Zadatak 321 (Nidko, gimnazija)

Faktoriziraj (rastavi na faktore): $\left(c^2-b^2+d^2-a^2\right)^2-4\cdot(a\cdot b-c\cdot d)^2$.

Rješenje 321

Ponovimo!

$$(x\cdot y)^n=x^n\cdot y^n \quad , \quad x^2-y^2=(x-y)\cdot(x+y) \quad , \quad (x-y)^2=x^2-2\cdot x\cdot y+y^2.$$

$$(x+y)^2=x^2+2\cdot x\cdot y+y^2.$$

Zakon distribucije množenja prema zbrajanju

$$x\cdot(y+z)=x\cdot y+x\cdot z \quad , \quad x\cdot y+x\cdot z=x\cdot(y+z).$$

$$\left(c^2-b^2+d^2-a^2\right)^2-4\cdot(a\cdot b-c\cdot d)^2=\left(c^2-b^2+d^2-a^2\right)^2-(2\cdot(a\cdot b-c\cdot d))^2=$$

$$=\left(\left(c^2-b^2+d^2-a^2\right)-2\cdot(a\cdot b-c\cdot d)\right)\cdot\left(\left(c^2-b^2+d^2-a^2\right)+2\cdot(a\cdot b-c\cdot d)\right)=$$

$$=\left(c^2-b^2+d^2-a^2-2\cdot a\cdot b+2\cdot c\cdot d\right)\cdot\left(c^2-b^2+d^2-a^2+2\cdot a\cdot b-2\cdot c\cdot d\right)=$$

$$=\left(c^2+2\cdot c\cdot d+d^2-a^2-2\cdot a\cdot b-b^2\right)\cdot\left(c^2-2\cdot c\cdot d+d^2-a^2+2\cdot a\cdot b-b^2\right)=$$

$$=\left(\left(c^2+2\cdot c\cdot d+d^2\right)-\left(a^2+2\cdot a\cdot b+b^2\right)\right)\cdot\left(\left(c^2-2\cdot c\cdot d+d^2\right)-\left(a^2-2\cdot a\cdot b+b^2\right)\right)=$$

$$=\left((c+d)^2-(a+b)^2\right)\cdot\left((c-d)^2-(a-b)^2\right)=$$

$$=((c+d)-(a+b))\cdot((c+d)+(a+b))\cdot((c-d)-(a-b))\cdot((c-d)+(a-b))=$$

$$=(c+d-a+b)\cdot(c+d+a+b)\cdot(c-d-a+b)\cdot(c-d+a-b).$$

Vježba 321

Faktoriziraj (rastavi na faktore): $a^2+b^2-c^2+2\cdot a\cdot b$.

Rezultat: $(a+b-c)\cdot(a+b+c)$.

Zadatak 322 (Valerija, gimnazija)

Pojednostavni: $\left(\frac{2}{x^2-4}+\frac{1}{2\cdot x-x^2}\right):\frac{1}{x^2+4\cdot x+4}$.

Rješenje 322

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a^n}{a^m}=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(\frac{2}{x^2-4}+\frac{1}{2\cdot x-x^2}\right):\frac{1}{x^2+4\cdot x+4}=\left(\frac{2}{x^2-4}+\frac{1}{-\left(x^2-2\cdot x\right)}\right):\frac{1}{x^2+4\cdot x+4}=$$

$$=\left(\frac{2}{x^2-4}-\frac{1}{x^2-2\cdot x}\right):\frac{1}{x^2+4\cdot x+4}=\left(\frac{2}{(x-2)\cdot(x+2)}-\frac{1}{x\cdot(x-2)}\right):\frac{1}{(x+2)^2}=$$

$$=\frac{2\cdot x-(x+2)}{x\cdot(x-2)\cdot(x+2)}:\frac{1}{(x+2)^2}=\frac{2\cdot x-x-2}{x\cdot(x-2)\cdot(x+2)}:\frac{1}{(x+2)^2}=\frac{x-2}{x\cdot(x-2)\cdot(x+2)}:\frac{1}{(x+2)^2}=$$

$$=\frac{x-2}{x\cdot(x-2)\cdot(x+2)}:\frac{1}{(x+2)^2}=\frac{1}{x\cdot(x+2)}:\frac{1}{(x+2)^2}=\frac{1}{x\cdot(x+2)}\cdot\frac{(x+2)^2}{1}=$$

$$=\frac{1}{x\cdot(x+2)}\cdot\frac{(x+2)^2}{1}=\frac{1}{x}\cdot\frac{x+2}{1}=\frac{x+2}{x}.$$

Vježba 322

Pojednostavni: $\left(\frac{1}{2\cdot x-x^2}-\frac{2}{4-x^2}\right):\frac{1}{x^2+4\cdot x+4}.$

Rezultat: $\frac{x+2}{x}.$

Zadatak 323 (Valerija, gimnazija)

Pojednostavni: $\left(2\cdot x+1-\frac{1}{1-2\cdot x}\right):\left(2\cdot x-\frac{4\cdot x^2}{2\cdot x-1}\right).$

Rješenje 323

Ponovimo!

$$n=\frac{n}{1}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad \frac{a^n}{a^m}=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(2\cdot x+1-\frac{1}{1-2\cdot x}\right):\left(2\cdot x-\frac{4\cdot x^2}{2\cdot x-1}\right)=\left(2\cdot x+1-\frac{1}{-(2\cdot x-1)}\right):\left(2\cdot x-\frac{4\cdot x^2}{2\cdot x-1}\right)=$$

$$=\left(2\cdot x+1+\frac{1}{2\cdot x-1}\right):\left(2\cdot x-\frac{4\cdot x^2}{2\cdot x-1}\right)=\left(\frac{2\cdot x+1}{1}+\frac{1}{2\cdot x-1}\right):\left(\frac{2\cdot x}{1}-\frac{4\cdot x^2}{2\cdot x-1}\right)=$$

$$=\frac{(2\cdot x+1)\cdot(2\cdot x-1)+1}{2\cdot x-1}:\frac{2\cdot x\cdot(2\cdot x-1)-4\cdot x^2}{2\cdot x-1}=\frac{4\cdot x^2-1+1}{2\cdot x-1}:\frac{4\cdot x^2-2\cdot x-4\cdot x^2}{2\cdot x-1}=$$

$$=\frac{4\cdot x^2-1+1}{2\cdot x-1}:\frac{4\cdot x^2-2\cdot x-4\cdot x^2}{2\cdot x-1}=\frac{4\cdot x^2}{2\cdot x-1}:\frac{-2\cdot x}{2\cdot x-1}=\frac{4\cdot x^2}{2\cdot x-1}\cdot\frac{2\cdot x-1}{-2\cdot x}=\frac{4\cdot x^2}{2\cdot x-1}\cdot\frac{2\cdot x-1}{-2\cdot x}=$$

$$=\frac{4\cdot x^2}{1}\cdot\frac{1}{-2\cdot x}=\frac{4\cdot x^2}{1}\cdot\frac{1}{-2\cdot x}=\frac{2\cdot x}{1}\cdot\frac{1}{-1}=-2\cdot x.$$

Vježba 323

Pojednostavni: $\left(2\cdot x+1+\frac{1}{2\cdot x-1}\right):\left(2\cdot x+\frac{4\cdot x^2}{1-2\cdot x}\right).$

Rezultat: $-2\cdot x.$

Zadatak 324 (Valerija, gimnazija)

Pojednostavni: $\left(\frac{a}{a^2-6\cdot a+9}-\frac{12}{a^2-3\cdot a}\right)\cdot\left(a+\frac{9}{a-6}\right).$

Rješenje 324

Ponovimo!

$$(x-y)^2=x^2-2\cdot x\cdot y+y^2\quad,\quad n=\frac{n}{1}\quad,\quad\frac{x}{y}-\frac{m}{n}=\frac{x\cdot n-y\cdot m}{y\cdot n}\quad,\quad\frac{x}{y}\cdot\frac{m}{n}=\frac{x\cdot m}{y\cdot n}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(\frac{a}{a^2-6\cdot a+9}-\frac{12}{a^2-3\cdot a}\right)\cdot\left(a+\frac{9}{a-6}\right)=\left(\frac{a}{(a-3)^2}-\frac{12}{a\cdot(a-3)}\right)\cdot\left(\frac{a}{1}+\frac{9}{a-6}\right)=$$

$$=\frac{a^2-12\cdot(a-3)}{a\cdot(a-3)^2}\cdot\frac{a\cdot(a-6)+9}{a-6}=\frac{a^2-12\cdot a+36}{a\cdot(a-3)^2}\cdot\frac{a^2-6\cdot a+9}{a-6}=\frac{(a-6)^2}{a\cdot(a-3)^2}\cdot\frac{(a-3)^2}{a-6}=$$

$$=\frac{(a-6)^2}{a\cdot(a-3)^2}\cdot\frac{(a-3)^2}{a-6}=\frac{(a-6)^2}{a}\cdot\frac{1}{a-6}=\frac{(a-6)^2}{a}\cdot\frac{1}{a-6}=\frac{a-6}{a}\cdot\frac{1}{1}=\frac{a-6}{a}.$$

Vježba 324

Pojednostavni: $\left(\frac{a}{a^2-6\cdot a+9}+-\frac{12}{3\cdot a-a^2}\right)\cdot\left(a-\frac{9}{6-a}\right).$

Rezultat: $\frac{a-6}{a}.$

Zadatak 325 (Valerija, gimnazija)

Pojednostavni: $\left(1-\frac{1}{1-\frac{a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}.$

Rješenje 325

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{x}{y}-\frac{m}{n}=\frac{x\cdot n-y\cdot m}{y\cdot n} \quad , \quad \frac{\frac{x}{y}}{\frac{m}{n}}=\frac{x\cdot n}{y\cdot m} \quad , \quad \frac{x}{y}+\frac{m}{n}=\frac{x\cdot n+y\cdot m}{y\cdot n}.$$

$$x^3+y^3=(x+y)\cdot\left(x^2-x\cdot y+y^2\right).$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left(1-\frac{1}{1-\frac{a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}=\left(1-\frac{1}{\frac{1}{1}-\frac{a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}=\left(1-\frac{1}{\frac{a-1-a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}=\left(1-\frac{\frac{1}{1}}{\frac{a-1-a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}=$$

$$=\left(1-\frac{a-1}{a-1-a^2}\right)\cdot\frac{a^3+1}{a^2}=\left(1-\frac{a-1}{-\left(a^2-a+1\right)}\right)\cdot\frac{a^3+1}{a^2}=\left(1+\frac{a-1}{a^2-a+1}\right)\cdot\frac{a^3+1}{a^2}=$$

$$=\left(\frac{1}{1}+\frac{a-1}{a^2-a+1}\right)\cdot\frac{a^3+1}{a^2}=\frac{a^2-a+1+a-1}{a^2-a+1}\cdot\frac{a^3+1}{a^2}=\frac{a^2-a+1+a-1}{a^2-a+1}\cdot\frac{a^3+1}{a^2}=$$

$$=\frac{a^2}{a^2-a+1}\cdot\frac{a^3+1}{a^2}=\frac{a^2}{a^2-a+1}\cdot\frac{(a+1)\cdot\left(a^2-a+1\right)}{a^2}=\frac{a^2}{a^2-a+1}\cdot\frac{(a+1)\cdot\left(a^2-a+1\right)}{a^2}=$$

$$=\frac{1}{1}\cdot\frac{a+1}{1}=a+1.$$

Vježba 325

Pojednostavni: $\left(1+\frac{1}{\frac{a^2}{a-1}-1}\right)\cdot\frac{a^3+1}{a^2}.$

Rezultat: $a+1.$

Zadatak 326 (Enna, gimnazija)

Pojednostavni: $\frac{1}{a-1}+\frac{2\cdot a+1}{a^2-1}-\frac{3\cdot a^2-5\cdot a-1}{1-a^3}.$

Rješenje 326

Ponovimo!

$$x^2-y^2=(x-y)\cdot(x+y) \quad , \quad x^3-y^3=(x-y)\cdot\left(x^2+x\cdot y+y^2\right) \quad , \quad \frac{x}{y}-\frac{m}{n}=\frac{x\cdot n-y\cdot m}{y\cdot n}.$$

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \frac{x}{y}+\frac{m}{n}=\frac{x\cdot n+y\cdot m}{y\cdot n}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$\frac{1}{a-1}+\frac{2\cdot a+1}{a^2-1}-\frac{3\cdot a^2-5\cdot a-1}{1-a^3}=\frac{1}{a-1}+\frac{2\cdot a+1}{a^2-1}-\frac{3\cdot a^2-5\cdot a-1}{-\left(a^3-1\right)}=\frac{1}{a-1}+\frac{2\cdot a+1}{a^2-1}+\frac{3\cdot a^2-5\cdot a-1}{a^3-1}=$$

$$=\frac{1}{a-1}+\frac{2\cdot a+1}{(a-1)\cdot(a+1)}+\frac{3\cdot a^2-5\cdot a-1}{(a-1)\cdot\left(a^2+a+1\right)}=$$

$$=\frac{(a+1)\cdot\left(a^2+a+1\right)+(2\cdot a+1)\cdot\left(a^2+a+1\right)+(a+1)\cdot\left(3\cdot a^2-5\cdot a-1\right)}{(a-1)\cdot(a+1)\cdot\left(a^2+a+1\right)}=$$

$$=\frac{a^3+a^2+a+a^2+a+1+2\cdot a^3+2\cdot a^2+2\cdot a+a^2+a+1+3\cdot a^3-5\cdot a^2-a+3\cdot a^2-5\cdot a-1}{(a-1)\cdot(a+1)\cdot\left(a^2+a+1\right)}=$$

$$=\frac{a^3+a^2+a+a^2+a+1+2\cdot a^3+2\cdot a^2+2\cdot a+a^2+a+1+3\cdot a^3-5\cdot a^2-a+3\cdot a^2-5\cdot a-1}{(a-1)\cdot(a+1)\cdot\left(a^2+a+1\right)}=$$

$$=\frac{a^3+1+2\cdot a^3+3\cdot a^3-a+3\cdot a^2}{(a-1)\cdot(a+1)\cdot\left(a^2+a+1\right)}=\frac{6\cdot a^3+3\cdot a^2-a+1}{(a-1)\cdot(a+1)\cdot\left(a^2+a+1\right)}.$$

Vježba 326

Pojednostavni: $\frac{1}{a-1}+\frac{2\cdot a+1}{1-a^2}+\frac{3\cdot a^2-5\cdot a-1}{a^3-1}.$

Rezultat: $\frac{6\cdot a^3+3\cdot a^2-a+1}{(a-1)\cdot(a+1)\cdot\left(a^2+a+1\right)}.$

Zadatak 327 (Iva, srednja škola)

Rastavi na faktore: $27\cdot a^4\cdot b-18\cdot a^3\cdot b^2.$

Rješenje 327

Ponovimo!

$$a^1=a \quad , \quad a^n : a^m=a^{n-m} \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$27\cdot a^4\cdot b-18\cdot a^3\cdot b^2=\left[\begin{array}{l}\text{kod brojeva izlučimo najveću zajedničku mjeru 9,}\\ \text{kod potencija izlučimo potenciju s manjim eksponentom}\end{array}\right]=$$

$$=9\cdot a^3\cdot b\cdot\left(3\cdot a^1-2\cdot b^1\right)=9\cdot a^3\cdot b\cdot(3\cdot a-2\cdot b).$$

2.inačica

$$27\cdot a^4\cdot b-18\cdot a^3\cdot b^2=9\cdot 3\cdot a^3\cdot a^1\cdot b-9\cdot 2\cdot a^3\cdot b^1\cdot b^1=9\cdot 3\cdot a^3\cdot a\cdot b-9\cdot 2\cdot a^3\cdot b\cdot b=$$

$$=9\cdot 3\cdot a^3\cdot a\cdot b-9\cdot 2\cdot a^3\cdot b\cdot b=9\cdot a^3\cdot b\cdot(3\cdot a-2\cdot b)=9\cdot a^3\cdot b\cdot(3\cdot a-2\cdot b).$$

Vježba 327

Rastavi na faktore: $27\cdot a^3\cdot b-18\cdot a^2\cdot b^2.$

Rezultat: $9\cdot a^2\cdot b\cdot(3\cdot a-2\cdot b).$

Zadatak 328 (MM, medicinska škola)

Koji je rezultat dijeljenja, $\left(\frac{3\cdot a-b}{b^2}+\frac{1}{b}\right):\frac{6\cdot a}{b}$, za $a\neq 0,\ b\neq 0$?

$A.\ \frac{2}{a}$ $B.\ \frac{2}{b}$ $C.\ \frac{1}{2\cdot a}$ $D.\ \frac{1}{2\cdot b}$

Rješenje 328

Ponovimo!

$$\frac{x}{y}+\frac{u}{v}=\frac{x\cdot v+y\cdot u}{y\cdot v}\quad,\quad\frac{x}{y}:\frac{u}{v}=\frac{x}{y}\cdot\frac{v}{u}=\frac{x\cdot v}{y\cdot u}\quad,\quad x^1=x\quad,\quad\frac{x^n}{x^m}=x^{n-m}.$$

$$\left(\frac{3\cdot a-b}{b^2}+\frac{1}{b}\right):\frac{6\cdot a}{b}=\frac{3\cdot a-b+b}{b^2}:\frac{6\cdot a}{b}=\frac{3\cdot a-b+b}{b^2}:\frac{6\cdot a}{b}=\frac{3\cdot a}{b^2}:\frac{6\cdot a}{b}=\frac{3\cdot a}{b^2}\cdot\frac{b}{6\cdot a}=$$

$$=\frac{3\cdot a}{b^2}\cdot\frac{b}{6\cdot a}=\frac{1}{b}\cdot\frac{1}{2}=\frac{1}{2\cdot b}.$$

Odgovor je pod D.

Vježba 328

Koji je rezultat dijeljenja, $\left(\frac{3\cdot a-b}{b^2}+\frac{1}{b}\right):\frac{3\cdot a}{b}$, za $a\neq 0,\ b\neq 0$?

$A.\ \frac{1}{a}$ $B.\ \frac{1}{b}$ $C.\ \frac{2}{a}$ $D.\ \frac{2}{b}$

Rezultat: B.

Zadatak 329 (Domagoj, tehnička škola)

Pojednostavni izraz: $\frac{\sqrt[3]{a^2\cdot b}+\sqrt[3]{a\cdot b^2}}{a\cdot\sqrt[3]{b}-b\cdot\sqrt[3]{a}}$.

Rješenje 329

Ponovimo!

$$\sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}\quad,\quad a\cdot\sqrt[n]{b}=\sqrt[n]{a^n\cdot b}\quad,\quad a^2-b^2=(a-b)\cdot(a+b).$$

$$a^n\cdot a^m=a^{n+m}\quad,\quad\left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{\sqrt[3]{a^2\cdot b}+\sqrt[3]{a\cdot b^2}}{a\cdot\sqrt[3]{b}-b\cdot\sqrt[3]{a}}=\frac{\sqrt[3]{a^2\cdot b}+\sqrt[3]{a\cdot b^2}}{\sqrt[3]{a^3\cdot b}-\sqrt[3]{a\cdot b^3}}=\frac{\sqrt[3]{a\cdot a\cdot b}+\sqrt[3]{a\cdot b\cdot b}}{\sqrt[3]{a^2\cdot a\cdot b}-\sqrt[3]{a\cdot b\cdot b^2}}=\frac{\sqrt[3]{a}\cdot\sqrt[3]{a\cdot b}+\sqrt[3]{a\cdot b}\cdot\sqrt[3]{b}}{\sqrt[3]{a^2}\cdot\sqrt[3]{a\cdot b}-\sqrt[3]{a\cdot b}\cdot\sqrt[3]{b^2}}=$$

$$=\frac{\sqrt[3]{a\cdot b}\cdot\left(\sqrt[3]{a}+\sqrt[3]{b}\right)}{\sqrt[3]{a\cdot b}\cdot\left(\sqrt[3]{a^2}-\sqrt[3]{b^2}\right)}=\frac{\sqrt[3]{a\cdot b}\cdot\left(\sqrt[3]{a}+\sqrt[3]{b}\right)}{\sqrt[3]{a\cdot b}\cdot\left(\sqrt[3]{a^2}-\sqrt[3]{b^2}\right)}=\frac{\sqrt[3]{a}+\sqrt[3]{b}}{\sqrt[3]{a^2}-\sqrt[3]{b^2}}=\frac{\sqrt[3]{a}+\sqrt[3]{b}}{\left(\sqrt[3]{a}\right)^2-\left(\sqrt[3]{b}\right)^2}=$$

$$=\frac{\sqrt[3]{a}+\sqrt[3]{b}}{\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\cdot\left(\sqrt[3]{a}+\sqrt[3]{b}\right)}=\frac{\sqrt[3]{a}+\sqrt[3]{b}}{\left(\sqrt[3]{a}-\sqrt[3]{b}\right)\cdot\left(\sqrt[3]{a}+\sqrt[3]{b}\right)}=\frac{1}{\sqrt[3]{a}-\sqrt[3]{b}}.$$

Vježba 329

Pojednostavni izraz: $\dfrac{a\cdot\sqrt[3]{b}-b\cdot\sqrt[3]{a}}{\sqrt[3]{a^2\cdot b}+\sqrt[3]{a\cdot b^2}}.$

Rezultat: $\sqrt[3]{a}-\sqrt[3]{b}.$

Zadatak 330 (Vlatka, srednja škola)

Provjeri jednakost: $\sqrt{4+2\cdot\sqrt{3}}=1+\sqrt{3}.$

Rješenje 330

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad\sqrt{a^2}=a\;,\;a\ge 0\quad,\quad a\cdot\sqrt[n]{b}=\sqrt[n]{a^n\cdot b}.$$

$$\sqrt{A\pm\sqrt{B}}=\sqrt{\frac{A+\sqrt{A^2-B}}{2}}\pm\sqrt{\frac{A-\sqrt{A^2-B}}{2}}\;,\;A\ge 0\;,\;B\ge 0\;,\;A^2\ge B.$$

1.inačica

$$\sqrt{4+2\cdot\sqrt{3}}=1+\sqrt{3}\;\Rightarrow\;\sqrt{4+2\cdot\sqrt{3}}=1+\sqrt{3}\;/^2\;\Rightarrow\;\left(\sqrt{4+2\cdot\sqrt{3}}\right)^2=\left(1+\sqrt{3}\right)^2\;\Rightarrow$$

$$\Rightarrow\;4+2\cdot\sqrt{3}=1+2\cdot\sqrt{3}+\left(\sqrt{3}\right)^2\;\Rightarrow\;4+2\cdot\sqrt{3}=1+2\cdot\sqrt{3}+3\;\Rightarrow\;4+2\cdot\sqrt{3}=4+2\cdot\sqrt{3}.$$

2.inačica

$$\sqrt{4+2\cdot\sqrt{3}}=\sqrt{1+2\cdot\sqrt{3}+3}=\sqrt{1+2\cdot\sqrt{3}+\left(\sqrt{3}\right)^2}=\sqrt{\left(1+\sqrt{3}\right)^2}=1+\sqrt{3}.$$

3.inačica

$$1+\sqrt{3}=\sqrt{\left(1+\sqrt{3}\right)^2}=\sqrt{1+2\cdot\sqrt{3}+\left(\sqrt{3}\right)^2}=\sqrt{1+2\cdot\sqrt{3}+3}=\sqrt{4+2\cdot\sqrt{3}}.$$

4.inačica

$$\sqrt{4+2\cdot\sqrt{3}}=\sqrt{4+\sqrt{2^2\cdot 3}}=\sqrt{4+\sqrt{4\cdot 3}}=\sqrt{4+\sqrt{12}}=$$

$$=\sqrt{\frac{4+\sqrt{4^2-12}}{2}}+\sqrt{\frac{4-\sqrt{4^2-12}}{2}}=\sqrt{\frac{4+\sqrt{16-12}}{2}}+\sqrt{\frac{4-\sqrt{16-12}}{2}}=\sqrt{\frac{4+\sqrt{4}}{2}}+\sqrt{\frac{4-\sqrt{4}}{2}}=$$

$$=\sqrt{\frac{4+2}{2}}+\sqrt{\frac{4-2}{2}}=\sqrt{\frac{6}{2}}+\sqrt{\frac{2}{2}}=\sqrt{3}+\sqrt{1}=\sqrt{3}+1=1+\sqrt{3}.$$

Vježba 330

Provjeri jednakost: $\sqrt{3+2\cdot\sqrt{2}}=1+\sqrt{2}.$

Rezultat: Točno je.

Zadatak 331 (Ana, srednja škola)

Pojednostavni: $\frac{2\cdot x^2\cdot y-6\cdot x\cdot y^2}{x^3-9\cdot x\cdot y^2}$.

Rješenje 331

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{2\cdot x^2\cdot y-6\cdot x\cdot y^2}{x^3-9\cdot x\cdot y^2}=\frac{2\cdot x\cdot y\cdot(x-3\cdot y)}{x\cdot\left(x^2-9\cdot y^2\right)}=\frac{2\cdot x\cdot y\cdot(x-3\cdot y)}{x\cdot\left(x^2-(3\cdot y)^2\right)}=\frac{2\cdot x\cdot y\cdot(x-3\cdot y)}{x\cdot(x-3\cdot y)\cdot(x+3\cdot y)}=$$

$$=\frac{2\cdot x\cdot y\cdot(x-3\cdot y)}{x\cdot(x-3\cdot y)\cdot(x+3\cdot y)}=\frac{2\cdot y}{x+3\cdot y}.$$

Vježba 331

Pojednostavni: $\frac{x^3-9\cdot x\cdot y^2}{2\cdot x^2\cdot y-6\cdot x\cdot y^2}$.

Rezultat: $\frac{x+3\cdot y}{2\cdot y}$.

Zadatak 332 (Dora, srednja škola)

Ako je $m+n=5$ i $m\cdot n=3$, nađi m^2+n^2.

Rješenje 332

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$\left.\begin{array}{l} m+n=5 \\ m\cdot n=3 \end{array}\right\} \Rightarrow \left.\begin{array}{l} m+n=5 \;/^2 \\ m\cdot n=3 \end{array}\right\} \Rightarrow \left.\begin{array}{l} (m+n)^2=5^2 \\ m\cdot n=3 \end{array}\right\} \Rightarrow \left.\begin{array}{l} m^2+2\cdot m\cdot n+n^2=25 \\ m\cdot n=3 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{supstitucije} \end{array}\right] \Rightarrow m^2+2\cdot 3+n^2=25 \Rightarrow$$

$$\Rightarrow m^2+6+n^2=25 \Rightarrow m^2+n^2=25-6 \Rightarrow m^2+n^2=19.$$

Vježba 332

Ako je $m+n=5$ i $m\cdot n=6$, nađi m^2+n^2.

Rezultat: 13.

Zadatak 333 (Dora, srednja škola)

Nađi x iz izraza : $y=\frac{x+a}{1-a\cdot x}$.

Rješenje 333

Ponovimo!

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$y=\frac{x+a}{1-a\cdot x} \Rightarrow y=\frac{x+a}{1-a\cdot x} \,/\cdot(1-a\cdot x) \Rightarrow y\cdot(1-a\cdot x)=x+a \Rightarrow y-y\cdot a\cdot x=x+a \Rightarrow$$

$$\Rightarrow -y\cdot a\cdot x-x=a-y \Rightarrow -y\cdot a\cdot x-x=a-y \,/\cdot(-1) \Rightarrow y\cdot a\cdot x+x=-a+y \Rightarrow$$

$$\Rightarrow y\cdot a\cdot x+x=y-a \Rightarrow x\cdot(y\cdot a+1)=y-a \Rightarrow x\cdot(y\cdot a+1)=y-a \,/\cdot\frac{1}{y\cdot a+1} \Rightarrow$$

$$\Rightarrow x=\frac{y-a}{y\cdot a+1} \Rightarrow x=\frac{y-a}{1+a\cdot y}.$$

Vježba 333

Nađi a iz izraza : $y=\frac{x+a}{1-a\cdot x}$.

Rezultat: $\frac{y-x}{1+x\cdot y}$.

Zadatak 334 (Nikolina, srednja škola)

Dokaži jednakost : $\left(2\cdot\sqrt{2}+\sqrt{6}\right)\cdot\sqrt{7-4\cdot\sqrt{3}}=\sqrt{2}$.

Rješenje 334

Ponovimo!

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$a\cdot\sqrt{b}=\sqrt{a^2\cdot b} \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \left(a\cdot\sqrt{b}\right)^2=a^2\cdot b \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}.$$

$$(a+b)\cdot(a-b)=a^2-b^2 \quad , \quad \left(\sqrt{a}\right)^2=a \quad , \quad a=b \Rightarrow a^2=b^2 \quad , \quad (a\cdot b)^2=a^2\cdot b^2.$$

$$\sqrt{a^2\cdot b}=a\cdot\sqrt{b} \,,\; a>0 \quad , \quad \sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b}.$$

1.inačica

Lijevu stranu jednakosti transformiramo tako da dobijemo desnu stranu.

$$\left(2\cdot\sqrt{2}+\sqrt{6}\right)\cdot\sqrt{7-4\cdot\sqrt{3}}=\sqrt{\left(2\cdot\sqrt{2}+\sqrt{6}\right)^2\cdot\left(7-4\cdot\sqrt{3}\right)}=$$

$$=\sqrt{\left(\left(2\cdot\sqrt{2}\right)^2+2\cdot2\cdot\sqrt{2}\cdot\sqrt{6}+\left(\sqrt{6}\right)^2\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=$$

$$=\sqrt{\left(2^2\cdot\left(\sqrt{2}\right)^2+4\cdot\sqrt{2\cdot6}+6\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=\sqrt{\left(4\cdot2+4\cdot\sqrt{12}+6\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=$$

$$=\sqrt{\left(8+4\cdot\sqrt{4\cdot3}+6\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=\sqrt{\left(14+4\cdot\sqrt{4}\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=$$

$$=\sqrt{\left(14+4\cdot2\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=\sqrt{\left(14+8\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=\sqrt{2\cdot\left(7+4\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=$$

$$=\sqrt{2\cdot\left(7^2-\left(4\cdot\sqrt{3}\right)^2\right)}=\sqrt{2\cdot\left(49-4^2\cdot\left(\sqrt{3}\right)^2\right)}=\sqrt{2\cdot(49-16\cdot3)}=\sqrt{2\cdot(49-48)}=$$

$$=\sqrt{2\cdot(49-16\cdot3)}=\sqrt{2\cdot(49-48)}=\sqrt{2\cdot1}=\sqrt{2}.$$

2.inačica

Kvadriramo lijevu i desnu stranu jednakosti.

- Kvadrat lijeve strane

$$\left(\left(2\cdot\sqrt{2}+\sqrt{6}\right)\cdot\sqrt{7-4\cdot\sqrt{3}}\right)^2=\left(2\cdot\sqrt{2}+\sqrt{6}\right)^2\cdot\left(\sqrt{7-4\cdot\sqrt{3}}\right)^2=$$

$$=\left(\left(2\cdot\sqrt{2}\right)^2+2\cdot2\cdot\sqrt{2}\cdot\sqrt{6}+\left(\sqrt{6}\right)^2\right)\cdot\left(7-4\cdot\sqrt{3}\right)=$$

$$=\left(2^2\cdot\left(\sqrt{2}\right)^2+4\cdot\sqrt{2\cdot6}+6\right)\cdot\left(7-4\cdot\sqrt{3}\right)=\left(4\cdot2+4\cdot\sqrt{12}+6\right)\cdot\left(7-4\cdot\sqrt{3}\right)=$$

$$=\left(8+4\cdot\sqrt{4\cdot3}+6\right)\cdot\left(7-4\cdot\sqrt{3}\right)=\left(14+4\cdot\sqrt{4}\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)=\left(14+4\cdot2\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)=$$

$$=\left(14+8\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)=2\cdot\left(7+4\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)=2\cdot\left(7^2-\left(4\cdot\sqrt{3}\right)^2\right)=$$

$$=2\cdot\left(49-4^2\cdot\left(\sqrt{3}\right)^2\right)=2\cdot\left(49-16\cdot3\right)=2\cdot\left(49-48\right)=2\cdot1=2.$$

- Kvadrat desne strane

$$\left(\sqrt{2}\right)^2=2.$$

Uočimo da su oba izraza u zadanoj jednakosti bila pozitivna pa iz jednakosti kvadrata lijeve i desne strane slijedi da su korijeni tih kvadrata jednaki. Dakle, vrijedi zadana jednakost.

Vježba 334

Dokaži jednakost: $\sqrt{3+2\cdot\sqrt{2}}=1+\sqrt{2}.$

Rezultat: Točna je.

Zadatak 335 (Ivan, gimnazija)

Dokaži identitet: $a\cdot(a+1)^2-a\cdot(a-1)^2=4\cdot a^2.$

Rješenje 335

Ponovimo!

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$(x+y)^2=x^2+2\cdot x\cdot y+y^2 \quad , \quad (x-y)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad x=x^1.$$

$$x^m\cdot x^n=x^{m+n} \quad , \quad x^2-y^2=(x-y)\cdot(x+y).$$

1.inačica

Lijevu stranu jednakosti transformiramo tako da dobijemo desnu stranu.

$$a\cdot(a+1)^2-a\cdot(a-1)^2=a\cdot\left(a^2+2\cdot a+1\right)-a\cdot\left(a^2-2\cdot a+1\right)=$$

$$=a^3+2\cdot a^2+a-a^3+2\cdot a^2-a=a^3+2\cdot a^2+a-a^3+2\cdot a^2-a=4\cdot a^2.$$

2.inačica

Lijevu stranu jednakosti transformiramo tako da dobijemo desnu stranu.

$$a\cdot(a+1)^2-a\cdot(a-1)^2=a\cdot\left((a+1)^2-(a-1)^2\right)=a\cdot\left(a^2+2\cdot a+1-\left(a^2-2\cdot a+1\right)\right)=$$

$$=a\cdot\left(a^{2}+2\cdot a+1-a^{2}+2\cdot a-1\right)=a\cdot\left(a^{2}+2\cdot a+1-a^{2}+2\cdot a-1\right)=a\cdot 4\cdot a=4\cdot a^{2}.$$

3.inačica

Lijevu stranu jednakosti transformiramo tako da dobijemo desnu stranu.

$$a\cdot(a+1)^{2}-a\cdot(a-1)^{2}=a\cdot\left((a+1)^{2}-(a-1)^{2}\right)=a\cdot\left((a+1)-(a-1)\right)\cdot\left((a+1)+(a-1)\right)=$$

$$=a\cdot(a+1-a+1)\cdot(a+1+a-1)=a\cdot(a+1-a+1)\cdot(a+1+a-1)=a\cdot 2\cdot 2\cdot a=4\cdot a^{2}.$$

Vježba 335

Dokaži identitet : $a\cdot(1-a)^{2}-a\cdot(a+1)^{2}=-4\cdot a^{2}.$

Rezultat: Točan je.

Zadatak 336 (Ivan, gimnazija)

Racionaliziraj nazivnik razlomka: $\dfrac{\sqrt{7-4\cdot\sqrt{3}}}{\sqrt{7+4\cdot\sqrt{3}}}.$

Rješenje 336

Ponovimo!

$$\left(\sqrt{a}\right)^{2}=a\quad,\quad\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}\quad,\quad(a-b)\cdot(a+b)=a^{2}-b^{2}\quad,\quad\left(a\cdot\sqrt{b}\right)^{2}=a^{2}\cdot b.$$

$$n=\frac{n}{1}\quad,\quad(a\cdot b)^{2}=a^{2}\cdot b^{2}\quad,\quad\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}\quad,\quad\sqrt{a^{2}}=|a|.$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x|=\begin{cases}x, & x\geq 0\\ -x, & x<0.\end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x\geq 0$, vrijedi $|x|=x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, $x<0$, je $|x|=-x$.

1.inačica

$$\frac{\sqrt{7-4\cdot\sqrt{3}}}{\sqrt{7+4\cdot\sqrt{3}}}=\left[\begin{matrix}\text{racionalizacija}\\ \text{nazivnika}\end{matrix}\right]=\frac{\sqrt{7-4\cdot\sqrt{3}}}{\sqrt{7+4\cdot\sqrt{3}}}\cdot\frac{\sqrt{7+4\cdot\sqrt{3}}}{\sqrt{7+4\cdot\sqrt{3}}}=\frac{\sqrt{7-4\cdot\sqrt{3}}\cdot\sqrt{7+4\cdot\sqrt{3}}}{\left(\sqrt{7+4\cdot\sqrt{3}}\right)^{2}}=$$

$$=\frac{\sqrt{\left(7-4\cdot\sqrt{3}\right)\cdot\left(7+4\cdot\sqrt{3}\right)}}{7+4\cdot\sqrt{3}}=\frac{\sqrt{7^{2}-\left(4\cdot\sqrt{3}\right)^{2}}}{7+4\cdot\sqrt{3}}=\frac{\sqrt{7^{2}-4^{2}\cdot\left(\sqrt{3}\right)^{2}}}{7+4\cdot\sqrt{3}}=\frac{\sqrt{49-16\cdot 3}}{7+4\cdot\sqrt{3}}=$$

$$=\frac{\sqrt{49-48}}{7+4\cdot\sqrt{3}}=\frac{\sqrt{1}}{7+4\cdot\sqrt{3}}=\frac{1}{7+4\cdot\sqrt{3}}=\left[\begin{matrix}\text{racionalizacija}\\ \text{nazivnika}\end{matrix}\right]=\frac{1}{7+4\cdot\sqrt{3}}\cdot\frac{7-4\cdot\sqrt{3}}{7-4\cdot\sqrt{3}}=$$

$$=\frac{7-4\cdot\sqrt{3}}{\left(7+4\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=\frac{7-4\cdot\sqrt{3}}{7^{2}-\left(4\cdot\sqrt{3}\right)^{2}}=\frac{7-4\cdot\sqrt{3}}{7^{2}-4^{2}\cdot\left(\sqrt{3}\right)^{2}}=\frac{7-4\cdot\sqrt{3}}{49-16\cdot 3}=$$

$$=\frac{7-4\cdot\sqrt{3}}{49-48}=\frac{7-4\cdot\sqrt{3}}{1}=7-4\cdot\sqrt{3}.$$

2.inačica

$$\frac{\sqrt{7-4\cdot\sqrt{3}}}{\sqrt{7+4\cdot\sqrt{3}}}=\sqrt{\frac{7-4\cdot\sqrt{3}}{7+4\cdot\sqrt{3}}}=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=\sqrt{\frac{7-4\cdot\sqrt{3}}{7+4\cdot\sqrt{3}}\cdot\frac{7-4\cdot\sqrt{3}}{7-4\cdot\sqrt{3}}}=\sqrt{\frac{\left(7-4\cdot\sqrt{3}\right)^2}{\left(7+4\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)}}=$$

$$=\sqrt{\frac{\left(7-4\cdot\sqrt{3}\right)^2}{7^2-\left(4\cdot\sqrt{3}\right)^2}}=\sqrt{\frac{\left(7-4\cdot\sqrt{3}\right)^2}{7^2-4^2\cdot\left(\sqrt{3}\right)^2}}=\sqrt{\frac{\left(7-4\cdot\sqrt{3}\right)^2}{49-16\cdot 3}}=\sqrt{\frac{\left(7-4\cdot\sqrt{3}\right)^2}{49-48}}=$$

$$=\sqrt{\frac{\left(7-4\cdot\sqrt{3}\right)^2}{1}}=\sqrt{\left(7-4\cdot\sqrt{3}\right)^2}=\left|7-4\cdot\sqrt{3}\right|=\left[\,7-4\cdot\sqrt{3}>0\,\right]=7-4\cdot\sqrt{3}.$$

Vježba 336

Racionaliziraj nazivnik razlomka: $\dfrac{\sqrt{\sqrt{2}-1}}{\sqrt{\sqrt{2}+1}}$.

Rezultat: $\sqrt{2}-1.$

Zadatak 337 (Barbara, srednja škola)

Racionaliziraj nazivnik razlomka: $\dfrac{1}{\sqrt{10}+\sqrt{15}+\sqrt{14}+\sqrt{21}}$.

Rješenje 337

Ponovimo!

$$\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}\quad,\quad(a-b)\cdot(a+b)=a^2-b^2\quad,\quad\left(\sqrt{a}\right)^2=a.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{1}{\sqrt{10}+\sqrt{15}+\sqrt{14}+\sqrt{21}}=\frac{1}{\sqrt{2\cdot 5}+\sqrt{3\cdot 5}+\sqrt{2\cdot 7}+\sqrt{3\cdot 7}}=$$

$$=\frac{1}{\sqrt{2}\cdot\sqrt{5}+\sqrt{3}\cdot\sqrt{5}+\sqrt{2}\cdot\sqrt{7}+\sqrt{3}\cdot\sqrt{7}}=\frac{1}{\sqrt{5}\cdot\left(\sqrt{2}+\sqrt{3}\right)+\sqrt{7}\cdot\left(\sqrt{2}+\sqrt{3}\right)}=$$

$$=\frac{1}{\left(\sqrt{2}+\sqrt{3}\right)\cdot\left(\sqrt{5}+\sqrt{7}\right)}=\frac{1}{\left(\sqrt{3}+\sqrt{2}\right)\cdot\left(\sqrt{7}+\sqrt{5}\right)}=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=$$

$$=\frac{1}{\left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{7}+\sqrt{5}\right)}\cdot\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}\right)\cdot\left(\sqrt{7}+\sqrt{5}\right)\cdot\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}=$$

$$=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}\right)\cdot\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}+\sqrt{5}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{\left(\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2\right)\cdot\left(\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2\right)}=$$

$$=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{(3-2)\cdot(7-5)}=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{1\cdot 2}=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{2}.$$

Vježba 337

Racionaliziraj nazivnik razlomka: $\frac{1}{2+\sqrt{2}+\sqrt{3}+\sqrt{6}}$.

Rezultat: $\left(\sqrt{2}-1\right)\cdot\left(\sqrt{3}-\sqrt{2}\right)$.

Zadatak 338 (Ante, srednja škola)

Pojednostavni izraz: $\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}$.

Rješenje 338

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \left(\sqrt{x}\right)^2=x.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=\frac{(a-b)\cdot\left(\sqrt{a}+\sqrt{b}\right)-(a-b)\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}=$$

$$=\frac{a\cdot\sqrt{a}+a\cdot\sqrt{b}-b\cdot\sqrt{a}-b\cdot\sqrt{b}-\left(a\cdot\sqrt{a}-a\cdot\sqrt{b}-b\cdot\sqrt{a}+b\cdot\sqrt{b}\right)}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}=$$

$$=\frac{a\cdot\sqrt{a}+a\cdot\sqrt{b}-b\cdot\sqrt{a}-b\cdot\sqrt{b}-a\cdot\sqrt{a}+a\cdot\sqrt{b}+b\cdot\sqrt{a}-b\cdot\sqrt{b}}{a-b}=$$

$$=\frac{a\cdot\sqrt{a}+a\cdot\sqrt{b}-b\cdot\sqrt{a}-b\cdot\sqrt{b}-a\cdot\sqrt{a}+a\cdot\sqrt{b}+b\cdot\sqrt{a}-b\cdot\sqrt{b}}{a-b}=$$

$$=\frac{a\cdot\sqrt{b}-b\cdot\sqrt{b}+a\cdot\sqrt{b}-b\cdot\sqrt{b}}{a-b}=\frac{2\cdot a\cdot\sqrt{b}-2\cdot b\cdot\sqrt{b}}{a-b}=\frac{2\cdot\sqrt{b}\cdot(a-b)}{a-b}=\frac{2\cdot\sqrt{b}\cdot(a-b)}{a-b}=2\cdot\sqrt{b}.$$

2.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=\frac{(a-b)\cdot\left(\sqrt{a}+\sqrt{b}\right)-(a-b)\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}=$$

$$=\frac{(a-b)\cdot\left(\left(\sqrt{a}+\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)\right)}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}=\frac{(a-b)\cdot\left(\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}\right)}{a-b}=$$

$$=\frac{(a-b)\cdot\left(\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}\right)}{a-b}=\frac{(a-b)\cdot 2\cdot\sqrt{b}}{a-b}=\frac{(a-b)\cdot 2\cdot\sqrt{b}}{a-b}=2\cdot\sqrt{b}.$$

3.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=\frac{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}{\sqrt{a}-\sqrt{b}}-\frac{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}{\sqrt{a}+\sqrt{b}}=$$

$$=\frac{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}-\sqrt{b}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}+\sqrt{b}}=$$

$$=\frac{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}-\sqrt{b}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}+\sqrt{b}}=\left(\sqrt{a}+\sqrt{b}\right)-\left(\sqrt{a}-\sqrt{b}\right)=$$

$$=\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}=\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}=2\cdot\sqrt{b}.$$

4.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=(a-b)\cdot\left(\frac{1}{\sqrt{a}-\sqrt{b}}-\frac{1}{\sqrt{a}+\sqrt{b}}\right)=(a-b)\cdot\frac{\sqrt{a}+\sqrt{b}-\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}=$$

$$=(a-b)\cdot\frac{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}=(a-b)\cdot\frac{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}}{a-b}=$$

$$=(a-b)\cdot\frac{2\cdot\sqrt{b}}{a-b}=(a-b)\cdot\frac{2\cdot\sqrt{b}}{a-b}=2\cdot\sqrt{b}.$$

Vježba 338

Pojednostavni izraz: $\frac{a-b}{\sqrt{a}-\sqrt{b}}+\frac{b-a}{\sqrt{a}+\sqrt{b}}.$

Rezultat: $2\cdot\sqrt{b}.$

Zadatak 339 (Ivana, gimnazija)

Ako je $a=3^{40}$, $b=4^{30}$, $c=2^{70}$, tada je:

$A.\ b<a<c \qquad B.\ b<c<a \qquad C.\ a<b<c \qquad D.\ a<c<b$

Rješenje 339

Ponovimo!

$\left(a^n\right)^m=a^{n\cdot m}$, $0<b<a\ \Rightarrow\ b^n<a^n$, $\log a^n=n\cdot\log a$, $\log a<\log b\ \Rightarrow\ a<b.$

1.inačica

Transformiramo zadane potencije na isti eksponent.

$$\left.\begin{matrix}a=3^{40}\\ b=4^{30}\\ c=2^{70}\end{matrix}\right\}\Rightarrow\left.\begin{matrix}a=\left(3^4\right)^{10}\\ b=\left(4^3\right)^{10}\\ c=\left(2^7\right)^{10}\end{matrix}\right\}\Rightarrow\left.\begin{matrix}a=81^{10}\\ b=64^{10}\\ c=128^{10}\end{matrix}\right\}.$$

Sada je:

$$64^{10}<81^{10}<128^{10}\ \Rightarrow\ b<a<c.$$

Odgovor je pod A.

2.inačica

Logaritmiramo zadane potencije i usporedimo njihove vrijednosti.

$$\left.\begin{array}{l} a=3^{40} \\ b=4^{30} \\ c=2^{70} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=3^{40} \quad /\log \\ b=4^{30} \quad /\log \\ c=2^{70} \quad /\log \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log a=\log 3^{40} \\ \log b=\log 4^{30} \\ \log c=\log 2^{70} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log a=40\cdot\log 3 \\ \log b=30\cdot\log 4 \\ \log c=70\cdot\log 2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} \log a=40\cdot 0.47712 \\ \log b=30\cdot 0.60206 \\ \log c=70\cdot 0.30103 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log a=19.0848 \\ \log b=18.0618 \\ \log c=21.0721 \end{array}\right\}.$$

Sada je:

$$\log b<\log a<\log c \;\Rightarrow\; b<a<c.$$

Vježba 339

Ako je $a=3^{40}$, $b=4^{30}$, $c=2^{50}$, tada je:

A. $c<b<a$ *B.* $b<c<a$ *C.* $c<a<b$ *D.* $a<c<b$

Rezultat: A.

Zadatak 340 (Snk, srednja škola)

Izračunaj: $3\cdot(2\cdot x+5)-(x-4)\cdot(2\cdot x+5)$.

Rješenje 340

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$3\cdot(2\cdot x+5)-(x-4)\cdot(2\cdot x+5)=6\cdot x+15-\left(2\cdot x^2+5\cdot x-8\cdot x-20\right)=$$

$$=6\cdot x+15-2\cdot x^2-5\cdot x+8\cdot x+20=\underline{6\cdot x}\underline{\underline{+15}}-2\cdot x^2\underline{-5\cdot x}\underline{+8\cdot x}\underline{\underline{+20}}=-2\cdot x^2+9\cdot x+35.$$

2.inačica

$$3\cdot(2\cdot x+5)-(x-4)\cdot(2\cdot x+5)=3\cdot\underline{(2\cdot x+5)}-(x-4)\cdot\underline{(2\cdot x+5)}=(2\cdot x+5)\cdot(3-(x-4))=$$

$$=(2\cdot x+5)\cdot(3-x+4)=(2\cdot x+5)\cdot(7-x)=14\cdot x-2\cdot x^2+35-5\cdot x=\underline{14\cdot x}-2\cdot x^2+35\underline{-5\cdot x}=$$

$$=-2\cdot x^2+9\cdot x+35.$$

Vježba 340

Izračunaj: $3\cdot(2\cdot x+5)+(4-x)\cdot(2\cdot x+5)$.

Rezultat: $-2\cdot x^2+9\cdot x+35.$