Zadatak 401 (Ana, gimnazija)

Čemu je, nakon sređivanja, jednak izraz $\left[\left(\frac{a}{b}-\frac{b}{a}\right):(a+b)+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}$, za sve a, b za koje je izraz definiran?

$$A.\ \frac{a-b}{a}\qquad B.\ \frac{a+b}{a}\qquad C.\ \frac{a}{a-b}\qquad D.\ \frac{a}{a+b}$$

Rješenje 401

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad \frac{a}{c}\cdot\frac{b}{d}=\frac{a\cdot b}{c\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left[\left(\frac{a}{b}-\frac{b}{a}\right):(a+b)+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}=\left[\frac{a^2-b^2}{a\cdot b}:(a+b)+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}=$$

$$=\left[\frac{a^2-b^2}{a\cdot b}:\frac{a+b}{1}+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}=\left[\frac{a^2-b^2}{a\cdot b}\cdot\frac{1}{a+b}+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}=$$

$$=\left[\frac{(a-b)\cdot(a+b)}{a\cdot b}\cdot\frac{1}{a+b}+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}=\left[\frac{(a-b)\cdot(a+b)}{a\cdot b}\cdot\frac{1}{a+b}+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}=$$

$$=\left[\frac{a-b}{a\cdot b}\cdot\frac{1}{1}+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}=\left[\frac{a-b}{a\cdot b}+\frac{a}{b}-1\right]\cdot\frac{b}{1+a}=\frac{a-b+a^2-a\cdot b}{a\cdot b}\cdot\frac{b}{1+a}=\frac{a-b+a^2-a\cdot b}{a\cdot b}\cdot\frac{b}{1+a}=$$

$$=\frac{a-b+a^2-a\cdot b}{a}\cdot\frac{1}{1+a}=\left[\begin{matrix}\text{brojnik prvog razlomka rastavimo}\\ \text{na faktore metodom grupiranja}\end{matrix}\right]=\frac{(a-b)+\left(a^2-a\cdot b\right)}{a}\cdot\frac{1}{1+a}=$$

$$=\frac{(a-b)+a\cdot(a-b)}{a}\cdot\frac{1}{1+a}=\frac{(a-b)\cdot(1+a)}{a}\cdot\frac{1}{1+a}=\frac{(a-b)\cdot(1+a)}{a}\cdot\frac{1}{1+a}=\frac{a-b}{a}\cdot\frac{1}{1}=\frac{a-b}{a}.$$

Odgovor je pod A.

Vježba 401

Čemu je, nakon sređivanja, jednak izraz $\left[\left(\frac{a}{b}-\frac{b}{a}\right):(a+b)-1+\frac{a}{b}\right]:\frac{1+a}{b}$, za sve a, b za koje je izraz definiran?

$$A.\ \frac{a-b}{a}\qquad B.\ \frac{a+b}{a}\qquad C.\ \frac{a}{a-b}\qquad D.\ \frac{a}{a+b}$$

Rezultat: A.

Zadatak 402 (Mira, gimnazija)

Psiholozi su razvili model koji pokazuje kako uspješnost izvođenja neke operacije ovisi o broju ponavljanja te operacije. Model je zadan formulom $p(n)=\frac{5+9\cdot(n-1)}{10+9\cdot(n-1)}$, $n>0$, gdje je n broj ponavljanja, a p(n) uspješnost nakon n ponavljanja. Za koliko je veća uspješnost nakon 2 · n ponavljanja od uspješnosti nakon n ponavljanja?

$$A.\ \frac{45\cdot n}{(9\cdot n+1)\cdot(18\cdot n+1)} \qquad B.\ \frac{27\cdot n}{(9\cdot n-1)\cdot(18\cdot n-1)}$$

$$C.\ \frac{109\cdot n}{(9\cdot n+1)\cdot(18\cdot n+1)} \qquad D.\ \frac{135\cdot n}{(9\cdot n-1)\cdot(18\cdot n-1)}$$

Rješenje 402

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a=a^{1} \quad , \quad a^{n}\cdot a^{m}=a^{n+m}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Kako se računa za koliko je broj a veći od broja b?

$$a-b.$$

Uspješnost nakon 2 · n ponavljanja od uspješnosti nakon n ponavljanja iznosi:

$$p(2\cdot n)-p(n)=\frac{5+9\cdot(2\cdot n-1)}{10+9\cdot(2\cdot n-1)}-\frac{5+9\cdot(n-1)}{10+9\cdot(n-1)}=\frac{5+18\cdot n-9}{10+18\cdot n-9}-\frac{5+9\cdot n-9}{10+9\cdot n-9}=$$

$$=\frac{18\cdot n-4}{18\cdot n+1}-\frac{9\cdot n-4}{9\cdot n+1}=\frac{(18\cdot n-4)\cdot(9\cdot n+1)-(9\cdot n-4)\cdot(18\cdot n+1)}{(18\cdot n+1)\cdot(9\cdot n+1)}=$$

$$=\frac{162\cdot n^{2}+18\cdot n-36\cdot n-4-\left(162\cdot n^{2}+9\cdot n-72\cdot n-4\right)}{(18\cdot n+1)\cdot(9\cdot n+1)}=$$

$$=\frac{162\cdot n^{2}+18\cdot n-36\cdot n-4-162\cdot n^{2}-9\cdot n+72\cdot n+4}{(18\cdot n+1)\cdot(9\cdot n+1)}=$$

$$=\frac{162\cdot n^{2}+18\cdot n-36\cdot n-4-162\cdot n^{2}-9\cdot n+72\cdot n+4}{(18\cdot n+1)\cdot(9\cdot n+1)}=\frac{18\cdot n-36\cdot n-9\cdot n+72\cdot n}{(18\cdot n+1)\cdot(9\cdot n+1)}=$$

$$=\frac{45\cdot n}{(18\cdot n+1)\cdot(9\cdot n+1)}=\frac{45\cdot n}{(9\cdot n+1)\cdot(18\cdot n+1)}.$$

Odgovor je pod A.

Vježba 402

Psiholozi su razvili model koji pokazuje kako uspješnost izvođenja neke operacije ovisi o broju ponavljanja te operacije. Model je zadan formulom $p(n)=\frac{5+9\cdot(n-1)}{10+9\cdot(n-1)}$, $n>0$, gdje je n broj ponavljanja, a p(n) uspješnost nakon n ponavljanja. Za koliko je veća uspješnost nakon 3 · n ponavljanja od uspješnosti nakon n ponavljanja?

$$A.\ \frac{100\cdot n}{(9\cdot n+1)\cdot(27\cdot n+1)} \qquad B.\ \frac{81\cdot n}{(9\cdot n-1)\cdot(27\cdot n-1)}$$

$$C.\ \frac{90 \cdot n}{(9 \cdot n+1) \cdot (27 \cdot n+1)} \qquad D.\ \frac{90 \cdot n}{(9 \cdot n-1) \cdot (27 \cdot n-1)}$$

Rezultat: C.

Zadatak 403 (Ivan, strukovna škola)

Napiši izraz $a^2 \cdot b - 2 \cdot a^2 - 3 \cdot a \cdot b + 6 \cdot a + 2 \cdot b - 4$ u obliku faktora.

Rješenje 403

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Izraz rastavimo na faktore metodom grupiranja.

$$a^2 \cdot b - 2 \cdot a^2 - 3 \cdot a \cdot b + 6 \cdot a + 2 \cdot b - 4 = \left(a^2 \cdot b - 2 \cdot a^2\right) + (-3 \cdot a \cdot b + 6 \cdot a) + (2 \cdot b - 4) =$$

$$= a^2 \cdot (b-2) - 3 \cdot a \cdot (b-2) + 2 \cdot (b-2) = a^2 \cdot (b-2) - 3 \cdot a \cdot (b-2) + 2 \cdot (b-2) =$$

$$= (b-2) \cdot \left(a^2 - 3 \cdot a + 2\right) = (b-2) \cdot \left(a^2 - 3 \cdot a + 2\right) = (b-2) \cdot \left(a^2 - 3 \cdot a + 2\right) = (b-2) \cdot \left(a^2 - a - 2 \cdot a + 2\right) =$$

$$= (b-2) \cdot \left(\left(a^2 - a\right) + (-2 \cdot a + 2)\right) = (b-2) \cdot (a \cdot (a-1) - 2 \cdot (a-1)) = (b-2) \cdot (a \cdot (a-1) - 2 \cdot (a-1)) =$$

$$= (b-2) \cdot (a-1) \cdot (a-2) = (b-2) \cdot (a-1) \cdot (a-2) = (a-1) \cdot (a-2) \cdot (b-2).$$

Vježba 403

Napiši izraz $a \cdot b^2 - 2 \cdot b^2 - 3 \cdot a \cdot b + 6 \cdot b + 2 \cdot a - 4$ u obliku faktora.

Rezultat: $(a-2) \cdot (b-1) \cdot (b-2).$

Zadatak 404 (Krešo, gimnazija)

Koliko je $z^{101} + \frac{1}{z^{101}}$, ako je $z + \frac{1}{z} = 1$?

Rješenje 404

Ponovimo!

$$x^3 + y^3 = (x+y) \cdot \left(x^2 - x \cdot y + y^2\right) \quad , \quad \left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^n \cdot a^m = a^{n+m}.$$

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Transformiramo zadanu jednakost.

$$z + \frac{1}{z} = 1 \Rightarrow z + \frac{1}{z} = 1 \ / \cdot z \Rightarrow z^2 + 1 = z \Rightarrow z^2 - z + 1 = 0.$$

Dobiveni izraz nadopunimo na zbroj kubova tako da ga pomnožimo sa z + 1.

$$z^2 - z + 1 = 0 \Rightarrow z^2 - z + 1 = 0 \ / \cdot (z+1) \Rightarrow (z+1) \cdot \left(z^2 - z + 1\right) = 0 \Rightarrow z^3 + 1 = 0 \Rightarrow z^3 = -1.$$

Uočimo da vrijedi:

$$z^{101} = z^{99} \cdot z^2 = \left(z^3\right)^{33} \cdot z^2 = (-1)^{33} \cdot z^2 = -1 \cdot z^2 = -z^2.$$

Sada je:

$$z^{101}+\frac{1}{z^{101}}=-z^2+\frac{1}{-z^2}=-z^2-\frac{1}{z^2}=-z^2-2-\frac{1}{z^2}+2=\left(-z^2-2-\frac{1}{z^2}\right)+2=$$

$$=-\left(z^2+2+\frac{1}{z^2}\right)+2=-\left(z+\frac{1}{z}\right)^2+2=-1^2+2=-1+2=1.$$

Vježba 404

Koliko je $z^{101}+\frac{1}{z^{101}}$, ako je $z+\frac{1}{z}=2$?

Rezultat: $-2.$

Zadatak 405 (Matija, gimnazija)

Što je rezultat sređivanja izraza $\left[\frac{x^3+8}{x^4-16}+\frac{2\cdot x}{x^3-2\cdot x^2+4\cdot x-8}\right]^{-2}$, za sve x za koje je izraz definiran?

$A.\ (x-2)^2$ $\quad B.\ \frac{1}{(x-2)^2}$ $\quad C.\ \frac{\left(x^2+4\right)^2}{(x-2)^2}$ $\quad D.\ \frac{16\cdot(x-2)^2}{\left(x^2+4\right)^2}$

Rješenje 405

Ponovimo!

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right) \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n} \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad n=\frac{n}{1}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left[\frac{x^3+8}{x^4-16}+\frac{2\cdot x}{x^3-2\cdot x^2+4\cdot x-8}\right]^{-2}=\left[\frac{x^3+2^3}{x^4-2^4}+\frac{2\cdot x}{\left(x^3-2\cdot x^2\right)+(4\cdot x-8)}\right]^{-2}=$$

$$=\left[\frac{(x+2)\cdot\left(x^2-2\cdot x+2^2\right)}{\left(x^2-2^2\right)\cdot\left(x^2+2^2\right)}+\frac{2\cdot x}{x^2\cdot(x-2)+4\cdot(x-2)}\right]^{-2}=$$

$$=\left[\frac{(x+2)\cdot\left(x^2-2\cdot x+4\right)}{(x-2)\cdot(x+2)\cdot\left(x^2+4\right)}+\frac{2\cdot x}{x^2\cdot(x-2)+4\cdot(x-2)}\right]^{-2}=$$

$$=\left[\frac{(x+2)\cdot\left(x^2-2\cdot x+4\right)}{(x-2)\cdot(x+2)\cdot\left(x^2+4\right)}+\frac{2\cdot x}{(x-2)\cdot\left(x^2+4\right)}\right]^{-2}=\left[\frac{x^2-2\cdot x+4}{(x-2)\cdot\left(x^2+4\right)}+\frac{2\cdot x}{(x-2)\cdot\left(x^2+4\right)}\right]^{-2}=$$

$$=\left[\frac{x^2-2\cdot x+4+2\cdot x}{(x-2)\cdot\left(x^2+4\right)}\right]^{-2}=\left[\frac{x^2-2\cdot x+4+2\cdot x}{(x-2)\cdot\left(x^2+4\right)}\right]^{-2}=\left[\frac{x^2+4}{(x-2)\cdot\left(x^2+4\right)}\right]^{-2}=$$

$$=\left[\frac{x^2+4}{(x-2)\cdot\left(x^2+4\right)}\right]^{-2}=\left[\frac{1}{x-2}\right]^{-2}=\left[\frac{x-2}{1}\right]^{2}=(x-2)^2.$$

Odgovor je pod A.

Vježba 405

Što je rezultat sređivanja izraza $\left[\frac{x^3+8}{x^4-16}+\frac{2\cdot x}{x^3-2\cdot x^2+4\cdot x-8}\right]^2$, za sve x za koje je izraz definiran?

$A.\ (x-2)^2 \qquad B.\ \frac{1}{(x-2)^2} \qquad C.\ \frac{\left(x^2+4\right)^2}{(x-2)^2} \qquad D.\ \frac{16\cdot(x-2)^2}{\left(x^2+4\right)^2}$

Rezultat: B.

Zadatak 406 (Max, gimnazija)

Skraćivanjem izraza $\frac{1-(a-3)^2}{4\cdot a-8}$ dobivamo:

$A.\ \frac{4+a}{4} \qquad B.\ 1 \qquad C.\ \frac{4-a}{4} \qquad D.\ \frac{a^2-2}{a-2}$

Rješenje 406

Ponovimo!

$$(x-y)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad x^2-y^2=(x-y)\cdot(x+y).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{1-(a-3)^2}{4\cdot a-8}=\frac{1-\left(a^2-6\cdot a+9\right)}{4\cdot(a-2)}=\frac{1-a^2+6\cdot a-9}{4\cdot(a-2)}=\frac{6\cdot a-a^2-8}{4\cdot(a-2)}=\frac{6\cdot a-a^2-8}{4\cdot(a-2)}=$$

$$=\frac{6\cdot a-12-a^2+4}{4\cdot(a-2)}=\left[\begin{matrix}\text{metoda}\\ \text{grupiranja}\end{matrix}\right]=\frac{(6\cdot a-12)-\left(a^2-4\right)}{4\cdot(a-2)}=\frac{6\cdot(a-2)-(a-2)\cdot(a+2)}{4\cdot(a-2)}=$$

$$=\frac{6\cdot(a-2)-(a-2)\cdot(a+2)}{4\cdot(a-2)}=\frac{(a-2)\cdot(6-(a+2))}{4\cdot(a-2)}=\frac{(a-2)\cdot(6-a-2)}{4\cdot(a-2)}=\frac{(a-2)\cdot(4-a)}{4\cdot(a-2)}=$$

$$=\frac{(a-2)\cdot(4-a)}{4\cdot(a-2)}=\frac{4-a}{4}.$$

Odgovor je pod C.

2.inačica

$$\frac{1-(a-3)^2}{4\cdot a-8}=\frac{(1-(a-3))\cdot(1+(a-3))}{4\cdot(a-2)}=\frac{(1-a+3)\cdot(1+a-3)}{4\cdot(a-2)}=\frac{(4-a)\cdot(a-2)}{4\cdot(a-2)}=$$

$$=\frac{(4-a)\cdot(a-2)}{4\cdot(a-2)}=\frac{4-a}{4}.$$

Odgovor je pod C.

Vježba 406

Skraćivanjem izraza $\frac{(a-3)^2-1}{8-4\cdot a}$ dobivamo:

$A.\ \frac{4+a}{4}$ $\quad B.\ 1$ $\quad C.\ \frac{4-a}{4}$ $\quad D.\ \frac{a^2-2}{a-2}$

Rezultat: C.

Zadatak 407 (Dado, gimnazija)

Pokazati da vrijednost izraza $\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{1}{b+\frac{1}{c}}}:\frac{1}{a+\frac{1}{b}}$, $(a\neq 0,\ b\neq 0,\ c\neq 0)$ ne ovisi od a, b i c.

Rješenje 407

Ponovimo!

$$n=\frac{n}{1}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}\quad,\quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{1}{b+\frac{1}{c}}}:\frac{1}{a+\frac{1}{b}}=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{1}{\frac{b}{1}+\frac{1}{c}}}:\frac{1}{\frac{a}{1}+\frac{1}{b}}=$$

$$=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{1}{\frac{b\cdot c+1}{c}}}:\frac{1}{\frac{a\cdot b+1}{b}}=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{\frac{1}{1}}{\frac{b\cdot c+1}{c}}}:\frac{\frac{1}{1}}{\frac{a\cdot b+1}{b}}=$$

$$=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{c}{b\cdot c+1}}:\frac{b}{a\cdot b+1}=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{\frac{a}{1}+\frac{c}{b\cdot c+1}}:\frac{b}{a\cdot b+1}=$$

$$=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{\frac{a\cdot(b\cdot c+1)+c}{b\cdot c+1}}:\frac{b}{a\cdot b+1}=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{\frac{a\cdot b\cdot c+a+c}{b\cdot c+1}}:\frac{b}{a\cdot b+1}=$$

$$=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{\frac{1}{1}}{\frac{a\cdot b\cdot c+a+c}{b\cdot c+1}}:\frac{b}{a\cdot b+1}=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{b\cdot c+1}{a\cdot b\cdot c+a+c}:\frac{b}{a\cdot b+1}=$$

$$=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{b\cdot c+1}{a\cdot b\cdot c+a+c}\cdot\frac{a\cdot b+1}{b}=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{(b\cdot c+1)\cdot(a\cdot b+1)}{b\cdot(a\cdot b\cdot c+a+c)}=$$

$$=\frac{1-(b\cdot c+1)\cdot(a\cdot b+1)}{b\cdot(a\cdot b\cdot c+a+c)}=\frac{1-\left(a\cdot b^2\cdot c+b\cdot c+a\cdot b+1\right)}{b\cdot(a\cdot b\cdot c+a+c)}=\frac{1-a\cdot b^2\cdot c-b\cdot c-a\cdot b-1}{b\cdot(a\cdot b\cdot c+a+c)}=$$

$$=\frac{1-a\cdot b^2\cdot c-b\cdot c-a\cdot b-1}{b\cdot(a\cdot b\cdot c+a+c)}=\frac{-a\cdot b^2\cdot c-b\cdot c-a\cdot b}{b\cdot(a\cdot b\cdot c+a+c)}=\frac{-b\cdot(a\cdot b\cdot c+c+a)}{b\cdot(a\cdot b\cdot c+a+c)}=\frac{-b\cdot(a\cdot b\cdot c+a+c)}{b\cdot(a\cdot b\cdot c+a+c)}=$$

$$=\frac{-b\cdot(a\cdot b\cdot c+a+c)}{b\cdot(a\cdot b\cdot c+a+c)}=-1.$$

Vježba 407

Pokazati da vrijednost izraza $\frac{1}{a+\frac{1}{b+\frac{1}{c}}}:\frac{1}{a+\frac{1}{b}}-\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}$, $(a\neq 0,\ b\neq 0,\ c\neq 0)$ ne ovisi od a, b i c.

Rezultat: 1.

Zadatak 408 (Iva, gimnazija)

Ako je

$$\begin{cases} a+b=x+y & (1) \\ a^2+b^2=x^2+y^2 & (2) \end{cases}$$

dokazati da je

$$a^3+b^3=x^3+y^3.$$

Rješenje 408

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a+b)^3=a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Ako prvu jednakost kvadriramo dobije se:

$$a+b=x+y \Rightarrow a+b=x+y\ /^2 \Rightarrow (a+b)^2=(x+y)^2 \Rightarrow a^2+2\cdot a\cdot b+b^2=x^2+2\cdot x\cdot y+y^2.$$

Zbog (2) slijedi:

$$\left.\begin{array}{l} a^2+2\cdot a\cdot b+b^2=x^2+2\cdot x\cdot y+y^2 \\ a^2+b^2=x^2+y^2 \end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{oduzmemo} \\ \text{jednakosti}\end{array}\right] \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b+b^2-\left(a^2+b^2\right)=x^2+2\cdot x\cdot y+y^2-\left(x^2+y^2\right) \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b+b^2-a^2-b^2=x^2+2\cdot x\cdot y+y^2-x^2-y^2 \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b+b^2-a^2-b^2=x^2+2\cdot x\cdot y+y^2-x^2-y^2 \Rightarrow$$

$$\Rightarrow 2\cdot a\cdot b=2\cdot x\cdot y \Rightarrow 2\cdot a\cdot b=2\cdot x\cdot y \;/:2 \Rightarrow a\cdot b=x\cdot y.$$

Ako sada prvu jednakost kubiramo dobije se:

$$a+b=x+y \Rightarrow a+b=x+y \;/^3 \Rightarrow (a+b)^3=(x+y)^3 \Rightarrow$$

$$\Rightarrow a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3=x^3+3\cdot x^2\cdot y+3\cdot x\cdot y^2+y^3 \Rightarrow$$

$$\Rightarrow a^3+b^3+3\cdot a\cdot b\cdot(a+b)=x^3+y^3+3\cdot x\cdot y\cdot(x+y) \Rightarrow \begin{bmatrix} a\cdot b=x\cdot y \\ a+b=x+y \end{bmatrix} \Rightarrow$$

$$\Rightarrow a^3+b^3+3\cdot x\cdot y\cdot(x+y)=x^3+y^3+3\cdot x\cdot y\cdot(x+y) \Rightarrow$$

$$\Rightarrow a^3+b^3+3\cdot x\cdot y\cdot(x+y)=x^3+y^3+3\cdot x\cdot y\cdot(x+y) \Rightarrow a^3+b^3=x^3+y^3.$$

Vježba 408

Ako je

$$\begin{cases} a-x=y-b & (1) \\ a^2-x^2=y^2-b^2 & (2) \end{cases}$$

dokazati da je

$$a^3-x^3=y^3-x^3.$$

Rezultat: Dokaz analogan.

Zadatak 409 (Iva, gimnazija)

Rastavi na faktore: $(a+b+c)\cdot(a\cdot b+b\cdot c+c\cdot a)-a\cdot b\cdot c.$

Rješenje 409

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$(a+b+c)\cdot(a\cdot b+b\cdot c+c\cdot a)-a\cdot b\cdot c=$$

$$=a^2\cdot b+a\cdot b\cdot c+a^2\cdot c+a\cdot b^2+b^2\cdot c+a\cdot b\cdot c+a\cdot b\cdot c+b\cdot c^2+a\cdot c^2-a\cdot b\cdot c=$$

$$=a^2\cdot b+a\cdot b\cdot c+a^2\cdot c+a\cdot b^2+b^2\cdot c+a\cdot b\cdot c+a\cdot b\cdot c+b\cdot c^2+a\cdot c^2-a\cdot b\cdot c=$$

$$=a^2\cdot b+a\cdot b\cdot c+a^2\cdot c+a\cdot b^2+b^2\cdot c+a\cdot b\cdot c+b\cdot c^2+a\cdot c^2=\begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix}=$$

$$=\left(a^2\cdot b+a\cdot b\cdot c\right)+\left(a^2\cdot c+a\cdot c^2\right)+\left(a\cdot b^2+b^2\cdot c\right)+\left(a\cdot b\cdot c+b\cdot c^2\right)=$$

$$=a\cdot b\cdot(a+c)+a\cdot c\cdot(a+c)+b^2\cdot(a+c)+b\cdot c\cdot(a+c)=$$

$$= a \cdot b \cdot (a+c) + a \cdot c \cdot (a+c) + b^2 \cdot (a+c) + b \cdot c \cdot (a+c) = (a+c) \cdot \left(a \cdot b + a \cdot c + b^2 + b \cdot c\right) =$$

$$= \begin{bmatrix} \text{metoda} \\ \text{grupiranja} \end{bmatrix} = (a+c) \cdot \left((a \cdot b + a \cdot c) + \left(b^2 + b \cdot c\right)\right) = (a+c) \cdot (a \cdot (b+c) + b \cdot (b+c)) =$$

$$= (a+c) \cdot (a \cdot (b+c) + b \cdot (b+c)) = (a+c) \cdot (b+c) \cdot (a+b) = (a+b) \cdot (b+c) \cdot (c+a).$$

Vježba 409

Rastavi na faktore: $a \cdot b \cdot c - (a+b+c) \cdot (a \cdot b + b \cdot c + c \cdot a)$.

Rezultat: $-(a+b) \cdot (b+c) \cdot (c+a)$.

Zadatak 410 (P2P, gimnazija)

Rastavi na faktore: $(x+y+z)^3 - x^3 - y^3 - z^3$.

Rješenje 410

Ponovimo!

$$(a+b)^3 = a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3 \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^3 - b^3 = (a-b) \cdot \left(a^2 + a \cdot b + b^2\right).$$

$$a^3 + b^3 = (a+b) \cdot \left(a^2 - a \cdot b + b^2\right) \quad , \quad (a+b+c)^2 = a^2 + b^2 + c^2 + 2 \cdot a \cdot b + 2 \cdot a \cdot c + 2 \cdot b \cdot c.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

$$(x+y+z)^3 - x^3 - y^3 - z^3 = ((x+y)+z)^3 - x^3 - y^3 - z^3 =$$

$$= (x+y)^3 + 3 \cdot (x+y)^2 \cdot z + 3 \cdot (x+y) \cdot z^2 + z^3 - x^3 - y^3 - z^3 =$$

$$= x^3 + 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + y^3 + 3 \cdot \left(x^2 + 2 \cdot x \cdot y + y^2\right) \cdot z + 3 \cdot (x+y) \cdot z^2 + z^3 - x^3 - y^3 - z^3 =$$

$$= x^3 + 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + y^3 + 3 \cdot x^2 \cdot z + 6 \cdot x \cdot y \cdot z + 3 \cdot y^2 \cdot z + 3 \cdot x \cdot z^2 + 3 \cdot y \cdot z^2 + z^3 - x^3 - y^3 - z^3 =$$

$$= x^3 + 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + y^3 + 3 \cdot x^2 \cdot z + 6 \cdot x \cdot y \cdot z + 3 \cdot y^2 \cdot z + 3 \cdot x \cdot z^2 + 3 \cdot y \cdot z^2 + z^3 - x^3 - y^3 - z^3 =$$

$$= 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + 3 \cdot x^2 \cdot z + 6 \cdot x \cdot y \cdot z + 3 \cdot y^2 \cdot z + 3 \cdot x \cdot z^2 + 3 \cdot y \cdot z^2 =$$

$$= 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + 3 \cdot x^2 \cdot z + 6 \cdot x \cdot y \cdot z + 3 \cdot y^2 \cdot z + 3 \cdot x \cdot z^2 + 3 \cdot y \cdot z^2 =$$

$$= 3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2 + 3 \cdot x^2 \cdot z + 3 \cdot x \cdot y \cdot z + 3 \cdot x \cdot y \cdot z + 3 \cdot y^2 \cdot z + 3 \cdot x \cdot z^2 + 3 \cdot y \cdot z^2 =$$

$$= \begin{bmatrix} \text{metoda} \\ \text{grupiranja} \end{bmatrix} = \left(3 \cdot x^2 \cdot y + 3 \cdot x \cdot y^2\right) + \left(3 \cdot x^2 \cdot z + 3 \cdot x \cdot y \cdot z\right) + \left(3 \cdot x \cdot y \cdot z + 3 \cdot y^2 \cdot z\right) + \left(3 \cdot x \cdot z^2 + 3 \cdot y \cdot z^2\right) =$$

$$= 3 \cdot x \cdot y \cdot (x+y) + 3 \cdot x \cdot z \cdot (x+y) + 3 \cdot y \cdot z \cdot (x+y) + 3 \cdot z^2 \cdot (x+y) =$$

$$= 3 \cdot x \cdot y \cdot (x+y) + 3 \cdot x \cdot z \cdot (x+y) + 3 \cdot y \cdot z \cdot (x+y) + 3 \cdot z^2 \cdot (x+y) =$$

$$= 3 \cdot (x+y) \cdot \left(x \cdot y + x \cdot z + y \cdot z + z^2\right) = \begin{bmatrix} \text{metoda} \\ \text{grupiranja} \end{bmatrix} = 3 \cdot (x+y) \cdot \left((x \cdot y + x \cdot z) + \left(y \cdot z + z^2\right)\right) =$$

$$= 3\cdot(x+y)\cdot\left(\left(x\cdot y + x\cdot z\right)+\left(y\cdot z + z^2\right)\right) = 3\cdot(x+y)\cdot\left(x\cdot(y+z)+z\cdot(y+z)\right) = 3\cdot(x+y)\cdot\left(x\cdot(y+z)+z\cdot(y+z)\right) =$$

$$= 3\cdot(x+y)\cdot(y+z)\cdot(x+z) = 3\cdot(x+y)\cdot(y+z)\cdot(z+x).$$

2.inačica

$$(x+y+z)^3 - x^3 - y^3 - z^3 = \left((x+y+z)^3 - x^3\right) - \left(y^3 + z^3\right) =$$

$$= \left((x+y+z)-x\right)\cdot\left((x+y+z)^2 + x\cdot(x+y+z) + x^2\right) - (y+z)\cdot\left(y^2 - y\cdot z + z^2\right) =$$

$$= (x+y+z-x)\cdot\left((x+y+z)^2 + x\cdot(x+y+z) + x^2\right) - (y+z)\cdot\left(y^2 - y\cdot z + z^2\right) =$$

$$= (x+y+z-x)\cdot\left((x+y+z)^2 + x\cdot(x+y+z) + x^2\right) - (y+z)\cdot\left(y^2 - y\cdot z + z^2\right) =$$

$$= (y+z)\cdot\left((x+y+z)^2 + x\cdot(x+y+z) + x^2\right) - (y+z)\cdot\left(y^2 - y\cdot z + z^2\right) =$$

$$= (y+z)\cdot\left((x+y+z)^2 + x\cdot(x+y+z) + x^2\right) - (y+z)\cdot\left(y^2 - y\cdot z + z^2\right) =$$

$$= (y+z)\cdot\left((x+y+z)^2 + x\cdot(x+y+z) + x^2 - \left(y^2 - y\cdot z + z^2\right)\right) =$$

$$= (y+z)\cdot\left(x^2 + y^2 + z^2 + 2\cdot x\cdot y + 2\cdot x\cdot z + 2\cdot y\cdot z + x^2 + x\cdot y + x\cdot z + x^2 - y^2 + y\cdot z - z^2\right) =$$

$$= (y+z)\cdot\left(x^2 + y^2 + z^2 + 2\cdot x\cdot y + 2\cdot x\cdot z + 2\cdot y\cdot z + x^2 + x\cdot y + x\cdot z + x^2 - y^2 + y\cdot z - z^2\right) =$$

$$= (y+z)\cdot\left(x^2 + 2\cdot x\cdot y + 2\cdot x\cdot z + 2\cdot y\cdot z + x^2 + x\cdot y + x\cdot z + x^2 + y\cdot z\right) =$$

$$= (y+z)\cdot\left(3\cdot x^2 + 3\cdot x\cdot y + 3\cdot x\cdot z + 3\cdot y\cdot z\right) = \begin{bmatrix}\text{metoda}\\ \text{grupiranja}\end{bmatrix} = (y+z)\cdot\left(\left(3\cdot x^2 + 3\cdot x\cdot y\right) + \left(3\cdot x\cdot z + 3\cdot y\cdot z\right)\right) =$$

$$= (y+z)\cdot\left(3\cdot x\cdot(x+y) + 3\cdot z\cdot(x+y)\right) = (y+z)\cdot\left(3\cdot x\cdot(x+y) + 3\cdot z\cdot(x+y)\right) =$$

$$= (y+z)\cdot 3\cdot(x+y)\cdot(x+z) = 3\cdot(x+y)\cdot(y+z)\cdot(z+x).$$

Vježba 410

Rastavi na faktore: $x^3 + y^3 + z^3 - (x+y+z)^3$.

Rezultat: $-3\cdot(x+y)\cdot(y+z)\cdot(z+x)$.

Zadatak 411 (Branka, gimnazija)

Ako je $a\cdot x^3 = b\cdot y^3 = c\cdot z^3$ i $\frac{1}{x}+\frac{1}{y}+\frac{1}{z} = 1$, onda je $\sqrt[3]{a\cdot x^2 + b\cdot y^2 + c\cdot z^2} = \sqrt[3]{a} + \sqrt[3]{b} + \sqrt[3]{c}$.

Dokažite.

Rješenje 411

Ponovimo!

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}$$

Neka su a, b pozitivni realni brojevi, a n prirodan broj, tada je

$$\sqrt[n]{a^n \cdot b} = a\cdot\sqrt[n]{b}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \;,\; n\neq 0 \;,\; n\neq 1.$$

Neka je

$$A=\sqrt[3]{a\cdot x^2+b\cdot y^2+c\cdot z^2}.$$

Dalje slijedi:

- $$A=\sqrt[3]{a\cdot x^2+b\cdot y^2+c\cdot z^2} \Rightarrow A=\sqrt[3]{\frac{a\cdot x^2\cdot x}{x}+\frac{b\cdot y^2\cdot y}{y}+\frac{c\cdot z^2\cdot z}{z}} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{\frac{a\cdot x^3}{x}+\frac{b\cdot y^3}{y}+\frac{c\cdot z^3}{z}} \Rightarrow \left[\begin{matrix}\text{uvjet}\\ a\cdot x^3=b\cdot y^3=c\cdot z^3\end{matrix}\right] \Rightarrow A=\sqrt[3]{\frac{a\cdot x^3}{x}+\frac{a\cdot x^3}{y}+\frac{a\cdot x^3}{z}} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{a\cdot x^3\cdot\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)} \Rightarrow \left[\begin{matrix}\text{uvjet}\\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1\end{matrix}\right] \Rightarrow A=\sqrt[3]{a\cdot x^3\cdot 1} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{a\cdot x^3} \Rightarrow A=x\cdot\sqrt[3]{a} \Rightarrow A=x\cdot\sqrt[3]{a} \;/\cdot\frac{1}{x} \Rightarrow \frac{A}{x}=\sqrt[3]{a}.$$

- $$A=\sqrt[3]{a\cdot x^2+b\cdot y^2+c\cdot z^2} \Rightarrow A=\sqrt[3]{\frac{a\cdot x^2\cdot x}{x}+\frac{b\cdot y^2\cdot y}{y}+\frac{c\cdot z^2\cdot z}{z}} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{\frac{a\cdot x^3}{x}+\frac{b\cdot y^3}{y}+\frac{c\cdot z^3}{z}} \Rightarrow \left[\begin{matrix}\text{uvjet}\\ a\cdot x^3=b\cdot y^3=c\cdot z^3\end{matrix}\right] \Rightarrow A=\sqrt[3]{\frac{b\cdot y^3}{x}+\frac{b\cdot y^3}{y}+\frac{b\cdot y^3}{z}} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{b\cdot y^3\cdot\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)} \Rightarrow \left[\begin{matrix}\text{uvjet}\\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1\end{matrix}\right] \Rightarrow A=\sqrt[3]{b\cdot y^3\cdot 1} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{b\cdot y^3} \Rightarrow A=y\cdot\sqrt[3]{b} \Rightarrow A=y\cdot\sqrt[3]{b} \;/\cdot\frac{1}{y} \Rightarrow \frac{A}{y}=\sqrt[3]{b}.$$

- $$A=\sqrt[3]{a\cdot x^2+b\cdot y^2+c\cdot z^2} \Rightarrow A=\sqrt[3]{\frac{a\cdot x^2\cdot x}{x}+\frac{b\cdot y^2\cdot y}{y}+\frac{c\cdot z^2\cdot z}{z}} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{\frac{a\cdot x^3}{x}+\frac{b\cdot y^3}{y}+\frac{c\cdot z^3}{z}} \Rightarrow \left[\begin{matrix}\text{uvjet}\\ a\cdot x^3=b\cdot y^3=c\cdot z^3\end{matrix}\right] \Rightarrow A=\sqrt[3]{\frac{c\cdot z^3}{x}+\frac{c\cdot z^3}{y}+\frac{c\cdot z^3}{z}} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{c\cdot z^3\cdot\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)} \Rightarrow \left[\begin{matrix}\text{uvjet}\\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1\end{matrix}\right] \Rightarrow A=\sqrt[3]{c\cdot z^3\cdot 1} \Rightarrow$$

$$\Rightarrow A=\sqrt[3]{c\cdot z^3} \Rightarrow A=z\cdot\sqrt[3]{c} \Rightarrow A=z\cdot\sqrt[3]{c} \;/\cdot\frac{1}{z} \Rightarrow \frac{A}{z}=\sqrt[3]{c}.$$

Iz sustava triju jednakosti dobije se istinitost tvrdnje.

$$\left.\begin{array}{l} \frac{A}{x}=\sqrt[3]{a} \\ \frac{A}{y}=\sqrt[3]{b} \\ \frac{A}{z}=\sqrt[3]{c} \end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{zbrojimo} \\ \text{jednakosti}\end{array}\right] \Rightarrow \frac{A}{x}+\frac{A}{y}+\frac{A}{z}=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c} \Rightarrow A\cdot\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{c}\text{uvjet} \\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1\end{array}\right] \Rightarrow A\cdot 1=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c} \Rightarrow A=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}.$$

Vježba 411

Ako je $a\cdot x^3=b\cdot y^3=c\cdot z^3$ i $\frac{x\cdot y\cdot z}{x\cdot y+y\cdot z+z\cdot x}=1$, onda je $\sqrt[3]{a\cdot x^2+b\cdot y^2+c\cdot z^2}=\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}.$

Dokažite.

Rezultat: Dokaz analogan.

Zadatak 412 (Alen, gimnazija)

Tri broja x, y, z zadovoljavaju relaciju $y^2=x\cdot z.$ Dokažite da je

$(x+y+z)\cdot(x-y+z)=x^2+y^2+z^2.$

Rješenje 412

Ponovimo!

$a^1=a$, $a^n\cdot a^m=a^{n+m}$, $a^2-b^2=(a-b)\cdot(a+b)$, $(a+b)^2=a^2+2\cdot a\cdot b+b^2.$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

1.inačica

$$(x+y+z)\cdot(x-y+z)=x^2-x\cdot y+x\cdot z+y\cdot x-y^2+y\cdot z+z\cdot x-z\cdot y+z^2=$$

$$=x^2-x\cdot y+x\cdot z+y\cdot x-y^2+y\cdot z+z\cdot x-z\cdot y+z^2=x^2+x\cdot z-y^2+z\cdot x+z^2=\left[\begin{array}{c}\text{uvjet} \\ y^2=x\cdot z\end{array}\right]=$$

$$=x^2+y^2-y^2+y^2+z^2=x^2+y^2-y^2+y^2+z^2=x^2+y^2+z^2.$$

2.inačica

$$(x+y+z)\cdot(x-y+z)=((x+z)+y)\cdot((x+z)-y)=\left[\begin{array}{c}\text{razlika} \\ \text{kvadrata}\end{array}\right]=(x+z)^2-y^2=$$

$$=x^2+2\cdot x\cdot z+z^2-y^2=\left[\begin{array}{c}\text{uvjet} \\ y^2=x\cdot z\end{array}\right]=x^2+2\cdot y^2+z^2-y^2=x^2+y^2+z^2.$$

Vježba 412

Tri broja x, y, z zadovoljavaju relaciju $z^2=x\cdot y.$ Dokažite da je

$(x+y+z)\cdot(x+y-z)=x^2+y^2+z^2.$

Rezultat: Dokaz analogan.

Zadatak 413 (Kruno, srednja škola)

Dokažite da je $a \cdot b + b \cdot c + c \cdot a \leq a^2 + b^2 + c^2$.

Rješenje 413

Ponovimo!

$$a \in R \Rightarrow a^2 \geq 0 \quad , \quad a \geq b \ , \ c < 0 \Rightarrow a \cdot c \leq b \cdot c \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

$$\left.\begin{array}{l} a \geq b \\ c \geq d \end{array}\right\} \Rightarrow a + c \geq b + d.$$

Pođimo od očiglednih nejednakosti:

$$\left.\begin{array}{l} (a-b)^2 \geq 0 \\ (a-c)^2 \geq 0 \\ (b-c)^2 \geq 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a^2 - 2 \cdot a \cdot b + b^2 \geq 0 \\ a^2 - 2 \cdot a \cdot c + c^2 \geq 0 \\ b^2 - 2 \cdot b \cdot c + c^2 \geq 0 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{nejednakosti} \end{array}\right] \Rightarrow$$

$$\Rightarrow a^2 - 2 \cdot a \cdot b + b^2 + a^2 - 2 \cdot a \cdot c + c^2 + b^2 - 2 \cdot b \cdot c + c^2 \geq 0 \Rightarrow$$

$$\Rightarrow -2 \cdot a \cdot b - 2 \cdot a \cdot c - 2 \cdot b \cdot c \geq -a^2 - b^2 - a^2 - c^2 - b^2 - c^2 \Rightarrow$$

$$\Rightarrow -2 \cdot a \cdot b - 2 \cdot a \cdot c - 2 \cdot b \cdot c \geq -2 \cdot a^2 - 2 \cdot b^2 - 2 \cdot c^2 \Rightarrow$$

$$\Rightarrow -2 \cdot a \cdot b - 2 \cdot a \cdot c - 2 \cdot b \cdot c \geq -2 \cdot a^2 - 2 \cdot b^2 - 2 \cdot c^2 \ \ /:(-2) \Rightarrow a \cdot b + a \cdot c + b \cdot c \leq a^2 + b^2 + c^2 \Rightarrow$$

$$\Rightarrow a \cdot b + b \cdot c + c \cdot a \leq a^2 + b^2 + c^2.$$

Vježba 413

Dokažite da je $a \cdot (a-b) + b \cdot (b-c) + c \cdot (c-a) \geq 0$.

Rezultat: Dokaz analogan.

Zadatak 414 (Kruno, srednja škola)

Izračunaj: $\sqrt[3]{\sqrt{9+\sqrt{17}} \cdot \sqrt{9-\sqrt{17}}}$.

A. 1 *B.* 2 *C.* 3 *D.* 4

Rješenje 414

Ponovimo!

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b} \quad , \quad a^2 - b^2 = (a-b) \cdot (a+b) \quad , \quad \left(\sqrt{a}\right)^2 = a.$$

$$\sqrt[n]{a^n} = a \ , \ a \geq 0 \quad , \quad \sqrt[n]{\sqrt[m]{a}} = \sqrt[n \cdot m]{a}.$$

$$\sqrt[3]{\sqrt{9+\sqrt{17}} \cdot \sqrt{9-\sqrt{17}}} = \sqrt[3]{\sqrt{\left(9+\sqrt{17}\right) \cdot \left(9-\sqrt{17}\right)}} = \sqrt[6]{\left(9+\sqrt{17}\right) \cdot \left(9-\sqrt{17}\right)} =$$

$$= \sqrt[6]{9^2 - \left(\sqrt{17}\right)^2} = \sqrt[6]{81-17} = \sqrt[6]{64} = \sqrt[6]{2^6} = 2.$$

Vježba 414

Izračunaj: $\sqrt{\sqrt[3]{11-\sqrt{57}} \cdot \sqrt[3]{11+\sqrt{57}}}$.

A. 1 *B.* 2 *C.* 3 *D.* 4

Rezultat: B.

Zadatak 415 (Ljilja, srednja škola)

Pojednostavnite: $\frac{a^{2}-1}{b^{2}+a\cdot b}\cdot\left(\frac{b}{b-1}-1\right)\cdot\frac{a-a\cdot b^{3}-b^{4}+b}{1-a^{2}}.$

Rješenje 415

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^{3}-b^{3}=(a-b)\cdot\left(a^{2}+a\cdot b+b^{2}\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{a^{2}-1}{b^{2}+a\cdot b}\cdot\left(\frac{b}{b-1}-1\right)\cdot\frac{a-a\cdot b^{3}-b^{4}+b}{1-a^{2}}=\frac{-\left(1-a^{2}\right)}{b\cdot(b+a)}\cdot\left(\frac{b}{b-1}-\frac{1}{1}\right)\cdot\frac{a\cdot\left(1-b^{3}\right)+b\cdot\left(1-b^{3}\right)}{1-a^{2}}=$$

$$=\frac{-\left(1-a^{2}\right)}{b\cdot(a+b)}\cdot\left(\frac{b}{b-1}-\frac{1}{1}\right)\cdot\frac{a\cdot\left(1-b^{3}\right)+b\cdot\left(1-b^{3}\right)}{1-a^{2}}=\frac{-1}{b\cdot(a+b)}\cdot\left(\frac{b}{b-1}-\frac{1}{1}\right)\cdot\frac{a\cdot\left(1-b^{3}\right)+b\cdot\left(1-b^{3}\right)}{1}=$$

$$=\frac{-1}{b\cdot(a+b)}\cdot\frac{b-(b-1)}{b-1}\cdot\frac{a\cdot\left(1-b^{3}\right)+b\cdot\left(1-b^{3}\right)}{1}=\frac{-1}{b\cdot(a+b)}\cdot\frac{b-b+1}{b-1}\cdot\frac{\left(1-b^{3}\right)\cdot(a+b)}{1}=$$

$$=\frac{-1}{b\cdot(a+b)}\cdot\frac{b-b+1}{b-1}\cdot\frac{\left(1-b^{3}\right)\cdot(a+b)}{1}=\frac{-1}{b}\cdot\frac{1}{b-1}\cdot\frac{1-b^{3}}{1}=\frac{b^{3}-1}{b\cdot(b-1)}=\frac{(b-1)\cdot\left(b^{2}+b+1\right)}{b\cdot(b-1)}=$$

$$=\frac{(b-1)\cdot\left(b^{2}+b+1\right)}{b\cdot(b-1)}=\frac{b^{2}+b+1}{b}.$$

Vježba 415

Pojednostavnite: $\frac{1-a^{2}}{b^{2}+a\cdot b}\cdot\left(1-\frac{b}{b-1}\right)\cdot\frac{b^{4}+a\cdot b^{3}-a-b}{a^{2}-1}.$

Rezultat: $\frac{b^{2}+b+1}{b}.$

Zadatak 416 (Ljilja, srednja škola)

Pojednostavnite: $\frac{(2\cdot x-3\cdot y)^{2}-x^{2}}{4\cdot x^{2}-(x+3\cdot y)^{2}}+\frac{4\cdot x^{2}-(3\cdot y-x)^{2}}{9\cdot\left(x^{2}-y^{2}\right)}+\frac{9\cdot y^{2}-x^{2}}{(2\cdot x+3\cdot y)^{2}-x^{2}}.$

Rješenje 416

Ponovimo!

$$a^{n}\cdot b^{n}=(a\cdot b)^{n}\quad,\quad a^{2}-b^{2}=(a-b)\cdot(a+b)\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ \ ,\ \ n\neq 0\ \ ,\ \ n\neq 1.$$

$$\frac{(2\cdot x-3\cdot y)^2-x^2}{4\cdot x^2-(x+3\cdot y)^2}+\frac{4\cdot x^2-(3\cdot y-x)^2}{9\cdot\left(x^2-y^2\right)}+\frac{9\cdot y^2-x^2}{(2\cdot x+3\cdot y)^2-x^2}=$$

$$=\frac{(2\cdot x-3\cdot y)^2-x^2}{(2\cdot x)^2-(x+3\cdot y)^2}+\frac{(2\cdot x)^2-(3\cdot y-x)^2}{9\cdot\left(x^2-y^2\right)}+\frac{(3\cdot y)^2-x^2}{(2\cdot x+3\cdot y)^2-x^2}=$$

$$=\frac{(2\cdot x-3\cdot y-x)\cdot(2\cdot x-3\cdot y+x)}{(2\cdot x-(x+3\cdot y))\cdot(2\cdot x+(x+3\cdot y))}+\frac{(2\cdot x-(3\cdot y-x))\cdot(2\cdot x+(3\cdot y-x))}{9\cdot(x-y)\cdot(x+y)}+$$

$$+\frac{(3\cdot y-x)\cdot(3\cdot y+x)}{(2\cdot x+3\cdot y-x)\cdot(2\cdot x+3\cdot y+x)}=$$

$$=\frac{(2\cdot x-3\cdot y-x)\cdot(2\cdot x-3\cdot y+x)}{(2\cdot x-x-3\cdot y)\cdot(2\cdot x+x+3\cdot y)}+\frac{(2\cdot x-3\cdot y+x)\cdot(2\cdot x+3\cdot y-x)}{9\cdot(x-y)\cdot(x+y)}+$$

$$+\frac{(3\cdot y-x)\cdot(3\cdot y+x)}{(2\cdot x+3\cdot y-x)\cdot(2\cdot x+3\cdot y+x)}=$$

$$=\frac{(x-3\cdot y)\cdot(3\cdot x-3\cdot y)}{(x-3\cdot y)\cdot(3\cdot x+3\cdot y)}+\frac{(3\cdot x-3\cdot y)\cdot(x+3\cdot y)}{9\cdot(x-y)\cdot(x+y)}+\frac{(3\cdot y-x)\cdot(3\cdot y+x)}{(x+3\cdot y)\cdot(3\cdot x+3\cdot y)}=$$

$$=\frac{(x-3\cdot y)\cdot 3\cdot(x-y)}{(x-3\cdot y)\cdot 3\cdot(x+y)}+\frac{3\cdot(x-y)\cdot(x+3\cdot y)}{9\cdot(x-y)\cdot(x+y)}+\frac{(3\cdot y-x)\cdot(3\cdot y+x)}{(x+3\cdot y)\cdot 3\cdot(x+y)}=$$

$$=\frac{(x-3\cdot y)\cdot 3\cdot(x-y)}{(x-3\cdot y)\cdot 3\cdot(x+y)}+\frac{3\cdot(x-y)\cdot(x+3\cdot y)}{9\cdot(x-y)\cdot(x+y)}+\frac{(3\cdot y-x)\cdot(3\cdot y+x)}{(x+3\cdot y)\cdot 3\cdot(x+y)}=$$

$$=\frac{x-y}{x+y}+\frac{x+3\cdot y}{3\cdot(x+y)}+\frac{3\cdot y-x}{3\cdot(x+y)}=\frac{3\cdot(x-y)+x+3\cdot y+3\cdot y-x}{3\cdot(x+y)}=\frac{3\cdot x-3\cdot y+x+3\cdot y+3\cdot y-x}{3\cdot(x+y)}=$$

$$=\frac{3\cdot x-3\cdot y+x+3\cdot y+3\cdot y-x}{3\cdot(x+y)}=\frac{3\cdot x+3\cdot y}{3\cdot(x+y)}=\frac{3\cdot(x+y)}{3\cdot(x+y)}=\frac{3\cdot(x+y)}{3\cdot(x+y)}=1.$$

Vježba 416

Pojednostavnite: $\frac{(2\cdot x-3\cdot y)^2-x^2}{4\cdot x^2-(x+3\cdot y)^2}-\frac{(3\cdot y-x)^2-4\cdot x^2}{9\cdot\left(x^2-y^2\right)}-\frac{x^2-9\cdot y^2}{(2\cdot x+3\cdot y)^2-x^2}.$

Rezultat: 1.

Zadatak 417 (Josip, strukovna škola)

Izrazite n iz formule $b=a+(n-1)\cdot d.$

Rješenje 417

Ponovimo!

$$x=y\ \Rightarrow\ y=x\quad,\quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$b=a+(n-1)\cdot d \;\Rightarrow\; b=a+n\cdot d-d \;\Rightarrow\; -n\cdot d=a-d-b \;\Rightarrow\; -n\cdot d=a-d-b \;/\cdot(-1) \;\Rightarrow$$

$$\Rightarrow\; n\cdot d=-a+d+b \;\Rightarrow\; n\cdot d=b-a+d \;\Rightarrow\; n\cdot d=b-a+d \;/:d \;\Rightarrow\; n=\frac{b-a+d}{d} \;\Rightarrow$$

$$\Rightarrow\; n=\frac{b-a}{d}+\frac{d}{d} \;\Rightarrow\; n=\frac{b-a}{d}+\frac{d}{d} \;\Rightarrow\; n=\frac{b-a}{d}+1.$$

2.inačica

$$b=a+(n-1)\cdot d \;\Rightarrow\; a+(n-1)\cdot d=b \;\Rightarrow\; (n-1)\cdot d=b-a \;\Rightarrow\; (n-1)\cdot d=b-a \;/:d \;\Rightarrow$$

$$\Rightarrow\; n-1=\frac{b-a}{d} \;\Rightarrow\; n=\frac{b-a}{d}+1.$$

Vježba 417

Izrazite d iz formule $b=a+(n-1)\cdot d$.

Rezultat: $\frac{b-a}{n-1}.$

Zadatak 418 (Valentina, ekonomska škola)

Pojednostavnite: $\sqrt[4]{\sqrt[6]{a^5}}\cdot\sqrt[12]{\sqrt{a^3}}\cdot\sqrt[3]{\sqrt[8]{a^9}}\cdot\sqrt{\sqrt[12]{a}}.$

Rješenje 418

Ponovimo!

$$\sqrt[n]{\sqrt[m]{a}}=\sqrt[n\cdot m]{a} \quad , \quad \sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b} \quad , \quad a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$\sqrt[n\cdot r]{a^{m\cdot r}}=\sqrt[n]{a^m}.$$

$$\sqrt[4]{\sqrt[6]{a^5}}\cdot\sqrt[12]{\sqrt{a^3}}\cdot\sqrt[3]{\sqrt[8]{a^9}}\cdot\sqrt{\sqrt[12]{a}}=\sqrt[24]{a^5}\cdot\sqrt[24]{a^3}\cdot\sqrt[24]{a^9}\cdot\sqrt[24]{a}=\sqrt[24]{a^5\cdot a^3\cdot a^9\cdot a}=$$

$$=\sqrt[24]{a^5\cdot a^3\cdot a^9\cdot a^1}=\sqrt[24]{a^{5+3+9+1}}=\sqrt[24]{a^{18}}=\sqrt[24]{a^{18}}=\sqrt[4]{a^3}.$$

Vježba 418

Pojednostavnite: $\sqrt[3]{\sqrt[8]{a}}\cdot\sqrt{\sqrt[12]{a^9}}\cdot\sqrt[12]{\sqrt{a^5}}\cdot\sqrt[6]{\sqrt[4]{a^3}}.$

Rezultat: $\sqrt[4]{a^3}.$

Zadatak 419 (Jure, gimnazija)

Dokazati da je $\left(a\cdot z^2+b\cdot z\right)\cdot\left(b\cdot z^2+a\cdot z\right)=a^2-a\cdot b+b^2$, pri čemu su a i b realni brojevi, a z rješenje jednadžbe $1+z+z^2=0.$

Rješenje 419

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

1.inačica

$$\left(a\cdot z^2+b\cdot z\right)\cdot\left(b\cdot z^2+a\cdot z\right)=z\cdot(a\cdot z+b)\cdot z\cdot(b\cdot z+a)=z^2\cdot(a\cdot z+b)\cdot(b\cdot z+a)=$$

$$=z^2\cdot\left(a\cdot b\cdot z^2+a^2\cdot z+b^2\cdot z+a\cdot b\right)=\begin{bmatrix}\text{uvjet}\\ 1+z+z^2=0\\ z^2=-1-z\end{bmatrix}=$$

$$=(-1-z)\cdot\left(a\cdot b\cdot(-1-z)+a^2\cdot z+b^2\cdot z+a\cdot b\right)=(-1-z)\cdot\left(-a\cdot b-a\cdot b\cdot z+a^2\cdot z+b^2\cdot z+a\cdot b\right)=$$

$$=(-1-z)\cdot\left(-a\cdot b-a\cdot b\cdot z+a^2\cdot z+b^2\cdot z+a\cdot b\right)=(-1-z)\cdot\left(-a\cdot b\cdot z+a^2\cdot z+b^2\cdot z\right)=$$

$$=(-1-z)\cdot z\cdot\left(-a\cdot b+a^2+b^2\right)=\left(-z-z^2\right)\cdot\left(a^2-a\cdot b+b^2\right)=\begin{bmatrix}\text{uvjet}\\ 1+z+z^2=0\\ 1=-z-z^2\end{bmatrix}=$$

$$=1\cdot\left(a^2-a\cdot b+b^2\right)=a^2-a\cdot b+b^2.$$

2.inačica

$$\left(a\cdot z^2+b\cdot z\right)\cdot\left(b\cdot z^2+a\cdot z\right)=\begin{bmatrix}\text{uvjet}\\ 1+z+z^2=0\\ z^2=-1-z\end{bmatrix}=(a\cdot(-1-z)+b\cdot z)\cdot(b\cdot(-1-z)+a\cdot z)=$$

$$=(-a-a\cdot z+b\cdot z)\cdot(-b-b\cdot z+a\cdot z)=$$

$$=a\cdot b+a\cdot b\cdot z-a^2\cdot z+a\cdot b\cdot z+a\cdot b\cdot z^2-a^2\cdot z^2-b^2\cdot z-b^2\cdot z^2+a\cdot b\cdot z^2=$$

$$=a\cdot b+2\cdot a\cdot b\cdot z+2\cdot a\cdot b\cdot z^2-a^2\cdot z-b^2\cdot z-a^2\cdot z^2-b^2\cdot z^2=\begin{bmatrix}\text{uvjet}\\ 1+z+z^2=0\\ z^2=-1-z\end{bmatrix}=$$

$$=a\cdot b+2\cdot a\cdot b\cdot z+2\cdot a\cdot b\cdot(-1-z)-a^2\cdot z-b^2\cdot z-a^2\cdot(-1-z)-b^2\cdot(-1-z)=$$

$$=a\cdot b+2\cdot a\cdot b\cdot z-2\cdot a\cdot b-2\cdot a\cdot b\cdot z-a^2\cdot z-b^2\cdot z+a^2+a^2\cdot z+b^2+b^2\cdot z=$$

$$=a\cdot b+2\cdot a\cdot b\cdot z-2\cdot a\cdot b-2\cdot a\cdot b\cdot z-a^2\cdot z-b^2\cdot z+a^2+a^2\cdot z+b^2+b^2\cdot z=$$

$$=a\cdot b-2\cdot a\cdot b+a^2+b^2=a^2-a\cdot b+b^2.$$

Vježba 419

Dokazati da je $\left(a\cdot z^2+b\cdot z\right)\cdot\left(b\cdot z^2+a\cdot z\right)=(a-b)^2+a\cdot b,$ pri čemu su a i b realni brojevi, a z rješenje jednadžbe $1+z+z^2=0.$

Rezultat: Dokaz analogan.

Zadatak 420 (Anita, strukovna škola)

Zapiši u obliku potencije s bazom 6: $2^{n+1} \cdot 3^{n} + 2^{n} \cdot 3^{n+1} + 2^{n} \cdot 3^{n}$.

Rješenje 420

Ponovimo!

$$a^{1} = a \quad , \quad a^{n} \cdot a^{m} = a^{n+m} \quad , \quad (a \cdot b)^{n} = a^{n} \cdot b^{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$2^{n+1} \cdot 3^{n} + 2^{n} \cdot 3^{n+1} + 2^{n} \cdot 3^{n} = 2^{n} \cdot 2^{1} \cdot 3^{n} + 2^{n} \cdot 3^{n} \cdot 3^{1} + 2^{n} \cdot 3^{n} =$$

$$= 2 \cdot 2^{n} \cdot 3^{n} + 3 \cdot 2^{n} \cdot 3^{n} + 2^{n} \cdot 3^{n} = 2 \cdot (2 \cdot 3)^{n} + 3 \cdot (2 \cdot 3)^{n} + (2 \cdot 3)^{n} = 2 \cdot 6^{n} + 3 \cdot 6^{n} + 6^{n} =$$

$$= 6^{n} \cdot (2 + 3 + 1) = 6^{n} \cdot 6 = 6^{n} \cdot 6^{1} = 6^{n+1}.$$

Vježba 420

Zapiši u obliku potencije s bazom 6: $2^{n+5} \cdot 3^{n} + 2^{n} \cdot 3^{n+1} + 2^{n+2} \cdot 3^{n}$.

Rezultat: 6^{n+2}.