\cdot a^2\cdot b^4=3\cdot a^2\cdot b^4.$$

Vježba 430

Izračunaj: $9\cdot a^2\cdot b^4-8\cdot a^2\cdot b^4+3\cdot a^2\cdot b^4$.

Rezultat: $4\cdot a^2\cdot b^4$.

Zadatak 431 (Ana, gimnazija)

Ako je $(4\cdot x-2)\cdot(3\cdot x-4)=9,$ koliko je $(3\cdot x-1)\cdot(2\cdot x-3)$?

Rješenje 431

Ponovimo!

$$a^{1}=a \quad , \quad a^{n}\cdot a^{m}=a^{n+m}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Transformiramo izraz

$$(4\cdot x-2)\cdot(3\cdot x-4)=9$$

na ovaj način:

$$(4\cdot x-2)\cdot(3\cdot x-4)=9 \Rightarrow 12\cdot x^{2}-16\cdot x-6\cdot x+8=9 \Rightarrow 12\cdot x^{2}-16\cdot x-6\cdot x=9-8 \Rightarrow$$

$$\Rightarrow 12\cdot x^{2}-22\cdot x=1 \Rightarrow 12\cdot x^{2}-22\cdot x=1 \;/:2 \Rightarrow 6\cdot x^{2}-11\cdot x=0.5.$$

Sada je

$$(3\cdot x-1)\cdot(2\cdot x-3)=6\cdot x^{2}-9\cdot x-2\cdot x+3=6\cdot x^{2}-11\cdot x+3=\left(6\cdot x^{2}-11\cdot x\right)+3=$$

$$=\left[\begin{array}{c}\text{uvjet}\\ 6\cdot x^{2}-11\cdot x=0.5\end{array}\right]=0.5+3=3.5.$$

2.inačica

Transformiramo izraz

$$(4\cdot x-2)\cdot(3\cdot x-4)=9$$

na ovaj način:

$$(4\cdot x-2)\cdot(3\cdot x-4)=9 \Rightarrow 12\cdot x^{2}-16\cdot x-6\cdot x+8=9 \Rightarrow 12\cdot x^{2}-16\cdot x-6\cdot x=9-8 \Rightarrow$$

$$\Rightarrow 12\cdot x^{2}-22\cdot x=1.$$

Sada je

$$(3\cdot x-1)\cdot(2\cdot x-3)=6\cdot x^{2}-9\cdot x-2\cdot x+3=6\cdot x^{2}-11\cdot x+3=\left(6\cdot x^{2}-11\cdot x\right)+3=$$

$$=\frac{1}{2}\cdot\left(12\cdot x^{2}-22\cdot x\right)+3=\left[\begin{array}{c}\text{uvjet}\\ 12\cdot x^{2}-22\cdot x=1\end{array}\right]=\frac{1}{2}\cdot 1+3=\frac{1}{2}+3=0.5+3=3.5.$$

Vježba 431

Ako je $(3\cdot x+2)\cdot(2\cdot x-3)=11,$ koliko je $(x-1)\cdot(6\cdot x+1)$?

Rezultat: 17.

Zadatak 432 (Josip, srednja škola)

Racionaliziraj nazivnik u razlomku: $\frac{2}{\sqrt[4]{5}-\sqrt[4]{3}}$.

Rješenje 432

Ponovimo!

$$(a-b)\cdot(a+b)=a^{2}-b^{2} \quad , \quad \left(\sqrt{a}\right)^{2}=a \quad , \quad \sqrt[n\cdot p]{a^{m\cdot p}}=\sqrt[n]{a^{m}} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{2}{\sqrt[4]{5}-\sqrt[4]{3}}=\left[\begin{matrix}\text{proširimo razlomak s}\\ \sqrt[4]{5}+\sqrt[4]{3}\end{matrix}\right]=\frac{2}{\sqrt[4]{5}-\sqrt[4]{3}}\cdot\frac{\sqrt[4]{5}+\sqrt[4]{3}}{\sqrt[4]{5}+\sqrt[4]{3}}=\frac{2\cdot\left(\sqrt[4]{5}+\sqrt[4]{3}\right)}{\left(\sqrt[4]{5}\right)^2-\left(\sqrt[4]{3}\right)^2}=$$

$$=\frac{2\cdot\left(\sqrt[4]{5}+\sqrt[4]{3}\right)}{\sqrt{5}-\sqrt{3}}=\left[\begin{matrix}\text{proširimo razlomak s}\\ \sqrt{5}+\sqrt{3}\end{matrix}\right]=\frac{2\cdot\left(\sqrt[4]{5}+\sqrt[4]{3}\right)}{\sqrt{5}-\sqrt{3}}\cdot\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}=$$

$$=\frac{2\cdot\left(\sqrt[4]{5}+\sqrt[4]{3}\right)\cdot\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}=\frac{2\cdot\left(\sqrt[4]{5}+\sqrt[4]{3}\right)\cdot\left(\sqrt{5}+\sqrt{3}\right)}{5-3}=\frac{2\cdot\left(\sqrt[4]{5}+\sqrt[4]{3}\right)\cdot\left(\sqrt{5}+\sqrt{3}\right)}{2}=$$

$$=\frac{2\cdot\left(\sqrt[4]{5}+\sqrt[4]{3}\right)\cdot\left(\sqrt{5}+\sqrt{3}\right)}{2}=\left(\sqrt[4]{5}+\sqrt[4]{3}\right)\cdot\left(\sqrt{5}+\sqrt{3}\right).$$

Vježba 432

Racionaliziraj nazivnik u razlomku: $\frac{1}{\sqrt[4]{5}-\sqrt[4]{3}}$.

Rezultat: $\frac{\left(\sqrt[4]{5}+\sqrt[4]{3}\right)\cdot\left(\sqrt{5}+\sqrt{3}\right)}{2}$.

Zadatak 433 (Dani4550, gimnazija)

Rastavi na faktore: $a^5\cdot b^5-a^3\cdot b^3$.

Rješenje 433

Ponovimo!

$$a^n : a^m = a^{n-m}\quad ,\quad a^2-b^2=(a-b)\cdot(a+b)\quad ,\quad a^n\cdot b^n=(a\cdot b)^n.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad ,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$a^5\cdot b^5-a^3\cdot b^3=\left[\begin{matrix}\text{izlučimo potencije s}\\ \text{manjim eksponentom}\\ a^3\cdot b^3\end{matrix}\right]=a^3\cdot b^3\cdot\left(a^2\cdot b^2-1\right)=$$

$$=a^3\cdot b^3\cdot\underbrace{\left(a^2\cdot b^2-1\right)}_{\text{razlika kvadrata}}=a^3\cdot b^3\cdot(a\cdot b-1)\cdot(a\cdot b+1).$$

Vježba 433

Rastavi na faktore: $a^5\cdot b^5\cdot c^5-a^3\cdot b^3\cdot c^3$.

Rezultat: $a^3\cdot b^3\cdot c^3\cdot(a\cdot b\cdot c-1)\cdot(a\cdot b\cdot c+1)$.

Zadatak 434 (Antonio, srednja škola)

Ako je $2^{10} \cdot 5^{12} = n \cdot 10^{8}$, onda je:

$A.\ n = 2.5 \qquad B.\ n = 25 \qquad C.\ n = 250 \qquad D.\ n = 2\,500$

Rješenje 434

Ponovimo!

$$a^{n} \cdot a^{m} = a^{n+m} \quad , \quad a^{n} \cdot b^{n} = (a \cdot b)^{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

$$2^{10} \cdot 5^{12} = n \cdot 10^{8} \Rightarrow 2^{2} \cdot 2^{8} \cdot 5^{4} \cdot 5^{8} = n \cdot 10^{8} \Rightarrow 4 \cdot 2^{8} \cdot 625 \cdot 5^{8} = n \cdot 10^{8} \Rightarrow$$

$$\Rightarrow (4 \cdot 625) \cdot \left(2^{8} \cdot 5^{8}\right) = n \cdot 10^{8} \Rightarrow 2\,500 \cdot (2 \cdot 5)^{8} = n \cdot 10^{8} \Rightarrow 2\,500 \cdot 10^{8} = n \cdot 10^{8} \Rightarrow$$

$$\Rightarrow 2\,500 \cdot 10^{8} = n \cdot 10^{8} \ / \cdot \frac{1}{10^{8}} \Rightarrow n = 2\,500.$$

Odgovor je pod D.

2.inačica

$$2^{10} \cdot 5^{12} = n \cdot 10^{8} \Rightarrow 2^{10} \cdot 5^{12} = n \cdot 10^{8} \ / \cdot \frac{1}{10^{8}} \Rightarrow n = \frac{2^{10} \cdot 5^{12}}{10^{8}} \Rightarrow n = \frac{2^{2} \cdot 2^{8} \cdot 5^{4} \cdot 5^{8}}{10^{8}} \Rightarrow$$

$$\Rightarrow n = \frac{2^{8} \cdot 5^{8} \cdot 2^{2} \cdot 5^{4}}{10^{8}} \Rightarrow n = \frac{(2 \cdot 5)^{8} \cdot 2^{2} \cdot 5^{4}}{10^{8}} \Rightarrow n = \frac{10^{8} \cdot 2^{2} \cdot 5^{4}}{10^{8}} \Rightarrow n = \frac{10^{8} \cdot 2^{2} \cdot 5^{4}}{10^{8}} \Rightarrow$$

$$\Rightarrow n = 2^{2} \cdot 5^{4} \Rightarrow n = 4 \cdot 625 \Rightarrow n = 2\,500.$$

Odgovor je pod D.

Vježba 434

Ako je $2^{10} \cdot 5^{12} = n \cdot 10^{10}$, onda je:

$A.\ n = 2.5 \qquad B.\ n = 25 \qquad C.\ n = 250 \qquad D.\ n = 2\,500$

Rezultat: B.

Zadatak 435 (Marko, srednja škola)

Pojednostavnite izraz $\frac{c^{2} - a \cdot b - a \cdot c + b \cdot c}{b + c}$.

Rješenje 435

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Metodom grupiranja dobije se:

1.inačica

$$\frac{c^2-a\cdot b-a\cdot c+b\cdot c}{b+c}=\frac{\left(c^2-a\cdot c\right)+\left(-a\cdot b+b\cdot c\right)}{b+c}=\frac{c\cdot(c-a)+b\cdot(-a+c)}{b+c}=$$

$$=\frac{c\cdot(c-a)+b\cdot(c-a)}{b+c}=\frac{c\cdot(c-a)+b\cdot(c-a)}{b+c}=\frac{(c-a)\cdot(c+b)}{b+c}=\frac{(c-a)\cdot(b+c)}{b+c}=c-a.$$

2.inačica

$$\frac{c^2-a\cdot b-a\cdot c+b\cdot c}{b+c}=\frac{\left(c^2+b\cdot c\right)+\left(-a\cdot b-a\cdot c\right)}{b+c}=\frac{c\cdot(c+b)-a\cdot(b+c)}{b+c}=$$

$$=\frac{c\cdot(b+c)-a\cdot(b+c)}{b+c}=\frac{c\cdot(b+c)-a\cdot(b+c)}{b+c}=\frac{(b+c)\cdot(c-a)}{b+c}=\frac{(b+c)\cdot(c-a)}{b+c}=c-a.$$

Vježba 435

Pojednostavnite izraz $\frac{b+c}{c^2-a\cdot b-a\cdot c+b\cdot c}$.

Rezultat: $\frac{1}{c-a}$.

Zadatak 436 (Ricky, srednja škola)

Ako su x i y pozitivni realni brojevi i $\frac{x+1}{y+1}>\frac{x}{y}$, onda je:

$A.\ x=y$ $B.\ x>y$ $C.\ x<y$ $D.\ x=-y$

Rješenje 436

Ponovimo!

$$a>b \text{ i } c>0 \Rightarrow a\cdot c>b\cdot c.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Budući da su x i y pozitivni realni brojevi, slijedi:

$$\frac{x+1}{y+1}>\frac{x}{y} \Rightarrow \frac{x+1}{y+1}>\frac{x}{y} \;/\cdot y\cdot(y+1) \Rightarrow y\cdot(x+1)>x\cdot(y+1) \Rightarrow x\cdot y+y>x\cdot y+x \Rightarrow$$

$$\Rightarrow x\cdot y+y>x\cdot y+x \Rightarrow y>x \Rightarrow x<y.$$

Odgovor je pod C.

Vježba 436

Ako su x i y pozitivni realni brojevi i $\frac{x+1}{y+1}<\frac{x}{y}$, onda je:

$A.\ x=y$ $B.\ x>y$ $C.\ x<y$ $D.\ x=-y$

Rezultat: B.

Zadatak 437 (Ricky, srednja škola)

Odredi x + y ako je $\frac{x+a}{y-a}=\frac{x}{y};\ y\neq 0,\ y\neq a,\ a\neq 0.$

$A.\ x+y=1$ $B.\ x+y=-1$ $C.\ x+y=0$ $D.\ x+y=a$

Rješenje 437

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a\cdot b=0 \Leftrightarrow a=0 \text{ ili } b=0 \text{ ili } a=b=0.$$

$$\frac{x+a}{y-a}=\frac{x}{y} \Rightarrow \frac{x+a}{y-a}=\frac{x}{y} \;/\cdot\; y\cdot(y-a) \Rightarrow y\cdot(x+a)=x\cdot(y-a) \Rightarrow x\cdot y+y\cdot a=x\cdot y-x\cdot a \Rightarrow$$

$$\Rightarrow x\cdot y+y\cdot a=x\cdot y-x\cdot a \Rightarrow y\cdot a=-x\cdot a \Rightarrow x\cdot a+y\cdot a=0 \Rightarrow a\cdot(x+y)=0.$$

Budući da je prema uvjetu zadatka a $\neq 0$, vrijedi:

$$a\cdot(x+y)=0 \Rightarrow x+y=0.$$

Odgovor je pod C.

Vježba 437

Odredi x + y ako je $\frac{x+1}{y-1}=\frac{x}{y};\; y\neq 0,\; y\neq 1.$

$A.\; x+y=1 \qquad B.\; x+y=-1 \qquad C.\; x+y=0 \qquad D.\; x+y=2$

Rezultat: C.

Zadatak 438 (Dora, turistička škola)

Pojednostavni: $3\cdot(3\cdot(3\cdot(3\cdot x-9)-9)-9)+27.$

Rješenje 438

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Zbroj brojeva b i c množimo brojem a tako da svaki pribrojnik tog zbroja pomnožimo brojem a i dobivene produkte zbrojimo.

Ako se u zadatku nalaze zagrade unutar zagrada, računamo od unutarnjih prema vanjskim zagradama.

1.inačica

Uporabom zakona distribucije množenja prema zbrajanju najprije izostavimo prvu zagradu, zatim drugu i na kraju treću zagradu

$$3\cdot(3\cdot(3\cdot(3\cdot x-9)-9)-9)+27=3\cdot\underbrace{\left(3\cdot\underbrace{\left(3\cdot\underbrace{(3\cdot x-9)}_{\text{prva zagrada}}-9\right)}_{\text{druga zagrada}}-9\right)}_{\text{treća zagrada}}+27=$$

$$=3\cdot\underbrace{\left(3\cdot\underbrace{(9\cdot x-27-9)}_{\text{druga zagrada}}-9\right)}_{\text{treća zagrada}}+27=3\cdot\underbrace{(27\cdot x-81-27-9)}_{\text{treća zagrada}}+27=81\cdot x-243-81-27+27=$$

$$=81\cdot x-243-81-27+27=81\cdot x-243-81=81\cdot x-324.$$

2.inačica

Uporabom zakona distribucije množenja prema zbrajanju najprije izostavimo prvu zagradu, zatim drugu i na kraju treću zagradu

$$3\cdot(3\cdot(3\cdot(3\cdot x-9)-9)-9)+27=3\cdot\underbrace{\left(3\cdot\underbrace{\left(3\cdot\underbrace{(3\cdot x-9)}_{\text{prva zagrada}}-9\right)}_{\text{druga zagrada}}-9\right)}_{\text{treća zagrada}}+27=$$

$$=3\cdot\underbrace{\left(3\cdot\underbrace{(9\cdot x-27-9)}_{\text{druga zagrada}}-9\right)}_{\text{treća zagrada}}+27=\left[\begin{array}{l}\text{u drugoj zagradi}\\ \text{zbrojimo iste veličine}\end{array}\right]=3\cdot\underbrace{\left(3\cdot\underbrace{(9\cdot x-36)}_{\text{druga zagrada}}-9\right)}_{\text{treća zagrada}}+27=$$

$$=3\cdot\underbrace{(27\cdot x-108-9)}_{\text{treća zagrada}}+27=\left[\begin{array}{l}\text{u trećoj zagradi}\\ \text{zbrojimo iste veličine}\end{array}\right]=3\cdot\underbrace{(27\cdot x-117)}_{\text{treća zagrada}}+27=81\cdot x-351+27=$$

$$=81\cdot x-324.$$

Vježba 438

Pojednostavni: $2\cdot(2\cdot(2\cdot(2\cdot x-4)-4)-4)+8.$

Rezultat: $16\cdot x-48.$

Zadatak 439 (Katarina, srednja škola)

Ako je $a_n = 2^n$ opći član niza (a_n) dokaži da tada za svaka tri uzastopna člana niza vrijedi

$$a_{n+2}+2\cdot a_n=3\cdot a_{n+1}.$$

Rješenje 439

Ponovimo!

$$a^n\cdot a^m=a^{n+m}\quad,\quad a^1=a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Uvrštavanjem pokazuje se točnost veze.

$$a_{n+2}+2\cdot a_n=3\cdot a_{n+1}\;\Rightarrow\;2^{n+2}+2\cdot 2^n=3\cdot 2^{n+1}\;\Rightarrow\;2^n\cdot 2^2+2\cdot 2^n=3\cdot 2^{n+1}\;\Rightarrow$$

$$\Rightarrow\;2^n\cdot\left(2^2+2\right)=3\cdot 2^{n+1}\;\Rightarrow\;2^n\cdot(4+2)=3\cdot 2^{n+1}\;\Rightarrow\;2^n\cdot 6=3\cdot 2^{n+1}\;\Rightarrow$$

$$\Rightarrow\;2^n\cdot 6=3\cdot 2^{n+1}\;/:3\;\Rightarrow\;2^n\cdot 2=2^{n+1}\;\Rightarrow\;2^n\cdot 2^1=3\cdot 2^{n+1}\;\Rightarrow\;2^{n+1}=2^{n+1}.$$

2.inačica

Uvrštavanjem pokazuje se točnost veze.

$$a_{n+2}+2\cdot a_n=3\cdot a_{n+1}\;\Rightarrow\;2^{n+2}+2\cdot 2^n=3\cdot 2^{n+1}\;\Rightarrow\;2^n\cdot 2^2+2\cdot 2^n=3\cdot 2^n\cdot 2^1\;\Rightarrow$$

$$\Rightarrow\;2^n\cdot\left(2^2+2\right)=3\cdot 2^n\cdot 2\;\Rightarrow\;2^n\cdot(4+2)=6\cdot 2^n\;\Rightarrow\;2^n\cdot 6=6\cdot 2^n.$$

Vježba 439

Ako je $a_n = 2^n$ opći član niza (a_n) dokaži da tada za svaka tri uzastopna člana niza vrijedi

$$a_{n+3} + 2 \cdot a_{n+1} = 3 \cdot a_{n+2}.$$

Rezultat: Dokaz analogan.

Zadatak 440 (Dino, srednja škola)

Izračunajte vrijednost izraza: $\frac{a^2}{a-b} + \frac{b^2}{b-a}$ za $a = \frac{1}{2}$, $b = -\frac{1}{3}$.

Rješenje 440

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \quad , \quad \frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

$$(a-b) \cdot (a+b) = a^2 - b^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \neq 0 \; , \; n \neq 1.$$

1.inačica

$$\frac{a^2}{a-b} + \frac{b^2}{b-a} = \begin{bmatrix} a = \frac{1}{2} \\ b = -\frac{1}{3} \end{bmatrix} = \frac{\left(\frac{1}{2}\right)^2}{\frac{1}{2} - \left(-\frac{1}{3}\right)} + \frac{\left(-\frac{1}{3}\right)^2}{-\frac{1}{3} - \frac{1}{2}} = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{3}} + \frac{\frac{1}{9}}{-\frac{1}{3} - \frac{1}{2}} = \frac{\frac{1}{4}}{\frac{3+2}{6}} + \frac{\frac{1}{9}}{\frac{-2-3}{6}} = \frac{\frac{1}{4}}{\frac{5}{6}} + \frac{\frac{1}{9}}{-\frac{5}{6}} =$$

$$= \frac{\frac{1}{4}}{\frac{5}{6}} - \frac{\frac{1}{9}}{\frac{5}{6}} = \frac{\frac{1}{4}}{\frac{5}{6}} - \frac{\frac{1}{9}}{\frac{5}{6}} = \frac{\frac{1}{2}}{\frac{5}{3}} - \frac{\frac{1}{3}}{\frac{5}{2}} = \frac{3}{10} - \frac{2}{15} = \frac{9-4}{30} = \frac{5}{30} = \frac{5}{30} = \frac{1}{6}.$$

2.inačica

Transformacijom zadanog izraza dobije se:

$$\frac{a^2}{a-b} + \frac{b^2}{b-a} = \frac{a^2}{a-b} + \frac{b^2}{-(a-b)} = \frac{a^2}{a-b} - \frac{b^2}{a-b} = \frac{a^2 - b^2}{a-b} = \frac{(a-b) \cdot (a+b)}{a-b} =$$

$$= \frac{(a-b) \cdot (a+b)}{a-b} = a + b = \begin{bmatrix} a = \frac{1}{2} \\ b = -\frac{1}{3} \end{bmatrix} = \frac{1}{2} + \left(-\frac{1}{3}\right) = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}.$$

Vježba 440

Izračunajte vrijednost izraza: $\frac{b^2}{a-b}-\frac{a^2}{b-a}$ za $a=\frac{1}{2}$, $b=-\frac{1}{3}$.

Rezultat: $\frac{13}{30}$.