**Zadatak 181 (Robertina kumica ☺, ekonomska škola)**

Izračunaj: $\sqrt[3]{2+\sqrt{3}}\cdot\sqrt[6]{7-4\cdot\sqrt{3}}.$

**Rješenje 181**

Ponovimo!

$$\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}\quad,\quad(x+y)^2=x^2+2\cdot x\cdot y+y^2\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad x^2-y^2=(x-y)\cdot(x+y).$$

$$\left(a\cdot\sqrt{b}\right)^2=a^2\cdot b.$$

$$\sqrt[3]{2+\sqrt{3}}\cdot\sqrt[6]{7-4\cdot\sqrt{3}}=\left[\begin{array}{l}\text{korijene svodimo na}\\ \text{zajednički korijen }\sqrt[6]{\ }\end{array}\right]=\sqrt[6]{\left(2+\sqrt{3}\right)^2}\cdot\sqrt[6]{7-4\cdot\sqrt{3}}=\sqrt[6]{\left(2+\sqrt{3}\right)^2\cdot\left(7-4\cdot\sqrt{3}\right)}=$$

$$=\sqrt[6]{\left(2^2+2\cdot2\cdot\sqrt{3}+\left(\sqrt{3}\right)^2\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=\sqrt[6]{\left(4+4\cdot\sqrt{3}+3\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=\sqrt[6]{\left(7+4\cdot\sqrt{3}\right)\cdot\left(7-4\cdot\sqrt{3}\right)}=$$

$$=\sqrt[6]{7^2-\left(4\cdot\sqrt{3}\right)^2}=\sqrt[6]{49-16\cdot3}=\sqrt[6]{49-48}=\sqrt[6]{1}=1.$$

**Vježba 181**

Izračunaj: $\sqrt[3]{2-\sqrt{3}}\cdot\sqrt[6]{7+4\cdot\sqrt{3}}.$

**Rezultat:** 1.

**Zadatak 182 (Robertina kumica ☺, ekonomska škola)**

Izračunaj: $\sqrt[3]{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{3}}.$

**Rješenje 182**

Ponovimo!

$$\sqrt[n]{a^m}=\sqrt[n\cdot p]{a^{m\cdot p}}\quad,\quad\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}\quad,\quad a^n\cdot a^m=a^{n\cdot m}\quad,\quad a^n\cdot b^n=(a\cdot b)^n.$$

$$\left(\sqrt{a}\right)^2=a\quad,\quad x^2-y^2=(x-y)\cdot(x+y).$$

$$\sqrt[3]{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{3}}=\left[\begin{array}{l}\text{korijene svodimo na}\\ \text{zajednički korijen }\sqrt[6]{\ }\end{array}\right]=\sqrt[6]{\left(2+\sqrt{3}\right)^2}\cdot\sqrt[6]{\left(2-\sqrt{3}\right)^3}=\sqrt[6]{\left(2+\sqrt{3}\right)^2\cdot\left(2-\sqrt{3}\right)^3}=$$

$$=\sqrt[6]{\left(2+\sqrt{3}\right)^2\cdot\left(2-\sqrt{3}\right)^2\cdot\left(2-\sqrt{3}\right)}=\sqrt[6]{\left(\left(2+\sqrt{3}\right)\cdot\left(2-\sqrt{3}\right)\right)^2\cdot\left(2-\sqrt{3}\right)}=\sqrt[6]{\left(2^2-\left(\sqrt{3}\right)^2\right)^2\cdot\left(2-\sqrt{3}\right)}=$$

$$=\sqrt[6]{(4-3)^2\cdot\left(2-\sqrt{3}\right)}=\sqrt[6]{1^2\cdot\left(2-\sqrt{3}\right)}=\sqrt[6]{1\cdot\left(2-\sqrt{3}\right)}=\sqrt[6]{2-\sqrt{3}}.$$

**Vježba 182**

Izračunaj: $\sqrt[3]{2+\sqrt{3}}\cdot\sqrt[3]{2-\sqrt{3}}.$

**Rezultat:** 1.

**Zadatak 183 (Robertina kumica ☺, ekonomska škola)**

Izračunaj: $\left(\sqrt[4]{a\cdot\sqrt[3]{a^2\cdot b\cdot\sqrt{a\cdot b}}}\right)^6.$

**Rješenje 183**

Ponovimo!

$$\left(\sqrt[n]{a}\right)^m=\sqrt[n]{a^m} \quad , \quad \sqrt[n]{a^m}=\sqrt[n\cdot p]{a^{m\cdot p}} \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \left(\sqrt[n]{a}\right)^n=a.$$

$$a^n\cdot a^m=a^{n+m} \quad , \quad a\cdot\sqrt{b}=\sqrt{a^2\cdot b} \quad , \quad \sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b}.$$

$$\left(\sqrt[4]{a\cdot\sqrt[3]{a^2\cdot b\cdot\sqrt{a\cdot b}}}\right)^6=\sqrt[4]{\left(a\cdot\sqrt[3]{a^2\cdot b\cdot\sqrt{a\cdot b}}\right)^6}=\sqrt[4]{\left(a\cdot\sqrt[3]{a^2\cdot b\cdot\sqrt{a\cdot b}}\right)^6}=\sqrt{\left(a\cdot\sqrt[3]{a^2\cdot b\cdot\sqrt{a\cdot b}}\right)^3}=$$

$$=\sqrt{a^3\cdot\left(\sqrt[3]{a^2\cdot b\cdot\sqrt{a\cdot b}}\right)^3}=\sqrt{a^3\cdot\left(\sqrt[3]{a^2\cdot b\cdot\sqrt{a\cdot b}}\right)^3}=\sqrt{a^3\cdot a^2\cdot b\cdot\sqrt{a\cdot b}}=\sqrt{a^5\cdot b\cdot\sqrt{a\cdot b}}=$$

$$=\sqrt{\sqrt{\left(a^5\cdot b\right)^2\cdot a\cdot b}}=\sqrt{\sqrt{a^{10}\cdot b^2\cdot a\cdot b}}=\sqrt{\sqrt{a^{11}\cdot b^3}}=\sqrt[4]{a^{11}\cdot b^3}=\left[\begin{matrix}\text{djelomično}\\ \text{korjenovanje}\end{matrix}\right]=$$

$$=\sqrt[4]{a^8\cdot a^3\cdot b^3}=\sqrt[4]{a^8}\cdot\sqrt[4]{a^3\cdot b^3}=\sqrt[4]{a^8}\cdot\sqrt[4]{a^3\cdot b^3}=a^2\cdot\sqrt[4]{a^3\cdot b^3}.$$

**Vježba 183**

Izračunaj: $\left(\sqrt{a\cdot\sqrt[3]{a^2\cdot b\cdot\sqrt{a\cdot b}}}\right)^3.$

**Rezultat:** $a^2\cdot\sqrt[4]{a^3\cdot b^3}.$

**Zadatak 184 (Robertina kumica ☺, ekonomska škola)**

Izračunaj: $\dfrac{\sqrt[n]{\sqrt[4]{a}}}{\sqrt[2\cdot n]{\sqrt[n]{a^{3\cdot n}}}}.$

**Rješenje 184**

Ponovimo!

$$\sqrt[n]{\sqrt[m]{a}}=\sqrt[n\cdot m]{a} \quad , \quad \sqrt[n]{a^m}=\sqrt[n\cdot p]{a^{m\cdot p}} \quad , \quad \frac{a^n}{a^m}=a^{n-m} \quad , \quad \frac{\sqrt[n]{a}}{\sqrt[n]{b}}=\sqrt[n]{\frac{a}{b}}.$$

$$\frac{\sqrt[n]{\sqrt[4]{a}}}{\sqrt[2\cdot n]{\sqrt[n]{a^{3\cdot n}}}}=\frac{\sqrt[4\cdot n]{a}}{\sqrt[2\cdot n]{\sqrt[n]{a^{3\cdot n}}}}=\frac{\sqrt[4\cdot n]{a}}{\sqrt[2\cdot n]{\sqrt[n]{a^{3\cdot n}}}}=\frac{\sqrt[4\cdot n]{a}}{\sqrt[2\cdot n]{a^3}}=\frac{\sqrt[4\cdot n]{a}}{\sqrt[4\cdot n]{\left(a^3\right)^2}}=\frac{\sqrt[4\cdot n]{a}}{\sqrt[4\cdot n]{a^6}}=\sqrt[4\cdot n]{\frac{a}{a^6}}=$$

$$=\sqrt[4\cdot n]{\frac{1}{a^5}}=\frac{1}{\sqrt[4\cdot n]{a^5}}.$$

**Vježba 184**

Izračunaj: $\dfrac{\sqrt[n]{\sqrt[4]{a}}}{\sqrt[2\cdot n]{\sqrt[3]{a^9}}}.$

**Rezultat:** $\dfrac{1}{\sqrt[4\cdot n]{a^5}}.$

**Zadatak 185 (Tea, gimnazija)**

Skrati razlomak: $\frac{x^5+x+1}{x^3-1}$.

**Rješenje 185**

Ponovimo!

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right).$$

$$\frac{x^5+x+1}{x^3-1}=\frac{x^5-x^2+x^2+x+1}{x^3-1}=\frac{\left(x^5-x^2\right)+\left(x^2+x+1\right)}{x^3-1}=\frac{x^2\cdot\left(x^3-1\right)+\left(x^2+x+1\right)}{x^3-1}=$$

$$=\frac{x^2\cdot(x-1)\cdot\left(x^2+x+1\right)+\left(x^2+x+1\right)}{(x-1)\cdot\left(x^2+x+1\right)}=\left[\begin{array}{c}\text{u brojniku izlučimo}\\ x^2+x+1\end{array}\right]=\frac{\left(x^2+x+1\right)\cdot\left(x^2\cdot(x-1)+1\right)}{(x-1)\cdot\left(x^2+x+1\right)}=$$

$$=\frac{\left(x^2+x+1\right)\cdot\left(x^3-x^2+1\right)}{(x-1)\cdot\left(x^2+x+1\right)}=\frac{\left(x^2+x+1\right)\cdot\left(x^3-x^2+1\right)}{(x-1)\cdot\left(x^2+x+1\right)}=\frac{x^3-x^2+1}{x-1}.$$

**Vježba 185**

Skrati razlomak: $\frac{x^3-1}{x^5+x+1}$.

**Rezultat:** $\frac{x-1}{x^3-x^2+1}$.



**Zadatak 186 (Tea, gimnazija)**

Izračunaj vrijednost izraza: $A=\frac{1}{x^2+4\cdot x+4}-\frac{4}{x^4+4\cdot x^3+4\cdot x^2}+\frac{4}{x^3+2\cdot x^2}$, za $x=0.5$.

**Rješenje 186**

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

Najprije pojednostavimo zadani izraz:

$$A=\frac{1}{x^2+4\cdot x+4}-\frac{4}{x^4+4\cdot x^3+4\cdot x^2}+\frac{4}{x^3+2\cdot x^2}=$$

$$=\frac{1}{x^2+4\cdot x+4}-\frac{4}{x^2\cdot\left(x^2+4\cdot x+4\right)}+\frac{4}{x^2\cdot(x+2)}=$$

$$=\frac{1}{(x+2)^2}-\frac{4}{x^2\cdot(x+2)^2}+\frac{4}{x^2\cdot(x+2)}=\frac{x^2-4+4\cdot(x+2)}{x^2\cdot(x+2)^2}=\frac{x^2-4+4\cdot x+8}{x^2\cdot(x+2)^2}=\frac{x^2+4\cdot x+4}{x^2\cdot(x+2)^2}=$$

$$=\frac{(x+2)^2}{x^2\cdot(x+2)^2}=\frac{(x+2)^2}{x^2\cdot(x+2)^2}=\frac{1}{x^2}.$$

Sada je:

$$\left.\begin{array}{l} A=\frac{1}{x^2} \\ x=0.5 \end{array}\right\} \Rightarrow A=\frac{1}{0.5^2}=\frac{1}{0.25}=\left[\begin{array}{l}\text{proširujemo} \\ \text{razlomak sa } 100\end{array}\right]=\frac{100}{25}=4.$$

**Vježba 186**

Izračunaj vrijednost izraza: $A=\frac{1}{x^2+4\cdot x+4}-\frac{4}{x^4+4\cdot x^3+4\cdot x^2}+\frac{4}{x^3+2\cdot x^2}$, za $x=1$.

**Rezultat:** 1.

**Zadatak 187 (Boris, maturant)**

Pojednostavni: $\left(\sqrt{x}-\frac{\sqrt{x\cdot y}+y}{\sqrt{x}+\sqrt{y}}\right)\cdot\left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}+\frac{2\cdot\sqrt{x\cdot y}}{x-y}\right)$ za $x\neq y,\ x\geq 0,\ y\geq 0.$

**Rješenje 187**

Ponovimo!

$$\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad a-b=\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right).$$

$$\left(\sqrt{a}\right)^2=a \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$\left(\sqrt{x}-\frac{\sqrt{x\cdot y}+y}{\sqrt{x}+\sqrt{y}}\right)\cdot\left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}+\frac{2\cdot\sqrt{x\cdot y}}{x-y}\right)=$$

$$=\frac{\sqrt{x}\cdot\left(\sqrt{x}+\sqrt{y}\right)-\left(\sqrt{x\cdot y}+y\right)}{\sqrt{x}+\sqrt{y}}\cdot\left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}+\frac{2\cdot\sqrt{x\cdot y}}{\underbrace{x-y}_{\left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\sqrt{x}+\sqrt{y}\right)}}\right)=$$

$$=\frac{\left(\sqrt{x}\right)^2+\sqrt{x\cdot y}-\sqrt{x\cdot y}-y}{\sqrt{x}+\sqrt{y}}\cdot\frac{\sqrt{x}\cdot\left(\sqrt{x}-\sqrt{y}\right)+\sqrt{y}\cdot\left(\sqrt{x}+\sqrt{y}\right)+2\cdot\sqrt{x\cdot y}}{x-y}=$$

$$=\frac{x+\sqrt{x\cdot y}-\sqrt{x\cdot y}-y}{\sqrt{x}+\sqrt{y}}\cdot\frac{\left(\sqrt{x}\right)^2-\sqrt{x\cdot y}+\sqrt{x\cdot y}-\left(\sqrt{y}\right)^2+2\cdot\sqrt{x\cdot y}}{x-y}=$$

$$=\frac{x-y}{\sqrt{x}+\sqrt{y}}\cdot\frac{x-\sqrt{x\cdot y}+\sqrt{x\cdot y}-y+2\cdot\sqrt{x\cdot y}}{x-y}=\frac{x-y}{\sqrt{x}+\sqrt{y}}\cdot\frac{x-y+2\cdot\sqrt{x\cdot y}}{x-y}=$$

$$=\frac{x-y}{\sqrt{x}+\sqrt{y}}\cdot\frac{x-y+2\cdot\sqrt{x\cdot y}}{x-y}=\frac{x-y+2\cdot\sqrt{x\cdot y}}{\sqrt{x}+\sqrt{y}}=\frac{\left(\sqrt{x}+\sqrt{y}\right)^2}{\sqrt{x}+\sqrt{y}}=\frac{\left(\sqrt{x}+\sqrt{y}\right)^2}{\sqrt{x}+\sqrt{y}}=\sqrt{x}+\sqrt{y}.$$

**Vježba 187**

Pojednostavni: $\left(\frac{1}{\sqrt{x}}+\frac{1}{\sqrt{y}}\right)\cdot\frac{\sqrt{x\cdot y}}{\sqrt{x}+\sqrt{y}}$ za $x\geq 0,\ y\geq 0.$

**Rezultat:** 1.

**Zadatak 188 (Josip, gimnazija)**

Pojednostavni: $\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}\,.$

**Rješenje 188**

Ponovimo!

$$\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \left(\sqrt{a}\right)^2=a.$$

$$\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{3}}}}\cdot\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{3}}}}=$$

$$=\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{\left(2+\sqrt{2+\sqrt{2+\sqrt{3}}}\right)\cdot\left(2-\sqrt{2+\sqrt{2+\sqrt{3}}}\right)}=$$

$$=\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2^2-\left(\sqrt{2+\sqrt{2+\sqrt{3}}}\right)^2}=\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{4-\left(2+\sqrt{2+\sqrt{3}}\right)}=$$

$$=\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{4-2-\sqrt{2+\sqrt{3}}}=\sqrt{2+\sqrt{3}}\cdot\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2-\sqrt{2+\sqrt{3}}}=$$

$$=\sqrt{2+\sqrt{3}}\cdot\sqrt{\left(2+\sqrt{2+\sqrt{3}}\right)\cdot\left(2-\sqrt{2+\sqrt{3}}\right)}=\sqrt{2+\sqrt{3}}\cdot\sqrt{2^2-\left(\sqrt{2+\sqrt{3}}\right)^2}=\sqrt{2+\sqrt{3}}\cdot\sqrt{4-\left(2+\sqrt{3}\right)}=$$

$$=\sqrt{2+\sqrt{3}}\cdot\sqrt{4-2-\sqrt{3}}=\sqrt{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{3}}=\sqrt{2^2-\left(\sqrt{3}\right)^2}=\sqrt{4-3}=\sqrt{1}=1.$$

**Vježba 188**

Pojednostavni: $\sqrt{4+\sqrt{7}}\cdot\sqrt{4-\sqrt{7}}.$

**Rezultat:** 3.



**Zadatak 189 (Leda, gimnazija)**

Ako je $\left(a+\frac{1}{a}\right)^2=3$, koliko je $a^3+\frac{1}{a^3}$?

**Rješenje 189**

Ponovimo!

$$(x+y)^3=x^3+3\cdot x^2\cdot y+3\cdot x\cdot y^2+y^3 \quad , \quad \left(\sqrt{a}\right)^n=\sqrt{a^n}.$$

$$\left(a+\frac{1}{a}\right)^2=3 \Rightarrow \left(a+\frac{1}{a}\right)^2=3\ /\sqrt{\ } \Rightarrow a+\frac{1}{a}=\sqrt{3}.$$

Sada uporabimo formulu za kub zbroja:

$$\left(a+\frac{1}{a}\right)^3=a^3+3\cdot a^2\cdot\frac{1}{a}+3\cdot a\cdot\frac{1}{a^2}+\frac{1}{a^3} \Rightarrow \left(a+\frac{1}{a}\right)^3=a^3+3\cdot a+3\cdot\frac{1}{a}+\frac{1}{a^3} \Rightarrow$$

$$\Rightarrow \left(a+\frac{1}{a}\right)^3=a^3+3\cdot\left(a+\frac{1}{a}\right)+\frac{1}{a^3} \Rightarrow \left(a+\frac{1}{a}\right)^3-3\cdot\left(a+\frac{1}{a}\right)=a^3+\frac{1}{a^3} \Rightarrow$$

$$\Rightarrow a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)^3-3\cdot\left(a+\frac{1}{a}\right) \Rightarrow \left[a+\frac{1}{a}=\sqrt{3}\right] \Rightarrow a^3+\frac{1}{a^3}=\left(\sqrt{3}\right)^3-3\cdot\sqrt{3} \Rightarrow$$

$$\Rightarrow a^3+\frac{1}{a^3}=\sqrt{3^3}-3\cdot\sqrt{3} \Rightarrow \left[\begin{array}{l}\text{djelomično}\\ \text{korjenovanje}\end{array}\right] \Rightarrow a^3+\frac{1}{a^3}=\sqrt{3^2\cdot 3}-3\cdot\sqrt{3} \Rightarrow$$

$$\Rightarrow a^3+\frac{1}{a^3}=3\cdot\sqrt{3}-3\cdot\sqrt{3} \Rightarrow a^3+\frac{1}{a^3}=0.$$

**Vježba 189**

$$\text{Ako je } \left(a+\frac{1}{a}\right)^2=2, \text{ koliko je } a^3+\frac{1}{a^3}?$$

**Rezultat:** $-\sqrt{2}.$

**Zadatak 190 (Leda, gimnazija)**

Neka je $x^2 = 1 + x$. Ako je $x^{10} = a + b \cdot x$, koliko je $a + b$?

**Rješenje 190**

Ponovimo!

$$(x+y)^2=x^2+2\cdot x\cdot y+y^2.$$

Kvadriramo i sređujemo:

$$x^2=1+x \Rightarrow x^2=1+x\ /^2 \Rightarrow x^4=(1+x)^2 \Rightarrow x^4=1+2\cdot x+x^2 \Rightarrow \begin{bmatrix}\text{supstitucija}\\ x^2=1+x\end{bmatrix} \Rightarrow$$

$$\Rightarrow x^4=1+2\cdot x+1+x \Rightarrow x^4=2+3\cdot x.$$

Ponovno kvadriramo i sređujemo:

$$x^4=2+3\cdot x \Rightarrow x^4=2+3\cdot x\ /^2 \Rightarrow x^8=(2+3\cdot x)^2 \Rightarrow x^8=4+12\cdot x+9\cdot x^2 \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{supstitucija}\\ x^2=1+x\end{bmatrix} \Rightarrow x^8=4+12\cdot x+9\cdot(1+x) \Rightarrow x^8=4+12\cdot x+9+9\cdot x \Rightarrow x^8=13+21\cdot x.$$

Množimo s $x^2$ i sređujemo:

$$x^8=13+21\cdot x \Rightarrow x^8=13+21\cdot x\ /\cdot x^2 \Rightarrow x^{10}=x^2\cdot(13+21\cdot x) \Rightarrow \begin{bmatrix}\text{supstitucija}\\ x^2=1+x\end{bmatrix} \Rightarrow$$

$$\Rightarrow x^{10}=(1+x)\cdot(13+21\cdot x) \Rightarrow x^{10}=13+21\cdot x+13\cdot x+21\cdot x^2 \Rightarrow x^{10}=13+34\cdot x+21\cdot x^2 \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{supstitucija}\\ x^2=1+x\end{bmatrix} \Rightarrow x^{10}=13+34\cdot x+21\cdot(1+x) \Rightarrow x^{10}=13+34\cdot x+21+21\cdot x \Rightarrow x^{10}=34+55\cdot x.$$

Budući da je iz uvjeta zadatka $x^{10} = a + b \cdot x$, slijedi:

$$\left.\begin{array}{l} x^{10}=a+b\cdot x \\ x^{10}=34+55\cdot x \end{array}\right\} \Rightarrow \left.\begin{array}{l} a=34 \\ b=55 \end{array}\right\} \Rightarrow a+b=34+55 \Rightarrow a+b=89.$$

**Vježba 190**

Neka je $x^2 = 1 + x$. Ako je $x^{10} = 2 \cdot a + 5 \cdot b \cdot x$, koliko je $a + b$?

**Rezultat:** 28.

**Zadatak 191 (Leda, gimnazija)**

$$\text{Pojednostavni: } \frac{a^4-(a-1)^2}{\left(a^2+1\right)^2-a^2}+\frac{a^2-\left(a^2-1\right)^2}{a^2\cdot(a+1)^2-1}+\frac{a^2\cdot(a-1)^2-1}{a^4-(a+1)^2}.$$

**Rješenje 191**

Ponovimo!

$$x^2-y^2=(x-y)\cdot(x+y).$$

$$\frac{a^4-(a-1)^2}{\left(a^2+1\right)^2-a^2}+\frac{a^2-\left(a^2-1\right)^2}{a^2\cdot(a+1)^2-1}+\frac{a^2\cdot(a-1)^2-1}{a^4-(a+1)^2}=$$

$$=\frac{\left(a^2-(a-1)\right)\cdot\left(a^2+(a-1)\right)}{\left(a^2+1-a\right)\cdot\left(a^2+1+a\right)}+\frac{\left(a-\left(a^2-1\right)\right)\cdot\left(a+\left(a^2-1\right)\right)}{\left(a\cdot(a+1)-1\right)\cdot\left(a\cdot(a+1)+1\right)}+\frac{\left(a\cdot(a-1)-1\right)\cdot\left(a\cdot(a-1)+1\right)}{\left(a^2-(a+1)\right)\cdot\left(a^2+(a+1)\right)}=$$

$$=\frac{\left(a^2-a+1\right)\cdot\left(a^2+a-1\right)}{\left(a^2+1-a\right)\cdot\left(a^2+1+a\right)}+\frac{\left(a-a^2+1\right)\cdot\left(a+a^2-1\right)}{\left(a^2+a-1\right)\cdot\left(a^2+a+1\right)}+\frac{\left(a^2-a-1\right)\cdot\left(a^2-a+1\right)}{\left(a^2-a-1\right)\cdot\left(a^2+a+1\right)}=$$

$$=\frac{\left(a^2-a+1\right)\cdot\left(a^2+a-1\right)}{\left(a^2+1-a\right)\cdot\left(a^2+1+a\right)}+\frac{\left(a-a^2+1\right)\cdot\left(a+a^2-1\right)}{\left(a^2+a-1\right)\cdot\left(a^2+a+1\right)}+\frac{\left(a^2-a-1\right)\cdot\left(a^2-a+1\right)}{\left(a^2-a-1\right)\cdot\left(a^2+a+1\right)}=$$

$$=\frac{a^2+a-1}{a^2+1+a}+\frac{a-a^2+1}{a^2+a+1}+\frac{a^2-a+1}{a^2+a+1}=\frac{a^2+a-1}{a^2+a+1}+\frac{a-a^2+1}{a^2+a+1}+\frac{a^2-a+1}{a^2+a+1}=$$

$$=\frac{a^2+a-1+a-a^2+1+a^2-a+1}{a^2+a+1}=\frac{a^2+a-1+a-a^2+1+a^2-a+1}{a^2+a+1}=\frac{a+a^2+1}{a^2+a+1}=\frac{a^2+a+1}{a^2+a+1}=1.$$

**Vježba 191**

Pojednostavni: $\dfrac{a^4-(a-1)^2}{\left(a^2+1\right)^2-a^2}-\dfrac{\left(a^2-1\right)^2-a^2}{a^2\cdot(a+1)^2-1}-\dfrac{1-a^2\cdot(a-1)^2}{a^4-(a+1)^2}.$

**Rezultat:** 1.

**Zadatak 192 (3A, TUPŠ)**

Ako je $x=\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}$, onda je:

$A.\ x^3+2\cdot x-3=0\ ,\ B.\ x^3-2\cdot x+3=0\ ,\ C.\ x^3-3\cdot x+2=0\ ,\ D.\ x^3+3\cdot x-2=0.$

**Rješenje 192**

Ponovimo!

$$(a-b)^3=a^3-3\cdot a^2\cdot b+3\cdot a\cdot b^2-b^3\quad,\quad\left(\sqrt[n]{a}\right)^n=a\quad,\quad\sqrt[n]{a}\cdot\sqrt[n]{b}=\sqrt[n]{a\cdot b}$$

$$a^2-b^2=(a-b)\cdot(a+b).$$

Najprije kubiramo zadanu jednakost:

$$x=\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}\ \Rightarrow\ x=\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}\ /^3\ \Rightarrow\ x^3=\left(\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}\right)^3\ \Rightarrow$$

$$\Rightarrow\ x^3=\left(\sqrt[3]{\sqrt{2}+1}\right)^3-3\cdot\left(\sqrt[3]{\sqrt{2}+1}\right)^2\cdot\sqrt[3]{\sqrt{2}-1}+3\cdot\sqrt[3]{\sqrt{2}+1}\cdot\left(\sqrt[3]{\sqrt{2}-1}\right)^2-\left(\sqrt[3]{\sqrt{2}-1}\right)^3\ \Rightarrow$$

$$\Rightarrow\ x^3=\sqrt{2}+1-3\cdot\sqrt[3]{\sqrt{2}+1}\cdot\sqrt[3]{\sqrt{2}+1}\cdot\sqrt[3]{\sqrt{2}-1}+3\cdot\sqrt[3]{\sqrt{2}+1}\cdot\sqrt[3]{\sqrt{2}-1}\cdot\sqrt[3]{\sqrt{2}-1}-\left(\sqrt{2}-1\right)\ \Rightarrow$$

$$\Rightarrow x^3=\sqrt{2}+1-3\cdot\sqrt[3]{\sqrt{2}+1}\cdot\sqrt[3]{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}-1\right)}+3\cdot\sqrt[3]{\left(\sqrt{2}+1\right)\cdot\left(\sqrt{2}-1\right)}\cdot\sqrt[3]{\sqrt{2}-1}-\left(\sqrt{2}-1\right)\Rightarrow$$

$$\Rightarrow x^3=\sqrt{2}+1-3\cdot\sqrt[3]{\sqrt{2}+1}\cdot\sqrt[3]{\left(\sqrt{2}\right)^2-1}+3\cdot\sqrt[3]{\left(\sqrt{2}\right)^2-1}\cdot\sqrt[3]{\sqrt{2}-1}-\sqrt{2}+1\Rightarrow$$

$$\Rightarrow x^3=\sqrt{2}+1-3\cdot\sqrt[3]{\sqrt{2}+1}\cdot\sqrt[3]{2-1}+3\cdot\sqrt[3]{2-1}\cdot\sqrt[3]{\sqrt{2}-1}-\sqrt{2}+1\Rightarrow$$

$$\Rightarrow x^3=2-3\cdot\sqrt[3]{\sqrt{2}+1}\cdot\sqrt[3]{1}+3\cdot\sqrt[3]{1}\cdot\sqrt[3]{\sqrt{2}-1}\Rightarrow x^3=2-3\cdot\sqrt[3]{\sqrt{2}+1}\cdot 1+3\cdot 1\cdot\sqrt[3]{\sqrt{2}-1}\Rightarrow$$

$$\Rightarrow x^3=2-3\cdot\sqrt[3]{\sqrt{2}+1}+3\cdot\sqrt[3]{\sqrt{2}-1}\Rightarrow x^3=2-3\cdot\left(\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}\right)\Rightarrow$$

$$\Rightarrow\left[x=\sqrt[3]{\sqrt{2}+1}-\sqrt[3]{\sqrt{2}-1}\right]\Rightarrow x^3=2-3\cdot x\Rightarrow x^3+3\cdot x-2=0.$$

Odgovor je pod D.

**Vježba 192**

Ako je $x=\sqrt[3]{\sqrt{2}-1}-\sqrt[3]{\sqrt{2}+1}$, onda je:

$A.\ x^3+3\cdot x+2=0\ ,\ B.\ x^3-3\cdot x+2=0\ ,\ C.\ x^3+3\cdot x-2=0\ ,\ D.\ x^3-2\cdot x+3.$

**Rezultat:** Odgovor pod A.

**Zadatak 193 (Mario, srednja škola)**

Neka je $x^2 - y^2 = 75$ i $x + y = 15$.

a) Koliko je $x - y$?

b) Koliko je $2 \cdot x - 2 \cdot y + 1$?

**Rješenje 193**

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)\Rightarrow a-b=\frac{a^2-b^2}{a+b}.$$



$$a)\ x-y=\frac{x^2-y^2}{x+y}\Rightarrow x-y=\frac{75}{15}\Rightarrow x-y=5.$$

$$b)\ 2\cdot x-2\cdot y+1=2\cdot(x-y)+1=2\cdot\frac{x^2-y^2}{x+y}+1=2\cdot\frac{75}{15}+1=2\cdot 5+1=11.$$

**Vježba 193**

Neka je $x^2 - y^2 = 50$ i $x + y = 10$. Koliko je $x - y$?

**Rezultat:** 5.

**Zadatak 194 (Emy, gimnazija)**

Racionaliziraj nazivnik razlomka: $\dfrac{1}{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}$.

**Rješenje 194**

Ponovimo!

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right)\quad,\quad\left(\sqrt[n]{a}\right)^n=a.$$

$$\frac{1}{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}=\frac{1}{\sqrt[3]{3^2}-\sqrt[3]{3\cdot 2}+\sqrt[3]{2^2}}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{1}{\sqrt[3]{3^2}-\sqrt[3]{3\cdot 2}+\sqrt[3]{2^2}}\cdot\frac{\sqrt[3]{3}+\sqrt[3]{2}}{\sqrt[3]{3}+\sqrt[3]{2}}=$$

$$=\frac{\sqrt[3]{3}+\sqrt[3]{2}}{\left(\sqrt[3]{3}\right)^3+\left(\sqrt[3]{2}\right)^3}=\frac{\sqrt[3]{3}+\sqrt[3]{2}}{3+2}=\frac{\sqrt[3]{3}+\sqrt[3]{2}}{5}.$$

**Vježba 194**

Racionaliziraj nazivnik razlomka: $\frac{1}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}}$.

**Rezultat:** $\sqrt[3]{3}-\sqrt[3]{2}.$

**Zadatak 195 (Zoran, gimnazija)**

Neka je $x=\frac{b^2+c^2-a^2}{2\cdot b\cdot c}$ i $y=\frac{(a-b+c)\cdot(a+b-c)}{(a+b+c)\cdot(b+c-a)}$. Dokaži da je $(x+1)\cdot(y+1)=2.$

**Rješenje 195**

Ponovimo!

$$x^2-y^2=(x-y)\cdot(x+y)\quad,\quad(x-y)^2=x^2-2\cdot x\cdot y+y^2\quad,\quad(x+y)^2=x^2+2\cdot x\cdot y+y^2.$$

Izračunamo svaki faktor x + 1 i y + 1 posebno:

- $x+1=\frac{b^2+c^2-a^2}{2\cdot b\cdot c}+1=\frac{b^2+c^2-a^2+2\cdot b\cdot c}{2\cdot b\cdot c}=\frac{-a^2+b^2+c^2+2\cdot b\cdot c}{2\cdot b\cdot c}.$
- $y+1=\frac{(a-b+c)\cdot(a+b-c)}{(a+b+c)\cdot(b+c-a)}+1=\frac{(a-b+c)\cdot(a+b-c)+(a+b+c)\cdot(b+c-a)}{(a+b+c)\cdot(b+c-a)}=$

$$=\frac{(a-b+c)\cdot(a+b-c)+(b+c+a)\cdot(b+c-a)}{(b+c+a)\cdot(b+c-a)}=$$

$$=\frac{(a-(b-c))\cdot(a+(b-c))+((b+c)+a)\cdot((b+c)-a)}{((b+c)+a)\cdot((b+c)-a)}=\frac{\left[a^2-(b-c)^2\right]+\left[(b+c)^2-a^2\right]}{(b+c)^2-a^2}=$$

$$=\frac{\left[a^2-\left(b^2-2\cdot b\cdot c+c^2\right)\right]+\left[b^2+2\cdot b\cdot c+c^2-a^2\right]}{b^2+2\cdot b\cdot c+c^2-a^2}=$$

$$=\frac{\left[a^2-b^2+2\cdot b\cdot c-c^2\right]+\left[b^2+2\cdot b\cdot c+c^2-a^2\right]}{b^2+2\cdot b\cdot c+c^2-a^2}=\frac{a^2-b^2+2\cdot b\cdot c-c^2+b^2+2\cdot b\cdot c+c^2-a^2}{b^2+2\cdot b\cdot c+c^2-a^2}=$$

$$=\frac{a^2-b^2+2\cdot b\cdot c-c^2+b^2+2\cdot b\cdot c+c^2-a^2}{b^2+2\cdot b\cdot c+c^2-a^2}=\frac{4\cdot b\cdot c}{b^2+2\cdot b\cdot c+c^2-a^2}=\frac{4\cdot b\cdot c}{-a^2+b^2+2\cdot b\cdot c+c^2}.$$

Sada izravno slijedi jednakost:

$$(x+1)\cdot(y+1)=\frac{-a^2+b^2+c^2+2\cdot b\cdot c}{2\cdot b\cdot c}\cdot\frac{4\cdot b\cdot c}{-a^2+b^2+2\cdot b\cdot c+c^2}=$$

$$=\frac{-a^2+b^2+c^2+2\cdot b\cdot c}{2\cdot b\cdot c}\cdot\frac{4\cdot b\cdot c}{-a^2+b^2+2\cdot b\cdot c+c^2}=\frac{4\cdot b\cdot c}{2\cdot b\cdot c}=\frac{4\cdot b\cdot c}{2\cdot b\cdot c}=2.$$

**Vježba 195**

Neka je $x=\frac{b^2+c^2-a^2}{2\cdot b\cdot c}$ i $y=\frac{(a-b+c)\cdot(a+b-c)}{(a+b+c)\cdot(b+c-a)}$. Dokaži da je $x\cdot y+x+y=1$.

**Rezultat:** Dokaz analogan.

**Zadatak 196 (Zoran, gimnazija)**

Ako vrijede jednakosti $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ i $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$, dokaži da vrijedi $\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=1$.

**Rješenje 196**

Ponovimo!

$$(a+b+c)^2=a^2+b^2+c^2+2\cdot a\cdot b+2\cdot a\cdot c+2\cdot b\cdot c \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}.$$

Kvadriranjem druge jednakosti dobije se:

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1\ /^2 \Rightarrow \left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)^2=1^2 \Rightarrow \left(\frac{a}{x}\right)^2+\left(\frac{b}{y}\right)^2+\left(\frac{c}{z}\right)^2+2\cdot\frac{a}{x}\cdot\frac{b}{y}+2\cdot\frac{a}{x}\cdot\frac{c}{z}+2\cdot\frac{b}{y}\cdot\frac{c}{z}=1 \Rightarrow$$

$$\Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+2\cdot\frac{a\cdot b}{x\cdot y}+2\cdot\frac{a\cdot c}{x\cdot z}+2\cdot\frac{b\cdot c}{y\cdot z}=1 \Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+\left(2\cdot\frac{a\cdot b}{x\cdot y}+2\cdot\frac{a\cdot c}{x\cdot z}+2\cdot\frac{b\cdot c}{y\cdot z}\right)=1 \Rightarrow$$

$$\Rightarrow \left[\begin{matrix}\text{iz zagrade izlučimo}\\ 2\cdot\frac{a\cdot b\cdot c}{x\cdot y\cdot z}\end{matrix}\right] \Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+2\cdot\frac{a\cdot b\cdot c}{x\cdot y\cdot z}\cdot\left(\frac{z}{c}+\frac{y}{b}+\frac{x}{a}\right)=1 \Rightarrow$$

$$\Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+2\cdot\frac{a\cdot b\cdot c}{x\cdot y\cdot z}\cdot\left(\frac{x}{a}+\frac{y}{b}+\frac{z}{c}\right)=1 \Rightarrow \left[\begin{matrix}\text{uvjet}\\ \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0\end{matrix}\right] \Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+2\cdot\frac{a\cdot b\cdot c}{x\cdot y\cdot z}\cdot 0=1 \Rightarrow$$

$$\Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}+0=1 \Rightarrow \frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=1.$$ Dokaz gotov



**Vježba 196**

Ako vrijede jednakosti $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ i $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$, dokaži da vrijedi $\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=4$.

**Rezultat:** Dokaz analogan.

**Zadatak 197 (Ivana, gimnazija)**

Dokaži da je $\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ ...\ +\frac{1}{\sqrt{2\cdot n-1}+\sqrt{2\cdot n+1}}=\frac{n-1}{\sqrt{3}+\sqrt{2\cdot n+1}}$.

**Rješenje 197**

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \left(\sqrt{a}\right)^2=a.$$

Računamo lijevu stranu jednakosti:

$$\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ ...\ +\frac{1}{\sqrt{2\cdot n-1}+\sqrt{2\cdot n+1}}=\left[\begin{matrix}\text{racionalizacija nazivnika}\\ \text{svakog razlomka}\end{matrix}\right]=$$

$$=\frac{1}{\sqrt{3}+\sqrt{5}}\cdot\frac{\sqrt{3}-\sqrt{5}}{\sqrt{3}-\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}\cdot\frac{\sqrt{5}-\sqrt{7}}{\sqrt{5}-\sqrt{7}}+\ \dots\ +\frac{1}{\sqrt{2\cdot n-1}+\sqrt{2\cdot n+1}}\cdot\frac{\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}}{\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}}=$$

$$=\frac{\sqrt{3}-\sqrt{5}}{\left(\sqrt{3}\right)^2-\left(\sqrt{5}\right)^2}+\frac{\sqrt{5}-\sqrt{7}}{\left(\sqrt{5}\right)^2-\left(\sqrt{7}\right)^2}+\ \dots\ +\frac{\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}}{\left(\sqrt{2\cdot n-1}\right)^2-\left(\sqrt{2\cdot n+1}\right)^2}=$$

$$=\frac{\sqrt{3}-\sqrt{5}}{3-5}+\frac{\sqrt{5}-\sqrt{7}}{5-7}+\ \dots\ +\frac{\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}}{2\cdot n-1-\left(2\cdot n+1\right)}=\frac{\sqrt{3}-\sqrt{5}}{3-5}+\frac{\sqrt{5}-\sqrt{7}}{5-7}+\ \dots\ +\frac{\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}}{2\cdot n-1-2\cdot n-1}=$$

$$=\frac{\sqrt{3}-\sqrt{5}}{-2}+\frac{\sqrt{5}-\sqrt{7}}{-2}+\ \dots\ +\frac{\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}}{2\cdot n-1-2\cdot n-1}=\frac{\sqrt{3}-\sqrt{5}}{-2}+\frac{\sqrt{5}-\sqrt{7}}{-2}+\ \dots\ +\frac{\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}}{-2}=$$

$$=-\frac{1}{2}\cdot\left(\sqrt{3}-\sqrt{5}+\sqrt{5}-\sqrt{7}+\ \dots\ +\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}\right)=$$

$$=-\frac{1}{2}\cdot\left(\sqrt{3}-\sqrt{5}+\sqrt{5}-\sqrt{7}+\ \dots\ +\sqrt{2\cdot n-1}-\sqrt{2\cdot n+1}\right)=-\frac{1}{2}\cdot\left(\sqrt{3}-\sqrt{2\cdot n+1}\right)=\frac{\sqrt{2\cdot n+1}-\sqrt{3}}{2}=$$

$$=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{\sqrt{2\cdot n+1}-\sqrt{3}}{2}\cdot\frac{\sqrt{2\cdot n+1}+\sqrt{3}}{\sqrt{2\cdot n+1}+\sqrt{3}}=\frac{\left(\sqrt{2\cdot n+1}\right)^2-\left(\sqrt{3}\right)^2}{2\cdot\left(\sqrt{2\cdot n+1}+\sqrt{3}\right)}=$$

$$=\frac{2\cdot n+1-3}{2\cdot\left(\sqrt{2\cdot n+1}+\sqrt{3}\right)}=\frac{2\cdot n-2}{2\cdot\left(\sqrt{2\cdot n+1}+\sqrt{3}\right)}=\frac{2\cdot\left(n-1\right)}{2\cdot\left(\sqrt{2\cdot n+1}+\sqrt{3}\right)}=$$

$$=\frac{2\cdot\left(n-1\right)}{2\cdot\left(\sqrt{2\cdot n+1}+\sqrt{3}\right)}=\frac{n-1}{2\cdot\left(\sqrt{2\cdot n+1}+\sqrt{3}\right)}.$$



**Vježba 197**

Dokaži da je $\frac{1}{\sqrt{3}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{7}}+\ \dots\ +\frac{1}{\sqrt{19}+\sqrt{21}}=\frac{9}{\sqrt{3}+\sqrt{21}}.$

**Rezultat:** Dokaz analogan.

**Zadatak 198 (Anita, prehrambena škola)**

Izračunaj: $\frac{2}{5}a^7:\frac{2}{3}a^3.$

**Rješenje 198**

Ponovimo!

Dijeljenje racionalnih brojeva

Za svaka dva racionalna broja $\frac{a}{b}$ i $\frac{c}{d}$, $\frac{c}{d}\neq 0$, vrijedi:

$$\frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{d}{c}=\frac{a\cdot d}{b\cdot c}.$$

Dijeljenje potencija jednakih baza (osnovica)

Potencije jednakih baza se dijele tako da se zajednička baza potencira razlikom njihovih eksponenata, tj.

$$a^n:a^m=a^{n-m},$$

pri čemu je $a\neq 0,\ n,\ m\in Z.$

$$\frac{2}{5}a^7 : \frac{2}{3}a^3 = \left[\begin{array}{l}\text{posebno dijelimo razlomke,}\\ \text{a posebno potencije}\end{array}\right] = \left(\frac{2}{5} : \frac{2}{3}\right)a^7 : a^3 = \left(\frac{2}{5} \cdot \frac{3}{2}\right)a^{7-3} = \left(\frac{2}{5} \cdot \frac{3}{2}\right)a^4 = \frac{3}{5}a^4.$$

**Vježba 198**

Izračunaj: $\frac{3}{5}a^9 : \frac{3}{7}a^7.$

**Rezultat:** $\frac{7}{5}a^2.$

**Zadatak 199 (Anita, prehrambena škola)**

Izračunaj: $\frac{2}{3}a^8b^9 : \frac{5}{7}a^3b^5.$

**Rješenje 199**

Ponovimo!

Dijeljenje racionalnih brojeva

Za svaka dva racionalna broja $\frac{a}{b}$ i $\frac{c}{d}$, $\frac{c}{d} \neq 0$, vrijedi:

$$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}.$$

Dijeljenje potencija jednakih baza (osnovica)

Potencije jednakih baza se dijele tako da se zajednička baza potencira razlikom njihovih eksponenata, tj.

$$a^n : a^m = a^{n-m},$$

pri čemu je $a \neq 0,\ n,\ m \in Z.$

$$\frac{2}{3}a^8b^9 : \frac{5}{7}a^3b^5 = \left[\begin{array}{l}\text{posebno dijelimo razlomke,}\\ \text{a posebno potencije istih baza}\end{array}\right] = \left(\frac{2}{3} : \frac{5}{7}\right)a^8b^9 : a^3b^5 =$$

$$= \left(\frac{2}{3} \cdot \frac{7}{5}\right)a^{8-3}b^{9-5} = \frac{14}{15}a^5b^4.$$



**Vježba 199**

Izračunaj: $\frac{2}{9}a^7b^6 : \frac{5}{7}a^3b^4.$

**Rezultat:** $\frac{14}{45}a^4b^2.$

**Zadatak 200 (Nely, gimnazija)**

Ako je $a \cdot y = b \cdot x$, dokaži $\frac{x^2}{x^2 + y^2} + \frac{b^2}{a^2 + b^2} = 1.$

**Rješenje 200**

Ponovimo!

$$(a \cdot b)^n = a^n \cdot b^n \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad \frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

1.inačica

$$\frac{x^2}{x^2 + y^2} + \frac{b^2}{a^2 + b^2} = \frac{x^2 \cdot \left(a^2 + b^2\right) + b^2 \cdot \left(x^2 + y^2\right)}{\left(x^2 + y^2\right) \cdot \left(a^2 + b^2\right)} = \frac{a^2 \cdot x^2 + b^2 \cdot x^2 + b^2 \cdot x^2 + b^2 \cdot y^2}{a^2 \cdot x^2 + b^2 \cdot x^2 + a^2 \cdot y^2 + b^2 \cdot y^2} =$$

$$=\frac{(a\cdot x)^2+(b\cdot x)^2+(b\cdot x)^2+(b\cdot y)^2}{(a\cdot x)^2+(b\cdot x)^2+(a\cdot y)^2+(b\cdot y)^2}=\left[\,a\cdot y=b\cdot x\,\right]=\frac{(a\cdot x)^2+(b\cdot x)^2+(b\cdot x)^2+(b\cdot y)^2}{(a\cdot x)^2+(b\cdot x)^2+(b\cdot x)^2+(b\cdot y)^2}=$$

$$=\frac{(a\cdot x)^2+2\cdot(b\cdot x)^2+(b\cdot y)^2}{(a\cdot x)^2+2\cdot(b\cdot x)^2+(b\cdot y)^2}=\frac{(a\cdot x)^2+2\cdot(b\cdot x)^2+(b\cdot y)^2}{(a\cdot x)^2+2\cdot(b\cdot x)^2+(b\cdot y)^2}=1.$$

2.inačica

Iz uvjeta

$$a\cdot y=b\cdot x$$

izračunamo, na primjer, a

$$a\cdot y=b\cdot x\ /:y\ \Rightarrow\ a=\frac{b\cdot x}{y}.$$

Dalje slijedi:

$$\frac{x^2}{x^2+y^2}+\frac{b^2}{a^2+b^2}=\frac{x^2}{x^2+y^2}+\frac{b^2}{\left(\frac{b\cdot x}{y}\right)^2+b^2}=\frac{x^2}{x^2+y^2}+\frac{b^2}{\frac{(b\cdot x)^2}{y^2}+b^2}=$$

$$=\frac{x^2}{x^2+y^2}+\frac{b^2}{\frac{b^2\cdot x^2}{y^2}+b^2}=\frac{x^2}{x^2+y^2}+\frac{b^2}{\frac{b^2\cdot x^2+b^2\cdot y^2}{y^2}}=\frac{x^2}{x^2+y^2}+\frac{b^2\cdot y^2}{b^2\cdot x^2+b^2\cdot y^2}=$$

$$=\frac{x^2}{x^2+y^2}+\frac{b^2\cdot y^2}{b^2\cdot\left(x^2+y^2\right)}=\frac{x^2}{x^2+y^2}+\frac{b^2\cdot y^2}{b^2\cdot\left(x^2+y^2\right)}=\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}=$$

$$=\frac{x^2+y^2}{x^2+y^2}=\frac{x^2+y^2}{x^2+y^2}=1.$$



**Vježba 200**

Ako je $\frac{a}{x}=\frac{b}{y}$, dokaži $\frac{x^2}{x^2+y^2}=\frac{a^2}{a^2+b^2}$.

**Rezultat:** Dokaz analogan.