Zadatak 301 (Ivana, gimnazija)

Pojednostavnite: $\left(\sqrt[3]{\frac{a^2}{b^2}}+\sqrt[3]{\frac{b^2}{a^2}}+1\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right).$

Rješenje 301

Ponovimo!

$$\left(\frac{x}{y}\right)^n=\frac{x^n}{y^n}\quad,\quad x^3-y^3=(x-y)\cdot\left(x^2+x\cdot y+y^2\right)\quad,\quad\left(\sqrt[3]{x}\right)^3=x\quad,\quad x^1=x.$$

$$\sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}\quad,\quad\frac{x}{y}-\frac{y}{x}=\frac{x^2-y^2}{x\cdot y}\quad,\quad x^n\cdot x^m=x^{n+m}\quad,\quad\frac{x}{y}\cdot\frac{y}{x}=1.$$

$$\left(\sqrt[3]{\frac{a^2}{b^2}}+\sqrt[3]{\frac{b^2}{a^2}}+1\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right)=\left(\sqrt[3]{\left(\frac{a}{b}\right)^2}+\sqrt[3]{\left(\frac{b}{a}\right)^2}+1\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right)=$$

$$=\left(\sqrt[3]{\left(\frac{a}{b}\right)^2}+1+\sqrt[3]{\left(\frac{b}{a}\right)^2}\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right)=\left(\sqrt[3]{\left(\frac{a}{b}\right)^2}+\sqrt[3]{\frac{a}{b}\cdot\frac{b}{a}}+\sqrt[3]{\left(\frac{b}{a}\right)^2}\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right)=$$

$$=\left(\sqrt[3]{\left(\frac{a}{b}\right)^2}+\sqrt[3]{\frac{a}{b}}\cdot\sqrt[3]{\frac{b}{a}}+\sqrt[3]{\left(\frac{b}{a}\right)^2}\right)\cdot\left(\sqrt[3]{\frac{a}{b}}-\sqrt[3]{\frac{b}{a}}\right)=$$

$$=\left(\sqrt[3]{\frac{a}{b}}\right)^3-\left(\sqrt[3]{\frac{b}{a}}\right)^3=\left(\sqrt[3]{\frac{a}{b}}\right)^3-\left(\sqrt[3]{\frac{b}{a}}\right)^3=\frac{a}{b}-\frac{b}{a}=\frac{a^2-b^2}{a\cdot b}.$$

Vježba 301

Pojednostavnite: $\left(\sqrt[3]{\frac{a^2}{b^2}}+\sqrt[3]{\frac{b^2}{a^2}}-1\right)\cdot\left(\sqrt[3]{\frac{a}{b}}+\sqrt[3]{\frac{b}{a}}\right).$

Rezultat: $\frac{a^2+b^2}{a\cdot b}.$

Zadatak 302 (Matija, gimnazija)

Jednostavniji zapis razlomka $\frac{\left(x^{-1}-y^{-1}\right)^{-2}}{\left(x^{-2}-y^{-2}\right)^{-1}}$ je:

$A)\ \frac{1}{y+x}\qquad B)\ \frac{x-y}{y+x}\qquad C)\ \frac{x+y}{y-x}\qquad D)\ 1$

Rješenje 302

Ponovimo!

$$a^{-n}=\frac{1}{a^n}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n\quad,\quad a^1=a\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$\frac{n\cdot a}{n\cdot b}=\frac{a}{b}\ ,\ n\neq 0\quad ,\quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad ,\quad (a\cdot b)^n=a^n\cdot b^n\quad ,\quad a^2-b^2=(a-b)\cdot(a+b).$

1.inačica

$$\frac{\left(x^{-1}-y^{-1}\right)^{-2}}{\left(x^{-2}-y^{-2}\right)^{-1}}=\frac{\left(x^{-2}-y^{-2}\right)^{1}}{\left(x^{-1}-y^{-1}\right)^{2}}=\frac{x^{-2}-y^{-2}}{\left(x^{-1}-y^{-1}\right)^{2}}=\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\left(\frac{1}{x}-\frac{1}{y}\right)^2}=\frac{\frac{y^2-x^2}{x^2\cdot y^2}}{\left(\frac{y-x}{x\cdot y}\right)^2}=$$

$$=\frac{\frac{y^2-x^2}{x^2\cdot y^2}}{\frac{(y-x)^2}{(x\cdot y)^2}}=\frac{\frac{y^2-x^2}{x^2\cdot y^2}}{\frac{(y-x)^2}{x^2\cdot y^2}}=\frac{\frac{y^2-x^2}{x^2\cdot y^2}}{\frac{(y-x)^2}{x^2\cdot y^2}}=\frac{\frac{y^2-x^2}{1}}{\frac{(y-x)^2}{1}}=\frac{y^2-x^2}{(y-x)^2}=\frac{(y-x)\cdot(y+x)}{(y-x)\cdot(y-x)}=$$

$$=\frac{(y-x)\cdot(y+x)}{(y-x)\cdot(y-x)}=\frac{y+x}{y-x}=\frac{x+y}{y-x}.$$

Odgovor je pod C.

2.inačica

$$\frac{\left(x^{-1}-y^{-1}\right)^{-2}}{\left(x^{-2}-y^{-2}\right)^{-1}}=\frac{\left(\frac{1}{x}-\frac{1}{y}\right)^{-2}}{\left(\frac{1}{x^2}-\frac{1}{y^2}\right)^{-1}}=\frac{\left(\frac{y-x}{x\cdot y}\right)^{-2}}{\left(\frac{y^2-x^2}{x^2\cdot y^2}\right)^{-1}}=\frac{\left(\frac{x\cdot y}{y-x}\right)^{2}}{\left(\frac{x^2\cdot y^2}{y^2-x^2}\right)^{1}}=\frac{\left(\frac{x\cdot y}{y-x}\right)^{2}}{\frac{x^2\cdot y^2}{y^2-x^2}}=$$

$$=\frac{\frac{(x\cdot y)^2}{(y-x)^2}}{\frac{x^2\cdot y^2}{y^2-x^2}}=\frac{\frac{x^2\cdot y^2}{(y-x)^2}}{\frac{x^2\cdot y^2}{y^2-x^2}}=\frac{\frac{x^2\cdot y^2}{(y-x)^2}}{\frac{x^2\cdot y^2}{y^2-x^2}}=\frac{\frac{1}{(y-x)^2}}{\frac{1}{y^2-x^2}}=\frac{y^2-x^2}{(y-x)^2}=\frac{(y-x)\cdot(y+x)}{(y-x)\cdot(y-x)}=$$

$$=\frac{(y-x)\cdot(y+x)}{(y-x)\cdot(y-x)}=\frac{y+x}{y-x}=\frac{x+y}{y-x}.$$

Odgovor je pod C.

3.inačica

$$\frac{\left(x^{-1}-y^{-1}\right)^{-2}}{\left(x^{-2}-y^{-2}\right)^{-1}}=\frac{\left(x^{-2}-y^{-2}\right)^{1}}{\left(x^{-1}-y^{-1}\right)^{2}}=\frac{x^{-2}-y^{-2}}{\left(x^{-1}-y^{-1}\right)^{2}}=\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\left(\frac{1}{x}-\frac{1}{y}\right)^2}=\frac{\left(\frac{1}{x}-\frac{1}{y}\right)\cdot\left(\frac{1}{x}+\frac{1}{y}\right)}{\left(\frac{1}{x}-\frac{1}{y}\right)\cdot\left(\frac{1}{x}-\frac{1}{y}\right)}=$$

$$=\frac{\left(\frac{1}{x}-\frac{1}{y}\right)\cdot\left(\frac{1}{x}+\frac{1}{y}\right)}{\left(\frac{1}{x}-\frac{1}{y}\right)\cdot\left(\frac{1}{x}-\frac{1}{y}\right)}=\frac{\frac{1}{x}+\frac{1}{y}}{\frac{1}{x}-\frac{1}{y}}=\frac{\frac{y+x}{x\cdot y}}{\frac{y-x}{x\cdot y}}=\frac{\frac{y+x}{x\cdot y}}{\frac{y-x}{x\cdot y}}=\frac{\frac{y+x}{1}}{\frac{y-x}{1}}=\frac{y+x}{y-x}.$$

Odgovor je pod C.

Vježba 302

Jednostavniji zapis razlomka $\dfrac{\left(x^{-2}-y^{-2}\right)^{-1}}{\left(x^{-1}-y^{-1}\right)^{-2}}$ je:

$A)\ y+x \qquad B)\ \dfrac{y+x}{x-y} \qquad C)\ \dfrac{y-x}{x+y} \qquad D)\ 1$

Rezultat: C.

Zadatak 303 (Vedrana, gimnazija)

Pojednostavnite izraz: $\left(x^2-1\right)\cdot\left(1-\dfrac{1}{x+1}+\dfrac{1}{x-1}\right)$.

Rješenje 303

Ponovimo!

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$-a+a=0 \quad , \quad n\cdot\frac{a}{n}=a.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left(x^2-1\right)\cdot\left(1-\frac{1}{x+1}+\frac{1}{x-1}\right)=\left(x^2-1\right)\cdot\left(\frac{1}{1}-\frac{1}{x+1}+\frac{1}{x-1}\right)=\left(x^2-1\right)\cdot\frac{(x+1)\cdot(x-1)-(x-1)+(x+1)}{(x+1)\cdot(x-1)}=$$

$$=\left(x^2-1\right)\cdot\frac{x^2-1-x+1+x+1}{x^2-1}=\left(x^2-1\right)\cdot\frac{x^2-1-x+1+x+1}{x^2-1}=\left(x^2-1\right)\cdot\frac{x^2+1}{x^2-1}=$$

$$=\left(x^2-1\right)\cdot\frac{x^2+1}{x^2-1}=x^2+1.$$

2.inačica

$$\left(x^2-1\right)\cdot\left(1-\frac{1}{x+1}+\frac{1}{x-1}\right)=(x-1)\cdot(x+1)\cdot\left(1-\frac{1}{x+1}+\frac{1}{x-1}\right)=$$

$$=(x-1)\cdot(x+1)-(x-1)\cdot(x+1)\cdot\frac{1}{x+1}+(x-1)\cdot(x+1)\cdot\frac{1}{x-1}=$$

$$=(x-1)\cdot(x+1)-(x-1)\cdot(x+1)\cdot\frac{1}{x+1}+(x-1)\cdot(x+1)\cdot\frac{1}{x-1}=(x-1)\cdot(x+1)-(x-1)+(x+1)=$$

$$=x^2-1-x+1+x+1=x^2-1-x+1+x+1=x^2+1.$$

3.inačica

$$\left(x^2-1\right)\cdot\left(1-\frac{1}{x+1}+\frac{1}{x-1}\right)=\left(x^2-1\right)\cdot\left(1-\left(\frac{1}{x+1}+\frac{1}{x-1}\right)\right)=\left(x^2-1\right)\cdot\left(1-\frac{x-1-(x+1)}{(x+1)\cdot(x-1)}\right)=$$

$$=\left(x^2-1\right)\cdot\left(1-\frac{x-1-x-1}{(x+1)\cdot(x-1)}\right)=\left(x^2-1\right)\cdot\left(1-\frac{x-1-x-1}{(x+1)\cdot(x-1)}\right)=\left(x^2-1\right)\cdot\left(1-\frac{-2}{x^2-1}\right)=$$

$$=\left(x^2-1\right)\cdot\left(1+\frac{2}{x^2-1}\right)=\left(x^2-1\right)\cdot\left(\frac{1}{1}+\frac{2}{x^2-1}\right)=\left(x^2-1\right)\cdot\frac{x^2-1+2}{x^2-1}=\left(x^2-1\right)\cdot\frac{x^2+1}{x^2-1}=$$

$$=\left(x^2-1\right)\cdot\frac{x^2+1}{x^2-1}=x^2+1.$$

Vježba 303

Pojednostavnite izraz: $\left(1-x^2\right)\cdot\left(\frac{1}{x+1}-\frac{1}{x-1}-1\right)$.

Rezultat: x^2+1.

Zadatak 304 (Josipa, gimnazija)

Izraz $\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(a^{-1}+b^{-1}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)$ jednak je :

A) $\frac{1}{a\cdot b}$ B) $\sqrt{a\cdot b}$ C) $\frac{1}{\sqrt{a\cdot b}}$ D) $a\cdot b$

Rješenje 304

Ponovimo!

$x^{-n}=\frac{1}{x^n}$, $\frac{x}{y}+\frac{m}{n}=\frac{x\cdot n+y\cdot m}{y\cdot n}$, $\sqrt{x}\cdot\sqrt{y}=\sqrt{x\cdot y}$, $\left(\sqrt{x}\right)^2=x$.

$x^2+2\cdot x\cdot y+y^2=(x+y)^2$, $\frac{x\cdot n}{y\cdot n}=\frac{x}{y}$, $n\neq 0$, $\frac{x}{y}\cdot\frac{m}{n}=\frac{x\cdot m}{y\cdot n}$.

Zakon distribucije množenja prema zbrajanju

$a\cdot(b+c)=a\cdot b+a\cdot c$, $a\cdot b+a\cdot c=a\cdot(b+c)$.

1.inačica

$$\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(a^{-1}+b^{-1}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)=$$

$$=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)=$$

$$=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{b+a}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\frac{\sqrt{b}+\sqrt{a}}{\sqrt{a}\cdot\sqrt{b}}=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{a+b}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a\cdot b}}=$$

$$=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{a+b}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a\cdot b}}=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{a+b}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{1}{\sqrt{a\cdot b}}=$$

$$=\frac{a+b}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{2}{\sqrt{a\cdot b}\cdot\left(\sqrt{a}+\sqrt{b}\right)^2}=\frac{a+b}{\left(\sqrt{a\cdot b}\right)^2\cdot\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{2}{\sqrt{a\cdot b}\cdot\left(\sqrt{a}+\sqrt{b}\right)^2}=$$

$$= \frac{a+b+2\cdot\sqrt{a\cdot b}}{\left(\sqrt{a\cdot b}\right)^2\cdot\left(\sqrt{a}+\sqrt{b}\right)^2} = \frac{a+2\cdot\sqrt{a\cdot b}+b}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^2} = \frac{\left(\sqrt{a}\right)^2+2\cdot\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^2} =$$

$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^2} = \frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^2} = \frac{1}{a\cdot b}.$$

Odgovor je pod A.

2.inačica

$$\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(a^{-1}+b^{-1}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right) =$$

$$= \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right) =$$

$$= \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{b+a}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\frac{\sqrt{b}+\sqrt{a}}{\sqrt{a}\cdot\sqrt{b}} = \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{a+b}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a\cdot b}} =$$

$$= \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{a+b}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a\cdot b}} = \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{a+b}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{1}{\sqrt{a\cdot b}} =$$

$$= \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\left(\frac{a+b}{a\cdot b}+\frac{2}{\sqrt{a\cdot b}}\right) = \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\left(\frac{a+b}{\left(\sqrt{a\cdot b}\right)^2}+\frac{2}{\sqrt{a\cdot b}}\right) =$$

$$= \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{a+b+2\cdot\sqrt{a\cdot b}}{\left(\sqrt{a\cdot b}\right)^2} = \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{a+2\cdot\sqrt{a\cdot b}+b}{a\cdot b} =$$

$$= \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{\left(\sqrt{a}\right)^2+2\cdot\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2}{a\cdot b} = \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{a\cdot b} =$$

$$= \frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\frac{\left(\sqrt{a}+\sqrt{b}\right)^2}{a\cdot b} = \frac{1}{a\cdot b}.$$

Odgovor je pod A.

Vježba 304

Izraz $\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)$ jednak je :

A) $\frac{1}{\sqrt{a\cdot b}\cdot\left(\sqrt{a}+\sqrt{b}\right)}$ B) $\frac{\sqrt{a\cdot b}}{\sqrt{a}+\sqrt{b}}$ C) $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a\cdot b}}$ D) $a\cdot b$

Rezultat: A.

Zadatak 305 (Marijan, gimnazija)

Pojednostavni izraz: $\dfrac{\dfrac{a^2}{a^3+8}-\dfrac{1}{a}}{\dfrac{a+2}{a}-\dfrac{a+4}{a+2}}+\dfrac{a^2+4}{a^3+8}-\dfrac{1}{a+2}$.

Rješenje 305

Ponovimo!

$x^3+y^3=(x+y)\cdot\left(x^2-x\cdot y+y^2\right)$, $\dfrac{x}{y}-\dfrac{m}{n}=\dfrac{x\cdot n-y\cdot m}{y\cdot n}$, $\dfrac{x}{y}+\dfrac{m}{n}=\dfrac{x\cdot n+y\cdot m}{y\cdot n}$.

$-x+x=0$, $x=\dfrac{x}{1}$, $\dfrac{\dfrac{x}{y}}{\dfrac{m}{n}}=\dfrac{x\cdot n}{y\cdot m}$, $x^n\cdot x^m=x^{n+m}$, $(x+y)^2=x^2+2\cdot x\cdot y+y^2$.

Zakon distribucije množenja prema zbrajanju

$a\cdot(b+c)=a\cdot b+a\cdot c$, $a\cdot b+a\cdot c=a\cdot(b+c)$.

$$\frac{\dfrac{a^2}{a^3+8}-\dfrac{1}{a}}{\dfrac{a+2}{a}-\dfrac{a+4}{a+2}}+\frac{a^2+4}{a^3+8}-\frac{1}{a+2}=\frac{\dfrac{a^2}{a^3+2^3}-\dfrac{1}{a}}{\dfrac{a+2}{a}-\dfrac{a+4}{a+2}}+\frac{a^2+4}{a^3+2^3}-\frac{1}{a+2}=$$

$$=\frac{\dfrac{a^2}{(a+2)\cdot\left(a^2-2\cdot a+4\right)}-\dfrac{1}{a}}{\dfrac{a+2}{a}-\dfrac{a+4}{a+2}}+\frac{a^2+4}{(a+2)\cdot\left(a^2-2\cdot a+4\right)}-\frac{1}{a+2}=$$

$$=\frac{\dfrac{a^3-(a+2)\cdot\left(a^2-2\cdot a+4\right)}{a\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}}{\dfrac{(a+2)^2-a\cdot(a+4)}{a\cdot(a+2)}}+\frac{a^2+4-\left(a^2-2\cdot a+4\right)}{(a+2)\cdot\left(a^2-2\cdot a+4\right)}=$$

$$=\frac{\dfrac{a^3-\left(a^3+2^3\right)}{a\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}}{\dfrac{a^2+4\cdot a+4-a^2-4\cdot a}{a\cdot(a+2)}}+\frac{a^2+4-a^2+2\cdot a-4}{(a+2)\cdot\left(a^2-2\cdot a+4\right)}=$$

$$=\frac{\dfrac{a^3-\left(a^3+8\right)}{a\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}}{\dfrac{a^2+4\cdot a+4-a^2-4\cdot a}{a\cdot(a+2)}}+\frac{a^2+4-a^2+2\cdot a-4}{(a+2)\cdot\left(a^2-2\cdot a+4\right)}=$$

$$= \frac{\dfrac{a^3 - a^3 - 8}{a\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}}{\dfrac{4}{a\cdot(a+2)}} + \frac{2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} =$$

$$= \frac{\dfrac{a^3 - a^3 - 8}{a\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}}{\dfrac{4}{a\cdot(a+2)}} + \frac{2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} =$$

$$= \frac{\dfrac{-8}{a\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}}{\dfrac{4}{a\cdot(a+2)}} + \frac{2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} =$$

$$= \frac{\dfrac{-8}{a\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}}{\dfrac{4}{a\cdot(a+2)}} + \frac{2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} = \frac{\dfrac{-2}{a^2-2\cdot a+4}}{\dfrac{1}{1}} + \frac{2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} =$$

$$= \frac{-2}{a^2-2\cdot a+4} + \frac{2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} = \frac{-2\cdot(a+2)+2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} =$$

$$= \frac{-2\cdot a-4+2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} = \frac{-2\cdot a-4+2\cdot a}{(a+2)\cdot\left(a^2-2\cdot a+4\right)} = \frac{-4}{a^3+2^3} = \frac{-4}{a^3+8}.$$

Vježba 305

Pojednostavni izraz: $\dfrac{\dfrac{1}{a} - \dfrac{a^2}{a^3+8}}{\dfrac{a+4}{a+2} - \dfrac{a+2}{a}} + \dfrac{a^2+4}{a^3+8} - \dfrac{1}{a+2}.$

Rezultat: $\dfrac{-4}{a^3+8}.$

Zadatak 306 (Ana, gimnazija)

Ako je $\dfrac{x}{y} + x = \dfrac{y}{x} + y$, $x \neq y$, koliko je $\dfrac{1}{x} + \dfrac{1}{y}$?

Rješenje 306

Ponovimo!

$\dfrac{a}{b} - \dfrac{c}{d} = \dfrac{a\cdot d - b\cdot c}{b\cdot d}$, $a^2 - b^2 = (a-b)\cdot(a+b)$, $\dfrac{a+b}{n} = \dfrac{a}{n} + \dfrac{b}{n}$, $\dfrac{a\cdot n}{b\cdot n} = \dfrac{a}{b}$, $n \neq 0$.

Zakon distribucije množenja prema zbrajanju

$a\cdot(b+c) = a\cdot b + a\cdot c$, $a\cdot b + a\cdot c = a\cdot(b+c)$.

$$\frac{x}{y}+x=\frac{y}{x}+y \Rightarrow \frac{x}{y}-\frac{y}{x}=y-x \Rightarrow \frac{x^2-y^2}{x\cdot y}=y-x \Rightarrow \frac{(x-y)\cdot(x+y)}{x\cdot y}=y-x \Rightarrow$$

$$\Rightarrow \frac{(x-y)\cdot(x+y)}{x\cdot y}=-(x-y) \Rightarrow \frac{(x-y)\cdot(x+y)}{x\cdot y}=-(x-y) \;/\cdot \frac{1}{x-y} \Rightarrow$$

$$\Rightarrow \frac{x+y}{x\cdot y}=-1 \Rightarrow \frac{x}{x\cdot y}+\frac{y}{x\cdot y}=-1 \Rightarrow \frac{x}{x\cdot y}+\frac{y}{x\cdot y}=-1 \Rightarrow \frac{1}{y}+\frac{1}{x}=-1 \Rightarrow \frac{1}{x}+\frac{1}{y}=-1.$$

Vježba 306

Ako je $\frac{x}{y}-x=\frac{y}{x}-y$, $x\neq y$, koliko je $\frac{1}{x}+\frac{1}{y}$?

Rezultat: 1.

Zadatak 307 (Ivan, gimnazija)

Izračunaj: $\frac{a^{-3}+a^{-2}}{a^{-2}-1}:\frac{1}{a^2}$.

Rješenje 307

Ponovimo!

$$x^{-n}=\frac{1}{x^n} \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a^n}{a^m}=a^{n-m}.$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad x^0=1 \quad , \quad n=\frac{n}{1}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{a^{-3}+a^{-2}}{a^{-2}-1}:\frac{1}{a^2}=\frac{\frac{1}{a^3}+\frac{1}{a^2}}{\frac{1}{a^2}-1}\cdot\frac{a^2}{1}=\frac{\frac{1+a}{a^3}}{\frac{1-a^2}{a^2}}\cdot\frac{a^2}{1}=\frac{a^2\cdot(1+a)}{a^3\cdot(1-a^2)}\cdot\frac{a^2}{1}=\frac{a^4\cdot(1+a)}{a^3\cdot(1-a^2)}=$$

$$=\frac{a^4\cdot(1+a)}{a^3\cdot(1-a)\cdot(1+a)}=\frac{a^4\cdot(1+a)}{a^3\cdot(1-a)\cdot(1+a)}=\frac{a}{1-a}.$$

2.inačica

$$\frac{a^{-3}+a^{-2}}{a^{-2}-1}:\frac{1}{a^2}=\frac{a^{-3}+a^{-2}}{a^{-2}-1}\cdot\frac{a^2}{1}=\frac{a^{-1}+a^0}{a^{-2}-1}=\frac{a^{-1}+1}{a^{-2}-1}=\frac{a^{-1}+1}{(a^{-1}-1)\cdot(a^{-1}+1)}=$$

$$=\frac{a^{-1}+1}{(a^{-1}-1)\cdot(a^{-1}+1)}=\frac{1}{a^{-1}-1}=\frac{1}{\frac{1}{a}-1}=\frac{1}{\frac{1-a}{a}}=\frac{\frac{1}{1}}{\frac{1-a}{a}}=\frac{a}{1-a}.$$

Vježba 307

Izračunaj: $\frac{1}{a^2} : \frac{a^{-3}+a^{-2}}{a^{-2}-1}$.

Rezultat: $\frac{1-a}{a}$.

Zadatak 308 (Mala, gimnazija)

Izračunaj: $\left(1+\frac{1}{a}\right)\cdot\left(1+\frac{1}{a^2}\right)\cdot a^3\cdot(a-1)$.

A) a^7-1 *B)* $\left(a^4-1\right)\cdot a^3$ *C)* $\frac{a-1}{a^2\cdot(a+1)}$ *D)* a^4-1

Rješenje 308

Ponovimo!

$x^{-n}=\frac{1}{x^n}$, $x^0=1$, $x^1=x$, $x^n\cdot x^m=x^{n+m}$, $\frac{x^n}{x^m}=x^{n-m}$.

$a^2-b^2=(a-b)\cdot(a+b)$, $\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}$, $n=\frac{n}{1}$.

Zakon distribucije množenja prema zbrajanju

$a\cdot(b+c)=a\cdot b+a\cdot c$, $a\cdot b+a\cdot c=a\cdot(b+c)$.

1.inačica

$$\left(1+\frac{1}{a}\right)\cdot\left(1+\frac{1}{a^2}\right)\cdot a^3\cdot(a-1)=\left(1+a^{-1}\right)\cdot\left(1+a^{-2}\right)\cdot a^3\cdot(a-1)=$$

$$=\left(1+a^{-1}\right)\cdot\left(1+a^{-2}\right)\cdot a^1\cdot a^2\cdot(a-1)=\left(1+a^{-1}\right)\cdot\left(1+a^{-2}\right)\cdot a\cdot a^2\cdot(a-1)=$$

$$=a\cdot\left(1+a^{-1}\right)\cdot a^2\cdot\left(1+a^{-2}\right)\cdot(a-1)=\left(a+a^0\right)\cdot\left(a^2+a^0\right)\cdot(a-1)=(a+1)\cdot\left(a^2+1\right)\cdot(a-1)=$$

$$=(a+1)\cdot(a-1)\cdot\left(a^2+1\right)=\left(a^2-1\right)\cdot\left(a^2+1\right)=a^4-1.$$

Odgovor je pod D.

2.inačica

$$\left(1+\frac{1}{a}\right)\cdot\left(1+\frac{1}{a^2}\right)\cdot a^3\cdot(a-1)=\left(\frac{1}{1}+\frac{1}{a}\right)\cdot\left(\frac{1}{1}+\frac{1}{a^2}\right)\cdot a^3\cdot(a-1)=$$

$$=\frac{a+1}{a}\cdot\frac{a^2+1}{a^2}\cdot a^3\cdot(a-1)=\frac{(a+1)\cdot\left(a^2+1\right)}{a^3}\cdot a^3\cdot(a-1)=\frac{(a+1)\cdot\left(a^2+1\right)}{a^3}\cdot a^3\cdot(a-1)=$$

$$=(a+1)\cdot\left(a^2+1\right)\cdot(a-1)=(a+1)\cdot(a-1)\cdot\left(a^2+1\right)=\left(a^2-1\right)\cdot\left(a^2+1\right)=a^4-1.$$

Odgovor je pod D.

3.inačica

$$\left(1+\frac{1}{a}\right)\cdot\left(1+\frac{1}{a^2}\right)\cdot a^3\cdot(a-1)=\left(1+\frac{1}{a^2}+\frac{1}{a}+\frac{1}{a^3}\right)\cdot a^3\cdot(a-1)=$$

$$=\left(a^3+\frac{a^3}{a^2}+\frac{a^3}{a}+\frac{a^3}{a^3}\right)\cdot(a-1)=\left(a^3+\frac{a^3}{a^2}+\frac{a^3}{a}+\frac{a^3}{a^3}\right)\cdot(a-1)=$$

$$=\left(a^3+a+a^2+1\right)\cdot(a-1)=a^4-a^3+a^2-a+a^3-a^2+a-1=$$

$$=a^4-a^3+a^2-a+a^3-a^2+a-1=a^4-1.$$

Odgovor je pod D.

4.inačica

$$\left(1+\frac{1}{a}\right)\cdot\left(1+\frac{1}{a^2}\right)\cdot a^3\cdot(a-1)=\left(1+\frac{1}{a^2}+\frac{1}{a}+\frac{1}{a^3}\right)\cdot a^3\cdot(a-1)=$$

$$=\left(1+\frac{1}{a^2}+\frac{1}{a}+\frac{1}{a^3}\right)\cdot\left(a^4-a^3\right)=a^4-a^3+\frac{a^4}{a^2}-\frac{a^3}{a^2}+\frac{a^4}{a}-\frac{a^3}{a}+\frac{a^4}{a^3}-\frac{a^3}{a^3}=$$

$$=a^4-a^3+\frac{a^4}{a^2}-\frac{a^3}{a^2}+\frac{a^4}{a}-\frac{a^3}{a}+\frac{a^4}{a^3}-\frac{a^3}{a^3}=a^4-a^3+a^2-a+a^3-a^2+a-1=$$

$$=a^4-a^3+a^2-a+a^3-a^2+a-1=a^4-1.$$

Odgovor je pod D.

Vježba 308

Izračunaj: $\left(1+\frac{1}{a}\right)\cdot a^2\cdot\left(1-\frac{1}{a}\right)$.

A) a^3-1 *B)* $\left(a^2-1\right)\cdot a$ *C)* $\frac{a-1}{a\cdot(a+1)}$ *D)* a^2-1

Rezultat: D.

Zadatak 309 (Vjeverica, gimnazija)

Izračunaj: $\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}$.

Rješenje 309

Ponovimo!

$$(x+y)^2=x^2+2\cdot x\cdot y+y^2\quad,\quad (x-y)^2=x^2-2\cdot x\cdot y+y^2\quad,\quad x^2-y^2=(x-y)\cdot(x+y).$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}=\left[\begin{array}{l}\text{u oba nazivnika}\\ \text{izlučimo } a\cdot b\end{array}\right]=\frac{a+b}{a\cdot b\cdot(a-b)}-\frac{a-b}{a\cdot b\cdot(a+b)}=$$

$$=\left[\begin{array}{l}\text{razlomke oduzimamo da ih}\\ \text{svedemo na zajednički nazivnik}\end{array}\right]=\frac{(a+b)^2-(a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}=$$

$$= \frac{a^2 + 2\cdot a\cdot b + b^2 - \left(a^2 - 2\cdot a\cdot b + b^2\right)}{a\cdot b\cdot (a-b)\cdot (a+b)} = \frac{a^2 + 2\cdot a\cdot b + b^2 - a^2 + 2\cdot a\cdot b - b^2}{a\cdot b\cdot (a-b)\cdot (a+b)} =$$

$$= \frac{a^2 + 2\cdot a\cdot b + b^2 - a^2 + 2\cdot a\cdot b - b^2}{a\cdot b\cdot (a-b)\cdot (a+b)} = \frac{4\cdot a\cdot b}{a\cdot b\cdot (a-b)\cdot (a+b)} = \frac{4\cdot a\cdot b}{a\cdot b\cdot (a-b)\cdot (a+b)} =$$

$$= \frac{4}{(a-b)\cdot (a+b)} = \frac{4}{a^2 - b^2}.$$

2.inačica

$$\frac{a+b}{a^2\cdot b - a\cdot b^2} - \frac{a-b}{a^2\cdot b + a\cdot b^2} = \left[\begin{array}{l}\text{u oba nazivnika}\\ \text{izlučimo } a\cdot b\end{array}\right] = \frac{a+b}{a\cdot b\cdot (a-b)} - \frac{a-b}{a\cdot b\cdot (a+b)} =$$

$$= \left[\begin{array}{l}\text{razlomke oduzimamo da ih}\\ \text{svedemo na zajednički nazivnik}\end{array}\right] = \frac{(a+b)^2 - (a-b)^2}{a\cdot b\cdot (a-b)\cdot (a+b)} =$$

$$= \left[\begin{array}{l}\text{u brojniku uočimo}\\ \text{razliku kvadrata}\end{array}\right] = \frac{\big((a+b)-(a-b)\big)\cdot\big((a+b)+(a-b)\big)}{a\cdot b\cdot (a-b)\cdot (a+b)} =$$

$$= \frac{(a+b-a+b)\cdot (a+b+a-b)}{a\cdot b\cdot (a-b)\cdot (a+b)} = \frac{(a+b-a+b)\cdot (a+b+a-b)}{a\cdot b\cdot (a-b)\cdot (a+b)} = \frac{2\cdot b\cdot 2\cdot a}{a\cdot b\cdot (a-b)\cdot (a+b)} =$$

$$= \frac{4\cdot a\cdot b}{a\cdot b\cdot (a-b)\cdot (a+b)} = \frac{4\cdot a\cdot b}{a\cdot b\cdot (a-b)\cdot (a+b)} = \frac{4}{(a-b)\cdot (a+b)} = \frac{4}{a^2 - b^2}.$$

Vježba 309

Izračunaj: $\frac{a+b}{a^2\cdot b - a\cdot b^2} + \frac{b-a}{a^2\cdot b - a\cdot b^2}$.

Rezultat: $\frac{4}{a^2 - b^2}$.

Zadatak 310 (Vjeverica, gimnazija)

Izračunaj: $\frac{1}{x} - \frac{x-9}{x^2 - 9} + \frac{3}{3\cdot x - x^2}$.

Rješenje 310

Ponovimo!

$$x^2 - y^2 = (x-y)\cdot (x+y).$$

$$\frac{a}{b} - \frac{c}{d} = \frac{a\cdot d - b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a\cdot d + b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot (b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot (b+c).$$

$$\frac{1}{x}-\frac{x-9}{x^2-9}+\frac{3}{3\cdot x-x^2}=\frac{1}{x}-\frac{x-9}{(x-3)\cdot(x+3)}+\frac{3}{-\left(x^2-3\cdot x\right)}=\frac{1}{x}-\frac{x-9}{(x-3)\cdot(x+3)}-\frac{3}{x^2-3\cdot x}=$$

$$=\frac{1}{x}-\frac{x-9}{(x-3)\cdot(x+3)}-\frac{3}{x\cdot(x-3)}=\left[\begin{array}{l}\text{razlomke oduzimamo da ih}\\ \text{svedemo na zajednički nazivnik}\end{array}\right]=$$

$$=\frac{(x-3)\cdot(x+3)-x\cdot(x-9)-3\cdot(x+3)}{x\cdot(x-3)\cdot(x+3)}=\frac{x^2-9-x^2+9\cdot x-3\cdot x-9}{x\cdot(x-3)\cdot(x+3)}=$$

$$=\frac{x^2-9-x^2+9\cdot x-3\cdot x-9}{x\cdot(x-3)\cdot(x+3)}=\frac{-9+9\cdot x-3\cdot x-9}{x\cdot(x-3)\cdot(x+3)}=\frac{6\cdot x-18}{x\cdot(x-3)\cdot(x+3)}=\frac{6\cdot(x-3)}{x\cdot(x-3)\cdot(x+3)}=$$

$$=\frac{6\cdot(x-3)}{x\cdot(x-3)\cdot(x+3)}=\frac{6}{x\cdot(x+3)}.$$

Vježba 310

Izračunaj: $\frac{1}{x}+\frac{9-x}{x^2-9}-\frac{3}{x^2-3\cdot x}$.

Rezultat: $\frac{6}{x\cdot(x+3)}$.

Zadatak 311 (Snježana, gimnazija)

Izračunaj: $\left(\frac{a^{-3}}{3\cdot b^{-2}}\right)^{-3}\cdot\left(9\cdot a^4\cdot b^{-1}\right)^{-2}$.

Rješenje 311

Ponovimo!

$$a^{-n}=\frac{1}{a^n}\quad,\quad\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n\quad,\quad a^1=a\quad,\quad n=\frac{n}{1}\quad,\quad\frac{a^m}{a^n}=a^{m-n}.$$

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

1.inačica

$$\left(\frac{a^{-3}}{3\cdot b^{-2}}\right)^{-3}\cdot\left(9\cdot a^4\cdot b^{-1}\right)^{-2}=\left(\frac{3\cdot b^{-2}}{a^{-3}}\right)^3\cdot\left(\frac{1}{9\cdot a^4\cdot b^{-1}}\right)^2=\left(\frac{3\cdot a^3}{b^2}\right)^3\cdot\left(\frac{b^1}{9\cdot a^4}\right)^2=$$

$$=\frac{3^3\cdot a^9}{b^6}\cdot\frac{b^2}{9^2\cdot a^8}=\frac{3^3\cdot a^9}{b^6}\cdot\frac{b^2}{\left(3^2\right)^2\cdot a^8}=\frac{3^3\cdot a^9}{b^6}\cdot\frac{b^2}{3^4\cdot a^8}=\frac{3^3\cdot a^9\cdot b^2}{3^4\cdot b^6\cdot a^8}=$$

$$=\frac{3^3\cdot a^8\cdot a^1\cdot b^2}{3^3\cdot 3^1\cdot b^2\cdot b^4\cdot a^8}=\frac{3^3\cdot a^8\cdot a^1\cdot b^2}{3^3\cdot 3^1\cdot b^2\cdot b^4\cdot a^8}=\frac{a^1}{3^1\cdot b^4}=\frac{a}{3\cdot b^4}.$$

2.inačica

$$\left(\frac{a^{-3}}{3\cdot b^{-2}}\right)^{-3}\cdot\left(9\cdot a^4\cdot b^{-1}\right)^{-2}=\left(\frac{b^2}{3\cdot a^3}\right)^{-3}\cdot\left(\frac{9\cdot a^4}{b^1}\right)^{-2}=\left(\frac{3\cdot a^3}{b^2}\right)^3\cdot\left(\frac{b^1}{9\cdot a^4}\right)^2=$$

$$= \frac{3^3 \cdot a^9}{b^6} \cdot \frac{b^2}{9^2 \cdot a^8} = \frac{3^3 \cdot a^9}{b^6} \cdot \frac{b^2}{\left(3^2\right)^2 \cdot a^8} = \frac{3^3 \cdot a^9}{b^6} \cdot \frac{b^2}{3^4 \cdot a^8} = \frac{3^3 \cdot a^9 \cdot b^2}{3^4 \cdot b^6 \cdot a^8} =$$

$$= \frac{3^3 \cdot a^8 \cdot a^1 \cdot b^2}{3^3 \cdot 3^1 \cdot b^2 \cdot b^4 \cdot a^8} = \frac{3^3 \cdot a^8 \cdot a^1 \cdot b^2}{3^3 \cdot 3^1 \cdot b^2 \cdot b^4 \cdot a^8} = \frac{a^1}{3^1 \cdot b^4} = \frac{a}{3 \cdot b^4}.$$

3.inačica

$$\left(\frac{a^{-3}}{3 \cdot b^{-2}}\right)^{-3} \cdot \left(9 \cdot a^4 \cdot b^{-1}\right)^{-2} = \frac{a^9}{3^{-3} \cdot b^6} \cdot 9^{-2} \cdot a^{-8} \cdot b^2 = \frac{3^3 \cdot a^9}{b^6} \cdot \frac{b^2}{9^2 \cdot a^8} = \frac{3^3 \cdot a^9}{b^6} \cdot \frac{b^2}{\left(3^2\right)^2 \cdot a^8} =$$

$$= \frac{3^3 \cdot a^9}{b^6} \cdot \frac{b^2}{3^4 \cdot a^8} = \frac{3^3 \cdot a^9 \cdot b^2}{3^4 \cdot b^6 \cdot a^8} = \frac{3^3 \cdot a^8 \cdot a^1 \cdot b^2}{3^3 \cdot 3^1 \cdot b^2 \cdot b^4 \cdot a^8} = \frac{3^3 \cdot a^8 \cdot a^1 \cdot b^2}{3^3 \cdot 3^1 \cdot b^2 \cdot b^4 \cdot a^8} = \frac{a^1}{3^1 \cdot b^4} = \frac{a}{3 \cdot b^4}.$$

Vježba 311

Izračunaj: $\left(\frac{9 \cdot a^4}{b}\right)^{-2} \cdot \left(\frac{a^{-3}}{3 \cdot b^{-2}}\right)^{-3}$.

Rezultat: $\frac{a}{3 \cdot b^4}$.

Zadatak 312 (Snježana, gimnazija)

Izračunaj: $\frac{2 \cdot 5^{n+2} + 3 \cdot 5^{n+1}}{3 \cdot 5^{n-1} - 2 \cdot 5^{n-2}}$.

Rješenje 312

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a^m}{a^n} = a^{m-n}.$$

$$\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

Zakon distribucije množenja prema zbrajanju

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$\frac{2 \cdot 5^{n+2} + 3 \cdot 5^{n+1}}{3 \cdot 5^{n-1} - 2 \cdot 5^{n-2}} = \frac{2 \cdot 5^n \cdot 5^2 + 3 \cdot 5^n \cdot 5^1}{3 \cdot 5^n \cdot 5^{-1} - 2 \cdot 5^n \cdot 5^{-2}} = \frac{5^n \cdot \left(2 \cdot 5^2 + 3 \cdot 5^1\right)}{5^n \cdot \left(3 \cdot 5^{-1} - 2 \cdot 5^{-2}\right)} =$$

$$= \frac{5^n \cdot \left(2 \cdot 5^2 + 3 \cdot 5^1\right)}{5^n \cdot \left(3 \cdot 5^{-1} - 2 \cdot 5^{-2}\right)} = \frac{2 \cdot 5^2 + 3 \cdot 5^1}{3 \cdot 5^{-1} - 2 \cdot 5^{-2}} = \frac{2 \cdot 25 + 3 \cdot 5}{3 \cdot \frac{1}{5} - 2 \cdot \frac{1}{5^2}} = \frac{50 + 15}{\frac{3}{5} - \frac{2}{25}} = \frac{65}{\frac{15 - 2}{25}} = \frac{65}{\frac{1}{13}} =$$

$$= \frac{\frac{65}{1}}{\frac{13}{25}} = \frac{\frac{65}{1}}{\frac{13}{25}} = \frac{\frac{5}{1}}{\frac{1}{25}} = \frac{125}{1} = 125.$$

2.inačica

$$\frac{2\cdot 5^{n+2}+3\cdot 5^{n+1}}{3\cdot 5^{n-1}-2\cdot 5^{n-2}} = \frac{2\cdot 5^{n+1}\cdot 5^{1}+3\cdot 5^{n+1}}{3\cdot 5^{n-2}\cdot 5^{1}-2\cdot 5^{n-2}} = \frac{2\cdot 5^{n+1}\cdot 5+3\cdot 5^{n+1}}{3\cdot 5^{n-2}\cdot 5-2\cdot 5^{n-2}} = \frac{10\cdot 5^{n+1}+3\cdot 5^{n+1}}{15\cdot 5^{n-2}-2\cdot 5^{n-2}} =$$

$$= \frac{(10+3)\cdot 5^{n+1}}{(15-2)\cdot 5^{n-2}} = \frac{13\cdot 5^{n+1}}{13\cdot 5^{n-2}} = \frac{13\cdot 5^{n+1}}{13\cdot 5^{n-2}} = \frac{5^{n+1}}{5^{n-2}} = 5^{n+1-(n-2)} = 5^{n+1-n+2} =$$

$$= 5^{n+1-n+2} = 5^{3} = 125.$$

Vježba 312

Izračunaj: $\frac{3\cdot 5^{n-1}-2\cdot 5^{n-2}}{10\cdot 5^{n+1}+3\cdot 5^{n+1}}$.

Rezultat: $\frac{1}{125}$.

Zadatak 313 (Tonino, strukovna škola)

Skrati razlomak: $\frac{5\cdot a+5\cdot b}{a+b}$.

Rješenje 313

Ponovimo!

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \quad , \quad n = \frac{n}{1}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

$$\frac{5\cdot a+5\cdot b}{a+b} = \left[\begin{array}{l}\text{u brojniku}\\ \text{izlučimo } 5\end{array}\right] = \frac{5\cdot(a+b)}{a+b} = \frac{5\cdot(a+b)}{a+b} = \left[\begin{array}{l}\text{brojnik i nazivnik}\\ \text{dijelimo sa } a+b\end{array}\right] = \frac{5}{1} = 5.$$

Vježba 313

Skrati razlomak: $\frac{7\cdot a+7\cdot b}{a+b}$.

Rezultat: 7.

Zadatak 314 (Tonino, strukovna škola)

Skrati razlomak: $\frac{x^2-y^2}{x-y}$.

Rješenje 314

Ponovimo!

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \quad , \quad n = \frac{n}{1}.$$

Razlika kvadrata:

$$a^2 - b^2 = (a-b)\cdot(a+b).$$

$$\frac{x^2-y^2}{x-y}=\left[\begin{array}{l}\text{u brojniku prepoznajemo}\\ \text{razliku kvadrata}\end{array}\right]=\frac{(x-y)\cdot(x+y)}{x-y}=\frac{(x-y)\cdot(x+y)}{x-y}=$$

$$=\left[\begin{array}{l}\text{brojnik i nazivnik}\\ \text{dijelimo sa } x-y\end{array}\right]=\frac{x+y}{1}=x+y.$$

Vježba 314

Skrati razlomak: $\frac{x^2-y^2}{x+y}$.

Rezultat: x – y.

Zadatak 315 (Crnka, gimnazija)

Riješi kvadratnu jednadžbu : $a\cdot b\cdot x^2-(a+b)\cdot x+1=0.$

Rješenje 315

Ponovimo!

$(-x)^2=x^2$, $(x+y)^2=x^2+2\cdot x\cdot y+y^2$, $(x-y)^2=x^2-2\cdot x\cdot y+y^2$.

$$\sqrt{x^2}=x\ ,\ x\ge 0.$$

$$a\cdot b\cdot x^2-(a+b)\cdot x+1=0\ \Rightarrow\ \left.\begin{array}{l}a\cdot b\cdot x^2-(a+b)\cdot x+1=0\\ a=a\cdot b\ ,\ b=-(a+b)\ ,\ c=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}a=a\cdot b\ ,\ b=-(a+b)\ ,\ c=1\\ x_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\}\Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{-(-(a+b))\pm\sqrt{(-(a+b))^2-4\cdot a\cdot b\cdot 1}}{2\cdot a\cdot b}\Rightarrow x_{1,2}=\frac{a+b\pm\sqrt{(a+b)^2-4\cdot a\cdot b}}{2\cdot a\cdot b}\Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{a+b\pm\sqrt{a^2+2\cdot a\cdot b+b^2-4\cdot a\cdot b}}{2\cdot a\cdot b}\Rightarrow x_{1,2}=\frac{a+b\pm\sqrt{a^2-2\cdot a\cdot b+b^2}}{2\cdot a\cdot b}\Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{a+b\pm\sqrt{(a-b)^2}}{2\cdot a\cdot b}\Rightarrow x_{1,2}=\frac{a+b\pm(a-b)}{2\cdot a\cdot b}\Rightarrow\left.\begin{array}{l}x_1=\frac{a+b+(a-b)}{2\cdot a\cdot b}\\ x_2=\frac{a+b-(a-b)}{2\cdot a\cdot b}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}x_1=\frac{a+b+a-b}{2\cdot a\cdot b}\\ x_2=\frac{a+b-a+b}{2\cdot a\cdot b}\end{array}\right\}\Rightarrow\left.\begin{array}{l}x_1=\frac{a+b+a-b}{2\cdot a\cdot b}\\ x_2=\frac{a+b-a+b}{2\cdot a\cdot b}\end{array}\right\}\Rightarrow\left.\begin{array}{l}x_1=\frac{2\cdot a}{2\cdot a\cdot b}\\ x_2=\frac{2\cdot b}{2\cdot a\cdot b}\end{array}\right\}\Rightarrow\left.\begin{array}{l}x_1=\frac{2\cdot a}{2\cdot a\cdot b}\\ x_2=\frac{2\cdot b}{2\cdot a\cdot b}\end{array}\right\}\Rightarrow\left.\begin{array}{l}x_1=\frac{1}{b}\\ x_2=\frac{1}{a}\end{array}\right\}.$$

Vježba 315

Riješi kvadratnu jednadžbu : $x^2-(a+b)\cdot x+a\cdot b=0.$

Rezultat: $x_1 = a$, $x_2 = b$.

Zadatak 316 (Nidko, gimnazija)

Faktoriziraj : $0.008\cdot a^9\cdot b^{6\cdot n+3}+c^{9\cdot n-6}$.

Rješenje 316

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad (a\cdot b)^{n}=a^{n}\cdot b^{n}\quad,\quad a^{3}+b^{3}=(a+b)\cdot\left(a^{2}-a\cdot b+b^{2}\right).$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$0.008\cdot a^{9}\cdot b^{6\cdot n+3}+c^{9\cdot n-6}=\left(0.2\cdot a^{3}\cdot b^{2\cdot n+1}\right)^{3}+\left(c^{3\cdot n-2}\right)^{3}=$$

$$=\left(0.2\cdot a^{3}\cdot b^{2\cdot n+1}+c^{3\cdot n-2}\right)\cdot\left(\left(0.2\cdot a^{3}\cdot b^{2\cdot n+1}\right)^{2}-0.2\cdot a^{3}\cdot b^{2\cdot n+1}\cdot c^{3\cdot n-2}+\left(c^{3\cdot n-2}\right)^{2}\right)=$$

$$=\left(0.2\cdot a^{3}\cdot b^{2\cdot n+1}+c^{3\cdot n-2}\right)\cdot\left(0.04\cdot a^{6}\cdot b^{4\cdot n+2}-0.2\cdot a^{3}\cdot b^{2\cdot n+1}\cdot c^{3\cdot n-2}+c^{6\cdot n-4}\right).$$

Vježba 316

Faktoriziraj: $a^{9}\cdot b^{6\cdot n+3}+c^{9\cdot n-6}$.

Rezultat: $\left(a^{3}\cdot b^{2\cdot n+1}+c^{3\cdot n-2}\right)\cdot\left(a^{6}\cdot b^{4\cdot n+2}-a^{3}\cdot b^{2\cdot n+1}\cdot c^{3\cdot n-2}+c^{6\cdot n-4}\right)$.

Zadatak 317 (Nidko, gimnazija)

Faktoriziraj: $\frac{27}{64}\cdot a^{6}-\frac{27}{125}\cdot b^{3}$.

Rješenje 317

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad (a\cdot b)^{n}=a^{n}\cdot b^{n}\quad,\quad a^{3}-b^{3}=(a-b)\cdot\left(a^{2}+a\cdot b+b^{2}\right).$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$\frac{27}{64}\cdot a^{6}-\frac{27}{125}\cdot b^{3}=\left(\frac{3}{4}\cdot a^{2}\right)^{3}-\left(\frac{3}{5}\cdot b\right)^{3}=\left(\frac{3}{4}\cdot a^{2}-\frac{3}{5}\cdot b\right)\cdot\left(\left(\frac{3}{4}\cdot a^{2}\right)^{2}+\frac{3}{4}\cdot a^{2}\cdot\frac{3}{5}\cdot b+\left(\frac{3}{5}\cdot b\right)^{2}\right)=$$

$$=\left(\frac{3}{4}\cdot a^{2}-\frac{3}{5}\cdot b\right)\cdot\left(\frac{9}{16}\cdot a^{4}+\frac{9}{20}\cdot a^{2}\cdot b+\frac{9}{25}\cdot b^{2}\right).$$

Vježba 317

Faktoriziraj: $a^{6}-b^{3}$.

Rezultat: $\left(a^{2}-b\right)\cdot\left(a^{4}+a^{2}\cdot b+b^{2}\right)$.

Zadatak 318 (Nidko, gimnazija)

Rastavi na faktore: $x^{2}-4\cdot x\cdot y+4\cdot y^{2}-4\cdot x^{2}\cdot y^{2}$.

Rješenje 318

Ponovimo!

$$(a-b)^{2}=a^{2}-2\cdot a\cdot b+b^{2}\quad,\quad a^{2}-b^{2}=(a-b)\cdot(a+b)\quad,\quad (a\cdot b)^{n}=a^{n}\cdot b^{n}.$$

$$x^{2}-4\cdot x\cdot y+4\cdot y^{2}-4\cdot x^{2}\cdot y^{2}=\left(x^{2}-4\cdot x\cdot y+4\cdot y^{2}\right)-4\cdot x^{2}\cdot y^{2}=(x-2\cdot y)^{2}-4\cdot x^{2}\cdot y^{2}=$$

$$=(x-2\cdot y)^{2}-(2\cdot x\cdot y)^{2}=(x-2\cdot y-2\cdot x\cdot y)\cdot(x-2\cdot y+2\cdot x\cdot y).$$

Vježba 318

Rastavi na faktore: $x^2-2\cdot x\cdot y+y^2-4\cdot x^2\cdot y^2$.

Rezultat: $(x-y-2\cdot x\cdot y)\cdot(x-y+2\cdot x\cdot y)$.

Zadatak 319 (Nidko, gimnazija)

Rastavi na faktore: $27\cdot a^3+21\cdot a^2-7\cdot a-1$.

Rješenje 319

Ponovimo!

$$(a^n)^m=a^{n\cdot m}\quad,\quad (a\cdot b)^n=a^n\cdot b^n\quad,\quad a^3-b^3=(a-b)\cdot(a^2+a\cdot b+b^2).$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$27\cdot a^3+21\cdot a^2-7\cdot a-1=27\cdot a^3-1+21\cdot a^2-7\cdot a=\left(27\cdot a^3-1\right)+\left(21\cdot a^2-7\cdot a\right)=$$

$$=\left((3\cdot a)^3-1^3\right)+7\cdot a\cdot(3\cdot a-1)=(3\cdot a-1)\cdot\left((3\cdot a)^2+3\cdot a\cdot 1+1^2\right)+7\cdot a\cdot(3\cdot a-1)=$$

$$=(3\cdot a-1)\cdot\left(9\cdot a^2+3\cdot a+1\right)+7\cdot a\cdot(3\cdot a-1)=(3\cdot a-1)\cdot\left(9\cdot a^2+3\cdot a+1+7\cdot a\right)=$$

$$=(3\cdot a-1)\cdot\left(9\cdot a^2+10\cdot a+1\right).$$

Vježba 319

Rastavi na faktore: $27\cdot a^3+21\cdot a-8$.

Rezultat: $(3\cdot a-1)\cdot\left(9\cdot a^2+3\cdot a+8\right)$.

Zadatak 320 (Iva, gimnazija)

Ako je $(4\cdot x-2)\cdot(3\cdot x-4)=9$, koliko je $(3\cdot x-1)\cdot(2\cdot x-3)$?

Rješenje 320

Ponovimo!

$$a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Transformiramo zadanu jednakost.

$$(4\cdot x-2)\cdot(3\cdot x-4)=9\ \Rightarrow\ 12\cdot x^2-16\cdot x-6\cdot x+8=9\ \Rightarrow\ 12\cdot x^2-16\cdot x-6\cdot x=9-8\ \Rightarrow$$

$$\Rightarrow\ 12\cdot x^2-22\cdot x=1.$$

Sada računamo vrijednost zadanog izraza.

$$(3\cdot x-1)\cdot(2\cdot x-3)=6\cdot x^2-9\cdot x-2\cdot x+3=6\cdot x^2-11\cdot x+3=\frac{1}{2}\cdot\left(12\cdot x^2-22\cdot x\right)+3=$$

$$=\left[12\cdot x^2-22\cdot x=1\right]=\frac{1}{2}\cdot 1+3=\frac{1}{2}+3=3\frac{1}{2}=3.5.$$

2.inačica

Transformiramo zadanu jednakost.

$$(4\cdot x-2)\cdot(3\cdot x-4)=9 \Rightarrow 12\cdot x^2-16\cdot x-6\cdot x+8=9 \Rightarrow 12\cdot x^2-16\cdot x-6\cdot x=9-8 \Rightarrow$$

$$\Rightarrow 12\cdot x^2-22\cdot x=1 \Rightarrow 2\cdot\left(6\cdot x^2-11\cdot x\right)=1 \Rightarrow 2\cdot\left(6\cdot x^2-11\cdot x\right)=1\ /:2 \Rightarrow 6\cdot x^2-11\cdot x=\frac{1}{2}.$$

Sada računamo vrijednost zadanog izraza.

$$(3\cdot x-1)\cdot(2\cdot x-3)=6\cdot x^2-9\cdot x-2\cdot x+3=6\cdot x^2-11\cdot x+3=\left(6\cdot x^2-11\cdot x\right)+3=$$

$$=\left[6\cdot x^2-11\cdot x=\frac{1}{2}\right]=\frac{1}{2}+3=3\frac{1}{2}=3.5.$$

Vježba 320

Ako je $(2-4\cdot x)\cdot(4-3\cdot x)=9$, koliko je $(1-3\cdot x)\cdot(3-2\cdot x)$?

Rezultat: 3.5.