Zadatak 201 (Vlado, maturant)

Ako je $a \geq 0$, za koji a izraz $\frac{1-a}{1+a}$ ima najveću vrijednost?

Rješenje 201

Ponovimo!

Među razlomcima jednakih brojnika najveći je onaj koji ima najmanji nazivnik.

$$x < y < z \Rightarrow \frac{b}{x} > \frac{b}{y} > \frac{b}{z}.$$

Među razlomcima jednakih nazivnika najveći je onaj koji ima najveći brojnik.

$$x > y > z \Rightarrow \frac{x}{n} > \frac{y}{n} > \frac{z}{n}.$$

Budući da je $a \geq 0$, za razlomak $\frac{1-a}{1+a}$ vrijedi:

- brojnik, razlika 1 – a je najveća kada je umanjitelj a najmanji, tj. a = 0
- nazivnik, zbroj 1+ a je najmanji kada je pribrojnik a najmanji, tj. a = 0.

Razlomak $\frac{1-a}{1+a}$ ima najveću vrijednost za a = 0 i ona iznosi:

$$\frac{1-a}{1+a} = \frac{1-0}{1+0} = \frac{1}{1} = 1.$$

Vježba 201

Ako je $a \geq 0$, za koji a izraz $\frac{2-a}{2+a}$ ima najveću vrijednost?

Rezultat: a = 0.

Zadatak 202 (Mia, gimnazija)

Izračunaj vrijednost razlomka $\frac{x+2\cdot y}{x-2\cdot y}$, ako je $x^2 + 4\cdot y^2 = 5\cdot x\cdot y$ i $0 < x < y$.

Rješenje 202

Ponovimo!

$$(a+b)^2 = a^2 + 2\cdot a\cdot b + b^2 \quad , \quad (a-b)^2 = a^2 - 2\cdot a\cdot b + b^2 \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

$$\left.\begin{array}{l} x^2 + 4\cdot y^2 = 5\cdot x\cdot y \\ x^2 + 4\cdot y^2 = 5\cdot x\cdot y \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{jednakosti pribrojimo } 4\cdot x\cdot y \\ \text{jednakosti pribrojimo } -4\cdot x\cdot y \end{array}\right] \Rightarrow \left.\begin{array}{l} x^2 + 4\cdot y^2 = 5\cdot x\cdot y \;/ + 4\cdot x\cdot y \\ x^2 + 4\cdot y^2 = 5\cdot x\cdot y \;/ - 4\cdot x\cdot y \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x^2 + 4\cdot y^2 + 4\cdot x\cdot y = 5\cdot x\cdot y + 4\cdot x\cdot y \\ x^2 + 4\cdot y^2 - 4\cdot x\cdot y = 5\cdot x\cdot y - 4\cdot x\cdot y \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + 4\cdot x\cdot y + 4\cdot y^2 = 9\cdot x\cdot y \\ x^2 - 4\cdot x\cdot y + 4\cdot y^2 = x\cdot y \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} (x+2\cdot y)^2 = 9\cdot x\cdot y \\ (x-2\cdot y)^2 = x\cdot y \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{podijelimo} \\ \text{jednakosti} \end{array}\right] \Rightarrow \frac{(x+2\cdot y)^2}{(x-2\cdot y)^2} = \frac{9\cdot x\cdot y}{x\cdot y} \Rightarrow \frac{(x+2\cdot y)^2}{(x-2\cdot y)^2} = \frac{9\cdot \cancel{x\cdot y}}{\cancel{x\cdot y}} \Rightarrow$$

$$\Rightarrow \frac{(x+2\cdot y)^2}{(x-2\cdot y)^2} = 9 \Rightarrow \left(\frac{x+2\cdot y}{x-2\cdot y}\right)^2 = 9 \;/\sqrt{} \Rightarrow \frac{x+2\cdot y}{x-2\cdot y} = \pm\sqrt{9} \Rightarrow \frac{x+2\cdot y}{x-2\cdot y} = \pm 3.$$

Vježba 202

Izračunaj vrijednost razlomka $\frac{x-2\cdot y}{x+2\cdot y}$, ako je $x^2 + 4 \cdot y^2 = 5 \cdot x \cdot y$ i $0 < x < y$.

Rezultat: $\pm\frac{1}{3}.$

Zadatak 203 (Mia, gimnazija)

Pojednostavnite: $a^4-\left(a^2-1\right)\cdot\left[\left(a-1\right)^2+2\cdot a\right].$

Rješenje 203

Ponovimo!

$$\left(x-y\right)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad \left(x-y\right)\cdot\left(x+y\right)=x^2-y^2 \quad , \quad \left(x^n\right)^m=x^{n\cdot m}.$$

$$a^4-\left(a^2-1\right)\cdot\left[\left(a-1\right)^2+2\cdot a\right]=a^4-\left(a^2-1\right)\cdot\left[a^2-2\cdot a+1+2\cdot a\right]=$$

$$=a^4-\left(a^2-1\right)\cdot\left[a^2-2\cdot a+1+2\cdot a\right]=a^4-\left(a^2-1\right)\cdot\left(a^2+1\right)=a^4-\underbrace{\left(a^2-1\right)\cdot\left(a^2+1\right)}_{\text{razlika kvadrata}}=$$

$$=a^4-\left(\left(a^2\right)^2-1^2\right)=a^4-\left(a^4-1\right)=a^4-a^4+1=a^4-a^4+1=1.$$

Vježba 203

Pojednostavnite: $a^4-\left(a^2+1\right)\cdot\left[\left(a+1\right)^2-2\cdot\left(a+1\right)\right].$

Rezultat: 1.

Zadatak 204 (Dada, gimnazija)

Rastavi na faktore: $a^2+10\cdot a\cdot b-70\cdot b-49.$

Rješenje 204

Ponovimo!

$$x^2-y^2=\left(x-y\right)\cdot\left(x+y\right).$$

$$a^2+10\cdot a\cdot b-70\cdot b-49=a^2-49+10\cdot a\cdot b-70\cdot b=\underbrace{\left(a^2-49\right)}_{\text{razlika kvadrata}}+\underbrace{\left(10\cdot a\cdot b-70\cdot b\right)}_{\text{izluči se }10\cdot b}=$$

$$=\left(a-7\right)\cdot\left(a+7\right)+10\cdot b\cdot\left(a-7\right)=\left[\begin{matrix}\text{izluči se}\\ a-7\end{matrix}\right]=\left(a-7\right)\cdot\left(a+7+10\cdot b\right)=\left(a-7\right)\cdot\left(a+10\cdot b+7\right).$$

Vježba 204

Rastavi na faktore: $a^2+5\cdot a\cdot b-25\cdot b-25.$

Rezultat: $\left(a-5\right)\cdot\left(a+5\cdot b+5\right).$

Zadatak 205 (Dada, gimnazija)

Dokazati da je $\frac{a}{b}+\frac{b}{a}\geq 2,$ ako je $a \cdot b > 0.$

Rješenje 205

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad x^2 \geq 0 \text{ za svaki realni broj } x.$$

$$\frac{a}{b}+\frac{b}{a} \geq 2 \Rightarrow \frac{a}{b}+\frac{b}{a} \geq 2 \; / \cdot a \cdot b \Rightarrow a^2+b^2 \geq 2 \cdot a \cdot b \Rightarrow a^2+b^2-2 \cdot a \cdot b \geq 0 \Rightarrow$$

$$\Rightarrow a^2-2 \cdot a \cdot b+b^2 \geq 0 \Rightarrow (a-b)^2 \geq 0. \text{ To je uvijek točno.}$$

Znak jednakosti vrijedi ako i samo ako je

$$a = b.$$

Vježba 205

Dokazati da je $x+\frac{1}{x} \geq 2,$ ako je $x > 0$.

Rezultat: Dokaz analogan.

Zadatak 206 (Miro, gimnazija)

Dokazati da je $a < \frac{a+b}{2} < b,$ ako je $a < b$.

Rješenje 206

Ponovimo!

$$x < y \Rightarrow x+z < y+z \text{ za svaki realni broj } z \quad , \quad x < y \Rightarrow x \cdot z < y \cdot z \;,\; z > 0.$$

$$x < y \text{ i } y < z \Rightarrow x < z \text{ tranzitivnost relacije } < \quad , \quad \frac{x}{n}+\frac{y}{n}=\frac{x+y}{n}.$$

Budući da je $a < b$, slijedi:

$a < b$	$a < b$
nejednakost pomnožimo sa $\frac{1}{2}$	nejednakost pomnožimo sa $\frac{1}{2}$
$a < b \; / \cdot \frac{1}{2}$	$a < b \; / \cdot \frac{1}{2}$
$\frac{a}{2} < \frac{b}{2}$	$\frac{a}{2} < \frac{b}{2}$
nejednakosti pribrojimo $\frac{a}{2}$	nejednakosti pribrojimo $\frac{b}{2}$
$\frac{a}{2} < \frac{b}{2} \; / + \frac{a}{2}$	$\frac{a}{2} < \frac{b}{2} \; / + \frac{b}{2}$
$\frac{a}{2}+\frac{a}{2} < \frac{b}{2}+\frac{a}{2}$	$\frac{a}{2}+\frac{b}{2} < \frac{b}{2}+\frac{b}{2}$
$2 \cdot \frac{a}{2} < \frac{b+a}{2}$	$\frac{a+b}{2} < 2 \cdot \frac{b}{2}$
$2 \cdot \frac{a}{2} < \frac{a+b}{2}$	$\frac{a+b}{2} < 2 \cdot \frac{b}{2}$
$a < \frac{a+b}{2}$	$\frac{a+b}{2} < b$

Sada je:

$$a < \frac{a+b}{2} < b.$$

Vježba 206

Dokazati da je $1<\frac{1+x}{2}<x,$ ako je 1 < x.

Rezultat: Dokaz analogan.

Zadatak 207 (Miro, gimnazija)

Dokazati da je $a<\frac{2\cdot a\cdot b}{a+b}<b,$ ako je 0 < a < b.

Rješenje 207

Ponovimo!

$$x<y\ \Rightarrow\ x+z<y+z\ \text{za svaki realni broj}\ z\quad,\quad x<y\ \Rightarrow\ x\cdot z<y\cdot z\ ,\ z>0.$$

$$x<y\ \text{i}\ y<z\ \Rightarrow\ x<z\ \text{tranzitivnost relacije} <.$$

Budući da je 0 < a < b, slijedi:

$a<b$ nejednakost pomnožimo sa a $a<b\ /\cdot a$ $a^2<a\cdot b$ nejednakosti pribrojimo $a\cdot b$ $a^2<a\cdot b\ /+a\cdot b$ $a^2+a\cdot b<a\cdot b+a\cdot b$ $a\cdot(a+b)<2\cdot a\cdot b$ nejednakost pomnožimo sa $\frac{1}{a+b}$ $a\cdot(a+b)<2\cdot a\cdot b\ /\cdot\frac{1}{a+b}$ $a<\frac{2\cdot a\cdot b}{a+b}$	$a<b$ nejednakost pomnožimo sa b $a<b\ /\cdot b$ $a\cdot b<b^2$ nejednakosti pribrojimo $a\cdot b$ $a\cdot b<b^2\ /+a\cdot b$ $a\cdot b+a\cdot b<b^2+a\cdot b$ $2\cdot a\cdot b<b\cdot(a+b)$ nejednakost pomnožimo sa $\frac{1}{a+b}$ $2\cdot a\cdot b<b\cdot(a+b)\ /\cdot\frac{1}{a+b}$ $\frac{2\cdot a\cdot b}{a+b}<b$

Sada je:

$$a<\frac{2\cdot a\cdot b}{a+b}<b.$$

Vježba 207

Dokazati da je $1<\frac{2\cdot x}{1+x}<x,$ ako je 1 < x.

Rezultat: Dokaz analogan.

Zadatak 208 (Miro, gimnazija)

Dokazati da je $a<\sqrt{a\cdot b}<b,$ ako je 0 < a < b.

Rješenje 208

Ponovimo!

$$x<y\ \Rightarrow\ x\cdot z<y\cdot z\ ,\ z>0\quad,\quad 0<x<y\ \Rightarrow\ \sqrt{x}<\sqrt{y}.$$

$$x<y\ \text{i}\ y<z\ \Rightarrow\ x<z\ \text{tranzitivnost relacije} <.$$

Budući da je 0 < a < b, slijedi:

$a < b$ nejednakost pomnožimo sa a $a < b \; / \cdot a$ $a^2 < a \cdot b$ korjenujemo nejednakost $a^2 < a \cdot b \; / \sqrt{}$ $a < \sqrt{a \cdot b}$	$a < b$ nejednakost pomnožimo sa b $a < b \; / \cdot b$ $a \cdot b < b^2$ korjenujemo nejednakost $a \cdot b < b^2 \; / \sqrt{}$ $\sqrt{a \cdot b} < b$

Sada je:

$$a < \sqrt{a \cdot b} < b.$$

Vježba 208

Dokazati da je $1 < \sqrt{x} < x$, ako je $1 < x$.

Rezultat: Dokaz analogan.

Zadatak 209 (1B, TUPŠ)

Racionaliziraj razlomak: $\sqrt{\dfrac{\sqrt{6}+\sqrt{2}}{3}}$.

Rješenje 209

Ponovimo!

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad , \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b} \, , \quad \sqrt{a} \cdot \sqrt{a} = \left(\sqrt{a}\right)^2 = a.$$

1.inačica

$$\sqrt{\frac{\sqrt{6}+\sqrt{2}}{3}} = \frac{\sqrt{\sqrt{6}+\sqrt{2}}}{\sqrt{3}} = \left[\begin{array}{l}\text{racionalizacija} \\ \text{nazivnika}\end{array}\right] = \frac{\sqrt{\sqrt{6}+\sqrt{2}}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\left(\sqrt{\sqrt{6}+\sqrt{2}}\right) \cdot \sqrt{3}}{\left(\sqrt{3}\right)^2} =$$

$$= \frac{\sqrt{3 \cdot \left(\sqrt{6}+\sqrt{2}\right)}}{3} = \frac{\sqrt{3 \cdot \sqrt{6} + 3 \cdot \sqrt{2}}}{3}.$$

2.inačica

$$\sqrt{\frac{\sqrt{6}+\sqrt{2}}{3}} = \sqrt{\frac{\sqrt{6}+\sqrt{2}}{3} \cdot \frac{3}{3}} = \sqrt{\frac{\left(\sqrt{6}+\sqrt{2}\right) \cdot 3}{9}} = \sqrt{\frac{3 \cdot \sqrt{6} + 3 \cdot \sqrt{2}}{9}} = \frac{\sqrt{3 \cdot \sqrt{6} + 3 \cdot \sqrt{2}}}{\sqrt{9}} = \frac{\sqrt{3 \cdot \sqrt{6} + 3 \cdot \sqrt{2}}}{3}.$$

Vježba 209

Racionaliziraj razlomak: $\sqrt{\dfrac{\sqrt{3}+\sqrt{2}}{2}}$.

Rezultat: $\dfrac{\sqrt{2 \cdot \sqrt{3} + 2 \cdot \sqrt{2}}}{2}$.

Zadatak 210 (Mira, gimnazija)

Reduciraj: $\left(\dfrac{a}{a-b} - \dfrac{b}{a+b} + \dfrac{2 \cdot a \cdot b}{a^2 - b^2}\right) \cdot \left(\dfrac{a}{a-b} - \dfrac{b}{a+b} - \dfrac{2 \cdot a \cdot b}{a^2 - b^2}\right)$.

Rješenje 210

Ponovimo!

$$x^2-y^2=(x-y)\cdot(x+y) \quad , \quad (x+y)^2=x^2+2\cdot x\cdot y+y^2 \quad , \quad (x-y)^2=x^2-2\cdot x\cdot y+y^2.$$

$$x^n\cdot y^n=(x\cdot y)^n.$$

$$\left(\frac{a}{a-b}-\frac{b}{a+b}+\frac{2\cdot a\cdot b}{a^2-b^2}\right)\cdot\left(\frac{a}{a-b}-\frac{b}{a+b}-\frac{2\cdot a\cdot b}{a^2-b^2}\right)=$$

$$=\frac{a\cdot(a+b)-b\cdot(a-b)+2\cdot a\cdot b}{a^2-b^2}\cdot\frac{a\cdot(a+b)-b\cdot(a-b)-2\cdot a\cdot b}{a^2-b^2}=$$

$$=\frac{a^2+a\cdot b-a\cdot b+b^2+2\cdot a\cdot b}{a^2-b^2}\cdot\frac{a^2+a\cdot b-a\cdot b+b^2-2\cdot a\cdot b}{a^2-b^2}=$$

$$=\frac{a^2+a\cdot b-a\cdot b+b^2+2\cdot a\cdot b}{a^2-b^2}\cdot\frac{a^2+a\cdot b-a\cdot b+b^2-2\cdot a\cdot b}{a^2-b^2}=\frac{a^2+b^2+2\cdot a\cdot b}{a^2-b^2}\cdot\frac{a^2+b^2-2\cdot a\cdot b}{a^2-b^2}=$$

$$=\frac{a^2+2\cdot a\cdot b+b^2}{a^2-b^2}\cdot\frac{a^2-2\cdot a\cdot b+b^2}{a^2-b^2}=\frac{(a+b)^2}{a^2-b^2}\cdot\frac{(a-b)^2}{a^2-b^2}=\frac{(a+b)^2\cdot(a-b)^2}{\left(a^2-b^2\right)^2}=$$

$$=\frac{((a+b)\cdot(a-b))^2}{\left(a^2-b^2\right)^2}=\frac{\left(a^2-b^2\right)^2}{\left(a^2-b^2\right)^2}=\frac{\left(a^2-b^2\right)^2}{\left(a^2-b^2\right)^2}=1.$$

Vježba 210

Reduciraj: $\left(\frac{a}{a-b}-\frac{b}{a+b}+\frac{2\cdot a\cdot b}{a^2-b^2}\right):\left(\frac{a}{a-b}-\frac{b}{a+b}-\frac{2\cdot a\cdot b}{a^2-b^2}\right).$

Rezultat: $\left(\frac{a+b}{a-b}\right)^2.$

Zadatak 211 (Nena, gimnazija)

Odredi $x+y$ ako je $\frac{x+a}{y-a}=\frac{x}{y}$; $y\neq 0,\ y\neq a,\ a\neq 0.$

Rješenje 211

Računamo x + y:

$$\frac{x+a}{y-a}=\frac{x}{y} \Rightarrow \frac{x+a}{y-a}=\frac{x}{y}\ /\cdot\ y\cdot(y-a) \Rightarrow y\cdot(x+a)=x\cdot(y-a) \Rightarrow y\cdot x+y\cdot a=x\cdot y-x\cdot a \Rightarrow$$

$$\Rightarrow y\cdot x+y\cdot a=x\cdot y-x\cdot a \Rightarrow y\cdot a=-x\cdot a \Rightarrow x\cdot a+y\cdot a=0 \Rightarrow x\cdot a+y\cdot a=0\ /:a \Rightarrow x+y=0.$$

Vježba 211

Odredi $x+y$ ako je $\frac{x+1}{y-1}=\frac{x}{y}$; $y\neq 0,\ y\neq 1.$

Rezultat: 0.

Zadatak 212 (Vinko, srednja škola)

Pojednostavnite: $\frac{(-4\cdot a)^{3}\cdot 2\cdot a^{2}}{\left(8\cdot a^{2}\right)^{2}}$.

Rješenje 212

Ponovimo!

$$(-a)^{2\cdot n+1}=-a^{2\cdot n+1}\quad,\quad a^{1}=a\quad,\quad \left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad a^{n}\cdot a^{m}=a^{n+m}.$$

$$\frac{a^{n}}{a^{m}}=a^{n-m}.$$

$$\frac{(-4\cdot a)^{3}\cdot 2\cdot a^{2}}{\left(8\cdot a^{2}\right)^{2}}=\frac{\left(-4^{1}\cdot a^{1}\right)^{3}\cdot 2\cdot a^{2}}{\left(8^{1}\cdot a^{2}\right)^{2}}=\frac{-4^{3}\cdot a^{3}\cdot 2\cdot a^{2}}{8^{2}\cdot a^{4}}=\frac{-\left(2^{2}\right)^{3}\cdot a^{3}\cdot 2\cdot a^{2}}{\left(2^{3}\right)^{2}\cdot a^{4}}=\frac{-2^{6}\cdot a^{3}\cdot 2\cdot a^{2}}{2^{6}\cdot a^{4}}=$$

$$=\frac{-2^{6}\cdot a^{3}\cdot 2\cdot a^{2}}{2^{6}\cdot a^{4}}=\frac{-2^{6}\cdot 2\cdot a^{5}}{2^{6}\cdot a^{4}}=\frac{-2^{6}\cdot 2\cdot a^{5}}{2^{6}\cdot a^{4}}=\frac{-2\cdot a^{5}}{a^{4}}=-2\cdot a^{5-4}=-2\cdot a^{1}=-2\cdot a.$$

Vježba 212

Pojednostavnite: $\frac{(4\cdot a)^{3}\cdot a^{2}}{\left(8\cdot a^{2}\right)^{2}}$.

Rezultat: a.

Zadatak 213 (Zaljubljena ☺, gimnazija)

Pojednostavnite: $\left(\frac{1}{a^{2}+b^{2}+2\cdot a\cdot b}-\frac{1}{b^{2}-a^{2}}\right):\frac{4\cdot a\cdot b}{a^{2}-b^{2}}$.

Rješenje 213

Ponovimo!

$$(a+b)^{2}=a^{2}+2\cdot a\cdot b+b^{2}\quad,\quad a^{2}-b^{2}=(a-b)\cdot(a+b).$$

$$\left(\frac{1}{a^{2}+b^{2}+2\cdot a\cdot b}-\frac{1}{b^{2}-a^{2}}\right):\frac{4\cdot a\cdot b}{a^{2}-b^{2}}=\left(\frac{1}{a^{2}+2\cdot a\cdot b+b^{2}}-\frac{1}{-\left(a^{2}-b^{2}\right)}\right)\cdot\frac{a^{2}-b^{2}}{4\cdot a\cdot b}=$$

$$=\left(\frac{1}{(a+b)^{2}}+\frac{1}{a^{2}-b^{2}}\right)\cdot\frac{a^{2}-b^{2}}{4\cdot a\cdot b}=\left(\frac{1}{(a+b)\cdot(a+b)}+\frac{1}{(a-b)\cdot(a+b)}\right)\cdot\frac{(a-b)\cdot(a+b)}{4\cdot a\cdot b}=$$

$$=\frac{a-b+a+b}{(a+b)\cdot(a+b)\cdot(a-b)}\cdot\frac{(a-b)\cdot(a+b)}{4\cdot a\cdot b}=\frac{a-b+a+b}{(a+b)\cdot(a+b)\cdot(a-b)}\cdot\frac{(a-b)\cdot(a+b)}{4\cdot a\cdot b}=$$

$$=\frac{2\cdot a}{(a+b)\cdot(a+b)\cdot(a-b)}\cdot\frac{(a-b)\cdot(a+b)}{4\cdot a\cdot b}=\frac{2\cdot a}{(a+b)\cdot(a+b)\cdot(a-b)}\cdot\frac{(a-b)\cdot(a+b)}{4\cdot a\cdot b}=$$

$$=\frac{2}{(a+b)}\cdot\frac{1}{4\cdot b}=\frac{2}{(a+b)}\cdot\frac{1}{4\cdot b}=\frac{1}{(a+b)}\cdot\frac{1}{2\cdot b}=\frac{1}{2\cdot b\cdot(a+b)}.$$

Vježba 213

Pojednostavnite: $\left(\frac{1}{a^2-b^2}+\frac{1}{a^2+b^2+2\cdot a\cdot b}\right):\frac{4\cdot a\cdot b}{a^2-b^2}.$

Rezultat: $\frac{1}{2\cdot b\cdot(a+b)}.$

Zadatak 214 (Zaljubljena ☺, gimnazija)

Pojednostavnite: $\frac{a^{-1}}{b^{-2}}\cdot\left[\left(\frac{a^2}{b^2}+\frac{b}{a}\right):\left(\frac{a}{b^2}-\frac{1}{b}+\frac{1}{a}\right)\right]:\frac{(a-b)^2+4\cdot a\cdot b}{1+\frac{a}{b}}.$

Rješenje 214

Ponovimo!

$$a^{-n}=\frac{1}{a^n}\quad,\quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right)\quad,\quad(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$\frac{a^{-1}}{b^{-2}}\cdot\left[\left(\frac{a^2}{b^2}+\frac{b}{a}\right):\left(\frac{a}{b^2}-\frac{1}{b}+\frac{1}{a}\right)\right]:\frac{(a-b)^2+4\cdot a\cdot b}{1+\frac{a}{b}}=$$

$$=\frac{b^2}{a}\cdot\left[\frac{a^3+b^3}{a\cdot b^2}:\frac{a^2-a\cdot b+b^2}{a\cdot b^2}\right]:\frac{a^2-2\cdot a\cdot b+b^2+4\cdot a\cdot b}{\frac{b+a}{b}}=$$

$$=\frac{b^2}{a}\cdot\left[\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{a\cdot b^2}\cdot\frac{a\cdot b^2}{a^2-a\cdot b+b^2}\right]:\frac{a^2+2\cdot a\cdot b+b^2}{\frac{a+b}{b}}=$$

$$=\frac{b^2}{a}\cdot\left[\frac{(a+b)\cdot\left(a^2-a\cdot b+b^2\right)}{a\cdot b^2}\cdot\frac{a\cdot b^2}{a^2-a\cdot b+b^2}\right]:\frac{(a+b)^2}{\frac{a+b}{b}}=$$

$$=\frac{b^2}{a}\cdot(a+b):\frac{b\cdot(a+b)^2}{a+b}=\frac{b^2}{a}\cdot(a+b)\cdot\frac{a+b}{b\cdot(a+b)^2}=\frac{b^2}{a}\cdot\frac{(a+b)^2}{b\cdot(a+b)^2}=\frac{b^2}{a}\cdot\frac{(a+b)^2}{b\cdot(a+b)^2}=$$

$$=\frac{b^2}{a\cdot b}=\frac{b^2}{a\cdot b}=\frac{b}{a}.$$

Vježba 214

Pojednostavnite: $\left[\left(\frac{a^2}{b^2}+\frac{b}{a}\right):\left(\frac{a}{b^2}-\frac{1}{b}+\frac{1}{a}\right)\right]\cdot\frac{a^{-1}}{b^{-2}}:\frac{(a-b)^2+4\cdot a\cdot b}{a+b}\cdot b.$

Rezultat: $\frac{b}{a}.$

Zadatak 215 (Mario, srednja škola)

Nađi x tako da vrijedi: $a^2 \cdot a^x = a^7$.

Rješenje 215

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

$$a^2 \cdot a^x = a^7 \Rightarrow a^{2+x} = a^7 \Rightarrow 2 + x = 7 \Rightarrow x = 7 - 2 \Rightarrow x = 5.$$

Vježba 215

Nađi x tako da vrijedi: $a^3 \cdot a^x = a^8$.

Rezultat: x = 5.

Zadatak 216 (Mario, srednja škola)

Pojednostavnite: $\left(\left(x^2\right)^3\right)^a \cdot \left(\left(x^{-a}\right)^{-1}\right)^2 : \left(\left(x^{-1}\right)^{-2}\right)^a$.

Rješenje 216

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^n : a^m = a^{n-m}.$$

$$\left(\left(x^2\right)^3\right)^a \cdot \left(\left(x^{-a}\right)^{-1}\right)^2 : \left(\left(x^{-1}\right)^{-2}\right)^a = \left(x^6\right)^a \cdot \left(x^a\right)^2 : \left(x^2\right)^a =$$

$$= x^{6 \cdot a} \cdot x^{2 \cdot a} : x^{2 \cdot a} = x^{6 \cdot a + 2 \cdot a} : x^{2 \cdot a} = x^{8 \cdot a} : x^{2 \cdot a} = x^{8 \cdot a - 2 \cdot a} = x^{6 \cdot a}.$$

Vježba 216

Pojednostavnite: $\left(\left(x^3\right)^2\right)^a \cdot \left(\left(x^{-1}\right)^{-a}\right)^2 : \left(\left(x^{-2}\right)^{-1}\right)^a$.

Rezultat: $x^{6 \cdot a}$.

Zadatak 217 (Mario, srednja škola)

Pojednostavnite: $\frac{1}{a + \sqrt{a^2 - b}} + \frac{1}{a - \sqrt{a^2 - b}}$.

Rješenje 217

Ponovimo!

$$a^2 - b^2 = (a - b) \cdot (a + b) \quad , \quad \left(\sqrt{a}\right)^2 = a.$$

$$\frac{1}{a + \sqrt{a^2 - b}} + \frac{1}{a - \sqrt{a^2 - b}} = \frac{a - \sqrt{a^2 - b} + a + \sqrt{a^2 - b}}{\left(a + \sqrt{a^2 - b}\right) \cdot \left(a - \sqrt{a^2 - b}\right)} = \frac{a - \sqrt{a^2 - b} + a + \sqrt{a^2 - b}}{\left(a + \sqrt{a^2 - b}\right) \cdot \left(a - \sqrt{a^2 - b}\right)} =$$

$$= \frac{2 \cdot a}{a^2 - \left(\sqrt{a^2 - b}\right)^2} = \frac{2 \cdot a}{a^2 - \left(a^2 - b\right)} = \frac{2 \cdot a}{a^2 - a^2 + b} = \frac{2 \cdot a}{a^2 - a^2 + b} = \frac{2 \cdot a}{b} = 2 \cdot \frac{a}{b}.$$

Vježba 217

Pojednostavnite: $\dfrac{1}{\sqrt{a}+\sqrt{a-b}}+\dfrac{1}{\sqrt{a}-\sqrt{a-b}}$.

Rezultat: $2\cdot\dfrac{\sqrt{a}}{b}$.

Zadatak 218 (Ivana, srednja škola)

Izračunaj: $\dfrac{x^2}{x^2-x\cdot y}+\dfrac{y^2}{x\cdot y-y^2}-\dfrac{2\cdot x}{x-y}$.

Rješenje 218

$$\frac{x^2}{x^2-x\cdot y}+\frac{y^2}{x\cdot y-y^2}-\frac{2\cdot x}{x-y}=\frac{x^2}{x\cdot(x-y)}+\frac{y^2}{y\cdot(x-y)}-\frac{2\cdot x}{x-y}=\frac{x^2}{x\cdot(x-y)}+\frac{y^2}{y\cdot(x-y)}-\frac{2\cdot x}{x-y}=$$

$$=\frac{x}{x-y}+\frac{y}{x-y}-\frac{2\cdot x}{x-y}=\frac{x+y-2\cdot x}{x-y}=\frac{y-x}{x-y}=\frac{-(x-y)}{x-y}=\frac{-(x-y)}{x-y}=-1.$$

Vježba 218

Izračunaj: $\dfrac{x^2}{x^2-x\cdot y}+\dfrac{y^2}{x\cdot y-y^2}-\dfrac{2\cdot y}{x-y}$.

Rezultat: 1.

Zadatak 219 (Matko, srednja škola)

Racionaliziraj nazivnik razlomka: $\dfrac{1}{\sqrt[3]{2}}$.

Rješenje 219

Ponovimo!

$$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad\left(\sqrt[n]{a}\right)^n=\sqrt[n]{a^n}=a.$$

$$\frac{1}{\sqrt[3]{2}}=\frac{1}{\sqrt[3]{2}}\cdot\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}=\frac{\sqrt[3]{2^2}}{\sqrt[3]{2\cdot 2^2}}=\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^1\cdot 2^2}}=\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^3}}=\frac{\sqrt[3]{4}}{2}=\frac{1}{2}\cdot\sqrt[3]{4}.$$

Vježba 219

Racionaliziraj nazivnik razlomka: $\dfrac{1}{\sqrt[3]{5}}$.

Rezultat: $\dfrac{1}{5}\cdot\sqrt[3]{25}$.

Zadatak 220 (Matko, srednja škola)

Racionaliziraj nazivnik razlomka: $\dfrac{8}{\sqrt[4]{64}}$.

Rješenje 220

Ponovimo!

$$\sqrt[n\cdot m]{a^{n\cdot p}}=\sqrt[m]{a^p}\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^1=a\quad,\quad\sqrt[n]{a^n\cdot b}=a\cdot\sqrt[n]{b}.$$

$$\left(\sqrt[n]{a}\right)^{n} = \sqrt[n]{a^{n}} = a.$$

$$\frac{8}{\sqrt[4]{64}} = \frac{8}{\sqrt[4]{2^{6}}} = \left[\begin{array}{l}\text{kratimo eksponent korijena} \\ \text{i eksponent radikanda sa 2}\end{array}\right] = \frac{8}{\sqrt{2^{3}}} = \frac{8}{\sqrt{2^{2} \cdot 2^{1}}} = \frac{8}{\sqrt{2^{2} \cdot 2}} = \frac{8}{\sqrt{2^{2}} \cdot \sqrt{2}} =$$

$$= \frac{8}{2 \cdot \sqrt{2}} = \frac{8}{2 \cdot \sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4 \cdot \sqrt{2}}{\left(\sqrt{2}\right)^{2}} = \frac{4 \cdot \sqrt{2}}{2} = \frac{4 \cdot \sqrt{2}}{2} = 2 \cdot \sqrt{2}.$$

Vježba 220

Racionaliziraj nazivnik razlomka: $\frac{4}{\sqrt[4]{64}}$.

Rezultat: $\sqrt{2}$.