**Zadatak 381 (Marija, strukovna škola)**

$$\text{Pojednostavnite}: \left(\frac{a^3+b^3}{a-b}\right)^n \cdot \left(\frac{a^2-b^2}{a^2-a\cdot b+b^2}\right)^n.$$

$A.\ (a+b)^{2\cdot n}$ $\quad B.\ (a-b)^{2\cdot n}$ $\quad C.\ (a-b)^{-n}$ $\quad D.\ (a+b)^{-n}$

**Rješenje 381**

Ponovimo!

$$a^n \cdot b^n = (a\cdot b)^n \quad , \quad a^3+b^3 = (a+b)\cdot\left(a^2-a\cdot b+b^2\right) \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \left(a^n\right)^m = a^{n\cdot m}.$$

$$\left(\frac{a^3+b^3}{a-b}\right)^n \cdot \left(\frac{a^2-b^2}{a^2-a\cdot b+b^2}\right)^n = \left(\frac{a^3+b^3}{a-b}\cdot\frac{a^2-b^2}{a^2-a\cdot b+b^2}\right)^n =$$

$$= \left(\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{a-b}\cdot\frac{(a-b)\cdot(a+b)}{a^2-a\cdot b+b^2}\right)^n = \left(\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{a-b}\cdot\frac{(a-b)\cdot(a+b)}{a^2-a\cdot b+b^2}\right)^n =$$

$$= \left((a+b)\cdot(a+b)\right)^n = \left((a+b)^2\right)^n = (a+b)^{2\cdot n}.$$

Odgovor je pod A.

**Vježba 381**

$$\text{Pojednostavnite}: \left(\frac{a^3+b^3}{a+b}\right)^n \cdot \left(\frac{a-b}{a^2-a\cdot b+b^2}\right)^n.$$

$A.\ (a+b)^{2\cdot n}$ $\quad B.\ (a-b)^{2\cdot n}$ $\quad C.\ (a-b)^{n}$ $\quad D.\ (a+b)^{-n}$

**Rezultat:** C.



**Zadatak 382 (Marija, strukovna škola)**

$$\text{Pojednostavnite}: \frac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}-2}.$$

$A.\ \frac{x-y}{x+y}$ $\quad B.\ \frac{x+y}{x-y}$ $\quad C.\ \frac{1}{x+y}$ $\quad D.\ \frac{1}{x-y}$

**Rješenje 382**

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$\frac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}-2}=\frac{\frac{x}{y}-\frac{y}{x}}{\frac{x}{y}+\frac{y}{x}-\frac{2}{1}}=\frac{\frac{x^2-y^2}{x\cdot y}}{\frac{x^2+y^2-2\cdot x\cdot y}{x\cdot y}}=\frac{\frac{x^2-y^2}{x\cdot y}}{\frac{x^2-2\cdot x\cdot y+y^2}{x\cdot y}}=\frac{\frac{x^2-y^2}{x\cdot y}}{\frac{x^2-2\cdot x\cdot y+y^2}{x\cdot y}}=$$

$$=\frac{\frac{x^2-y^2}{1}}{\frac{x^2-2\cdot x\cdot y+y^2}{1}}=\frac{x^2-y^2}{x^2-2\cdot x\cdot y+y^2}=\frac{(x-y)\cdot(x+y)}{(x-y)^2}=\frac{(x-y)\cdot(x+y)}{(x-y)^2}=\frac{x+y}{x-y}.$$

Odgovor je pod B.

**Vježba 382**

$$\text{Pojednostavnite}:\frac{\frac{x}{y}+\frac{y}{x}-2}{\frac{x}{y}-\frac{y}{x}}.$$

$$A.\ \frac{x-y}{x+y}\qquad B.\ \frac{x+y}{x-y}\qquad C.\ \frac{1}{x+y}\qquad D.\ \frac{1}{x-y}$$

**Rezultat:** A.

**Zadatak 383 (XY, strukovna škola)**

$$\text{Pojednostavnite}:\frac{1-\frac{a^3}{b^3}}{1-\frac{a^2}{b^2}}.$$

**Rješenje 383**

Ponovimo!

$$n=\frac{n}{1}\quad,\quad a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right)\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad a^n:a^m=a^{n-m}\quad,\quad\frac{a^n}{a^m}=a^{n-m}.$$

$$\frac{1-\frac{a^3}{b^3}}{1-\frac{a^2}{b^2}}=\frac{\frac{1}{1}-\frac{a^3}{b^3}}{\frac{1}{1}-\frac{a^2}{b^2}}=\frac{\frac{b^3-a^3}{b^3}}{\frac{b^2-a^2}{b^2}}=\frac{\frac{b^3-a^3}{b^3}}{\frac{b^2-a^2}{b^2}}=\frac{\frac{b^3-a^3}{b}}{\frac{b^2-a^2}{1}}=\frac{b^3-a^3}{b\cdot\left(b^2-a^2\right)}=$$

$$=\frac{(b-a)\cdot\left(b^2+b\cdot a+a^2\right)}{b\cdot(b-a)\cdot(b+a)}=\frac{(b-a)\cdot\left(b^2+b\cdot a+a^2\right)}{b\cdot(b-a)\cdot(b+a)}=\frac{b^2+b\cdot a+a^2}{b\cdot(b+a)}=\frac{a^2+a\cdot b+b^2}{b\cdot(a+b)}.$$

**Vježba 383**

$$\text{Pojednostavnite}: \frac{1-\frac{a^2}{b^2}}{1-\frac{a^3}{b^3}}.$$

**Rezultat:** $\frac{b\cdot(a+b)}{a^2+a\cdot b+b^2}.$

**Zadatak 384 (July, gimnazija)**

$$\text{Pojednostavnite}: \frac{1}{x^2+x\cdot y}+\frac{1}{x\cdot y+y^2}-\frac{1}{x\cdot y}.$$

$$A.\ x \qquad B.\ y \qquad C.\ 0 \qquad D.\ 1$$

**Rješenje 384**

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{1}{x^2+x\cdot y}+\frac{1}{x\cdot y+y^2}-\frac{1}{x\cdot y}=\begin{bmatrix}\text{u prvom nazivniku izlučimo } x\\ \text{u drugom nazivniku izlučimo } y\end{bmatrix}=\frac{1}{x\cdot(x+y)}+\frac{1}{y\cdot(x+y)}-\frac{1}{x\cdot y}=$$

$$=\begin{bmatrix}\text{zajednički nazivnik je}\\ x\cdot y\cdot(x+y)\end{bmatrix}=\frac{y+x-(x+y)}{x\cdot y\cdot(x+y)}=\frac{y+x-x-y}{x\cdot y\cdot(x+y)}=\frac{y+x-x-y}{x\cdot y\cdot(x+y)}=\frac{0}{x\cdot y\cdot(x+y)}=0.$$

Odgovor je pod C.

**Vježba 384**

$$\text{Pojednostavnite}: \frac{1}{x\cdot y}-\frac{1}{x^2+x\cdot y}-\frac{1}{x\cdot y+y^2}.$$

$$A.\ x \qquad B.\ y \qquad C.\ 0 \qquad D.\ 1$$

**Rezultat:** C.

**Zadatak 385 (July, gimnazija)**

$$\text{Pojednostavnite}: \frac{x}{x^2+y^2}-\frac{y\cdot(x-y)^2}{x^4-y^4}.$$

$$A.\ \frac{1}{x-y} \qquad B.\ \frac{1}{x+y} \qquad C.\ \frac{x}{y} \qquad D.\ \frac{y}{x}$$

**Rješenje 385**

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$(a-b)^2 = a^2 - 2\cdot a\cdot b + b^2.$$

1.inačica

$$\frac{x}{x^2+y^2} - \frac{y\cdot(x-y)^2}{x^4-y^4} = \frac{x}{x^2+y^2} - \frac{y\cdot(x-y)^2}{\left(x^2\right)^2 - \left(y^2\right)^2} = \frac{x}{x^2+y^2} - \frac{y\cdot(x-y)^2}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} =$$

$$= \frac{x\cdot\left(x^2-y^2\right) - y\cdot(x-y)^2}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{x\cdot(x-y)\cdot(x+y) - y\cdot(x-y)^2}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{(x-y)\cdot\left[x\cdot(x+y) - y\cdot(x-y)\right]}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} =$$

$$= \frac{(x-y)\cdot\left[x^2 + x\cdot y - x\cdot y + y^2\right]}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{(x-y)\cdot\left[x^2 + x\cdot y - x\cdot y + y^2\right]}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{(x-y)\cdot\left(x^2+y^2\right)}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} =$$

$$= \frac{(x-y)\cdot\left(x^2+y^2\right)}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{x-y}{x^2-y^2} = \frac{x-y}{(x-y)\cdot(x+y)} = \frac{x-y}{(x-y)\cdot(x+y)} = \frac{1}{x+y}.$$

Odgovor je pod B.

2.inačica

$$\frac{x}{x^2+y^2} - \frac{y\cdot(x-y)^2}{x^4-y^4} = \frac{x}{x^2+y^2} - \frac{y\cdot(x-y)^2}{\left(x^2\right)^2 - \left(y^2\right)^2} = \frac{x}{x^2+y^2} - \frac{y\cdot(x-y)^2}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} =$$

$$= \frac{x\cdot\left(x^2-y^2\right) - y\cdot\left(x^2 - 2\cdot x\cdot y + y^2\right)}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{x^3 - x\cdot y^2 - x^2\cdot y + 2\cdot x\cdot y^2 - y^3}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} =$$

$$= \frac{x^3 + x\cdot y^2 - x^2\cdot y - y^3}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{x\cdot\left(x^2+y^2\right) - y\cdot\left(x^2+y^2\right)}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{\left(x^2+y^2\right)\cdot(x-y)}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} =$$

$$= \frac{\left(x^2+y^2\right)\cdot(x-y)}{\left(x^2-y^2\right)\cdot\left(x^2+y^2\right)} = \frac{x-y}{x^2-y^2} = \frac{x-y}{(x-y)\cdot(x+y)} = \frac{x-y}{(x-y)\cdot(x+y)} = \frac{1}{x+y}.$$



Odgovor je pod B.

**Vježba 385**

Pojednostavnite: $\dfrac{y\cdot(x-y)^2}{x^4-y^4} - \dfrac{x}{x^2+y^2}.$

$A.\ \dfrac{-1}{y-x} \qquad B.\ \dfrac{1}{y-x} \qquad C.\ -\dfrac{x}{y} \qquad D.\ -\dfrac{y}{x}$

**Rezultat:** B.

**Zadatak 386 (Uporna Katarina ☺, gimnazija)**

$$\text{Pojednostavnite}: \left(\frac{a}{b}-\frac{b}{a}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right).$$

**Rješenje 386**

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad (a-b)\cdot(a+b)=a^2-b^2\quad,\quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}.$$

$$\left(\frac{a}{b}-\frac{b}{a}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)=\left(\frac{a^2-b^2}{a\cdot b}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)=$$

$$=\left(\frac{a^2-b^2}{a\cdot b}\right)^2+\underbrace{\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)}_{\text{razlika kvadrata}}=\left(\frac{a^2-b^2}{a\cdot b}\right)^2+a^2-\left(\frac{a^2-b^2}{a\cdot b}\right)^2=$$

$$=\left(\frac{a^2-b^2}{a\cdot b}\right)^2+a^2-\left(\frac{a^2-b^2}{a\cdot b}\right)^2=a^2.$$

**Vježba 386**

$$\text{Pojednostavnite}: \left(\frac{a}{b}-\frac{b}{a}\right)^2+\left(a+\frac{b^2-a^2}{a\cdot b}\right)\cdot\left(a-\frac{b^2-a^2}{a\cdot b}\right).$$

**Rezultat:** $a^2.$



**Zadatak 387 (Uporna Katarina ☺, gimnazija)**

$$\text{Pojednostavnite}: \left(\frac{a}{b}-\frac{b}{a}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right).$$

**Rješenje 387**

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad (a-b)\cdot(a+b)=a^2-b^2\quad,\quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad,\quad (a-b)^2=(b-a)^2.$$

$$\left(\frac{b}{a}-\frac{a}{b}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)=\left(\frac{b^2-a^2}{a\cdot b}\right)^2+\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)=$$

$$=\left(\frac{b^2-a^2}{a\cdot b}\right)^2+\underbrace{\left(a-\frac{a^2-b^2}{a\cdot b}\right)\cdot\left(a+\frac{a^2-b^2}{a\cdot b}\right)}_{\text{razlika kvadrata}}=\left(\frac{b^2-a^2}{a\cdot b}\right)^2+a^2-\left(\frac{a^2-b^2}{a\cdot b}\right)^2=$$

$$=\frac{\left(b^2-a^2\right)^2}{(a\cdot b)^2}+a^2-\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}=\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}+a^2-\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}=$$

$$=\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}+a^2-\frac{\left(a^2-b^2\right)^2}{(a\cdot b)^2}=a^2.$$

**Vježba 387**

Pojednostavnite: $\left(\frac{b}{a}-\frac{a}{b}\right)^2+\left(a+\frac{b^2-a^2}{a\cdot b}\right)\cdot\left(a-\frac{b^2-a^2}{a\cdot b}\right).$

**Rezultat:** $a^2.$

**Zadatak 388 (Dvije maturantice, gimnazija)**

Ako je $\left(a+b\cdot\sqrt{2}\right)^2=9+4\cdot\sqrt{2}$, onda je:

$A.\ a\cdot b=6 \qquad B.\ a\cdot b=4 \qquad C.\ a\cdot b=3 \qquad D.\ a\cdot b=2$

**Rješenje 388**

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad (a\cdot b)^2=a^2\cdot b^2.$$

$$a^2=b^2 \Rightarrow a=b \ , \ a, b>0.$$

Poučak o jednakosti polinoma:
Dva polinoma jednaka su ako i samo ako su istog stupnja i ako su im koeficijenti uz iste potencije jednaki.

1.inačica

$$\left(a+b\cdot\sqrt{2}\right)^2=9+4\cdot\sqrt{2} \Rightarrow \left[\begin{array}{l}\text{nadopunjavanje na}\\ \text{potpuni kvadrat}\end{array}\right] \Rightarrow \left(a+b\cdot\sqrt{2}\right)^2=1^2+2\cdot 2\cdot\sqrt{2}+\left(2\cdot\sqrt{2}\right)^2 \Rightarrow$$

$$\Rightarrow \left(a+b\cdot\sqrt{2}\right)^2=\left(1+2\cdot\sqrt{2}\right)^2 \Rightarrow a+b\cdot\sqrt{2}=1+2\cdot\sqrt{2} \Rightarrow \left.\begin{array}{l}a=1\\ b=2\end{array}\right\} \Rightarrow a\cdot b=1\cdot 2 \Rightarrow a\cdot b=2.$$



Odgovor je pod D.

2.inačica

$$\left(a+b\cdot\sqrt{2}\right)^2=9+4\cdot\sqrt{2} \Rightarrow a^2+2\cdot a\cdot b\cdot\sqrt{2}+\left(b\cdot\sqrt{2}\right)^2=9+4\cdot\sqrt{2} \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b\cdot\sqrt{2}+2\cdot b^2=9+4\cdot\sqrt{2} \Rightarrow \left(a^2+2\cdot b^2\right)+2\cdot a\cdot b\cdot\sqrt{2}=9+4\cdot\sqrt{2}.$$

Iz definicije jednakosti polinoma slijedi:

$$2\cdot a\cdot b\cdot\sqrt{2}=4\cdot\sqrt{2} \Rightarrow 2\cdot a\cdot b\cdot\sqrt{2}=4\cdot\sqrt{2} \ /\cdot\frac{1}{2\cdot\sqrt{2}} \Rightarrow a\cdot b=2.$$

Odgovor je pod D.

**Vježba 388**

Ako je $\left(a+b\cdot\sqrt{2}\right)^2=9+4\cdot\sqrt{2}$, onda je:

$A.\ a+b=6 \qquad B.\ a+b=4 \qquad C.\ a+b=3 \qquad D.\ a+b=2$

**Rezultat:** C.

**Zadatak 389 (Dvije maturantice, gimnazija)**

Koji od sljedećih dvočlanih izraza nije faktor polinoma $a^6 - b^6$?

$A.\ a^2 - b^2 \qquad B.\ a^2 + b^2 \qquad C.\ a - b \qquad D.\ a^2 - a \cdot b + b^2$

**Rješenje 389**

Ponovimo!

$$x^2 - y^2 = (x-y) \cdot (x+y) \quad , \quad \left(a^n\right)^m = a^{n \cdot m} \quad , \quad x^3 - y^3 = (x-y) \cdot \left(x^2 + x \cdot y + y^2\right).$$

$$x^3 + y^3 = (x+y) \cdot \left(x^2 - x \cdot y + y^2\right).$$

$$a^6 - b^6 = \left(a^3\right)^2 - \left(b^3\right)^2 = \left(a^3 - b^3\right) \cdot \left(a^3 + b^3\right) =$$

$$= (a-b) \cdot \left(a^2 + a \cdot b + b^2\right) \cdot (a+b) \cdot \left(a^2 - a \cdot b + b^2\right).$$

Diskusija

$A.\ a^2 - b^2 = (a-b) \cdot (a+b)$ je faktor zadanog polinoma

$$a^6 - b^6 = (a-b) \cdot \left(a^2 + a \cdot b + b^2\right) \cdot (a+b) \cdot \left(a^2 - a \cdot b + b^2\right)$$

$B.\ a^2 + b^2$ nije faktor zadanog polinoma

$$a^6 - b^6 = (a-b) \cdot \left(a^2 + a \cdot b + b^2\right) \cdot (a+b) \cdot \left(a^2 - a \cdot b + b^2\right)$$

$C.\ a - b$ je faktor zadanog polinoma

$$a^6 - b^6 = (a-b) \cdot \left(a^2 + a \cdot b + b^2\right) \cdot (a+b) \cdot \left(a^2 - a \cdot b + b^2\right)$$

$D.\ a^2 - a \cdot b + b^2$ je faktor zadanog polinoma

$$a^6 - b^6 = (a-b) \cdot \left(a^2 + a \cdot b + b^2\right) \cdot (a+b) \cdot \left(a^2 - a \cdot b + b^2\right)$$



Odgovor je pod B.

**Vježba 389**

Koji od sljedećih dvočlanih izraza nije faktor polinoma $a^6 - b^6$?

$A.\ a^3 - b^3 \qquad B.\ a^3 + b^3 \qquad C.\ a + 3 \qquad D.\ a^2 + a \cdot b + b^2$

**Rezultat:** C.

**Zadatak 390 (Tonka, srednja škola)**

Odredite s ako je $t = \frac{s+r}{s-r} \ (s \neq r,\ t \neq 1)$.

**Rješenje 390**

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$t = \frac{s+r}{s-r} \Rightarrow t = \frac{s+r}{s-r} \ / \cdot (s-r) \Rightarrow t \cdot (s-r) = s+r \Rightarrow t \cdot s - t \cdot r = s + r \Rightarrow$$

$$\Rightarrow t \cdot s - s = r + t \cdot r \Rightarrow s \cdot (t-1) = r \cdot (1+t) \Rightarrow s \cdot (t-1) = r \cdot (1+t) \ / \cdot \frac{1}{t-1} \Rightarrow$$

$$\Rightarrow s=\frac{r\cdot(1+t)}{t-1} \Rightarrow s=\frac{r\cdot(t+1)}{t-1}.$$

**Vježba 390**

Odredite r ako je $t=\frac{s+r}{s-r}$ $(s\neq r,\ t\neq -1)$.

**Rezultat:** $r=\frac{s\cdot(t-1)}{t+1}.$

**Zadatak 391 (Ante, srednja škola)**

Ako je $x+y=m$ i $x^2+y^2=n,$ onda je $x^3+y^3=\frac{1}{2}\cdot m\cdot\left(3\cdot n-m^2\right).$ Dokažite!

**Rješenje 391**

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

- $x^2+2\cdot x\cdot y+y^2=(x+y)^2 \Rightarrow 2\cdot x\cdot y=(x+y)^2-x^2-y^2 \Rightarrow$

$$\Rightarrow 2\cdot x\cdot y=(x+y)^2-\left(x^2+y^2\right) \Rightarrow 2\cdot x\cdot y=(x+y)^2-\left(x^2+y^2\right) /\cdot\frac{1}{2} \Rightarrow$$

$$\Rightarrow x\cdot y=\frac{1}{2}\cdot\left((x+y)^2-\left(x^2+y^2\right)\right). \qquad (1)$$

- $x^3+y^3=(x+y)\cdot\left(x^2-x\cdot y+y^2\right)=(x+y)\cdot\left(x^2+y^2-x\cdot y\right)=$

$$=(x+y)\cdot\left(\left(x^2+y^2\right)-x\cdot y\right)=\left[\begin{matrix}\text{zbog}\\(1)\end{matrix}\right]=(x+y)\cdot\left(\left(x^2+y^2\right)-\frac{1}{2}\cdot\left((x+y)^2-\left(x^2+y^2\right)\right)\right)=$$

$$=m\cdot\left(n-\frac{1}{2}\cdot\left(m^2-n\right)\right)=m\cdot\left(n-\frac{1}{2}\cdot m^2+\frac{1}{2}\cdot n\right)=m\cdot\left(\frac{n}{1}-\frac{1}{2}\cdot m^2+\frac{1}{2}\cdot n\right)=m\cdot\left(\frac{3}{2}\cdot n-\frac{1}{2}\cdot m^2\right)=$$

$$=\frac{1}{2}\cdot m\cdot\left(3\cdot n-m^2\right).$$

**Vježba 391**

Ako je $x+y=3$ i $x^2+y^2=5,$ onda je $x^3+y^3=9.$ Dokažite!

**Rezultat:** Točno je.

**Zadatak 392 (Anna, ekonomska škola)**

Ako je $a\cdot b+b\cdot c=33,$ $b\cdot c+a\cdot c=30,$ $a\cdot c+a\cdot b=15,$ izračunajte $a+b+c.$

**Rješenje 392**

Ponovimo!

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0.\end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x\geq 0$, vrijedi $|x|=x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, $x < 0$, je $|x| = -x$.

$$\sqrt{a^2} = |a|.$$

Napišemo sustav jednadžbi i izračunamo a · b, b · c i a · c.

$$\left.\begin{matrix} a\cdot b+b\cdot c=33 \\ b\cdot c+a\cdot c=30 \\ a\cdot c+a\cdot b=15 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{od prve jednadžbe oduzmemo} \\ \text{drugu jednadžbu} \end{bmatrix} \Rightarrow \left.\begin{matrix} a\cdot b+b\cdot c-b\cdot c-a\cdot c=33-30 \\ a\cdot c+a\cdot b=15 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a\cdot b+b\cdot c-b\cdot c-a\cdot c=33-30 \\ a\cdot c+a\cdot b=15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a\cdot b-a\cdot c=3 \\ a\cdot c+a\cdot b=15 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a\cdot b-a\cdot c=3 \\ a\cdot b+a\cdot c=15 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{metoda suprotnih} \\ \text{koeficijenata} \end{bmatrix} \Rightarrow 2\cdot a\cdot b=18 \Rightarrow 2\cdot a\cdot b=18 \ /:2 \Rightarrow a\cdot b=9.$$

Računamo b · c i a · c.

- $$\left.\begin{matrix} a\cdot b+b\cdot c=33 \\ a\cdot b=9 \end{matrix}\right\} \Rightarrow 9+b\cdot c=33 \Rightarrow b\cdot c=33-9 \Rightarrow b\cdot c=24$$
- $$\left.\begin{matrix} a\cdot c+a\cdot b=15 \\ a\cdot b=9 \end{matrix}\right\} \Rightarrow a\cdot c+9=15 \Rightarrow a\cdot c=15-9 \Rightarrow a\cdot c=6.$$

Dobije se novi sustav jednadžbi:

$$\left.\begin{matrix} a\cdot b=9 \\ b\cdot c=24 \\ a\cdot c=6 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{pomnožimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow a\cdot b\cdot b\cdot c\cdot a\cdot c=9\cdot 24\cdot 6 \Rightarrow a^2\cdot b^2\cdot c^2=1296 \Rightarrow$$

$$\Rightarrow (a\cdot b\cdot c)^2=1296 \Rightarrow (a\cdot b\cdot c)^2=1296 \ /\sqrt{\ } \Rightarrow a\cdot b\cdot c=\pm\sqrt{1296} \Rightarrow a\cdot b\cdot c=\pm 36.$$

Nepoznanice a, b i c iznose:

- $$\left.\begin{matrix} a\cdot b\cdot c=\pm 36 \\ a\cdot b=9 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{podijelimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow \frac{a\cdot b\cdot c}{a\cdot b}=\frac{\pm 36}{9} \Rightarrow \frac{a\cdot b\cdot c}{a\cdot b}=\frac{\pm 36}{9} \Rightarrow c=\pm 4.$$
- $$\left.\begin{matrix} a\cdot b\cdot c=\pm 36 \\ b\cdot c=24 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{podijelimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow \frac{a\cdot b\cdot c}{b\cdot c}=\frac{\pm 36}{24} \Rightarrow \frac{a\cdot b\cdot c}{b\cdot c}=\frac{\pm 36}{24} \Rightarrow a=\pm\frac{3}{2}.$$
- $$\left.\begin{matrix} a\cdot b\cdot c=\pm 36 \\ a\cdot c=6 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{podijelimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow \frac{a\cdot b\cdot c}{a\cdot c}=\frac{\pm 36}{6} \Rightarrow \frac{a\cdot b\cdot c}{a\cdot c}=\frac{\pm 36}{6} \Rightarrow b=\pm 6.$$

Tada je

$$\left.\begin{matrix} a=\frac{3}{2} \\ b=6 \\ c=4 \end{matrix}\right\} \Rightarrow a+b+c=\frac{3}{2}+6+4=1.5+6+4=11.5.$$

$$\left.\begin{array}{l} a=-\frac{3}{2} \\ b=-6 \\ c=-4 \end{array}\right\} \Rightarrow a+b+c=-\frac{3}{2}-6-4=-1.5-6-4=-11.5.$$

**Vježba 392**

Ako je $a \cdot b+b \cdot c=33,\ b \cdot c+a \cdot c=30,\ a \cdot c+a \cdot b=15$, izračunajte $a \cdot b \cdot c$.

**Rezultat:** $\pm 36$.

**Zadatak 393 (Anna, ekonomska škola)**

Izračunajte : $\frac{1}{0.1}+\frac{2}{0.2}+\frac{3}{0.3}+\frac{4}{0.4}+\frac{5}{0.5}+\frac{6}{0.6}+\frac{7}{0.7}+\frac{8}{0.8}+\frac{9}{0.9}$.

**Rješenje 393**

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a \cdot d}{b \cdot c}.$$

Konačni decimalni broj piše se u obliku decimalnog razlomka tako da se u brojnik napiše zadani decimalni broj bez decimalne točke, a u nazivnik se napiše dekadska jedinica koja ima toliko nula koliko decimalni broj ima decimala. Na primjer:

$$5.3=5.3=\frac{53}{10}=\frac{53}{10} \quad , \quad 3.49=3.49=\frac{349}{100}=\frac{349}{100} \quad , \quad 0.007=0.007=\frac{7}{1000}=\frac{7}{1000}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice.

$$\frac{a}{b}=\frac{a \cdot n}{b \cdot n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice.

$$\frac{a \cdot n}{b \cdot n}=\frac{a}{b}.$$

1.inačica

$$\frac{1}{0.1}+\frac{2}{0.2}+\frac{3}{0.3}+\frac{4}{0.4}+\frac{5}{0.5}+\frac{6}{0.6}+\frac{7}{0.7}+\frac{8}{0.8}+\frac{9}{0.9}=\frac{1}{\frac{1}{10}}+\frac{2}{\frac{2}{10}}+\frac{3}{\frac{3}{10}}+\frac{4}{\frac{4}{10}}+\frac{5}{\frac{5}{10}}+\frac{6}{\frac{6}{10}}+\frac{7}{\frac{7}{10}}+\frac{8}{\frac{8}{10}}+\frac{9}{\frac{9}{10}}=$$

$$=\frac{\frac{1}{1}}{\frac{1}{10}}+\frac{\frac{2}{1}}{\frac{2}{10}}+\frac{\frac{3}{1}}{\frac{3}{10}}+\frac{\frac{4}{1}}{\frac{4}{10}}+\frac{\frac{5}{1}}{\frac{5}{10}}+\frac{\frac{6}{1}}{\frac{6}{10}}+\frac{\frac{7}{1}}{\frac{7}{10}}+\frac{\frac{8}{1}}{\frac{8}{10}}+\frac{\frac{9}{1}}{\frac{9}{10}}=\frac{10}{1}+\frac{20}{2}+\frac{30}{3}+\frac{40}{4}+\frac{50}{5}+\frac{60}{6}+\frac{70}{7}+\frac{80}{8}+\frac{90}{9}=$$

$$=10+10+10+10+10+10+10+10+10=90.$$

2.inačica

$$\frac{1}{0.1}+\frac{2}{0.2}+\frac{3}{0.3}+\frac{4}{0.4}+\frac{5}{0.5}+\frac{6}{0.6}+\frac{7}{0.7}+\frac{8}{0.8}+\frac{9}{0.9}=\left[\begin{array}{l}\text{svaki razlomak} \\ \text{proširimo brojem 10}\end{array}\right]=$$

$$=\frac{1 \cdot 10}{0.1 \cdot 10}+\frac{2 \cdot 10}{0.2 \cdot 10}+\frac{3 \cdot 10}{0.3 \cdot 10}+\frac{4 \cdot 10}{0.4 \cdot 10}+\frac{5 \cdot 10}{0.5 \cdot 10}+\frac{6 \cdot 10}{0.6 \cdot 10}+\frac{7 \cdot 10}{0.7 \cdot 10}+\frac{8 \cdot 10}{0.8 \cdot 10}+\frac{9 \cdot 10}{0.9 \cdot 10}=$$

$$=\frac{10}{1}+\frac{20}{2}+\frac{30}{3}+\frac{40}{4}+\frac{50}{5}+\frac{60}{6}+\frac{70}{7}+\frac{80}{8}+\frac{90}{9}=10+10+10+10+10+10+10+10+10=90.$$

**Vježba 393**

Izračunajte : $\frac{1}{0.1}+\frac{2}{0.2}+\frac{3}{0.3}+\frac{4}{0.4}+\frac{5}{0.5}+\frac{6}{0.6}+\frac{7}{0.7}.$

**Rezultat:** 70.

**Zadatak 394 (Anna, ekonomska škola)**

Izračunajte : $1-2+3-4+5-6+7-8+\ \dots\ +99-100+101.$

**Rješenje 394**

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$1-2+3-4+5-6+7-8+\ \dots\ +99-100+101=(1-2)+(3-4)+(5-6)+(7-8)+\ \dots\ +(99-100)+101=$$

$$=\underbrace{(1-2)+(3-4)+(5-6)+(7-8)+\ \dots\ +(99-100)}_{50\text{ je članova}}+101=\underbrace{(-1)+(-1)+(-1)+(-1)+\ \dots\ +(-1)}_{50\text{ je članova}}+101=$$

$$=50\cdot(-1)+101=-50+101=51.$$

2.inačica

$$1-2+3-4+5-6+7-8+\ \dots\ +99-100+101=1+(-2+3)+(-4+5)+(-6+7)+\ \dots\ +(-100+101)=$$

$$=1+\underbrace{(-2+3)+(-4+5)+(-6+7)+\ \dots\ +(-100+101)}_{50\text{ je članova}}=1+\underbrace{1+1+1+\ \dots\ +1}_{50\text{ je članova}}=1+50\cdot 1=1+50=51.$$



**Vježba 394**

Izračunajte : $1-2+3-4+5-6+7-8+\ \dots\ +99-100+100.$

**Rezultat:** 50.

**Zadatak 395 (KNM, gimnazija)**

Koliko je $\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1+\sqrt[16]{a}\right)\cdot\left(1-\sqrt[16]{a}\right)$, za $a=2$?

**Rješenje 395**

Ponovimo!

$$\left(\sqrt[n\cdot p]{a}\right)^p=\sqrt[n]{a} \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad (a-b)\cdot(a+b)=a^2-b^2.$$

$$\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1+\sqrt[16]{a}\right)\cdot\left(1-\sqrt[16]{a}\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\underbrace{\left(1+\sqrt[16]{a}\right)\cdot\left(1-\sqrt[16]{a}\right)}_{\text{razlika kvadrata}}=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1-\left(\sqrt[16]{a}\right)^2\right)=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1-\sqrt[8]{a}\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\underbrace{\left(1+\sqrt[8]{a}\right)\cdot\left(1-\sqrt[8]{a}\right)}_{\text{razlika kvadrata}}=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1-\left(\sqrt[8]{a}\right)^2\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1-\sqrt[4]{a}\right)=\left(1+\sqrt{a}\right)\cdot\underbrace{\left(1+\sqrt[4]{a}\right)\cdot\left(1-\sqrt[4]{a}\right)}_{\text{razlika kvadrata}}=\left(1+\sqrt{a}\right)\cdot\left(1-\left(\sqrt[4]{a}\right)^2\right)=$$

$$=\left(1+\sqrt{a}\right)\cdot\left(1-\sqrt{a}\right)=\left[\begin{matrix}\text{razlika}\\ \text{kvadrata}\end{matrix}\right]=1-\left(\sqrt{a}\right)^2=1-a=\left[a=2\right]=1-2=-1.$$

**Vježba 395**

Koliko je $\left(1+\sqrt{a}\right)\cdot\left(1+\sqrt[4]{a}\right)\cdot\left(1+\sqrt[8]{a}\right)\cdot\left(1+\sqrt[16]{a}\right)\cdot\left(1-\sqrt[16]{a}\right)$, za $a=6$?

**Rezultat:** $-5$.

**Zadatak 396 (Viki, ekonomska škola)**

Koji je rezultat dijeljenja $\left(\frac{3\cdot a-b}{b^2}+\frac{1}{b}\right):\frac{6\cdot a}{b}$, za $a\neq 0,\ b\neq 0$?

$A.\ \frac{2}{a}$ $\quad B.\ \frac{2}{b}$ $\quad C.\ \frac{1}{2\cdot a}$ $\quad D.\ \frac{1}{2\cdot b}$

**Rješenje 396**

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$\left(\frac{3\cdot a-b}{b^2}+\frac{1}{b}\right):\frac{6\cdot a}{b}=\frac{3\cdot a-b+b}{b^2}:\frac{6\cdot a}{b}=\frac{3\cdot a-b+b}{b^2}:\frac{6\cdot a}{b}=\frac{3\cdot a}{b^2}:\frac{6\cdot a}{b}=\frac{3\cdot a}{b^2}\cdot\frac{b}{6\cdot a}=$$

$$=\frac{3\cdot a}{b^2}\cdot\frac{b}{6\cdot a}=\frac{1}{b}\cdot\frac{1}{2}=\frac{1}{2\cdot b}.$$

Odgovor je pod D.



**Vježba 396**

Koji je rezultat dijeljenja $\left(\frac{3\cdot a-b}{b^2}+\frac{1}{b}\right):\frac{3\cdot a}{b}$, za $a\neq 0,\ b\neq 0$?

$A.\ \frac{2}{a}$ $\quad B.\ \frac{2}{b}$ $\quad C.\ \frac{1}{a}$ $\quad D.\ \frac{1}{b}$

**Rezultat:** D.

**Zadatak 397 (Ofelija ☺, ekonomska škola)**

Koliki je rezultat umnoška $\left(\sqrt{3}-1\right)^2\cdot\left(\sqrt{3}+1\right)^2$?

$A.\ \sqrt{3}-1$ $\quad B.\ \sqrt{3}+1$ $\quad C.\ 4$ $\quad D.\ 8$

**Rješenje 397**

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad\left(\sqrt{a}\right)^2=a.$$

$$(a-b)\cdot(a+b)=a^2-b^2\quad,\quad(a\cdot b)^n=a^n\cdot b^n.$$

1.inačica

$$\left(\sqrt{3}-1\right)^2\cdot\left(\sqrt{3}+1\right)^2=\left(\left(\sqrt{3}\right)^2-2\cdot\sqrt{3}\cdot1+1^2\right)\cdot\left(\left(\sqrt{3}\right)^2+2\cdot\sqrt{3}\cdot1+1^2\right)=$$

$$= \left(3-2\cdot\sqrt{3}+1\right)\cdot\left(3+2\cdot\sqrt{3}+1\right) = \left(4-2\cdot\sqrt{3}\right)\cdot\left(4+2\cdot\sqrt{3}\right) = \underbrace{\left(4-2\cdot\sqrt{3}\right)\cdot\left(4+2\cdot\sqrt{3}\right)}_{\text{razlika kvadrata}} =$$

$$= 4^2 - \left(2\cdot\sqrt{3}\right)^2 = 4^2 - 2^2\cdot\left(\sqrt{3}\right)^2 = 16 - 4\cdot 3 = 16 - 12 = 4.$$

Odgovor je pod C.

2.inačica

$$\left(\sqrt{3}-1\right)^2\cdot\left(\sqrt{3}+1\right)^2 = \left(\left(\sqrt{3}-1\right)\cdot\left(\sqrt{3}+1\right)\right)^2 = \left(\underbrace{\left(\sqrt{3}-1\right)\cdot\left(\sqrt{3}+1\right)}_{\text{razlika kvadrata}}\right)^2 =$$

$$= \left(\left(\sqrt{3}\right)^2 - 1^2\right)^2 = (3-1)^2 = 2^2 = 4.$$

Odgovor je pod C.

**Vježba 397**

Koliki je rezultat umnoška $\left(\sqrt{5}-1\right)^2\cdot\left(\sqrt{5}+1\right)^2$ ?

$A.\ \sqrt{5}-1 \qquad B.\ \sqrt{5}+1 \qquad C.\ 8 \qquad D.\ 16$

**Rezultat:** D.



**Zadatak 398 (Nevzat, srednja škola)**

Izračunaj $\dfrac{\frac{1}{a}-\frac{1}{b}+\frac{1}{c}}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}$, ako je $a = 3\cdot b = 4\cdot c$.

**Rješenje 398**

Ponovimo!

$$n = \frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{n} - \frac{b}{n} = \frac{a-b}{n} \quad , \quad \frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

Iz jednakosti

$$a = 3\cdot b = 4\cdot c$$

dobije se

$$\left.\begin{matrix} a = 3\cdot b \\ a = 4\cdot c \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a = 3\cdot b \ /\cdot\frac{1}{3} \\ a = 4\cdot c \ /\cdot\frac{1}{4} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} b = \frac{a}{3} \\ c = \frac{a}{4} \end{matrix}\right\}.$$

Tada je

$$\frac{\frac{1}{a}-\frac{1}{b}+\frac{1}{c}}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}=\frac{\frac{1}{a}-\frac{1}{\frac{a}{3}}+\frac{1}{\frac{a}{4}}}{\frac{1}{a}+\frac{1}{\frac{a}{3}}+\frac{1}{\frac{a}{4}}}=\frac{\frac{1}{a}-\frac{\frac{1}{1}}{\frac{a}{3}}+\frac{\frac{1}{1}}{\frac{a}{4}}}{\frac{1}{a}+\frac{\frac{1}{1}}{\frac{a}{3}}+\frac{\frac{1}{1}}{\frac{a}{4}}}=\frac{\frac{1}{a}-\frac{3}{a}+\frac{4}{a}}{\frac{1}{a}+\frac{3}{a}+\frac{4}{a}}=\frac{\frac{1-3+4}{a}}{\frac{1+3+4}{a}}=\frac{\frac{2}{a}}{\frac{8}{a}}=\frac{\frac{2}{a}}{\frac{8}{a}}=\frac{\frac{2}{1}}{\frac{8}{1}}=\frac{2}{8}=\frac{2}{8}=\frac{1}{4}.$$

**Vježba 398**

Izračunaj $\frac{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}}{\frac{1}{a}-\frac{1}{b}+\frac{1}{c}}$, ako je $a=3\cdot b=4\cdot c$.

**Rezultat:** 4.

**Zadatak 399 (Mira, gimnazija)**

Pojednostavni $\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}+a-b\right)\cdot(a-b)\cdot\left(\sqrt{\frac{a+b}{a-b}}-1\right)$.

**Rješenje 399**

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad n=\frac{n}{1}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}, \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$



1.inačica

$$\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}+a-b\right)\cdot(a-b)\cdot\left(\sqrt{\frac{a+b}{a-b}}-1\right)=\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}+(a-b)\right)\cdot(a-b)\cdot\left(\sqrt{\frac{a+b}{a-b}}-1\right)=$$

$$=(a-b)\cdot\left(\sqrt{\frac{a+b}{a-b}}+1\right)\cdot(a-b)\cdot\left(\sqrt{\frac{a+b}{a-b}}-1\right)=(a-b)^2\cdot\left(\sqrt{\frac{a+b}{a-b}}+1\right)\cdot\left(\sqrt{\frac{a+b}{a-b}}-1\right)=\left[\begin{matrix}\text{razlika}\\ \text{kvadrata}\end{matrix}\right]=$$

$$=(a-b)^2\cdot\left(\left(\sqrt{\frac{a+b}{a-b}}\right)^2-1^2\right)=(a-b)^2\cdot\left(\frac{a+b}{a-b}-1\right)=(a-b)^2\cdot\left(\frac{a+b}{a-b}-\frac{1}{1}\right)=(a-b)^2\cdot\frac{a+b-(a-b)}{a-b}=$$

$$=(a-b)^2\cdot\frac{a+b-a+b}{a-b}=(a-b)^2\cdot\frac{a+b-a+b}{a-b}=(a-b)^2\cdot\frac{2\cdot b}{a-b}=(a-b)^2\cdot\frac{2\cdot b}{a-b}=$$

$$=(a-b)\cdot 2\cdot b=2\cdot b\cdot(a-b).$$

2.inačica

$$\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}+a-b\right)\cdot(a-b)\cdot\left(\sqrt{\frac{a+b}{a-b}}-1\right)=\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}+(a-b)\right)\cdot\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}-(a-b)\right)=$$

$$=\begin{bmatrix}\text{razlika}\\ \text{kvadrata}\end{bmatrix}=\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}\right)^2-(a-b)^2=(a-b)^2\cdot\left(\sqrt{\frac{a+b}{a-b}}\right)^2-(a-b)^2=$$

$$=(a-b)^2\cdot\frac{a+b}{a-b}-(a-b)^2=(a-b)^2\cdot\frac{a+b}{a-b}-(a-b)^2=(a-b)\cdot(a+b)-(a-b)^2=$$

$$=(a-b)\cdot((a+b)-(a-b))=(a-b)\cdot(a+b-a+b)=(a-b)\cdot(a+b-a+b)=$$

$$=(a-b)\cdot 2\cdot b=2\cdot b\cdot(a-b).$$

3.inačica

$$\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}+a-b\right)\cdot(a-b)\cdot\left(\sqrt{\frac{a+b}{a-b}}-1\right)=\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}+(a-b)\right)\cdot\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}-(a-b)\right)=$$

$$=\begin{bmatrix}\text{razlika}\\ \text{kvadrata}\end{bmatrix}=\left((a-b)\cdot\sqrt{\frac{a+b}{a-b}}\right)^2-(a-b)^2=(a-b)^2\cdot\left(\sqrt{\frac{a+b}{a-b}}\right)^2-(a-b)^2=$$

$$=(a-b)^2\cdot\frac{a+b}{a-b}-(a-b)^2=(a-b)^2\cdot\frac{a+b}{a-b}-(a-b)^2=(a-b)\cdot(a+b)-(a-b)^2=$$

$$=\begin{bmatrix}\text{razlika kvadrata}\\ \text{kvadrat razlike}\end{bmatrix}=a^2-b^2-\left(a^2-2\cdot a\cdot b+b^2\right)=a^2-b^2-a^2+2\cdot a\cdot b-b^2=$$

$$=a^2-b^2-a^2+2\cdot a\cdot b-b^2=2\cdot a\cdot b-2\cdot b^2=2\cdot b\cdot(a-b).$$



**Vježba 399**

Pojednostavni $\left((a-1)\cdot\sqrt{\frac{a+1}{a-1}}+a-1\right)\cdot(a-1)\cdot\left(\sqrt{\frac{a+1}{a-1}}-1\right).$

**Rezultat:** $2\cdot(a-1).$

**Zadatak 400 (Tony, srednja škola)**

Koliko je $5\cdot 2^{2010}-3\cdot 2^{2011}+14\cdot 2^{2009}$?

A. $9\cdot 2^{2009}$     B. $7\cdot 2^{2010}$     C. $3\cdot 2^{2011}$     D. $5\cdot 2^{2012}$

**Rješenje 400**

Ponovimo!

$$a^n\cdot a^m=a^{n+m}\quad,\quad a^1=a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$5\cdot 2^{2010}-3\cdot 2^{2011}+14\cdot 2^{2009}=5\cdot 2^{2009}\cdot 2^1-3\cdot 2^{2009}\cdot 2^2+14\cdot 2^{2009}=$$

$$=2^{2009}\cdot\left(5\cdot 2^1-3\cdot 2^2+14\right)=2^{2009}\cdot(5\cdot 2-3\cdot 4+14)=2^{2009}\cdot(10-12+14)=2^{2009}\cdot 12=$$

$$=12\cdot 2^{2009}=3\cdot 4\cdot 2^{2009}=3\cdot 2^2\cdot 2^{2009}=3\cdot 2^{2011}.$$

Odgovor je pod C.

**Vježba 400**

Koliko je $5 \cdot 2^{2010} + 3 \cdot 2^{2011} - 14 \cdot 2^{2009}$ ?

A. $2^{2009}$ *B*. $2^{2010}$ *C*. $2^{2011}$ *D*. $2^{2012}$

**Rezultat:** D.