Zadatak 281 (Danijela, ekonomska škola)

Ako je $10^{x}=\dfrac{\frac{1}{2}\cdot 10^{-3}+\frac{1}{2}\cdot 10^{-4}}{55\cdot 10^{-7}}$, odredite x.

Rješenje 281

Ponovimo!

$$a^{-n}=\frac{1}{a^{n}} \quad , \quad a^{n}\cdot a^{m}=a^{n+m} \quad , \quad \frac{a^{n}}{a^{m}}=a^{n-m} \quad , \quad a^{1}=a.$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$10^{x}=\frac{\frac{1}{2}\cdot 10^{-3}+\frac{1}{2}\cdot 10^{-4}}{55\cdot 10^{-7}} \Rightarrow 10^{x}=\frac{\frac{1}{2}\cdot\frac{1}{10^{3}}+\frac{1}{2}\cdot\frac{1}{10^{4}}}{55\cdot\frac{1}{10^{7}}} \Rightarrow 10^{x}=\frac{\frac{1}{2\cdot 10^{3}}+\frac{1}{2\cdot 10^{4}}}{\frac{55}{10^{7}}} \Rightarrow$$

$$\Rightarrow 10^{x}=\frac{\frac{10+1}{2\cdot 10^{4}}}{\frac{55}{10^{7}}} \Rightarrow 10^{x}=\frac{\frac{11}{2\cdot 10^{4}}}{\frac{55}{10^{7}}} \Rightarrow 10^{x}=\frac{11\cdot 10^{7}}{2\cdot 10^{4}\cdot 55} \Rightarrow 10^{x}=\frac{11\cdot 10^{7}}{2\cdot 10^{4}\cdot 55} \Rightarrow$$

$$\Rightarrow 10^{x}=\frac{10^{7}}{2\cdot 10^{4}\cdot 5} \Rightarrow 10^{x}=\frac{10^{7}}{10\cdot 10^{4}} \Rightarrow 10^{x}=\frac{10^{7}}{10^{1}\cdot 10^{4}} \Rightarrow 10^{x}=\frac{10^{7}}{10^{5}} \Rightarrow 10^{x}=10^{2} \Rightarrow x=2.$$

2.inačica

$$10^{x}=\frac{\frac{1}{2}\cdot 10^{-3}+\frac{1}{2}\cdot 10^{-4}}{55\cdot 10^{-7}} \Rightarrow 10^{x}=\frac{\frac{1}{2}\cdot 10^{-4}\cdot(10+1)}{55\cdot 10^{-7}} \Rightarrow 10^{x}=\frac{\frac{1}{2}\cdot 10^{-4}\cdot 11}{55\cdot 10^{-7}} \Rightarrow$$

$$\Rightarrow 10^{x}=\frac{10^{-4}\cdot 11}{2\cdot 55\cdot 10^{-7}} \Rightarrow 10^{x}=\frac{10^{-4}\cdot 11}{2\cdot 55\cdot 10^{-7}} \Rightarrow 10^{x}=\frac{10^{-4}}{2\cdot 5\cdot 10^{-7}} \Rightarrow 10^{x}=\frac{10^{-4}}{10\cdot 10^{-7}} \Rightarrow$$

$$\Rightarrow 10^{x}=\frac{10^{-4}}{10^{1}\cdot 10^{-7}} \Rightarrow 10^{x}=\frac{10^{-4}}{10^{-6}} \Rightarrow 10^{x}=10^{-4}\cdot 10^{6} \Rightarrow 10^{x}=10^{2} \Rightarrow x=2.$$

3.inačica

$$10^{x}=\frac{\frac{1}{2}\cdot 10^{-3}+\frac{1}{2}\cdot 10^{-4}}{55\cdot 10^{-7}} \Rightarrow 10^{x}=\frac{\left(\frac{1}{2}\cdot 10^{-3}+\frac{1}{2}\cdot 10^{-4}\right)\cdot 10^{7}}{55} \Rightarrow 10^{x}=\frac{\frac{1}{2}\cdot 10^{4}+\frac{1}{2}\cdot 10^{3}}{55} \Rightarrow$$

$$\Rightarrow 10^{x}=\frac{\frac{1}{2}\cdot 10^{3}\cdot(10+1)}{55} \Rightarrow 10^{x}=\frac{\frac{1}{2}\cdot 10^{3}\cdot 11}{55} \Rightarrow 10^{x}=\frac{10^{3}\cdot 11}{2\cdot 55} \Rightarrow 10^{x}=\frac{10^{3}\cdot 11}{2\cdot 55} \Rightarrow$$

$$\Rightarrow 10^{x}=\frac{10^{3}}{2\cdot 5} \Rightarrow 10^{x}=\frac{10^{3}}{10} \Rightarrow 10^{x}=\frac{10^{3}}{10^{1}} \Rightarrow 10^{x}=10^{2} \Rightarrow x=2.$$

Vježba 281

Ako je $10^{x}=\dfrac{10^{-2}+10^{-3}}{110\cdot 10^{-6}}$, odredite x.

Rezultat: 2.

Zadatak 282 (Josip, ekonomska škola)

Pojednostavni: $\left(1-\dfrac{3\cdot x^{2}}{1-x^{2}}\right):\left(\dfrac{x}{x-1}+1\right)$.

Rješenje 282

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{a}{1}=a\quad,\quad a^{2}-b^{2}=(a-b)\cdot(a+b)\quad,\quad a-b=-(b-a).$$

$$\left(1-\frac{3\cdot x^{2}}{1-x^{2}}\right):\left(\frac{x}{x-1}+1\right)=\left(\frac{1}{1}-\frac{3\cdot x^{2}}{1-x^{2}}\right):\left(\frac{x}{x-1}+\frac{1}{1}\right)=\frac{1-x^{2}-3\cdot x^{2}}{1-x^{2}}:\frac{x+x-1}{x-1}=$$

$$=\frac{1-4\cdot x^{2}}{1-x^{2}}:\frac{2\cdot x-1}{x-1}=\frac{1-4\cdot x^{2}}{1-x^{2}}\cdot\frac{x-1}{2\cdot x-1}=\frac{-\left(4\cdot x^{2}-1\right)}{-\left(x^{2}-1\right)}\cdot\frac{x-1}{2\cdot x-1}=\frac{4\cdot x^{2}-1}{x^{2}-1}\cdot\frac{x-1}{2\cdot x-1}=$$

$$=\frac{(2\cdot x-1)\cdot(2\cdot x+1)}{(x-1)\cdot(x+1)}\cdot\frac{x-1}{2\cdot x-1}=\frac{(2\cdot x-1)\cdot(2\cdot x+1)}{(x-1)\cdot(x+1)}\cdot\frac{x-1}{2\cdot x-1}=\frac{2\cdot x+1}{x+1}.$$

Vježba 282

Pojednostavni: $\left(\dfrac{x}{x-1}+1\right):\left(1-\dfrac{3\cdot x^{2}}{1-x^{2}}\right)$.

Rezultat: $\dfrac{x+1}{2\cdot x+1}$.

Zadatak 283 (Tonka, srednja škola)

Rastavi na faktore izraz: $a^{2}+10\cdot a\cdot b-70\cdot b-49$.

Rješenje 283

Ponovimo!

$$x^{2}-y^{2}=(x-y)\cdot(x+y).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$a^{2}+10\cdot a\cdot b-70\cdot b-49=\left[\begin{matrix}\text{metoda}\\ \text{grupiranja}\end{matrix}\right]=\left(a^{2}-49\right)+(10\cdot a\cdot b-70\cdot b)=$$

$$=\left(a^{2}-7^{2}\right)+(10\cdot a\cdot b-70\cdot b)=(a-7)\cdot(a+7)+10\cdot b\cdot(a-7)=(a-7)\cdot(a+7+10\cdot b).$$

Vježba 283

Rastavi na faktore izraz: $a^{2}+10\cdot a\cdot b-30\cdot b-9$.

Rezultat: $(a-3)\cdot(a+3+10\cdot b)$.

Zadatak 284 (Lidija, srednja škola)

$$\text{Pojednostavni}: \frac{y-\frac{a^2}{y}}{a-\frac{y^2}{a}}.$$

Rješenje 284

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad a-b=-(b-a).$$

$$\frac{y-\frac{a^2}{y}}{a-\frac{y^2}{a}}=\frac{\frac{y}{1}-\frac{a^2}{y}}{\frac{a}{1}-\frac{y^2}{a}}=\frac{\frac{y^2-a^2}{y}}{\frac{a^2-y^2}{a}}=\frac{a\cdot\left(y^2-a^2\right)}{y\cdot\left(a^2-y^2\right)}=\frac{-a\cdot\left(a^2-y^2\right)}{y\cdot\left(a^2-y^2\right)}=\frac{-a\cdot\left(a^2-y^2\right)}{y\cdot\left(a^2-y^2\right)}=-\frac{a}{y}.$$

Vježba 284

$$\text{Pojednostavni}: \frac{1-\frac{a}{y}}{1-\frac{y}{a}}.$$

Rezultat: $-\frac{a}{y}$.

Zadatak 285 (Lidija, srednja škola)

$$\text{Pojednostavni}: \frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{1}{b+\frac{1}{c}}}:\frac{1}{a+\frac{1}{b}}.$$

Rješenje 285

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad a-b=-(b-a).$$

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{1}{b+\frac{1}{c}}}:\frac{1}{a+\frac{1}{b}}=\frac{1}{b\cdot(a\cdot b\cdot c+a+c)}-\frac{1}{a+\frac{1}{\frac{b}{1}+\frac{1}{c}}}:\frac{1}{\frac{a}{1}+\frac{1}{b}}=$$

$$= \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{1}{a+\dfrac{1}{\dfrac{b\cdot c+1}{c}}} : \frac{1}{\dfrac{a\cdot b+1}{b}} = \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{1}{a+\dfrac{\dfrac{1}{1}}{\dfrac{b\cdot c+1}{c}}} : \frac{\dfrac{1}{1}}{\dfrac{a\cdot b+1}{b}} =$$

$$= \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{1}{a+\dfrac{c}{b\cdot c+1}} : \frac{b}{a\cdot b+1} = \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{1}{\dfrac{a}{1}+\dfrac{c}{b\cdot c+1}} : \frac{b}{a\cdot b+1} =$$

$$= \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{1}{\dfrac{a\cdot(b\cdot c+1)+c}{b\cdot c+1}} : \frac{b}{a\cdot b+1} = \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{1}{\dfrac{a\cdot b\cdot c+a+c}{b\cdot c+1}} : \frac{b}{a\cdot b+1} =$$

$$= \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{\dfrac{1}{1}}{\dfrac{a\cdot b\cdot c+a+c}{b\cdot c+1}} : \frac{b}{a\cdot b+1} = \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{b\cdot c+1}{a\cdot b\cdot c+a+c} : \frac{b}{a\cdot b+1} =$$

$$= \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{b\cdot c+1}{a\cdot b\cdot c+a+c}\cdot\frac{a\cdot b+1}{b} = \frac{1}{b\cdot(a\cdot b\cdot c+a+c)} - \frac{(b\cdot c+1)\cdot(a\cdot b+1)}{b\cdot(a\cdot b\cdot c+a+c)} =$$

$$= \frac{1-(b\cdot c+1)\cdot(a\cdot b+1)}{b\cdot(a\cdot b\cdot c+a+c)} = \frac{1-\left(a\cdot b^{2}\cdot c+b\cdot c+a\cdot b+1\right)}{b\cdot(a\cdot b\cdot c+a+c)} = \frac{1-a\cdot b^{2}\cdot c-b\cdot c-a\cdot b-1}{b\cdot(a\cdot b\cdot c+a+c)} =$$

$$= \frac{1-a\cdot b^{2}\cdot c-b\cdot c-a\cdot b-1}{b\cdot(a\cdot b\cdot c+a+c)} = \frac{-a\cdot b^{2}\cdot c-b\cdot c-a\cdot b}{b\cdot(a\cdot b\cdot c+a+c)} = \frac{-b\cdot(a\cdot b\cdot c+a+c)}{b\cdot(a\cdot b\cdot c+a+c)} =$$

$$= \frac{b\cdot(a\cdot b\cdot c+a+c)}{b\cdot(a\cdot b\cdot c+a+c)} = -1.$$

Vježba 285

Pojednostavni : $\dfrac{-1}{b\cdot(a\cdot b\cdot c+a+c)} + \dfrac{1}{a+\dfrac{1}{b+\dfrac{1}{c}}}\cdot\dfrac{a\cdot b+1}{b}$.

Rezultat: 1.

Zadatak 286 (Željka, gimnazija)

Tri broja x, y, z zadovoljavaju relaciju $y^2 = x \cdot z$. Dokaži da je

$$(x+y+z)\cdot(x-y+z) = x^{2}+y^{2}+z^{2}.$$

Rješenje 286

Ponovimo!

$$a^{2}-b^{2} = (a-b)\cdot(a+b) \quad , \quad (a+b)^{2} = a^{2}+2\cdot a\cdot b+b^{2}.$$

Množenje zagrada:

$$(a+b)\cdot(c+d) = a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

1.inačica

$$(x+y+z)\cdot(x-y+z) = x^{2}-x\cdot y+x\cdot z+y\cdot x-y^{2}+y\cdot z+z\cdot x-z\cdot y+z^{2} =$$

$$= x^2 - x\cdot y + x\cdot z + y\cdot x - y^2 + y\cdot z + z\cdot x - z\cdot y + z^2 = x^2 + x\cdot z - y^2 + z\cdot x + z^2 =$$

$$= \left[\, y^2 = x\cdot z \,\right] = x^2 + y^2 - y^2 + y^2 + z^2 = x^2 + y^2 - y^2 + y^2 + z^2 = x^2 + y^2 + z^2.$$

2.inačica

$$(x+y+z)\cdot(x-y+z) = \big((x+z)+y\big)\cdot\big((x+z)-y\big) = (x+z)^2 - y^2 =$$

$$= x^2 + 2\cdot x\cdot z + z^2 - y^2 = \left[\, y^2 = x\cdot z \,\right] = x^2 + 2\cdot y^2 + z^2 - y^2 = x^2 + y^2 + z^2.$$

Vježba 286

Tri broja x, y, z zadovoljavaju relaciju $z^2 = x \cdot y$. Dokaži da je

$$(x+y+z)\cdot(x+y-z) = x^2 + y^2 + z^2.$$

Rezultat: Dokaz analogan.

Zadatak 287 (Filip, gimnazija)

Rastavi na faktore $(a+b+c)\cdot(a\cdot b + b\cdot c + a\cdot c) - a\cdot b\cdot c.$

Rješenje 287

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Množenje zagrada:

$$(a+b)\cdot(c+d) = a\cdot c + a\cdot d + b\cdot c + b\cdot d.$$

$$(a+b+c)\cdot(a\cdot b + b\cdot c + a\cdot c) - a\cdot b\cdot c =$$

$$= a^2\cdot b + a\cdot b\cdot c + a^2\cdot c + a\cdot b^2 + b^2\cdot c + a\cdot b\cdot c + a\cdot b\cdot c + b\cdot c^2 + a\cdot c^2 - a\cdot b\cdot c =$$

$$= a^2\cdot b + a\cdot b\cdot c + a^2\cdot c + a\cdot b^2 + b^2\cdot c + a\cdot b\cdot c + a\cdot b\cdot c + b\cdot c^2 + a\cdot c^2 - a\cdot b\cdot c =$$

$$= a^2\cdot b + a\cdot b\cdot c + a^2\cdot c + a\cdot b^2 + b^2\cdot c + a\cdot b\cdot c + b\cdot c^2 + a\cdot c^2 = \left[\begin{matrix}\text{metoda}\\ \text{grupiranja}\end{matrix}\right] =$$

$$= \left(a^2\cdot b + a\cdot b\cdot c\right) + \left(a^2\cdot c + a\cdot c^2\right) + \left(a\cdot b^2 + b^2\cdot c\right) + \left(a\cdot b\cdot c + b\cdot c^2\right) =$$

$$= a\cdot b\cdot(a+c) + a\cdot c\cdot(a+c) + b^2\cdot(a+c) + b\cdot c\cdot(a+c) =$$

$$= (a+c)\cdot\left(a\cdot b + a\cdot c + b^2 + b\cdot c\right) = (a+c)\cdot\big(a\cdot(b+c) + b\cdot(b+c)\big) = (a+c)\cdot(b+c)\cdot(a+b).$$

Vježba 287

Rastavi na faktore $(a+b+c)\cdot\big(a\cdot b + (a+b)\cdot c\big) - a\cdot b\cdot c.$

Rezultat: $(a+c)\cdot(b+c)\cdot(a+b).$

Zadatak 288 (Filip, gimnazija)

Ako je $a+b+c=0$ i $a^2+b^2+c^2=1$, izračunaj $a^4+b^4+c^4$.

Rješenje 288

Ponovimo!

$$(x+y)^2 = x^2 + 2\cdot x\cdot y + y^2 \quad , \quad (x+y+z)^2 = x^2 + y^2 + z^2 + 2\cdot x\cdot y + 2\cdot x\cdot z + 2\cdot y\cdot z.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Kvadriramo prvu jednakost.

$$a+b+c=0 \Rightarrow a+b=-c \Rightarrow a+b=-c \;/^2 \Rightarrow (a+b)^2=(-c)^2 \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b+b^2=c^2 \Rightarrow a^2+b^2-c^2=-2\cdot a\cdot b.$$

Ponovno kvadriramo dobivenu jednakost.

$$a^2+b^2-c^2=-2\cdot a\cdot b \Rightarrow a^2+b^2-c^2=-2\cdot a\cdot b \;/^2 \Rightarrow \left(a^2+b^2-c^2\right)^2=(-2\cdot a\cdot b)^2 \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4+2\cdot a^2\cdot b^2-2\cdot a^2\cdot c^2-2\cdot b^2\cdot c^2=4\cdot a^2\cdot b^2 \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4=4\cdot a^2\cdot b^2-2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2 \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4=2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2 \Rightarrow$$

$$\Rightarrow 2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=a^4+b^4+c^4.$$

Kvadriramo drugu jednakost iz zadatka.

$$a^2+b^2+c^2=1 \Rightarrow a^2+b^2+c^2=1 \;/^2 \Rightarrow \left(a^2+b^2+c^2\right)^2=1^2 \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4+2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=1 \Rightarrow$$

$$\Rightarrow 2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=1-a^4-b^4-c^4.$$

Iz sustava jednakosti dobije se:

$$\left.\begin{aligned} 2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2&=a^4+b^4+c^4\\ 2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2&=1-a^4-b^4-c^4\end{aligned}\right\} \Rightarrow \left[\begin{matrix}\text{metoda}\\ \text{komparacije}\end{matrix}\right] \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4=1-a^4-b^4-c^4 \Rightarrow a^4+b^4+c^4+a^4+b^4+c^4=1 \Rightarrow$$

$$\Rightarrow 2\cdot a^4+2\cdot b^4+2\cdot c^4=1 \Rightarrow 2\cdot\left(a^4+b^4+c^4\right)=1 \Rightarrow 2\cdot\left(a^4+b^4+c^4\right)=1 \;/:2 \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4=\frac{1}{2}.$$

Vježba 288

Ako je $a+b+c=0$ i $a^2+b^2+c^2-1=0$, izračunaj $a^4+b^4+c^4$.

Rezultat: $a^4+b^4+c^4=\frac{1}{2}$.

Zadatak 289 (Božidar, srednja škola)

Rastavi na faktore: x^6-y^6.

Rješenje 289

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right) \quad , \quad \left(a^n\right)^m=a^{n\cdot m}.$$

$$(a+b)^2 = a^2 + 2\cdot a\cdot b + b^2 \quad , \quad a^3 + b^3 = (a+b)\cdot\left(a^2 - a\cdot b + b^2\right).$$

1.inačica

Zadani izraz raspišemo kao razliku kvadrata.

$$x^6 - y^6 = \left(x^3\right)^2 - \left(y^3\right)^2 = \left(x^3 - y^3\right)\cdot\left(x^3 + y^3\right) =$$

$$= (x-y)\cdot\left(x^2 + x\cdot y + y^2\right)\cdot(x+y)\cdot\left(x^2 - x\cdot y + y^2\right) =$$

$$= (x-y)\cdot(x+y)\cdot\left(x^2 - x\cdot y + y^2\right)\cdot\left(x^2 + x\cdot y + y^2\right).$$

2.inačica

Zadani izraz raspišemo kao razliku kubova.

$$x^6 - y^6 = \left(x^2\right)^3 - \left(y^2\right)^3 = \left(x^2 - y^2\right)\cdot\left(\left(x^2\right)^2 + x^2\cdot y^2 + \left(y^2\right)^2\right) =$$

$$= (x-y)\cdot(x+y)\cdot\left(x^4 + x^2\cdot y^2 + y^4\right).$$

Množenjem dviju zagrada ili uporabom formule za razliku kvadrata lako se uvjerimo da je

$$\left(x^2 - x\cdot y + y^2\right)\cdot\left(x^2 + x\cdot y + y^2\right) = x^4 + x^2\cdot y^2 + y^4.$$

Učinimo to!

- množenje zagrada

$$\left(x^2 - x\cdot y + y^2\right)\cdot\left(x^2 + x\cdot y + y^2\right) =$$

$$= x^4 + x^3\cdot y + x^2\cdot y^2 - x^3\cdot y - x^2\cdot y^2 - x\cdot y^3 + y^2\cdot x^2 + x\cdot y^3 + y^4 =$$

$$= x^4 + x^3\cdot y + x^2\cdot y^2 - x^3\cdot y - x^2\cdot y^2 - x\cdot y^3 + y^2\cdot x^2 + x\cdot y^3 + y^4 =$$

$$= x^4 + x^2\cdot y^2 + y^4.$$

- formule za razliku kvadrata i kvadrata zbroja

$$\left(x^2 - x\cdot y + y^2\right)\cdot\left(x^2 + x\cdot y + y^2\right) = \left(x^2 + y^2 - x\cdot y\right)\cdot\left(x^2 + y^2 + x\cdot y\right) =$$

$$= \left(\left(x^2 + y^2\right) - x\cdot y\right)\cdot\left(\left(x^2 + y^2\right) + x\cdot y\right) = \left(x^2 + y^2\right)^2 - (x\cdot y)^2 =$$

$$= \left(x^2\right)^2 + 2\cdot x^2\cdot y^2 + \left(y^2\right)^2 - x^2\cdot y^2 = x^4 + 2\cdot x^2\cdot y^2 + y^4 - x^2\cdot y^2 = x^4 + x^2\cdot y^2 + y^4.$$

Vježba 289

Rastavi na faktore: $x^8 - y^8$.

Rezultat: $(x-y)\cdot(x+y)\cdot\left(x^2 + y^2\right)\cdot\left(x^4 + y^4\right).$

Zadatak 290 (Božidar, srednja škola)

$$\text{Pojednostavnite: } \left(1-\frac{1}{1-\frac{a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}.$$

Rješenje 290

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$\left(1-\frac{1}{1-\frac{a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}=\left(1-\frac{1}{\frac{1}{1}-\frac{a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}=\left(1-\frac{1}{\frac{a-1-a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}=\left(1-\frac{\frac{1}{1}}{\frac{a-1-a^2}{a-1}}\right)\cdot\frac{a^3+1}{a^2}=$$

$$=\left(1-\frac{a-1}{a-1-a^2}\right)\cdot\frac{a^3+1}{a^2}=\left(1-\frac{a-1}{-\left(a^2-a+1\right)}\right)\cdot\frac{a^3+1}{a^2}=\left(1+\frac{a-1}{a^2-a+1}\right)\cdot\frac{a^3+1}{a^2}=$$

$$=\frac{a^2-a+1+a-1}{a^2-a+1}\cdot\frac{a^3+1}{a^2}=\frac{a^2-a+1+a-1}{a^2-a+1}\cdot\frac{a^3+1}{a^2}=\frac{a^2}{a^2-a+1}\cdot\frac{a^3+1}{a^2}=$$

$$=\frac{a^2}{a^2-a+1}\cdot\frac{a^3+1}{a^2}=\frac{1}{a^2-a+1}\cdot\frac{a^3+1}{1}=\frac{a^3+1}{a^2-a+1}=\frac{(a+1)\cdot\left(a^2-a+1\right)}{a^2-a+1}=$$

$$=\frac{(a+1)\cdot\left(a^2-a+1\right)}{a^2-a+1}=a+1.$$

Vježba 290

$$\text{Pojednostavnite: } \left(1-\frac{1}{1-\frac{a^2}{a-1}}\right):\frac{a^2}{a^3+1}.$$

Rezultat: a + 1.

Zadatak 291 (Nina, ekonomska škola)

Izrazite c iz jednadžbe $\frac{1}{a}=\frac{1}{b}+\frac{1}{c}$.

Rješenje 291

Ponovimo!

$$x=y \Rightarrow y=x \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c} \quad , \quad \frac{n}{1}=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{1}{a}=\frac{1}{b}+\frac{1}{c} \Rightarrow \frac{1}{b}+\frac{1}{c}=\frac{1}{a} \Rightarrow \frac{1}{c}=\frac{1}{a}-\frac{1}{b} \Rightarrow \frac{1}{c}=\frac{b-a}{a\cdot b} \Rightarrow \frac{c}{1}=\frac{a\cdot b}{b-a} \Rightarrow c=\frac{a\cdot b}{b-a}.$$

2.inačica

$$\frac{1}{a}=\frac{1}{b}+\frac{1}{c} \Rightarrow \frac{1}{a}=\frac{1}{b}+\frac{1}{c} \ / \cdot a\cdot b\cdot c \Rightarrow b\cdot c=a\cdot c+a\cdot b \Rightarrow b\cdot c-a\cdot c=a\cdot b \Rightarrow$$

$$\Rightarrow c\cdot(b-a)=a\cdot b \Rightarrow c\cdot(b-a)=a\cdot b \ / \cdot \frac{1}{b-a} \Rightarrow c=\frac{a\cdot b}{b-a}.$$

Vježba 291

Izrazite b iz jednadžbe $\frac{1}{a}=\frac{1}{b}+\frac{1}{c}$.

Rezultat: $\frac{a\cdot c}{c-a}$.

Zadatak 292 (Marija, opća gimnazija)

Izračunaj: $\frac{1}{a-3}-\frac{6}{a^2-9}$.

Rješenje 292

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{1}{a-3}-\frac{6}{a^2-9}=\frac{1}{a-3}-\frac{6}{a^2-3^2}=\frac{1}{a-3}-\frac{6}{(a-3)\cdot(a+3)}=\left[\begin{array}{c}\text{zajednički nazivnik}\\(a-3)\cdot(a+3)\end{array}\right]=$$

$$=\frac{a+3-6}{(a-3)\cdot(a+3)}=\frac{a-3}{(a-3)\cdot(a+3)}=\frac{a-3}{(a-3)\cdot(a+3)}=\frac{1}{a+3}.$$

Vježba 292

Izračunaj: $\frac{1}{a-3}+\frac{6}{9-a^2}$.

Rezultat: $\frac{1}{a+3}$.

Zadatak 293 (Marija, opća gimnazija)

Izračunaj: $\left(\frac{x-2}{x+2}-\frac{x+2}{x-2}\right):\frac{x}{x^2-4}$.

Rješenje 293

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

$$(a-b)^2 = a^2 - 2\cdot a\cdot b + b^2 \quad , \quad (a+b)^2 = a^2 + 2\cdot a\cdot b + b^2 .$$

1.inačica

$$\left(\frac{x-2}{x+2} - \frac{x+2}{x-2}\right) : \frac{x}{x^2-4} = \left[\begin{array}{c}\text{zajednički nazivnik u zagradi}\\ (x+2)\cdot(x-2)\end{array}\right] = \frac{(x-2)^2-(x+2)^2}{(x+2)\cdot(x-2)} \cdot \frac{x^2-4}{x} =$$

$$= \frac{x^2-4\cdot x+4-\left(x^2+4\cdot x+4\right)}{(x+2)\cdot(x-2)} \cdot \frac{x^2-2^2}{x} = \frac{x^2-4\cdot x+4-x^2-4\cdot x-4}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} =$$

$$= \frac{x^2-4\cdot x+4-x^2-4\cdot x-4}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} = \frac{-8\cdot x}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} =$$

$$= \frac{-8\cdot x}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} = \frac{-8\cdot x}{x} = \frac{-8\cdot x}{x} = -8.$$

2.inačica

$$\left(\frac{x-2}{x+2} - \frac{x+2}{x-2}\right) : \frac{x}{x^2-4} = \left[\begin{array}{c}\text{zajednički nazivnik u zagradi}\\ (x+2)\cdot(x-2)\end{array}\right] = \frac{(x-2)^2-(x+2)^2}{(x+2)\cdot(x-2)} \cdot \frac{x^2-4}{x} =$$

$$= \frac{\left((x-2)-(x+2)\right)\cdot\left((x-2)+(x+2)\right)}{(x+2)\cdot(x-2)} \cdot \frac{x^2-2^2}{x} = \frac{(x-2-x-2)\cdot(x-2+x+2)}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} =$$

$$= \frac{(x-2-x-2)\cdot(x-2+x+2)}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} = \frac{-4\cdot 2\cdot x}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} =$$

$$= \frac{-8\cdot x}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} = \frac{-8\cdot x}{(x+2)\cdot(x-2)} \cdot \frac{(x-2)\cdot(x+2)}{x} = \frac{-8\cdot x}{x} = \frac{-8\cdot x}{x} = -8.$$

Vježba 293

Izračunaj: $\left(\frac{x-2}{x+2} + \frac{x+2}{2-x}\right) : \frac{x}{x^2-4}$.

Rezultat: -8.

Zadatak 294 (Marija, opća gimnazija)

Izračunaj: $\left(6-3\cdot a+\frac{18\cdot a^2}{6+3\cdot a}\right) : \frac{9\cdot a^4-144}{6\cdot a^3+48}$.

Rješenje 294

Ponovimo!

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} : \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} = \frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a\cdot d - b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a\cdot d + b\cdot c}{b\cdot d}.$$

$$a^3 + b^3 = (a+b)\cdot\left(a^2 - a\cdot b + b^2\right) \quad , \quad a^2 - b^2 = (a-b)\cdot(a+b).$$

$$a^4 - b^4 = \left(a^2-b^2\right)\cdot\left(a^2+b^2\right) = (a-b)\cdot(a+b)\cdot\left(a^2+b^2\right) \quad , \quad \left(a^n\right)^m = a^{n\cdot m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

$$\left(6-3\cdot a+\frac{18\cdot a^2}{6+3\cdot a}\right):\frac{9\cdot a^4-144}{6\cdot a^3+48}=\left(6-3\cdot a+\frac{18\cdot a^2}{3\cdot(2+a)}\right)\cdot\frac{6\cdot a^3+48}{9\cdot a^4-144}=$$

$$=\left(6-3\cdot a+\frac{18\cdot a^2}{3\cdot(2+a)}\right)\cdot\frac{6\cdot\left(a^3+8\right)}{9\cdot\left(a^4-16\right)}=\left(6-3\cdot a+\frac{6\cdot a^2}{2+a}\right)\cdot\frac{6\cdot\left(a^3+8\right)}{9\cdot\left(a^4-16\right)}=$$

$$=\left(6-3\cdot a+\frac{6\cdot a^2}{2+a}\right)\cdot\frac{2\cdot\left(a^3+8\right)}{3\cdot\left(a^4-16\right)}=\left(6-3\cdot a+\frac{6\cdot a^2}{2+a}\right)\cdot\frac{2\cdot\left(a^3+2^3\right)}{3\cdot\left(\left(a^2\right)^2-\left(2^2\right)^2\right)}=$$

$$=\left(6-3\cdot a+\frac{6\cdot a^2}{2+a}\right)\cdot\frac{2\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}{3\cdot\left(a^2-2^2\right)\cdot\left(a^2+2^2\right)}=\left(\frac{6}{1}-\frac{3\cdot a}{1}+\frac{6\cdot a^2}{2+a}\right)\cdot\frac{2\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}{3\cdot(a-2)\cdot(a+2)\cdot\left(a^2+4\right)}=$$

$$=\frac{6\cdot(2+a)-3\cdot a\cdot(2+a)+6\cdot a^2}{2+a}\cdot\frac{2\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}{3\cdot(a-2)\cdot(a+2)\cdot\left(a^2+4\right)}=$$

$$=\frac{12+6\cdot a-6\cdot a-3\cdot a^2+6\cdot a^2}{2+a}\cdot\frac{2\cdot(a+2)\cdot\left(a^2-2\cdot a+4\right)}{3\cdot(a-2)\cdot(a+2)\cdot\left(a^2+4\right)}=$$

$$=\frac{12+6\cdot a-6\cdot a-3\cdot a^2+6\cdot a^2}{2+a}\cdot\frac{2\cdot\left(a^2-2\cdot a+4\right)}{3\cdot(a-2)\cdot\left(a^2+4\right)}=\frac{12+3\cdot a^2}{2+a}\cdot\frac{2\cdot\left(a^2-2\cdot a+4\right)}{3\cdot(a-2)\cdot\left(a^2+4\right)}=$$

$$=\frac{3\cdot\left(4+a^2\right)}{2+a}\cdot\frac{2\cdot\left(a^2-2\cdot a+4\right)}{3\cdot(a-2)\cdot\left(a^2+4\right)}=\frac{3\cdot\left(a^2+4\right)}{2+a}\cdot\frac{2\cdot\left(a^2-2\cdot a+4\right)}{3\cdot(a-2)\cdot\left(a^2+4\right)}=$$

$$=\frac{2\cdot\left(a^2-2\cdot a+4\right)}{(2+a)\cdot(a-2)}=\frac{2\cdot\left(a^2-2\cdot a+4\right)}{(a+2)\cdot(a-2)}=\frac{2\cdot\left(a^2-2\cdot a+4\right)}{a^2-4}.$$

Vježba 294

Izračunaj: $\frac{9\cdot a^4-144}{6\cdot a^3+48}:\left(6-3\cdot a+\frac{18\cdot a^2}{6+3\cdot a}\right)$.

Rezultat: $\frac{a^2-4}{2\cdot\left(a^2-2\cdot a+4\right)}$.

Zadatak 295 (Ivona, gimnazija)

Izračunaj: $\left(x^{2\cdot m}-y^n\right)\cdot\left(x^m+y^{2\cdot n}\right)$.

Rješenje 295

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad \left(a^n\right)^m = a^{n \cdot m} \quad , \quad (a \cdot b)^n = a^n \cdot b^n .$$

Množenje zagrada:

$$(a+b)\cdot(c+d) = a\cdot c + a\cdot d + b\cdot c + b\cdot d .$$

$$\left(x^{2\cdot m} - y^n\right)\cdot\left(x^m + y^{2\cdot n}\right) = x^{2\cdot m}\cdot x^m + x^{2\cdot m}\cdot y^{2\cdot n} - y^n \cdot x^m - y^n \cdot y^{2\cdot n} =$$

$$= x^{3\cdot m} + x^{2\cdot m}\cdot y^{2\cdot n} - x^m \cdot y^n - y^{3\cdot n} = x^{3\cdot m} + \left(x^m\right)^2 \cdot \left(y^n\right)^2 - x^m \cdot y^n - y^{3\cdot n} =$$

$$= x^{3\cdot m} + \left(x^m \cdot y^n\right)^2 - x^m \cdot y^n - y^{3\cdot n} .$$

Vježba 295

Izračunaj: $\left(x^m - y^n\right)\cdot\left(x^m + y^n\right)$.

Rezultat: $x^{2\cdot m} - y^{2\cdot n}$.

Zadatak 296 (Neven, tehnička škola)

Izračunajte: $\left(\dfrac{a+b}{a^2\cdot b - a\cdot b^2} - \dfrac{a-b}{a^2\cdot b + a\cdot b^2}\right)\cdot\dfrac{a^4 - b^4}{4}$.

Rješenje 296

Ponovimo!

$$x^2 - y^2 = (x-y)\cdot(x+y) \quad , \quad x^4 - y^4 = \left(x^2 - y^2\right)\cdot\left(x^2 + y^2\right) = (x-y)\cdot(x+y)\cdot\left(x^2 + y^2\right).$$

$$(x+y)^2 = x^2 + 2\cdot x\cdot y + y^2 \quad , \quad (x-y)^2 = x^2 - 2\cdot x\cdot y + y^2 \quad , \quad \frac{x}{n} - \frac{y}{n} = \frac{x-y}{n} .$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c) .$$

1.inačica

$$\left(\frac{a+b}{a^2\cdot b - a\cdot b^2} - \frac{a-b}{a^2\cdot b + a\cdot b^2}\right)\cdot\frac{a^4 - b^4}{4} = \left(\frac{a+b}{a\cdot b\cdot(a-b)} - \frac{a-b}{a\cdot b\cdot(a+b)}\right)\cdot\frac{\left(a^2 - b^2\right)\cdot\left(a^2 + b^2\right)}{4} =$$

$$= \frac{(a+b)^2 - (a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}\cdot\frac{(a-b)\cdot(a+b)\cdot\left(a^2 + b^2\right)}{4} = \frac{(a+b)^2 - (a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}\cdot\frac{(a-b)\cdot(a+b)\cdot\left(a^2 + b^2\right)}{4} =$$

$$= \frac{(a+b)^2 - (a-b)^2}{a\cdot b}\cdot\frac{a^2 + b^2}{4} = \frac{a^2 + 2\cdot a\cdot b + b^2 - \left(a^2 - 2\cdot a\cdot b + b^2\right)}{a\cdot b}\cdot\frac{a^2 + b^2}{4} =$$

$$= \frac{a^2 + 2\cdot a\cdot b + b^2 - a^2 + 2\cdot a\cdot b - b^2}{a\cdot b}\cdot\frac{a^2 + b^2}{4} =$$

$$= \frac{a^2 + 2\cdot a\cdot b + b^2 - a^2 + 2\cdot a\cdot b - b^2}{a\cdot b}\cdot\frac{a^2 + b^2}{4} = \frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2 + b^2}{4} = \frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2 + b^2}{4} = a^2 + b^2 .$$

2.inačica

$$\left(\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}\right)\cdot\frac{a^4-b^4}{4}=\left(\frac{a+b}{a\cdot b\cdot(a-b)}-\frac{a-b}{a\cdot b\cdot(a+b)}\right)\cdot\frac{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)}{4}=$$

$$=\frac{(a+b)^2-(a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}\cdot\frac{(a-b)\cdot(a+b)\cdot\left(a^2+b^2\right)}{4}=\frac{(a+b)^2-(a-b)^2}{a\cdot b\cdot(a-b)\cdot(a+b)}\cdot\frac{(a-b)\cdot(a+b)\cdot\left(a^2+b^2\right)}{4}=$$

$$=\frac{(a+b)^2-(a-b)^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{((a+b)-(a-b))\cdot((a+b)+(a-b))}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{(a+b-a+b)\cdot(a+b+a-b)}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{(a+b-a+b)\cdot(a+b+a-b)}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{2\cdot b\cdot 2\cdot a}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=a^2+b^2.$$

3.inačica

$$\left(\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}\right)\cdot\frac{a^4-b^4}{4}=\left(\frac{a+b}{a\cdot b\cdot(a-b)}-\frac{a-b}{a\cdot b\cdot(a+b)}\right)\cdot\frac{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)}{4}=$$

$$=\left(\frac{a+b}{a\cdot b\cdot(a-b)}-\frac{a-b}{a\cdot b\cdot(a+b)}\right)\cdot\frac{(a-b)\cdot(a+b)\cdot\left(a^2+b^2\right)}{4}=$$

$$=\left(\frac{a+b}{a\cdot b\cdot(a-b)}\cdot(a-b)\cdot(a+b)-\frac{a-b}{a\cdot b\cdot(a+b)}\cdot(a-b)\cdot(a+b)\right)\cdot\frac{a^2+b^2}{4}=$$

$$=\left(\frac{a+b}{a\cdot b\cdot(a-b)}\cdot(a-b)\cdot(a+b)-\frac{a-b}{a\cdot b\cdot(a+b)}\cdot(a-b)\cdot(a+b)\right)\cdot\frac{a^2+b^2}{4}=$$

$$=\left(\frac{(a+b)^2}{a\cdot b}-\frac{(a-b)^2}{a\cdot b}\right)\cdot\frac{a^2+b^2}{4}=\frac{(a+b)^2-(a-b)^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{a^2+2\cdot a\cdot b+b^2-\left(a^2-2\cdot a\cdot b+b^2\right)}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}=$$

$$=\frac{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=\frac{4\cdot a\cdot b}{a\cdot b}\cdot\frac{a^2+b^2}{4}=a^2+b^2.$$

4.inačica

$$\left(\frac{a+b}{a^2\cdot b-a\cdot b^2}-\frac{a-b}{a^2\cdot b+a\cdot b^2}\right)\cdot\frac{a^4-b^4}{4}=\left(\frac{a+b}{a\cdot b\cdot(a-b)}-\frac{a-b}{a\cdot b\cdot(a+b)}\right)\cdot\frac{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)}{4}=$$

$$=\left(\frac{a+b}{a\cdot b\cdot(a-b)}-\frac{a-b}{a\cdot b\cdot(a+b)}\right)\cdot\frac{(a-b)\cdot(a+b)\cdot\left(a^2+b^2\right)}{4}=$$

$$= \left(\frac{a+b}{a\cdot b\cdot (a-b)} \cdot (a-b)\cdot (a+b) - \frac{a-b}{a\cdot b\cdot (a+b)} \cdot (a-b)\cdot (a+b) \right) \cdot \frac{a^2+b^2}{4} =$$

$$= \left(\frac{a+b}{a\cdot b\cdot (a-b)} \cdot (a-b)\cdot (a+b) - \frac{a-b}{a\cdot b\cdot (a+b)} \cdot (a-b)\cdot (a+b) \right) \cdot \frac{a^2+b^2}{4} =$$

$$= \left(\frac{(a+b)^2}{a\cdot b} - \frac{(a-b)^2}{a\cdot b} \right) \cdot \frac{a^2+b^2}{4} = \frac{(a+b)^2-(a-b)^2}{a\cdot b} \cdot \frac{a^2+b^2}{4} =$$

$$= \frac{\big((a+b)-(a-b)\big)\cdot\big((a+b)+(a-b)\big)}{a\cdot b} \cdot \frac{a^2+b^2}{4} = \frac{(a+b-a+b)\cdot(a+b+a-b)}{a\cdot b} \cdot \frac{a^2+b^2}{4} =$$

$$= \frac{(a+b-a+b)\cdot(a+b+a-b)}{a\cdot b} \cdot \frac{a^2+b^2}{4} = \frac{2\cdot b\cdot 2\cdot a}{a\cdot b} \cdot \frac{a^2+b^2}{4} =$$

$$= \frac{4\cdot a\cdot b}{a\cdot b} \cdot \frac{a^2+b^2}{4} = \frac{4\cdot a\cdot b}{a\cdot b} \cdot \frac{a^2+b^2}{4} = a^2+b^2.$$

Vježba 296

Izračunajte: $\left(\frac{a+b}{a^2\cdot b - a\cdot b^2} + \frac{b-a}{a^2\cdot b + a\cdot b^2} \right) \cdot \frac{a^4-b^4}{4}.$

Rezultat: $a^2+b^2.$

Zadatak 297 (Tanja, srednja škola)

Ako je $a\cdot(a-b)=11$, a $b\cdot(a-b)=13$, tada je:

A) $a^2-b^2=143$, B) $a^2-b^2=2$, C) $a^2-b^2=24$, D) $a^2-b^2=48$

Rješenje 297

Ponovimo!

$$x^n\cdot x^m = x^{n+m} \quad , \quad x^1 = x \quad , \quad (x\cdot y)^n = x^n\cdot y^n.$$

$$(x+y)^2 = x^2+2\cdot x\cdot y+y^2 \quad , \quad (x-y)^2 = x^2-2\cdot x\cdot y+y^2 \quad , \quad \frac{x}{n}-\frac{y}{n}=\frac{x-y}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c = a\cdot(b+c).$$

1.inačica

$$\left.\begin{matrix} a\cdot(a-b)=11 \\ b\cdot(a-b)=13 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a^2-a\cdot b=11 \\ a\cdot b-b^2=13 \end{matrix}\right\} \Rightarrow \left[\begin{matrix}\text{zbrojimo} \\ \text{jednakosti}\end{matrix}\right] \Rightarrow a^2-a\cdot b+a\cdot b-b^2=11+13 \Rightarrow$$

$$\Rightarrow a^2-a\cdot b+a\cdot b-b^2=24 \Rightarrow a^2-b^2=24.$$

Odgovor je pod C.

2.inačica

$$\left.\begin{matrix} a\cdot(a-b)=11 \\ b\cdot(a-b)=13 \end{matrix}\right\} \Rightarrow \left[\begin{matrix}\text{podijelimo} \\ \text{jednakosti}\end{matrix}\right] \Rightarrow \frac{a\cdot(a-b)}{b\cdot(a-b)}=\frac{11}{13} \Rightarrow \frac{a\cdot(a-b)}{b\cdot(a-b)}=\frac{11}{13} \Rightarrow$$

$$\Rightarrow \frac{a}{b}=\frac{11}{13} \Rightarrow \frac{a}{b}=\frac{11}{13} \;/\cdot b \Rightarrow a=\frac{11}{13}\cdot b.$$

Odredimo b^2.

$$\left.\begin{array}{l} a=\frac{11}{13}\cdot b \\ a\cdot(a-b)=11 \end{array}\right\} \Rightarrow \frac{11}{13}\cdot b\cdot\left(\frac{11}{13}\cdot b-b\right)=11 \Rightarrow \frac{11}{13}\cdot b\cdot b\cdot\left(\frac{11}{13}-1\right)=11 \Rightarrow \frac{11}{13}\cdot b^2\cdot\left(\frac{11}{13}-\frac{1}{1}\right)=11 \Rightarrow$$

$$\Rightarrow \frac{11}{13}\cdot b^2\cdot\frac{11-13}{13}=11 \Rightarrow \frac{11}{13}\cdot b^2\cdot\left(-\frac{2}{13}\right)=11 \Rightarrow -\frac{22}{169}\cdot b^2=11 \Rightarrow -\frac{22}{169}\cdot b^2=11\ /\cdot\left(-\frac{169}{22}\right) \Rightarrow$$

$$\Rightarrow b^2=11\cdot\left(-\frac{169}{22}\right) \Rightarrow b^2=11\cdot\left(-\frac{169}{22}\right) \Rightarrow b^2=-\frac{169}{2}.$$

Računamo a^2.

$$\left.\begin{array}{l} a=\frac{11}{13}\cdot b \\ b^2=-\frac{169}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=\frac{11}{13}\cdot b\ /^2 \\ b^2=-\frac{169}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} a^2=\frac{121}{169}\cdot b^2 \\ b^2=-\frac{169}{2} \end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda} \\ \text{supstitucije}\end{array}\right] \Rightarrow a^2=\frac{121}{169}\cdot\left(-\frac{169}{2}\right) \Rightarrow$$

$$\Rightarrow a^2=\frac{121}{169}\cdot\left(-\frac{169}{2}\right) \Rightarrow a^2=-\frac{121}{2}.$$

Tada je

$$a^2-b^2=-\frac{121}{2}-\left(-\frac{169}{2}\right)=-\frac{121}{2}+\frac{169}{2}=\frac{-121+169}{2}=\frac{48}{2}=24.$$

Odgovor je pod C.

Vježba 297

Ako je $a\cdot(a-b)=10$, a $b\cdot(a-b)=12$, tada je:

A) $a^2-b^2=120$, *B)* $a^2-b^2=2$, *C)* $a^2-b^2=22$, *D)* $a^2-b^2=44$

Rezultat: C.

Zadatak 298 (Dalia, gimnazija)

Ako je $2\cdot x^2+5\cdot y^2+z^2-4\cdot x\cdot y+2\cdot x\cdot z+2\cdot y+1=0$, koliko je $x+y+z$?

Rješenje 298

Ponovimo!

$$(x-y)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad (x+y)^2=x^2+2\cdot x\cdot y+y^2.$$

$$a^2+b^2+c^2=0 \Leftrightarrow a=b=c=0.$$

Metodom grupiranja zadanu jednakost transformiramo na sljedeći oblik:

$$2\cdot x^2+5\cdot y^2+z^2-4\cdot x\cdot y+2\cdot x\cdot z+2\cdot y+1=0 \Rightarrow$$

$$\Rightarrow x^2-4\cdot x\cdot y+4\cdot y^2+x^2+2\cdot x\cdot z+z^2+y^2+2\cdot y+1=0 \Rightarrow$$

$$\Rightarrow \left(x^2-4\cdot x\cdot y+4\cdot y^2\right)+\left(x^2+2\cdot x\cdot z+z^2\right)+\left(y^2+2\cdot y+1\right)=0 \Rightarrow$$

$$\Rightarrow (x-2\cdot y)^2+(x+z)^2+(y+1)^2=0 \Rightarrow \left.\begin{array}{l} x-2\cdot y=0 \\ x+z=0 \\ y+1=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=2\cdot y \\ z=-x \\ y=-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=2\cdot(-1) \\ z=-x \\ y=-1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x=-2 \\ z=-x \\ y=-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=-2 \\ z=-(-2) \\ y=-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=-2 \\ z=2 \\ y=-1 \end{array}\right\}.$$

Tada je:

$$x+y+z=-2-1+2 \Rightarrow x+y+z=-2-1+2 \Rightarrow x+y+z=-1.$$

Vježba 298

Ako je $2\cdot x^2+5\cdot y^2+z^2-4\cdot x\cdot y-2\cdot x\cdot z+2\cdot y+1=0$, koliko je $x+y+z$?

Rezultat: -5.

Zadatak 299 (Dalia, gimnazija)

Ako je $x^2-x+2=0$, onda je $x^4-2\cdot x^3+x^2+7$ jednako:

A) 11 *B*) 13 *C*) 15 *D*) 17

Rješenje 299

Ponovimo!

Kvadrat trinoma

$$(a+b+c)^2=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c.$$

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad (a^n)^m=a^{n\cdot m} \quad , \quad -a+a=0.$$

Kvadriramo zadanu jednakost (kvadrat trinoma).

$$x^2-x+2=0 \Rightarrow x^2-x+2=0 \ /^2 \Rightarrow \left(x^2-x+2\right)^2=0 \Rightarrow$$

$$\Rightarrow \left(x^2\right)^2+(-x)^2+2^2+2\cdot x^2\cdot(-x)+2\cdot x^2\cdot 2+2\cdot(-x)\cdot 2=0 \Rightarrow$$

$$\Rightarrow x^4+x^2+4-2\cdot x^3+4\cdot x^2-4\cdot x=0 \Rightarrow$$

$$\Rightarrow x^4-2\cdot x^3+x^2+4\cdot x^2-4\cdot x+4=0 \Rightarrow x^4-2\cdot x^3+x^2+7-7+4\cdot x^2-4\cdot x+4=0 \Rightarrow$$

$$\Rightarrow x^4-2\cdot x^3+x^2+7-7+4\cdot x^2-4\cdot x+4=0 \Rightarrow x^4-2\cdot x^3+x^2+7=-4\cdot x^2+4\cdot x+7-4 \Rightarrow$$

$$\Rightarrow x^4-2\cdot x^3+x^2+7=-4\cdot x^2+4\cdot x+3 \Rightarrow x^4-2\cdot x^3+x^2+7=-4\cdot x^2+4\cdot x-8+8+3 \Rightarrow$$

$$\Rightarrow x^4-2\cdot x^3+x^2+7=\left(-4\cdot x^2+4\cdot x-8\right)+8+3 \Rightarrow x^4-2\cdot x^3+x^2+7=-4\cdot\left(x^2-x+2\right)+11 \Rightarrow$$

$$\Rightarrow x^4-2\cdot x^3+x^2+7=-4\cdot\underbrace{\left(x^2-x+2\right)}_{=0}+11 \Rightarrow x^4-2\cdot x^3+x^2+7=-4\cdot 0+11 \Rightarrow$$

$$\Rightarrow x^4-2\cdot x^3+x^2+7=0+11 \Rightarrow x^4-2\cdot x^3+x^2+7=11.$$

Odgovor je pod A.

Vježba 299

Ako je $x^2=x-2$, onda je $x^4-2\cdot x^3+x^2+7$ jednako:

A) 12 *B*) 11 *C*) 13 *D*) 14

Rezultat: B.

Zadatak 300 (Dalia, gimnazija)

Ako je $(x-1)\cdot(x-3)=4$, koliko je $(x-2)^2$?

Rješenje 300

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Transformiramo zadanu jednadžbu.

$$(x-1)\cdot(x-3)=4 \Rightarrow x^2-3\cdot x-x+3=4 \Rightarrow x^2-4\cdot x=4-3 \Rightarrow x^2-4\cdot x=1 \Rightarrow$$

$$\Rightarrow x^2-4\cdot x+4-4=1 \Rightarrow \left(x^2-4\cdot x+4\right)-4=1 \Rightarrow (x-2)^2-4=1 \Rightarrow (x-2)^2=1+4 \Rightarrow$$

$$\Rightarrow (x-2)^2=5.$$

2.inačica

Riješimo kvadratnu jednadžbu.

$$(x-1)\cdot(x-3)=4 \Rightarrow x^2-3\cdot x-x+3=4 \Rightarrow x^2-4\cdot x+3-4=0 \Rightarrow x^2-4\cdot x-1=0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x^2-4\cdot x-1=0 \\ a=1\ ,\ b=-4\ ,\ c=-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=1\ ,\ b=-4\ ,\ c=-1 \\ x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow x_{1,2}=\frac{4\pm\sqrt{16-4\cdot 1\cdot(-1)}}{2\cdot 1} \Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{4\pm\sqrt{16+4}}{2} \Rightarrow x_{1,2}=\frac{4\pm\sqrt{20}}{2} \Rightarrow x_{1,2}=\frac{4\pm\sqrt{4\cdot 5}}{2} \Rightarrow x_{1,2}=\frac{4\pm 2\cdot\sqrt{5}}{2} \Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{2\cdot\left(2\pm\sqrt{5}\right)}{2} \Rightarrow x_{1,2}=\frac{2\cdot\left(2\pm\sqrt{5}\right)}{2} \Rightarrow x_{1,2}=2\pm\sqrt{5} \Rightarrow \left.\begin{array}{l} x_1=2+\sqrt{5} \\ x_2=2-\sqrt{5} \end{array}\right\}.$$

Sada je:

$$(x-2)^2=\left[\begin{array}{l} x=2+\sqrt{5} \\ x=2-\sqrt{5} \end{array}\right]=\left.\begin{array}{l} \left(2+\sqrt{5}-2\right)^2=\left(2+\sqrt{5}-2\right)^2=\left(\sqrt{5}\right)^2=5 \\ \left(2-\sqrt{5}-2\right)^2=\left(2-\sqrt{5}-2\right)^2=\left(-\sqrt{5}\right)^2=5 \end{array}\right\}.$$

Vježba 300

Ako je $(1-x)\cdot(3-x)=4$, koliko je $(x-2)^2$?

Rezultat: 5.