Zadatak 261 (Kiki, ekonomska škola)

Kolika je vrijednost izraza $x \cdot (x + 2) + y \cdot (y - 2) - 2 \cdot x \cdot y$, ako je $x - y = 7$?

Rješenje 261

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$x \cdot (x+2) + y \cdot (y-2) - 2 \cdot x \cdot y = x^2 + 2 \cdot x + y^2 - 2 \cdot y - 2 \cdot x \cdot y = x^2 - 2 \cdot x \cdot y + y^2 + 2 \cdot x - 2 \cdot y =$$

$$= \left(x^2 - 2 \cdot x \cdot y + y^2\right) + 2 \cdot (x-y) = (x-y)^2 + 2 \cdot (x-y) = [\, x - y = 7 \,] = 7^2 + 2 \cdot 7 = 63.$$

Vježba 261

Kolika je vrijednost izraza $x \cdot (x + 2) + y \cdot (y - 2) - 2 \cdot x \cdot y$, ako je $x - y = 5$?

Rezultat: 35.

Zadatak 262 (Kiki, ekonomska škola)

Rastavi na faktore izraz: $x^2 \cdot \left(x^2 - 9\right) - x^2 + 9.$

Rješenje 262

Ponovimo!

$$a^2 - b^2 = (a-b) \cdot (a+b) \quad , \quad (a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

$$x^2 \cdot \left(x^2 - 9\right) - x^2 + 9 = x^2 \cdot \left(x^2 - 9\right) - \left(x^2 - 9\right) = \begin{bmatrix} \text{izlučimo} \\ x^2 - 9 \end{bmatrix} = \left(x^2 - 9\right) \cdot \left(x^2 - 1\right) =$$

$$= (x-3) \cdot (x+3) \cdot (x-1) \cdot (x+1).$$

2.inačica

$$x^2 \cdot \left(x^2 - 9\right) - x^2 + 9 = x^4 - 9 \cdot x^2 - x^2 + 9 = x^4 - 10 \cdot x^2 + 9 = x^4 - 10 \cdot x^2 + 5^2 - 5^2 + 9 =$$

$$= x^4 - 10 \cdot x^2 + 25 - 25 + 9 = \left(x^4 - 10 \cdot x^2 + 25\right) - 25 + 9 = \left(x^4 - 10 \cdot x^2 + 25\right) - 16 =$$

$$= \left(x^2 - 5\right)^2 - 4^2 = \left(x^2 - 5 - 4\right) \cdot \left(x^2 - 5 + 4\right) = \left(x^2 - 9\right) \cdot \left(x^2 - 1\right) = (x-3) \cdot (x+3) \cdot (x-1) \cdot (x+1).$$

Vježba 262

Rastavi na faktore izraz: $x^2 \cdot \left(x^2 - 4\right) - x^2 + 4$

Rezultat: $(x-2) \cdot (x+2) \cdot (x-1) \cdot (x+1).$

Zadatak 263 (Malecka, ekonomska škola)

Pojednostavni: $\dfrac{(2 \cdot x + 1)^2 - 8 \cdot x}{1 - 4 \cdot x^2}.$

Rješenje 263

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad a^2-b^2=(a-b)\cdot(a+b).$$

$$\frac{(2\cdot x+1)^2-8\cdot x}{1-4\cdot x^2}=\frac{4\cdot x^2+4\cdot x+1-8\cdot x}{(1-2\cdot x)\cdot(1+2\cdot x)}=\frac{4\cdot x^2-4\cdot x+1}{(1-2\cdot x)\cdot(1+2\cdot x)}=\frac{(2\cdot x-1)^2}{(1-2\cdot x)\cdot(1+2\cdot x)}=$$

$$=\frac{(1-2\cdot x)^2}{(1-2\cdot x)\cdot(1+2\cdot x)}=\frac{(1-2\cdot x)^2}{(1-2\cdot x)\cdot(1+2\cdot x)}=\frac{1-2\cdot x}{1+2\cdot x}=\frac{1-2\cdot x}{2\cdot x+1}.$$

Vježba 263

Pojednostavni: $\frac{1-4\cdot x^2}{(2\cdot x+1)^2-8\cdot x}$.

Rezultat: $\frac{2\cdot x+1}{1-2\cdot x}$.

Zadatak 264 (Malecka, ekonomska škola)

Iz formule $P=\frac{a+c}{2}\cdot v$ izrazi c.

Rješenje 264

Ponovimo!

$$\frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}.$$

Zakon distribucije množenja prema zbrajanju:

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$P=\frac{a+c}{2}\cdot v \Rightarrow P=\frac{a+c}{2}\cdot v \;/\cdot 2 \Rightarrow 2\cdot P=(a+c)\cdot v \Rightarrow 2\cdot P=(a+c)\cdot v \;/\cdot \frac{1}{v} \Rightarrow$$

$$\Rightarrow \frac{2\cdot P}{v}=a+c \Rightarrow -c=a-\frac{2\cdot P}{v} \Rightarrow -c=a-\frac{2\cdot P}{v} \;/\cdot(-1) \Rightarrow c=-a+\frac{2\cdot P}{v} \Rightarrow c=\frac{2\cdot P}{v}-a.$$

2.inačica

$$P=\frac{a+c}{2}\cdot v \Rightarrow P=\frac{a+c}{2}\cdot v \;/\cdot \frac{2}{v} \Rightarrow \frac{2\cdot P}{v}=a+c \Rightarrow a+c=\frac{2\cdot P}{v} \Rightarrow c=\frac{2\cdot P}{v}-a.$$

3.inačica

$$P=\frac{a+c}{2}\cdot v \Rightarrow P=\frac{(a+c)\cdot v}{2} \Rightarrow P=\frac{a\cdot v+c\cdot v}{2} \Rightarrow P=\frac{a\cdot v+c\cdot v}{2} \;/\cdot 2 \Rightarrow 2\cdot P=a\cdot v+c\cdot v \Rightarrow$$

$$\Rightarrow -c\cdot v=a\cdot v-2\cdot P \Rightarrow -c\cdot v=a\cdot v-2\cdot P \;/\cdot(-1) \Rightarrow c\cdot v=-a\cdot v+2\cdot P \Rightarrow$$

$$\Rightarrow c\cdot v=2\cdot P-a\cdot v \Rightarrow c\cdot v=2\cdot P-a\cdot v \;/\cdot \frac{1}{v} \Rightarrow c=\frac{2\cdot P-a\cdot v}{v} \Rightarrow c=\frac{2\cdot P}{v}-\frac{a\cdot v}{v} \Rightarrow$$

$$\Rightarrow c=\frac{2\cdot P}{v}-\frac{a\cdot v}{v} \Rightarrow c=\frac{2\cdot P}{v}-a.$$

Vježba 264

Iz formule $P=\frac{a+c}{2}\cdot v$ izrazi a.

Rezultat: $a=\frac{2\cdot P}{v}-c.$

Zadatak 265 (Malecka, ekonomska škola)

Iz formule $s=v\cdot t-\frac{1}{2}\cdot a\cdot t^2$ izrazi a.

Rješenje 265

Ponovimo!

Zakon distribucije množenja prema zbrajanju:

$$x\cdot(y+z)=x\cdot y+x\cdot z \quad , \quad x\cdot y+x\cdot z=x\cdot(y+z).$$

1.inačica

$$s=v\cdot t-\frac{1}{2}\cdot a\cdot t^2 \Rightarrow s=v\cdot t-\frac{1}{2}\cdot a\cdot t^2 \;/\cdot 2 \Rightarrow 2\cdot s=2\cdot v\cdot t-a\cdot t^2 \Rightarrow a\cdot t^2=2\cdot v\cdot t-2\cdot s \Rightarrow$$

$$\Rightarrow a\cdot t^2=2\cdot v\cdot t-2\cdot s \;/:t^2 \Rightarrow a=\frac{2\cdot v\cdot t-2\cdot s}{t^2} \Rightarrow a=\frac{2\cdot(v\cdot t-s)}{t^2}.$$

2.inačica

$$s=v\cdot t-\frac{1}{2}\cdot a\cdot t^2 \Rightarrow \frac{1}{2}\cdot a\cdot t^2=v\cdot t-s \Rightarrow \frac{1}{2}\cdot a\cdot t^2=v\cdot t-s \;/\cdot\frac{2}{t^2} \Rightarrow a=\frac{2\cdot(v\cdot t-s)}{t^2}.$$

Vježba 265

Iz formule $s=v\cdot t+\frac{1}{2}\cdot a\cdot t^2$ izrazi a.

Rezultat: $a=\frac{2\cdot(s-v\cdot t)}{t^2}.$

Zadatak 266 (Branko, srednja škola)

Izračunajte: $\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdot\left(1-\frac{1}{4^2}\right)\cdot\ \ldots\ \cdot\left(1-\frac{1}{98^2}\right)\cdot\left(1-\frac{1}{99^2}\right)\cdot\left(1-\frac{1}{100^2}\right).$

Rješenje 266

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b).$$

$$\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdot\ \ldots\ \cdot\left(1-\frac{1}{99^2}\right)\cdot\left(1-\frac{1}{100^2}\right)=$$

$$=\left(1-\frac{1}{2}\right)\cdot\left(1+\frac{1}{2}\right)\cdot\left(1-\frac{1}{3}\right)\cdot\left(1+\frac{1}{3}\right)\cdot\ \ldots\ \cdot\left(1-\frac{1}{99}\right)\cdot\left(1+\frac{1}{99}\right)\cdot\left(1-\frac{1}{100}\right)\cdot\left(1+\frac{1}{100}\right)=$$

$$=\frac{2-1}{2}\cdot\frac{2+1}{2}\cdot\frac{3-1}{3}\cdot\frac{3+1}{3}\cdot\ \ldots\ \cdot\frac{99-1}{99}\cdot\frac{99+1}{99}\cdot\frac{100-1}{100}\cdot\frac{100+1}{100}=$$

$$=\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\ \ldots\ \cdot\frac{98}{99}\cdot\frac{100}{99}\cdot\frac{99}{100}\cdot\frac{101}{100}=\frac{1}{2}\cdot\frac{3}{2}\cdot\frac{2}{3}\cdot\frac{4}{3}\cdot\ \ldots\ \cdot\frac{98}{99}\cdot\frac{100}{99}\cdot\frac{99}{100}\cdot\frac{101}{100}=\frac{1}{2}\cdot\frac{101}{100}=\frac{101}{200}.$$

Vježba 266

Izračunajte: $\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdot\ \ldots\ \cdot\left(1-\frac{1}{49^2}\right)\cdot\left(1-\frac{1}{50^2}\right).$

Rezultat: $\frac{51}{100}.$

Zadatak 267 (Marina, gimnazija)

Racionaliziraj nazivnik : $\frac{x-y}{x+y+2\cdot\sqrt{x\cdot y}}.$

Rješenje 267

Ponovimo!

$$\left(\sqrt{a}\right)^2=a \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad a\cdot a=a^2.$$

$$\frac{x-y}{x+y+2\cdot\sqrt{x\cdot y}}=\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\sqrt{x}+\sqrt{y}\right)}{x+2\cdot\sqrt{x\cdot y}+y}=\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\sqrt{x}+\sqrt{y}\right)}{\left(\sqrt{x}+\sqrt{y}\right)^2}=\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\sqrt{x}+\sqrt{y}\right)}{\left(\sqrt{x}+\sqrt{y}\right)^2}=$$

$$=\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\cdot\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}-\sqrt{y}}=\frac{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\sqrt{x}-\sqrt{y}\right)}{\left(\sqrt{x}\right)^2-\left(\sqrt{y}\right)^2}=$$

$$=\frac{\left(\sqrt{x}-\sqrt{y}\right)^2}{x-y}.$$

Vježba 267

Racionaliziraj nazivnik : $\frac{x-y}{x+y-2\cdot\sqrt{x\cdot y}}.$

Rezultat: $\frac{\left(\sqrt{x}+\sqrt{y}\right)^2}{x-y}.$

Zadatak 268 (Vedran, srednja škola)

Pojednostavni : $18\cdot\sqrt{98}-2\cdot\sqrt{7}+\sqrt{18}.$

Rješenje 268

Ponovimo!

$$\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b} \quad , \quad \sqrt{a^2}=a,\ a\ge 0.$$

Svaki korijen djelomično korjenujemo:

- $\sqrt{98}=\sqrt{49\cdot 2}=\sqrt{49}\cdot\sqrt{2}=\sqrt{7^2}\cdot\sqrt{2}=7\cdot\sqrt{2}$
- $\sqrt{18}=\sqrt{9\cdot 2}=\sqrt{9}\cdot\sqrt{2}=\sqrt{3^2}\cdot\sqrt{2}=3\cdot\sqrt{2}.$

Sada je:

$$18\cdot\sqrt{98}-2\cdot\sqrt{7}+\sqrt{18}=18\cdot 7\cdot\sqrt{2}-2\cdot\sqrt{7}+3\cdot\sqrt{2}=126\cdot\sqrt{2}-2\cdot\sqrt{7}+3\cdot\sqrt{2}=129\cdot\sqrt{2}-2\cdot\sqrt{7}.$$

Vježba 268

Pojednostavni : $18\cdot\sqrt{98}-2\cdot\sqrt{2}+\sqrt{18}.$

Rezultat: $127\cdot\sqrt{2}.$

Zadatak 269 (Jasna, srednja škola)

Ako je $a+b+c=0$ i $a^2+b^2+c^2=1$, izračunaj $a^4+b^4+c^4$.

Rješenje 269

Ponovimo!

$$(x\cdot y)^n=x^n\cdot y^n \quad , \quad \left(x^n\right)^m=x^{n\cdot m} \quad , \quad (x+y)^2=x^2+2\cdot x\cdot y+y^2.$$

$$(x+y+z)^2=x^2+y^2+z^2+2\cdot x\cdot y+2\cdot x\cdot z+2\cdot y\cdot z.$$

Kvadriramo prvu jednakost.

$$a+b+c=0 \Rightarrow a+b=-c \Rightarrow a+b=-c\ /^2 \Rightarrow (a+b)^2=(-c)^2 \Rightarrow a^2+2\cdot a\cdot b+b^2=c^2 \Rightarrow$$

$$\Rightarrow a^2+b^2-c^2=-2\cdot a\cdot b \Rightarrow a^2+b^2-c^2=-2\cdot a\cdot b\ /^2 \Rightarrow \left(a^2+b^2-c^2\right)^2=(-2\cdot a\cdot b)^2 \Rightarrow$$

$$\Rightarrow \left(a^2\right)^2+\left(b^2\right)^2+\left(-c^2\right)^2+2\cdot a^2\cdot b^2+2\cdot a^2\cdot(-c)^2+2\cdot b^2\cdot(-c)^2=4\cdot a^2\cdot b^2 \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4+2\cdot a^2\cdot b^2-2\cdot a^2\cdot c^2-2\cdot b^2\cdot c^2=4\cdot a^2\cdot b^2 \Rightarrow$$

$$\Rightarrow -2\cdot a^2\cdot b^2-2\cdot a^2\cdot c^2-2\cdot b^2\cdot c^2=-a^4-b^4-c^4 \Rightarrow$$

$$\Rightarrow -2\cdot a^2\cdot b^2-2\cdot a^2\cdot c^2-2\cdot b^2\cdot c^2=-a^4-b^4-c^4\ /\cdot(-1) \Rightarrow$$

$$\Rightarrow 2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=a^4+b^4+c^4. \qquad (1)$$

Kvadriramo drugu jednakost.

$$a^2+b^2+c^2=1 \Rightarrow a^2+b^2+c^2=1\ /^2 \Rightarrow \left(a^2+b^2+c^2\right)^2=1^2 \Rightarrow$$

$$\Rightarrow \left(a^2\right)^2+\left(b^2\right)^2+\left(c^2\right)^2+2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=1 \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4+2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=1 \Rightarrow$$

$$\Rightarrow 2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=1-a^4-b^4-c^4. \qquad (2)$$

Iz (1) i (2) dobije se:

$$\left.\begin{array}{l} 2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=a^4+b^4+c^4 \\ 2\cdot a^2\cdot b^2+2\cdot a^2\cdot c^2+2\cdot b^2\cdot c^2=1-a^4-b^4-c^4 \end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda} \\ \text{komparacije}\end{array}\right] \Rightarrow$$

$$\Rightarrow a^4+b^4+c^4=1-a^4-b^4-c^4 \Rightarrow a^4+b^4+c^4+a^4+b^4+c^4=1 \Rightarrow$$

$$\Rightarrow 2\cdot a^4+2\cdot b^4+2\cdot c^4=1 \Rightarrow 2\cdot a^4+2\cdot b^4+2\cdot c^4=1\ /:2 \Rightarrow a^4+b^4+c^4=\frac{1}{2}.$$

Vježba 269

Ako je $a+b+c=0$ i $a^2+b^2+c^2=2$, izračunaj $a^4+b^4+c^4$.

Rezultat: 2.

Zadatak 270 (Jasna, srednja škola)

Ako su x, y i z brojevi za koje vrijedi x + y + z = 0, dokaži da je zbroj kubova tih brojeva djeljiv sa 3.

Rješenje 270

Ponovimo!

$$(-a-b)^3=-(a+b)^3 \quad , \quad (a+b)^3=a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3.$$

Zakon distribucije množenja prema zbrajanju:

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Za cijeli broj a kažemo da je djeljiv s cijelim brojem b (b ≠ 0) ako postoji cijeli broj k, tako da vrijedi

$$a=k\cdot b.$$

$$x+y+z=0 \Rightarrow z=-x-y \Rightarrow z=-x-y \ /^3 \Rightarrow z^3=(-x-y)^3 \Rightarrow z^3=-(x+y)^3 \Rightarrow$$

$$\Rightarrow z^3=-\left(x^3+3\cdot x^2\cdot y+3\cdot x\cdot y^2+y^3\right) \Rightarrow z^3=-x^3-3\cdot x^2\cdot y-3\cdot x\cdot y^2-y^3 \Rightarrow$$

$$\Rightarrow x^3+y^3+z^3=-3\cdot x^2\cdot y-3\cdot x\cdot y^2 \Rightarrow x^3+y^3+z^3=3\cdot x\cdot y\cdot(-x-y) \Rightarrow$$

$$\Rightarrow [z=-x-y] \Rightarrow x^3+y^3+z^3=3\cdot x\cdot y\cdot z \Rightarrow x^3+y^3+z^3=3\cdot x\cdot y\cdot z.$$

Vježba 270

Ako su x, y i z brojevi za koje vrijedi x + y + z = 0, dokaži da je zbroj kvadrata tih brojeva djeljiv s 2.

Rezultat: Dokaz analogan.

Zadatak 271 (Nevenka, srednja škola)

Izračunaj: $3^{-1}+\left(\frac{8}{4}\right)^{-2}-\left(\frac{3}{4}\right)^{-3}.$

Rješenje 271

Ponovimo!

$$a^{-n}=\frac{1}{a^n} \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad a^1=1.$$

$$3^{-1}+\left(\frac{8}{4}\right)^{-2}-\left(\frac{3}{4}\right)^{-3}=\frac{1}{3^1}+\left(\frac{4}{8}\right)^2-\left(\frac{4}{3}\right)^3=\frac{1}{3}+\left(\frac{4}{8}\right)^2-\left(\frac{4}{3}\right)^3=\frac{1}{3}+\left(\frac{1}{2}\right)^2-\left(\frac{4}{3}\right)^3=$$

$$=\frac{1}{3}+\frac{1^2}{2^2}-\frac{4^3}{3^3}=\frac{1}{3}+\frac{1}{4}-\frac{64}{27}=\frac{36+27-256}{108}=-\frac{193}{108}.$$

Vježba 271

Izračunaj: $3^{-1}+\left(\frac{6}{3}\right)^{-2}-\left(\frac{3}{4}\right)^{-2}.$

Rezultat: $-\frac{193}{108}.$

Zadatak 272 (Nevenka, srednja škola)

Izračunaj: $81\cdot\left(\frac{3}{4}\right)^{-2}+\frac{3}{8}-\left(\frac{3}{2}\right)^3.$

Rješenje 272

Ponovimo!

$$\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}.$$

$$81\cdot\left(\frac{3}{4}\right)^{-2}+\frac{3}{8}-\left(\frac{3}{2}\right)^{3}=81\cdot\left(\frac{4}{3}\right)^{2}+\frac{3}{8}-\left(\frac{3}{2}\right)^{3}=81\cdot\frac{4^2}{3^2}+\frac{3}{8}-\frac{3^3}{2^3}=$$

$$=81\cdot\frac{16}{9}+\frac{3}{8}-\frac{27}{8}=81\cdot\frac{16}{9}+\frac{3}{8}-\frac{27}{8}=9\cdot 16+\frac{3}{8}-\frac{27}{8}=144+\frac{3}{8}-\frac{27}{8}=\frac{144\cdot 8+3-27}{8}=$$

$$=\frac{1128}{8}=141.$$

Vježba 272

Izračunaj: $\left(\frac{3}{2}\right)^{3}-81\cdot\left(\frac{3}{4}\right)^{-2}-\frac{3}{8}.$

Rezultat: $-141.$

Zadatak 273 (Kruno, srednja škola)

Pojednostavni : $\frac{a}{a\cdot b-b^2}+\frac{b}{a^2-a\cdot b}-\frac{a+b}{a\cdot b}.$

Rješenje 273

Ponovimo!

$$x^2-y^2=(x-y)\cdot(x+y).$$

Zakon distribucije množenja prema zbrajanju:

$$x\cdot(y+z)=x\cdot y+x\cdot z \quad , \quad x\cdot y+x\cdot z=x\cdot(y+z).$$

$$\frac{a}{a\cdot b-b^2}+\frac{b}{a^2-a\cdot b}-\frac{a+b}{a\cdot b}=\frac{a}{b\cdot(a-b)}+\frac{b}{a\cdot(a-b)}-\frac{a+b}{a\cdot b}=\frac{a^2+b^2-(a+b)\cdot(a-b)}{a\cdot b\cdot(a-b)}=$$

$$=\frac{a^2+b^2-\left(a^2-b^2\right)}{a\cdot b\cdot(a-b)}=\frac{a^2+b^2-a^2+b^2}{a\cdot b\cdot(a-b)}=\frac{a^2+b^2-a^2+b^2}{a\cdot b\cdot(a-b)}=\frac{2\cdot b^2}{a\cdot b\cdot(a-b)}=$$

$$=\frac{2\cdot b^2}{a\cdot b\cdot(a-b)}=\frac{2\cdot b}{a\cdot(a-b)}.$$

Vježba 273

Pojednostavni : $\frac{a}{a\cdot b-b^2}-\frac{a+b}{a\cdot b}-\frac{b}{a\cdot b-a^2}.$

Rezultat: $\frac{2\cdot b}{a\cdot(a-b)}.$

Zadatak 274 (Kruno, srednja škola)

Pojednostavni : $\left(\frac{x+z}{z}-\frac{x+y}{x}\right)\cdot\frac{z^2}{x^2-y\cdot z}.$

Rješenje 274

$$\left(\frac{x+z}{z}-\frac{x+y}{x}\right)\cdot\frac{z^2}{x^2-y\cdot z}=\frac{x\cdot(x+z)-z\cdot(x+y)}{x\cdot z}\cdot\frac{z^2}{x^2-y\cdot z}=\frac{x^2+x\cdot z-x\cdot z-y\cdot z}{x\cdot z}\cdot\frac{z^2}{x^2-y\cdot z}=$$

$$=\frac{x^2+x\cdot z-x\cdot z-y\cdot z}{x\cdot z}\cdot\frac{z^2}{x^2-y\cdot z}=\frac{x^2-y\cdot z}{x\cdot z}\cdot\frac{z^2}{x^2-y\cdot z}=\frac{x^2-y\cdot z}{x\cdot z}\cdot\frac{z^2}{x^2-y\cdot z}=$$

$$=\frac{z^2}{x\cdot z}=\frac{z^2}{x\cdot z}=\frac{z}{x}.$$

Vježba 274

$$\text{Pojednostavni}:\left(\frac{x+y}{x}-\frac{x+z}{z}\right)\cdot\frac{z^2}{y\cdot z-x^2}.$$

Rezultat: $\frac{z}{x}.$

Zadatak 275 (Nina, HTT)

$$\text{Čemu je jednak izraz}: 2\cdot x^2+12\cdot x+18?$$

A) $(2\cdot x+3)^2$ B) $\left(2\cdot x+\frac{3}{2}\right)^2$ C) $2\cdot(x+3)^2$ D) $2\cdot\left(x+\frac{3}{2}\right)^2$

Rješenje 275

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

Zakon distribucije množenja prema zbrajanju:

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$2\cdot x^2+12\cdot x+18=2\cdot\left(x^2+6\cdot x+9\right)=2\cdot\left(x^2+6\cdot x+3^2\right)=2\cdot(x+3)^2.$$

Odgovor je pod C.

2.inačica

Do rješenja se može doći i na jednostavniji način. Ako se dva izraza podudaraju za bilo koju vrijednost varijable x, oni se moraju podudarati i kad odaberemo konkretnu vrijednost. Tako na primjer, u početnom izrazu možemo uzeti da je x = 0. Za tu vrijednost od x dobije se:

$$\left.\begin{array}{l}2\cdot x^2+12\cdot x+18\\ x=0\end{array}\right\}\Rightarrow 2\cdot 0^2+12\cdot 0+18=2\cdot 0+0+18=0+0+18=18.$$

Za x = 0 računamo vrijednost izraza ponuđenih pod A, B, C i D.
Računamo vrijednost izraza pod A.

$$\left.\begin{array}{l}(2\cdot x+3)^2\\ x=0\end{array}\right\}\Rightarrow (2\cdot 0+3)^2=(0+3)^2=3^2=9\neq 18.$$

Računamo vrijednost izraza pod B.

$$\left.\begin{array}{l}\left(2\cdot x+\frac{3}{2}\right)^2\\ x=0\end{array}\right\}\Rightarrow \left(2\cdot 0+\frac{3}{2}\right)^2=\left(0+\frac{3}{2}\right)^2=\left(\frac{3}{2}\right)^2=\frac{9}{4}\neq 18.$$

Računamo vrijednost izraza pod C.

$$\left.\begin{array}{l} 2\cdot(x+3)^2 \\ x=0 \end{array}\right\} \Rightarrow 2\cdot(0+3)^2 = 2\cdot 3^2 = 2\cdot 9 = 18 = 18.$$

Dakle, odgovor je pod C.

Računanje vrijednosti izraza pod D nije potrebno jer je samo jedan odgovor točan.

Vježba 275

Čemu je jednak izraz: $2\cdot x^2 + 20\cdot x + 50$?

A) $(2\cdot x+5)^2$ B) $\left(2\cdot x+\frac{5}{2}\right)^2$ C) $2\cdot(x+5)^2$ D) $2\cdot\left(x+\frac{5}{2}\right)^2$

Rezultat: C.

Zadatak 276 (Nina, HTT)

Ako je $1 = 3\cdot a + 2\cdot b$, koliko je b?

A) $b = \frac{1}{2} - \frac{3}{2}\cdot a$ B) $b = \frac{1}{2} + \frac{3}{2}\cdot a$ C) $b = -\frac{1}{2} + 3\cdot a$ D) $b = -\frac{1}{2} - 3\cdot a$

Rješenje 276

Ponovimo!

$$\frac{x-y}{n} = \frac{x}{n} - \frac{y}{n}.$$

1.inačica

$$1 = 3\cdot a + 2\cdot b \Rightarrow -2\cdot b = 3\cdot a - 1 \Rightarrow -2\cdot b = 3\cdot a - 1 \;/\cdot(-1) \Rightarrow 2\cdot b = -3\cdot a + 1 \Rightarrow$$

$$\Rightarrow 2\cdot b = 1 - 3\cdot a \Rightarrow 2\cdot b = 1 - 3\cdot a \;/:2 \Rightarrow b = \frac{1-3\cdot a}{2} \Rightarrow b = \frac{1}{2} - \frac{3}{2}\cdot a.$$

Odgovor je pod A.

2.inačica

$$1 = 3\cdot a + 2\cdot b \Rightarrow 3\cdot a + 2\cdot b = 1 \Rightarrow 2\cdot b = 1 - 3\cdot a \Rightarrow$$

$$\Rightarrow 2\cdot b = 1 - 3\cdot a \;/:2 \Rightarrow b = \frac{1-3\cdot a}{2} \Rightarrow b = \frac{1}{2} - \frac{3}{2}\cdot a.$$

Odgovor je pod A.

Vježba 276

Ako je $1 = 2\cdot a + 3\cdot b$, koliko je a?

A) $a = \frac{1}{2} - \frac{3}{2}\cdot b$ B) $a = \frac{1}{2} + \frac{3}{2}\cdot b$ C) $a = -\frac{1}{2} + 3\cdot b$ D) $a = -\frac{1}{2} - 3\cdot b$

Rezultat: A.

Zadatak 277 (Nina, HTT)

Koliki je rezultat oduzimanja: $\frac{5\cdot x}{1+x} - 2$?

A) $\frac{3\cdot x-2}{1+x}$ B) $\frac{4\cdot x-2}{1+x}$ C) $\frac{6\cdot x-2}{1+x}$ D) $\frac{7\cdot x-2}{1+x}$

Rješenje 277

Ponovimo!

$$\frac{n}{1} = n.$$

Zakon distribucije množenja prema zbrajanju:

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{5\cdot x}{1+x}-2=\frac{5\cdot x}{1+x}-\frac{2}{1}=\frac{5\cdot x-2\cdot(1+x)}{1+x}=\frac{5\cdot x-2-2\cdot x}{1+x}=\frac{3\cdot x-2}{1+x}.$$

Odgovor je pod A.

2.inačica

Do rješenja se može doći i na jednostavniji način. Ako se dva izraza podudaraju za bilo koju vrijednost varijable x, oni se moraju podudarati i kad odaberemo konkretnu vrijednost. Tako na primjer, u početnom izrazu možemo uzeti da je x = 4. Za tu vrijednost od x dobije se:

$$\left.\begin{matrix}\frac{5\cdot x}{1+x}-2\\ x=4\end{matrix}\right\}\Rightarrow \frac{5\cdot 4}{1+4}-2=\frac{20}{5}-2=4-2=2.$$

Za x = 4 računamo vrijednost izraza ponuđenih pod A, B, C i D.
Računamo vrijednost izraza pod A.

$$\left.\begin{matrix}\frac{3\cdot x-2}{1+x}\\ x=4\end{matrix}\right\}\Rightarrow \frac{3\cdot 4-2}{1+4}=\frac{12-2}{5}=\frac{10}{5}=2=2.$$

Dakle, odgovor je pod A.
Računaje vrijednosti pod B, C i D nije potrebno jer je samo jedan odgovor točan.

Vježba 277

Koliki je rezultat oduzimanja: $\frac{5\cdot x}{1+x}-2$?

A) $\frac{3\cdot x-2}{1+x}$ B) $\frac{4\cdot x-2}{1+x}$ C) $\frac{6\cdot x-2}{1+x}$ D) $\frac{7\cdot x-2}{1+x}$

Rezultat: A.

Zadatak 278 (Sara, srednja škola)

Koji je rezultat sređivanja izraza $\left[\left(\frac{8}{9-x^2}-\frac{3-x}{2\cdot x+6}\right):\frac{x-7}{2\cdot(x+3)}-1\right]^{-1}$ za $x\neq\pm 3$?

A) $\frac{3-x}{2}$ B) $\frac{3-x}{4}$ C) $\frac{x-3}{2}$ D) $\frac{x-3}{4}$

Rješenje 278

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad a+b=b+a \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n}.$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

Zakon distribucije množenja prema zbrajanju:

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left[\left(\frac{8}{9-x^2}-\frac{3-x}{2\cdot x+6}\right):\frac{x-7}{2\cdot(x+3)}-1\right]^{-1}=\left[\left(\frac{8}{(3-x)\cdot(3+x)}-\frac{3-x}{2\cdot(x+3)}\right):\frac{x-7}{2\cdot(x+3)}-1\right]^{-1}=$$

$$=\left[\frac{16-(3-x)\cdot(3-x)}{2\cdot(3-x)\cdot(3+x)}:\frac{x-7}{2\cdot(x+3)}-1\right]^{-1}=\left[\frac{16-(3-x)^2}{2\cdot(3-x)\cdot(3+x)}\cdot\frac{2\cdot(x+3)}{x-7}-1\right]^{-1}=$$

$$=\left[\frac{16-\left(9-6\cdot x+x^2\right)}{2\cdot(3-x)\cdot(3+x)}\cdot\frac{2\cdot(x+3)}{x-7}-1\right]^{-1}=\left[\frac{16-9+6\cdot x-x^2}{3-x}\cdot\frac{1}{x-7}-1\right]^{-1}=$$

$$=\left[\frac{7+6\cdot x-x^2}{3-x}\cdot\frac{1}{x-7}-1\right]^{-1}=\left[\frac{7+6\cdot x-x^2}{(3-x)\cdot(x-7)}-1\right]^{-1}=\left[\frac{7+6\cdot x-x^2}{(3-x)\cdot(x-7)}-\frac{1}{1}\right]^{-1}=$$

$$=\left[\frac{7+6\cdot x-x^2-(3-x)\cdot(x-7)}{(3-x)\cdot(x-7)}\right]^{-1}=\left[\frac{7+6\cdot x-x^2-\left(3\cdot x-21-x^2+7\cdot x\right)}{(3-x)\cdot(x-7)}\right]^{-1}=$$

$$=\left[\frac{7+6\cdot x-x^2-3\cdot x+21+x^2-7\cdot x}{(3-x)\cdot(x-7)}\right]^{-1}=\left[\frac{7+6\cdot x-x^2-3\cdot x+21+x^2-7\cdot x}{(3-x)\cdot(x-7)}\right]^{-1}=$$

$$=\left[\frac{7+6\cdot x-3\cdot x+21-7\cdot x}{(3-x)\cdot(x-7)}\right]^{-1}=\left[\frac{-4\cdot x+28}{(3-x)\cdot(x-7)}\right]^{-1}=\frac{(3-x)\cdot(x-7)}{-4\cdot x+28}=\frac{(3-x)\cdot(x-7)}{-4\cdot(x-7)}=$$

$$=\frac{(3-x)\cdot(x-7)}{-4\cdot(x-7)}=\frac{3-x}{-4}=\frac{-(3-x)}{4}=\frac{-3+x}{4}=\frac{x-3}{4}.$$

Odgovor je pod D.

Vježba 278

Koji je rezultat sređivanja izraza $\left[\left(\frac{8}{9-x^2}+\frac{x-3}{2\cdot x+6}\right):\frac{x-7}{2\cdot(x+3)}-1\right]^{-1}$ za $x\neq\pm3$?

A) $\frac{3-x}{2}$ B) $\frac{3-x}{4}$ C) $\frac{x-3}{2}$ D) $\frac{x-3}{4}$

Rezultat: D.

Zadatak 279 (Ana, gimnazija)

Ako vrijede jednakosti $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ i $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1,$ dokaži da vrijedi $\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=1.$

Rješenje 279

Ponovimo!

$$(a+b+c)^2=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}.$$

Zakon distribucije množenja prema zbrajanju:

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Kvadriranjem jednakosti

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$$

dobije se:

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1 \Rightarrow \frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1 \;/^2 \Rightarrow \left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)^2=1^2 \Rightarrow$$

$$\Rightarrow \left(\frac{a}{x}\right)^2+\left(\frac{b}{y}\right)^2+\left(\frac{c}{z}\right)^2+2\cdot\frac{a}{x}\cdot\frac{b}{y}+2\cdot\frac{a}{x}\cdot\frac{c}{z}+2\cdot\frac{b}{y}\cdot\frac{c}{z}=1 \Rightarrow$$

$$\Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+2\cdot\frac{a\cdot b}{x\cdot y}+2\cdot\frac{a\cdot c}{x\cdot z}+2\cdot\frac{b\cdot c}{y\cdot z}=1 \Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+2\cdot\frac{a\cdot b\cdot c}{x\cdot y\cdot z}\cdot\left(\frac{z}{c}+\frac{y}{b}+\frac{x}{a}\right)=1 \Rightarrow$$

$$\Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+2\cdot\frac{a\cdot b\cdot c}{x\cdot y\cdot z}\cdot\underbrace{\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)}_{=0}=1 \Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+2\cdot\frac{a\cdot b\cdot c}{x\cdot y\cdot z}\cdot 0=1 \Rightarrow$$

$$\Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=1.$$

Vježba 279

Ako vrijede jednakosti $x\cdot b\cdot c+y\cdot a\cdot c+z\cdot a\cdot b=0$ i $a\cdot y\cdot z+b\cdot x\cdot z+c\cdot x\cdot y=x\cdot y\cdot z,$ dokaži da vrijedi $a^2\cdot y^2\cdot z^2+b^2\cdot x^2\cdot z^2+c^2\cdot x^2\cdot y^2=x^2\cdot y^2\cdot z^2.$

Rezultat: Dokaz analogan.

Zadatak 280 (Ana, gimnazija)

Ako je

$$a+b=x+y \qquad (1)$$

$$a^2+b^2=x^2+y^2 \qquad (2)$$

dokazati da je

$$a^3+b^3=x^3+y^3.$$

Rješenje 280

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a+b)^3=a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3.$$

Zakon distribucije množenja prema zbrajanju:

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Kvadriramo jednakost (1) i uporabimo (2).

$$a+b=x+y \Rightarrow a+b=x+y \;/^2 \Rightarrow (a+b)^2=(x+y)^2 \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b+b^2=x^2+2\cdot x\cdot y+y^2 \Rightarrow \left[\begin{array}{c}\text{uvjet } (2)\\ a^2+b^2=x^2+y^2\end{array}\right] \Rightarrow$$

$$\Rightarrow a^2+2\cdot a\cdot b+b^2=x^2+2\cdot x\cdot y+y^2 \Rightarrow 2\cdot a\cdot b=2\cdot x\cdot y \Rightarrow 2\cdot a\cdot b=2\cdot x\cdot y \;/:2 \Rightarrow$$

$$\Rightarrow a\cdot b=x\cdot y. \qquad (3)$$

Sada kubiramo jednakost (1) i uporabimo (3) i (1).

$$a+b=x+y \Rightarrow a+b=x+y \ /^3 \Rightarrow (a+b)^3=(x+y)^3 \Rightarrow$$

$$\Rightarrow a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3=x^3+3\cdot x^2\cdot y+3\cdot x\cdot y^2+y^3 \Rightarrow$$

$$\Rightarrow a^3+3\cdot a\cdot b\cdot(a+b)+b^3=x^3+3\cdot x\cdot y\cdot(x+y)+y^3 \Rightarrow \left[\begin{matrix}\text{uvjeti } (3) \text{ i } (1)\\ a\cdot b=x\cdot y\\ a+b=x+y\end{matrix}\right] \Rightarrow$$

$$\Rightarrow a^3+3\cdot x\cdot y\cdot(x+y)+b^3=x^3+3\cdot x\cdot y\cdot(x+y)+y^3 \Rightarrow$$

$$\Rightarrow a^3+3\cdot x\cdot y\cdot(x+y)+b^3=x^3+3\cdot x\cdot y\cdot(x+y)+y^3 \Rightarrow a^3+b^3=x^3+y^3.$$

Vježba 280

Ako je

$$a-x=y-b \qquad (1)$$

$$a^2-x^2=y^2-b^2 \qquad (2)$$

dokazati da je

$$a^3-x^3=y^3-b^3.$$

Rezultat: Dokaz analogan.