Zadatak 161 (Ana, gimnazija)

Dokažite da za sve realne brojeve x, y i z vrijedi nejednakost:

$$x^2 + y^2 + 3 \cdot z^2 - 2 \cdot x \cdot z - 2 \cdot y \cdot z - 2 \cdot z + 1 \geq 0.$$

Rješenje 161

Ponovimo!

$$a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2 \quad , \quad a^2 \geq 0 \ , \ a \in R.$$

$$x^2 + y^2 + 3 \cdot z^2 - 2 \cdot x \cdot z - 2 \cdot y \cdot z - 2 \cdot z + 1 = x^2 - 2 \cdot x \cdot z + z^2 + y^2 - 2 \cdot y \cdot z + z^2 + z^2 - 2 \cdot z + 1 =$$

$$= \left(x^2 - 2 \cdot x \cdot z + z^2\right) + \left(y^2 - 2 \cdot y \cdot z + z^2\right) + \left(z^2 - 2 \cdot z + 1\right) = (x-z)^2 + (y-z)^2 + (z-1)^2 \geq 0.$$

Dobivena nejednakost istinita je za sve realne brojeve x, y i z pa je i polazna nejednakost istinita za sve x, y i z. Jednakost vrijedi ako i samo ako je x = y = z = 1.

Vježba 161

Dokažite da za sve realne brojeve x, y i z vrijedi nejednakost: $x^2 + y^2 + 2 \cdot z^2 \geq 2 \cdot z \cdot (x + y)$.

Rezultat: $(x-z)^2 + (y-z)^2 \geq 0 \Rightarrow x^2 + y^2 + 2 \cdot z^2 \geq 2 \cdot z \cdot (x+y).$

Zadatak 162 (Miro, maturant gimnazija)

Izračunajte vrijednost izraza $x^{100} + 100 \cdot y$ ako je $x^2 + 4 \cdot y^2 + 2 \cdot x - 8 \cdot y + 5 = 0$.

Rješenje 162

Ponovimo!

$$a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2 \quad , \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2 \quad , \quad a^2 = 0 \Leftrightarrow a = 0$$

$$a^2 + b^2 = 0 \Leftrightarrow a = b = 0 \quad , \quad (-a)^{2 \cdot n} = a^{2 \cdot n}.$$

Zadani izraz

$$x^2 + 4 \cdot y^2 + 2 \cdot x - 8 \cdot y + 5 = 0$$

transformiramo pa dobijemo vrijednosti za x i y:

$$x^2 + 4 \cdot y^2 + 2 \cdot x - 8 \cdot y + 5 = 0 \Rightarrow x^2 + 2 \cdot x + 1 + 4 \cdot y^2 - 8 \cdot y + 4 = 0 \Rightarrow$$

$$\Rightarrow \left(x^2 + 2 \cdot x + 1\right) + \left(4 \cdot y^2 - 8 \cdot y + 4\right) = 0 \Rightarrow (x+1)^2 + (2 \cdot y - 2)^2 = 0 \Rightarrow \left.\begin{matrix} x + 1 = 0 \\ 2 \cdot y - 2 = 0 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x = -1 \\ 2 \cdot y = 2 \ /:2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = -1 \\ y = 1 \end{matrix}\right\}.$$

Na kraju dobivamo da je

$$x^{100} + 100 \cdot y = (-1)^{100} + 100 \cdot 1 = 1 + 100 = 101.$$

Vježba 162

Izračunajte vrijednost izraza $x^{2008} + 2008 \cdot y$ ako je $x^2 + 4 \cdot y^2 + 2 \cdot x - 8 \cdot y + 5 = 0$.

Rezultat: 2009.

Zadatak 163 (Karolina, gimnazija)

Izračunajte vrijednost izraza $x \cdot (x + 2) + y \cdot (y - 2) - 2 \cdot x \cdot y$, ako je $x - y = 5$.

Rješenje 163

Ponovimo!

$$a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2.$$

Zadani izraz

$$x \cdot (x + 2) + y \cdot (y - 2) - 2 \cdot x \cdot y$$

transformiramo da dobijemo članove x – y:

$$x \cdot (x+2) + y \cdot (y-2) - 2 \cdot x \cdot y = x^2 + 2 \cdot x + y^2 - 2 \cdot y - 2 \cdot x \cdot y = x^2 - 2 \cdot x \cdot y + y^2 + 2 \cdot x - 2 \cdot y =$$

$$=\left(x^2-2\cdot x\cdot y+y^2\right)+2\cdot(x-y)=(x-y)^2+2\cdot(x-y)=5^2+2\cdot 5=25+10=35.$$

Vježba 163

Izračunajte vrijednost izraza x · (x + 2) + y · (y – 2) – 2 · x · y, ako je x – y = 1.

Rezultat: 3.

Zadatak 164 (Tea, TUPŠ)

Ako je $x+\frac{1}{x}=3$, izračunajte $x^2+\frac{1}{x^2}$.

Rješenje 164

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}.$$

$$x+\frac{1}{x}=3 \Rightarrow x+\frac{1}{x}=3\ /^2 \Rightarrow \left(x+\frac{1}{x}\right)^2=3^2 \Rightarrow x^2+2\cdot x\cdot\frac{1}{x}+\left(\frac{1}{x}\right)^2=9 \Rightarrow x^2+2+\frac{1}{x^2}=9 \Rightarrow$$

$$\Rightarrow x^2+\frac{1}{x^2}=9-2 \Rightarrow x^2+\frac{1}{x^2}=7.$$

Vježba 164

Ako je $x+\frac{1}{x}=2$, izračunajte $x^2+\frac{1}{x^2}$.

Rezultat: 2.

Zadatak 165 (Tea, TUPŠ)

Ako je $\frac{a}{b}+\frac{b}{a}=3$, izračunajte $\frac{a^3}{b^3}+\frac{b^3}{a^3}$.

Rješenje 165

Ponovimo!

$$(x+y)^3=x^3+3\cdot x^2\cdot y+3\cdot x\cdot y^2+y^3 \quad , \quad \left(\frac{x}{y}\right)^n=\frac{x^n}{y^n}.$$

$$\frac{a}{b}+\frac{b}{a}=3 \Rightarrow \frac{a}{b}+\frac{b}{a}=3\ /^3 \Rightarrow \left(\frac{a}{b}+\frac{b}{a}\right)^3=3^3 \Rightarrow \left(\frac{a}{b}\right)^3+3\cdot\left(\frac{a}{b}\right)^2\cdot\frac{b}{a}+3\cdot\frac{a}{b}\cdot\left(\frac{b}{a}\right)^2+\left(\frac{b}{a}\right)^3=27 \Rightarrow$$

$$\Rightarrow \frac{a^3}{b^3}+3\cdot\frac{a^2}{b^2}\cdot\frac{b}{a}+3\cdot\frac{a}{b}\cdot\frac{b^2}{a^2}+\frac{b^3}{a^3}=27 \Rightarrow \frac{a^3}{b^3}+3\cdot\frac{a}{b}+3\cdot\frac{b}{a}+\frac{b^3}{a^3}=27 \Rightarrow \frac{a^3}{b^3}+3\cdot\left(\frac{a}{b}+\frac{b}{a}\right)+\frac{b^3}{a^3}=27 \Rightarrow$$

$$\Rightarrow \frac{a^3}{b^3}+3\cdot 3+\frac{b^3}{a^3}=27 \Rightarrow \frac{a^3}{b^3}+9+\frac{b^3}{a^3}=27 \Rightarrow \frac{a^3}{b^3}+\frac{b^3}{a^3}=27-9 \Rightarrow \frac{a^3}{b^3}+\frac{b^3}{a^3}=18.$$

Vježba 165

Ako je $\frac{a}{b}+\frac{b}{a}=2$, izračunajte $\frac{a^3}{b^3}+\frac{b^3}{a^3}$.

Rezultat: 2.

Zadatak 166 (Mario, gimnazija)

Pojednostavnite izraz: $\sqrt{5+\sqrt{24}}$.

Rješenje 166

Ponovimo!

$$\sqrt{A \pm \sqrt{B}} = \sqrt{\frac{A + \sqrt{A^2 - B}}{2}} \pm \sqrt{\frac{A - \sqrt{A^2 - B}}{2}}.$$

Sada računamo:

$$\sqrt{5+\sqrt{24}} = \sqrt{\frac{5+\sqrt{5^2-24}}{2}} + \sqrt{\frac{5-\sqrt{5^2-24}}{2}} = \sqrt{\frac{5+\sqrt{25-24}}{2}} + \sqrt{\frac{5-\sqrt{25-24}}{2}} =$$

$$= \sqrt{\frac{5+\sqrt{1}}{2}} + \sqrt{\frac{5-\sqrt{1}}{2}} = \sqrt{\frac{5+1}{2}} + \sqrt{\frac{5-1}{2}} = \sqrt{\frac{6}{2}} + \sqrt{\frac{4}{2}} = \sqrt{3} + \sqrt{2} = \sqrt{2} + \sqrt{3}.$$

Vježba 166

Pojednostavnite izraz: $\sqrt{6+\sqrt{20}}$.

Rezultat: $1+\sqrt{5}$.

Zadatak 167 (Mira, srednja škola)

Ako je $\frac{1}{a\cdot(b+1)} + \frac{1}{b\cdot(a+1)} = \frac{1}{(a+1)\cdot(b+1)}$, koliko je $\frac{1}{a} + \frac{1}{b}$?

Rješenje 167

Ponovimo!

$$\frac{x+y}{n} = \frac{x}{n} + \frac{y}{n}.$$

Sada računamo:

$$\frac{1}{a\cdot(b+1)} + \frac{1}{b\cdot(a+1)} = \frac{1}{(a+1)\cdot(b+1)} \Rightarrow \left[\begin{array}{l}\text{množimo jednakost} \\ \text{sa } (a+1)\cdot(b+1)\end{array}\right] \Rightarrow$$

$$\Rightarrow \frac{1}{a\cdot(b+1)} + \frac{1}{b\cdot(a+1)} = \frac{1}{(a+1)\cdot(b+1)} \; /\cdot(a+1)\cdot(b+1) \Rightarrow \frac{a+1}{a} + \frac{b+1}{b} = 1 \Rightarrow \frac{a}{a} + \frac{1}{a} + \frac{b}{b} + \frac{1}{b} = 1 \Rightarrow$$

$$\Rightarrow 1 + \frac{1}{a} + 1 + \frac{1}{b} = 1 \Rightarrow 1 + \frac{1}{a} + 1 + \frac{1}{b} = 1 \Rightarrow 1 + \frac{1}{a} + \frac{1}{b} = 0 \Rightarrow \frac{1}{a} + \frac{1}{b} = -1.$$

Vježba 167

Ako je $\frac{1}{a\cdot(b+1)} + \frac{1}{b\cdot(a+1)} = \frac{2}{(a+1)\cdot(b+1)}$, koliko je $\frac{1}{a} + \frac{1}{b}$?

Rezultat: 0.

Zadatak 168 (Deny, gimnazija)

Pojednostavnite izraz:

$$\frac{a-b}{a+b+\sqrt{(a+b)^2-(a-b)^2}} + \frac{2\cdot\left(a\cdot\sqrt{a}-b\cdot\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left[a+b-\sqrt{(a+b)^2-(a-b)^2}\right]}.$$

Rješenje 168

Ponovimo!

$$(x+y)^2 = x^2 + 2\cdot x\cdot y + y^2 \quad , \quad (x-y)^2 = x^2 - 2\cdot x\cdot y + y^2 \quad , \quad x^3 - y^3 = (x-y)\cdot\left(x^2 + x\cdot y + y^2\right)$$

$$x^2 - y^2 = (x-y)\cdot(x+y) \quad , \quad x - y = \left(\sqrt{x}-\sqrt{y}\right)\cdot\left(\sqrt{x}+\sqrt{y}\right) \quad , \quad \sqrt{a^2} = |a| \quad , \quad |a| = \begin{cases} a, & a \ge 0 \\ -a, & a < 0 \end{cases}$$

$$\sqrt{x}\cdot\sqrt{y} = \sqrt{x\cdot y}.$$

$$\frac{a-b}{a+b+\sqrt{(a+b)^2-(a-b)^2}} + \frac{2\cdot\left(a\cdot\sqrt{a}-b\cdot\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left[a+b-\sqrt{(a+b)^2-(a-b)^2}\right]} =$$

$$=\frac{a-b}{a+b+\sqrt{a^2+2\cdot a\cdot b+b^2-\left(a^2-2\cdot a\cdot b+b^2\right)}}+$$

$$+\frac{2\cdot\left(\sqrt{a^2\cdot a}-\sqrt{b^2\cdot b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left[a+b-\sqrt{a^2+2\cdot a\cdot b+b^2-\left(a^2-2\cdot a\cdot b+b^2\right)}\right]}=$$

$$=\frac{a-b}{a+b+\sqrt{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}}+$$

$$+\frac{2\cdot\left(\sqrt{a^3}-\sqrt{b^3}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left[a+b-\sqrt{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}\right]}=$$

$$=\frac{a-b}{a+b+\sqrt{4\cdot a\cdot b}}+\frac{2\cdot\left(\left(\sqrt{a}\right)^3-\left(\sqrt{b}\right)^3\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left[a+b-\sqrt{4\cdot a\cdot b}\right]}=\frac{a-b}{a+b+2\cdot\sqrt{a\cdot b}}+\frac{2\cdot\left(\left(\sqrt{a}\right)^3-\left(\sqrt{b}\right)^3\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left[a+b-2\cdot\sqrt{a\cdot b}\right]}=$$

$$=\frac{a-b}{a+2\cdot\sqrt{a\cdot b}+b}+\frac{2\cdot\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\left(\sqrt{a}\right)^2+\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left[a-2\cdot\sqrt{a\cdot b}+b\right]}=$$

$$=\frac{a-b}{\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{2\cdot\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(a+\sqrt{a\cdot b}+b\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left(\sqrt{a}-\sqrt{b}\right)^2}=\frac{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{2\cdot\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(a+\sqrt{a\cdot b}+b\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left(\sqrt{a}-\sqrt{b}\right)^2}=$$

$$=\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}+\frac{2\cdot\left(a+\sqrt{a\cdot b}+b\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left(\sqrt{a}-\sqrt{b}\right)}=\left[\begin{array}{l}\text{zbrajamo razlomke svodeći}\\ \text{ih na zajednički nazivnik}\end{array}\right]=\frac{\left(\sqrt{a}-\sqrt{b}\right)^2+2\cdot\left(a+\sqrt{a\cdot b}+b\right)}{\left(\sqrt{a}+\sqrt{b}\right)\cdot\left(\sqrt{a}-\sqrt{b}\right)}=$$

$$=\frac{\left(\sqrt{a}\right)^2-2\cdot\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2+2\cdot a+2\cdot\sqrt{a\cdot b}+2\cdot b}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}=\frac{a-2\cdot\sqrt{a\cdot b}+b+2\cdot a+2\cdot\sqrt{a\cdot b}+2\cdot b}{a-b}=$$

$$=\frac{3\cdot a+3\cdot b}{a-b}=\frac{3\cdot(a+b)}{a-b}.$$

Vježba 168

Pojednostavnite izraz:

$$\frac{a-b}{a+b+\sqrt{(a+b)^2-(a-b)^2}}.$$

Rezultat: $\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}+\sqrt{b}}.$

Zadatak 169 (Nena, gimnazija)

Zapiši u obliku potencije s bazom 6: $3^{2\cdot n-1}\cdot 4^{n+1}+9^{n+1}\cdot 2^{2\cdot n-1}+6^{2\cdot n-1}.$

Rješenje 169

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^n\cdot b^n=(a\cdot b)^n\quad,\quad a^{-n}=\frac{1}{a^n}.$$

1.inačica

$$3^{2\cdot n-1}\cdot 4^{n+1}+9^{n+1}\cdot 2^{2\cdot n-1}+6^{2\cdot n-1}=3^{2\cdot n-1}\cdot\left(2^2\right)^{n+1}+\left(3^2\right)^{n+1}\cdot 2^{2\cdot n-1}+6^{2\cdot n-1}=$$

$$=3^{2\cdot n-1}\cdot 2^{2\cdot n+2}+3^{2\cdot n+2}\cdot 2^{2\cdot n-1}+6^{2\cdot n-1}=3^{2\cdot n-1}\cdot 2^{2\cdot n-1}\cdot 2^3+3^{2\cdot n-1}\cdot 3^3\cdot 2^{2\cdot n-1}+6^{2\cdot n-1}=$$

$$=(3\cdot 2)^{2\cdot n-1}\cdot 2^3+(3\cdot 2)^{2\cdot n-1}\cdot 3^3+6^{2\cdot n-1}=6^{2\cdot n-1}\cdot 2^3+6^{2\cdot n-1}\cdot 3^3+6^{2\cdot n-1}=$$

$$=6^{2\cdot n-1}\cdot 8+6^{2\cdot n-1}\cdot 27+6^{2\cdot n-1}=6^{2\cdot n-1}\cdot(8+27+1)=6^{2\cdot n-1}\cdot 36=6^{2\cdot n-1}\cdot 6^2=6^{2\cdot n+1}.$$

2.inačica

$$3^{2\cdot n-1}\cdot 4^{n+1}+9^{n+1}\cdot 2^{2\cdot n-1}+6^{2\cdot n-1}=3^{2\cdot n}\cdot 3^{-1}\cdot 4^n\cdot 4+9^n\cdot 9\cdot 2^{2\cdot n}\cdot 2^{-1}+6^{2\cdot n}\cdot 6^{-1}=$$

$$=\left(3^2\right)^n\cdot\frac{1}{3}\cdot 4^n\cdot 4+9^n\cdot 9\cdot\left(2^2\right)^n\cdot\frac{1}{2}+\left(6^2\right)^n\cdot\frac{1}{6}=9^n\cdot 4^n\cdot\frac{4}{3}+9^n\cdot 4^n\cdot\frac{9}{2}+36^n\cdot\frac{1}{6}=$$

$$=(9\cdot 4)^n\cdot\frac{4}{3}+(9\cdot 4)^n\cdot\frac{9}{2}+36^n\cdot\frac{1}{6}=36^n\cdot\frac{4}{3}+36^n\cdot\frac{9}{2}+36^n\cdot\frac{1}{6}=36^n\cdot\left(\frac{4}{3}+\frac{9}{2}+\frac{1}{6}\right)=$$

$$=36^n\cdot\frac{8+27+1}{6}=36^n\cdot\frac{36}{6}=36^n\cdot 6=\left(6^2\right)^n\cdot 6=6^{2\cdot n}\cdot 6=6^{2\cdot n+1}.$$

Vježba 169

Prikaži u obliku potencije s bazom 10 sljedeći brojevni izraz: $2^6\cdot 5^4+6\cdot 10^4$.

Rezultat: 10^5.

Zadatak 170 (Ines, gimnazija)

Skratite: $\dfrac{a^{n+2}-2\cdot a^n+a^{n-2}}{a^{n+2}-a^{n+1}+a^{n-1}-a^{n-2}}$.

Rješenje 170

Ponovimo!

$$x^n\cdot x^m=x^{n+m}\quad,\quad(x-y)^2=x^2-2\cdot x\cdot y+y^2\quad,\quad x^2-y^2=(x-y)\cdot(x+y).$$

$$x^3+y^3=(x+y)\cdot\left(x^2-x\cdot y+y^2\right)\quad,\quad x^{-n}=\frac{1}{x^n}.$$

1.inačica

$$\frac{a^{n+2}-2\cdot a^n+a^{n-2}}{a^{n+2}-a^{n+1}+a^{n-1}-a^{n-2}}=\left[\begin{array}{l}\text{u brojniku i nazivniku}\\ \text{izlučimo } a^{n-2}\end{array}\right]=\frac{a^{n-2}\cdot\left(a^4-2\cdot a^2+1\right)}{a^{n-2}\cdot\left(a^4-a^3+a-1\right)}=$$

$$=\frac{a^{n-2}\cdot\left(a^4-2\cdot a^2+1\right)}{a^{n-2}\cdot\left(a^4-a^3+a-1\right)}=\frac{a^4-2\cdot a^2+1}{a^4-a^3+a-1}=\frac{\left(a^2-1\right)^2}{a^3\cdot(a-1)+(a-1)}=\frac{\left(a^2-1\right)^2}{(a-1)\cdot\left(a^3+1\right)}=$$

$$=\frac{\left(a^2-1\right)\cdot\left(a^2-1\right)}{(a-1)\cdot(a+1)\cdot\left(a^2-a+1\right)}=\frac{\left(a^2-1\right)\cdot\left(a^2-1\right)}{\left(a^2-1\right)\cdot\left(a^2-a+1\right)}=\frac{\left(a^2-1\right)\cdot\left(a^2-1\right)}{\left(a^2-1\right)\cdot\left(a^2-a+1\right)}=\frac{a^2-1}{a^2-a+1}.$$

2.inačica

$$\frac{a^{n+2}-2\cdot a^n+a^{n-2}}{a^{n+2}-a^{n+1}+a^{n-1}-a^{n-2}}=\frac{a^n\cdot a^2-2\cdot a^n+a^n\cdot a^{-2}}{a^n\cdot a^2-a^n\cdot a^1+a^n\cdot a^{-1}-a^n\cdot a^{-2}}=\left[\begin{array}{l}\text{u brojniki i nazivniku}\\ \text{izlučimo } a^n\end{array}\right]=$$

$$=\frac{a^{n}\cdot\left(a^{2}-2+a^{-2}\right)}{a^{n}\cdot\left(a^{2}-a+a^{-1}-a^{-2}\right)}=\frac{a^{n}\cdot\left(a^{2}-2+a^{-2}\right)}{a^{n}\cdot\left(a^{2}-a+a^{-1}-a^{-2}\right)}=\frac{a^{2}-2+a^{-2}}{a^{2}-a+a^{-1}-a^{-2}}=\frac{a^{2}-2+\frac{1}{a^{2}}}{a^{2}-a+\frac{1}{a}-\frac{1}{a^{2}}}=$$

$$=\frac{\frac{a^{4}-2\cdot a^{2}+1}{a^{2}}}{\frac{a^{4}-a^{3}+a-1}{a^{2}}}=\frac{\frac{a^{4}-2\cdot a^{2}+1}{a^{2}}}{\frac{a^{4}-a^{3}+a-1}{a^{2}}}=\frac{a^{4}-2\cdot a^{2}+1}{a^{4}-a^{3}+a-1}=\frac{\left(a^{2}-1\right)^{2}}{a^{3}\cdot(a-1)+(a-1)}=\frac{\left(a^{2}-1\right)^{2}}{(a-1)\cdot\left(a^{3}+1\right)}=$$

$$=\frac{\left(a^{2}-1\right)\cdot\left(a^{2}-1\right)}{(a-1)\cdot(a+1)\cdot\left(a^{2}-a+1\right)}=\frac{\left(a^{2}-1\right)\cdot\left(a^{2}-1\right)}{\left(a^{2}-1\right)\cdot\left(a^{2}-a+1\right)}=\frac{\left(a^{2}-1\right)\cdot\left(a^{2}-1\right)}{\left(a^{2}-1\right)\cdot\left(a^{2}-a+1\right)}=\frac{a^{2}-1}{a^{2}-a+1}.$$

Vježba 170

Skratite: $\frac{a^{n+2}-a^{n+1}+a^{n-1}-a^{n-2}}{a^{n+2}-2\cdot a^{n}+a^{n-2}}.$

Rezultat: $\frac{a^{2}-a+1}{a^{2}-1}.$

Zadatak 171 (Goran, gimnazija)

Pojednostavnite: $\frac{\sqrt{5-2\cdot\sqrt{6}}}{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt[4]{3}-\sqrt[4]{2}\right)}.$

Rješenje 171

Ponovimo!

$(a+b)\cdot(a-b)=a^{2}-b^{2}$, $\left(\sqrt{a}\right)^{2}=a\ ,\ a\geq 0$, $\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}$, $(a+b)^{2}=a^{2}+2\cdot a\cdot b+b^{2}.$

$a\cdot\sqrt{b}=\sqrt{a^{2}\cdot b}$, $\frac{n}{1}=n$, $\sqrt{A\pm\sqrt{B}}=\sqrt{\frac{A+\sqrt{A^{2}-B}}{2}}\pm\sqrt{\frac{A-\sqrt{A^{2}-B}}{2}}.$

1.inačica

$$\frac{\sqrt{5-2\cdot\sqrt{6}}}{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt[4]{3}-\sqrt[4]{2}\right)}=\frac{\sqrt{5-2\cdot\sqrt{6}}}{\left(\sqrt[4]{3}\right)^{2}-\left(\sqrt[4]{2}\right)^{2}}=\frac{\sqrt{5-2\cdot\sqrt{6}}}{\sqrt{3}-\sqrt{2}}=\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]=\frac{\sqrt{5-2\cdot\sqrt{6}}}{\sqrt{3}-\sqrt{2}}\cdot\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}=$$

$$=\frac{\sqrt{5-2\cdot\sqrt{6}}\cdot\left(\sqrt{3}+\sqrt{2}\right)}{\left(\sqrt{3}\right)^{2}-\left(\sqrt{2}\right)^{2}}=\frac{\sqrt{\left(5-2\cdot\sqrt{6}\right)\cdot\left(\sqrt{3}+\sqrt{2}\right)^{2}}}{3-2}=\frac{\sqrt{\left(5-2\cdot\sqrt{6}\right)\cdot\left(\left(\sqrt{3}\right)^{2}+2\cdot\sqrt{3}\cdot\sqrt{2}+\left(\sqrt{2}\right)^{2}\right)}}{1}=$$

$$=\sqrt{\left(5-2\cdot\sqrt{6}\right)\cdot\left(3+2\cdot\sqrt{6}+2\right)}=\sqrt{\left(5-2\cdot\sqrt{6}\right)\cdot\left(5+2\cdot\sqrt{6}\right)}=\sqrt{5^{2}-\left(2\cdot\sqrt{6}\right)^{2}}=\sqrt{25-24}=\sqrt{1}=1.$$

2.inačica

$$\frac{\sqrt{5-2\cdot\sqrt{6}}}{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt[4]{3}-\sqrt[4]{2}\right)}=\frac{\sqrt{5-\sqrt{2^{2}\cdot 6}}}{\left(\sqrt[4]{3}\right)^{2}-\left(\sqrt[4]{2}\right)^{2}}=\frac{\sqrt{5-\sqrt{24}}}{\sqrt{3}-\sqrt{2}}=$$

$$=\frac{\sqrt{\frac{5+\sqrt{5^2-24}}{2}}-\sqrt{\frac{5-\sqrt{5^2-24}}{2}}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{\frac{5+\sqrt{25-24}}{2}}-\sqrt{\frac{5-\sqrt{25-24}}{2}}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{\frac{5+\sqrt{1}}{2}}-\sqrt{\frac{5-\sqrt{1}}{2}}}{\sqrt{3}-\sqrt{2}}=$$

$$=\frac{\sqrt{\frac{5+1}{2}}-\sqrt{\frac{5-1}{2}}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{\frac{6}{2}}-\sqrt{\frac{4}{2}}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}=1.$$

Vježba 171

Pojednostavnite: $\dfrac{\left(\sqrt[4]{3}+\sqrt[4]{2}\right)\cdot\left(\sqrt[4]{3}-\sqrt[4]{2}\right)}{\sqrt{5-2\cdot\sqrt{6}}}.$

Rezultat: 1.

Zadatak 172 (Ema, gimnazija)

Skrati razlomak: $\dfrac{a^3\cdot c-2\cdot a^2\cdot c^2+a\cdot c^3-a\cdot b^2\cdot c}{\left(a^2+c^2-b^2\right)^2-4\cdot a^2\cdot c^2}.$

Rješenje 172

Ponovimo!

$$(x-y)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad (x+y)^2=x^2+2\cdot x\cdot y+y^2 \quad , \quad x^2-y^2=(x-y)\cdot(x+y).$$

$$\frac{a^3\cdot c-2\cdot a^2\cdot c^2+a\cdot c^3-a\cdot b^2\cdot c}{\left(a^2+c^2-b^2\right)^2-4\cdot a^2\cdot c^2}=\left[\begin{array}{l}\text{u brojniku izlučimo a}\cdot\text{c,}\\ \text{u nazivniku je razlika kvadrata}\end{array}\right]=$$

$$=\frac{a\cdot c\cdot\left(a^2-2\cdot a\cdot c+c^2-b^2\right)}{\left(a^2+c^2-b^2-2\cdot a\cdot c\right)\cdot\left(a^2+c^2-b^2+2\cdot a\cdot c\right)}=\frac{a\cdot c\cdot\left(a^2-2\cdot a\cdot c+c^2-b^2\right)}{\left(a^2-2\cdot a\cdot c+c^2-b^2\right)\cdot\left(a^2+2\cdot a\cdot c+c^2-b^2\right)}=$$

$$=\frac{a\cdot c\cdot\left(\left(a^2-2\cdot a\cdot c+c^2\right)-b^2\right)}{\left(\left(a^2-2\cdot a\cdot c+c^2\right)-b^2\right)\cdot\left(\left(a^2+2\cdot a\cdot c+c^2\right)-b^2\right)}=\frac{a\cdot c\cdot\left((a-c)^2-b^2\right)}{\left((a-c)^2-b^2\right)\cdot\left((a+c)^2-b^2\right)}=$$

$$=\frac{a\cdot c\cdot(a-c-b)\cdot(a-c+b)}{(a-c-b)\cdot(a-c+b)\cdot(a+c-b)\cdot(a+c+b)}=\frac{a\cdot c\cdot(a-c-b)\cdot(a-c+b)}{(a-c-b)\cdot(a-c+b)\cdot(a+c-b)\cdot(a+c+b)}=$$

$$=\frac{a\cdot c}{(a+c-b)\cdot(a+c+b)}=\frac{a\cdot c}{(a+c)^2-b^2}.$$

Vježba 172

Skrati razlomak: $\dfrac{4\cdot a^2\cdot c^2-\left(a^2+c^2-b^2\right)^2}{a\cdot b^2\cdot c-a\cdot c^3+2\cdot a^2\cdot c^2-a^3\cdot c}.$

Rezultat: $\dfrac{(a+c)^2-b^2}{a\cdot c}.$

Zadatak 173 (Miki, gimnazija)

Racionaliziraj nazivnik: $\frac{1}{\sqrt{10}+\sqrt{15}+\sqrt{14}+\sqrt{21}}$.

Rješenje 173

Ponovimo!

$$\sqrt{a\cdot b}=\sqrt{a}\cdot\sqrt{b} \quad , \quad (a+b)\cdot(a-b)=a^2-b^2 \quad , \quad \left(\sqrt{a}\right)^2=a\ ,\ a\ge 0.$$

$$\frac{1}{\sqrt{10}+\sqrt{15}+\sqrt{14}+\sqrt{21}}=\frac{1}{\sqrt{2\cdot 5}+\sqrt{3\cdot 5}+\sqrt{2\cdot 7}+\sqrt{3\cdot 7}}=\frac{1}{\sqrt{2}\cdot\sqrt{5}+\sqrt{3}\cdot\sqrt{5}+\sqrt{2}\cdot\sqrt{7}+\sqrt{3}\cdot\sqrt{7}}=$$

$$=\frac{1}{\sqrt{5}\cdot\left(\sqrt{2}+\sqrt{3}\right)+\sqrt{7}\cdot\left(\sqrt{2}+\sqrt{3}\right)}=\frac{1}{\left(\sqrt{2}+\sqrt{3}\right)\cdot\left(\sqrt{5}+\sqrt{7}\right)}=\frac{1}{\left(\sqrt{3}+\sqrt{2}\right)\cdot\left(\sqrt{7}+\sqrt{5}\right)}=$$

$$=\begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix}=\frac{1}{\left(\sqrt{3}+\sqrt{2}\right)\cdot\left(\sqrt{7}+\sqrt{5}\right)}\cdot\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}=$$

$$=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{\left(\sqrt{3}+\sqrt{2}\right)\cdot\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}+\sqrt{5}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{\left(\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2\right)\cdot\left(\left(\sqrt{7}\right)^2-\left(\sqrt{5}\right)^2\right)}=$$

$$=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{(3-2)\cdot(7-5)}=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{1\cdot 2}=\frac{\left(\sqrt{3}-\sqrt{2}\right)\cdot\left(\sqrt{7}-\sqrt{5}\right)}{2}.$$

Vježba 173

Racionaliziraj nazivnik: $\frac{1}{\sqrt{7}-\sqrt{6}}$.

Rezultat: $\sqrt{7}+\sqrt{6}.$

Zadatak 174 (Sučo, elektrostrojarska škola)

Izračunaj: $3\cdot(2\cdot a-3\cdot b)^2-5\cdot(a-3\cdot b)\cdot(a+3\cdot b)$.

Rješenje 174

Ponovimo!

$$(x-y)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad (x-y)\cdot(x+y)=x^2-y^2.$$

$$3\cdot(2\cdot a-3\cdot b)^2-5\cdot(a-3\cdot b)\cdot(a+3\cdot b)=\begin{bmatrix}\text{prvu zagradu kvadriramo,}\\ \text{druge dvije zagrade čine razliku kvadrata}\end{bmatrix}=$$

$$=3\cdot\left((2\cdot a)^2-2\cdot 2\cdot a\cdot 3\cdot b+(3\cdot b)^2\right)-5\cdot\left(a^2-(3\cdot b)^2\right)=3\cdot\left(4\cdot a^2-12\cdot a\cdot b+9\cdot b^2\right)-5\cdot\left(a^2-9\cdot b^2\right)=$$

$$=12\cdot a^2-36\cdot a\cdot b+27\cdot b^2-5\cdot a^2+45\cdot b^2=7\cdot a^2-36\cdot a\cdot b+72\cdot b^2.$$

Vježba 174

Izračunaj: $3\cdot(2\cdot a-3\cdot b)^2+5\cdot(a-3\cdot b)\cdot(a+3\cdot b)$.

Rezultat: $17\cdot a^2-36\cdot a\cdot b-18\cdot b^2.$

Zadatak 175 (Sučo, elektrostrojarska škola)

Izračunaj: $2\cdot(a-3\cdot b)^3-(2\cdot a+3\cdot b)\cdot\left(4\cdot a^2-6\cdot a\cdot b+9\cdot b^2\right)$.

Rješenje 175

Ponovimo!

$$(x-y)^3=x^3-3\cdot x^2\cdot y+3\cdot x\cdot y^2-y^3 \quad , \quad (x+y)\cdot\left(x^2-x\cdot y+y^2\right)=x^3-y^3.$$

$$2\cdot(a-3\cdot b)^3-(2\cdot a+3\cdot b)\cdot\left(4\cdot a^2-6\cdot a\cdot b+9\cdot b^2\right)=\left[\begin{array}{l}\text{prvu zagradu kubiramo,}\\ \text{druge dvije zagrade čine zbroj kubova}\end{array}\right]=$$

$$=2\cdot\left(a^3-3\cdot a^2\cdot 3\cdot b+3\cdot a\cdot(3\cdot b)^2-(3\cdot b)^3\right)-\left((2\cdot a)^3+(3\cdot b)^3\right)=$$

$$=2\cdot\left(a^3-9\cdot a^2\cdot b+3\cdot a\cdot 9\cdot b^2-27\cdot b^3\right)-\left(8\cdot a^3+27\cdot b^3\right)=$$

$$=2\cdot\left(a^3-9\cdot a^2\cdot b+27\cdot a\cdot b^2-27\cdot b^3\right)-\left(8\cdot a^3+27\cdot b^3\right)=$$

$$=2\cdot a^3-18\cdot a^2\cdot b+54\cdot a\cdot b^2-54\cdot b^3-8\cdot a^3-27\cdot b^3=-6\cdot a^3-18\cdot a^2\cdot b+54\cdot a\cdot b^2-81\cdot b^3=$$

$$=-3\cdot\left(2\cdot a^3+6\cdot a^2\cdot b-18\cdot a\cdot b^2+27\cdot b^3\right).$$

Vježba 175

Izračunaj: $2\cdot(a-3\cdot b)^3+(2\cdot a+3\cdot b)\cdot\left(4\cdot a^2-6\cdot a\cdot b+9\cdot b^2\right).$

Rezultat: $10\cdot a^3-18\cdot a^2\cdot b+54\cdot a\cdot b^2-27\cdot b^3.$

Zadatak 176 (Sučo, elektrostrojarska škola)

Rastavi na faktore: $8\cdot a^2-50\cdot b^2.$

Rješenje 176

Ponovimo!

$$(x+y)\cdot(x-y)=x^2-y^2.$$

$$8\cdot a^2-50\cdot b^2=2\cdot\left(4\cdot a^2-25\cdot b^2\right)=2\cdot(2\cdot a-5\cdot b)\cdot(2\cdot a+5\cdot b).$$

Vježba 176

Rastavi na faktore: $12\cdot a^2-75\cdot b^2.$

Rezultat: $3\cdot(2\cdot a-5\cdot b)\cdot(2\cdot a+5\cdot b).$

Zadatak 177 (Sučo, elektrostrojarska škola)

Skrati razlomak: $\dfrac{x\cdot y-y-x+1}{-1+2\cdot x-x^2}.$

Rješenje 177

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$\frac{x\cdot y-y-x+1}{-1+2\cdot x-x^2}=\frac{x\cdot y-y-x+1}{-x^2+2\cdot x-1}=\frac{x\cdot y-y-x+1}{-\left(x^2-2\cdot x+1\right)}=\frac{x\cdot y-y-x+1}{-(x-1)^2}=\frac{y\cdot(x-1)-(x-1)}{-(x-1)^2}=$$

$$=\frac{(x-1)\cdot(y-1)}{-(x-1)^2}=\frac{(x-1)\cdot(y-1)}{-(x-1)^2}=\frac{y-1}{-(x-1)}=\frac{y-1}{-x+1}=\frac{y-1}{1-x}.$$

Vježba 177

Skrati razlomak: $\dfrac{-1+2\cdot x-x^2}{x\cdot y-y-x+1}.$

Rezultat: $\dfrac{1-x}{y-1}.$

Zadatak 178 (Sučo, elektrostrojarska škola)

$$\text{Izračunaj}: \frac{\frac{a-b}{a+b}+\frac{a+b}{a-b}}{\frac{a+b}{a-b}-\frac{a-b}{a+b}}.$$

Rješenje 178

Ponovimo!

$$(x-y)^2=x^2-2\cdot x\cdot y+y^2 \quad , \quad (x+y)^2=x^2+2\cdot x\cdot y+y^2.$$

$$\frac{\frac{a-b}{a+b}+\frac{a+b}{a-b}}{\frac{a+b}{a-b}-\frac{a-b}{a+b}}=\frac{\frac{(a-b)\cdot(a-b)+(a+b)\cdot(a+b)}{(a+b)\cdot(a-b)}}{\frac{(a+b)\cdot(a+b)-(a-b)\cdot(a-b)}{(a-b)\cdot(a+b)}}=\frac{\frac{(a-b)^2+(a+b)^2}{(a+b)\cdot(a-b)}}{\frac{(a+b)^2-(a-b)^2}{(a-b)\cdot(a+b)}}=\frac{\frac{(a-b)^2+(a+b)^2}{(a+b)\cdot(a-b)}}{\frac{(a+b)^2-(a-b)^2}{(a-b)\cdot(a+b)}}=$$

$$=\frac{(a-b)^2+(a+b)^2}{(a+b)^2-(a-b)^2}=\frac{a^2-2\cdot a\cdot b+b^2+a^2+2\cdot a\cdot b+b^2}{a^2+2\cdot a\cdot b+b^2-\left(a^2-2\cdot a\cdot b+b^2\right)}=\frac{a^2-2\cdot a\cdot b+b^2+a^2+2\cdot a\cdot b+b^2}{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}=$$

$$=\frac{a^2-2\cdot a\cdot b+b^2+a^2+2\cdot a\cdot b+b^2}{a^2+2\cdot a\cdot b+b^2-a^2+2\cdot a\cdot b-b^2}=\frac{2\cdot a^2+2\cdot b^2}{4\cdot a\cdot b}=\frac{2\cdot\left(a^2+b^2\right)}{4\cdot a\cdot b}=\frac{2\cdot\left(a^2+b^2\right)}{4\cdot a\cdot b}=\frac{a^2+b^2}{2\cdot a\cdot b}.$$

Vježba 178

$$\text{Izračunaj}: \frac{\frac{a+b}{a-b}-\frac{a-b}{a+b}}{\frac{a-b}{a+b}+\frac{a+b}{a-b}}.$$

Rezultat: $\frac{2\cdot a\cdot b}{a^2+b^2}.$

Zadatak 179 (Robertina kumica ☺, ekonomska škola)

$$\text{Izračunaj}: \sqrt[3]{2\cdot\sqrt{2}}.$$

Rješenje 179

Ponovimo!

$$\sqrt[n]{a^m}=a^{\frac{m}{n}} \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^1=a \quad , \quad a\cdot\sqrt{b}=\sqrt{a^2\cdot b}.$$

$$\sqrt[n]{\sqrt[m]{a}}=\sqrt[n\cdot m]{a} \quad , \quad \sqrt[n]{a^m}=\sqrt[n\cdot p]{a^{m\cdot p}}.$$

1.inačica

Redom pretvaramo korijene u racionalni eksponent, te primjenom svojstava potencije pojednostavimo rezultat:

$$\sqrt[3]{2\cdot\sqrt{2}}=\left(2\cdot\sqrt{2}\right)^{\frac{1}{3}}=\left(2\cdot 2^{\frac{1}{2}}\right)^{\frac{1}{3}}=\left(2^1\cdot 2^{\frac{1}{2}}\right)^{\frac{1}{3}}=\left(2^{1+\frac{1}{2}}\right)^{\frac{1}{3}}=\left(2^{\frac{3}{2}}\right)^{\frac{1}{3}}=\left(2^{\frac{3}{2}}\right)^{\frac{1}{3}}=2^{\frac{1}{2}}=\sqrt{2}.$$

2.inačica

Korijene prvo svedemo na jedan korijen:

$$\sqrt[3]{2\cdot\sqrt{2}}=\sqrt[3]{\sqrt{2^2\cdot 2}}=\sqrt[3]{\sqrt{2^2\cdot 2^1}}=\sqrt[3]{\sqrt{2^3}}=\sqrt[6]{2^3}=\sqrt[6]{2^3}=\sqrt{2}.$$

Vježba 179

Izračunaj: $\sqrt[4]{7\cdot\sqrt{7}}$.

Rezultat: $\sqrt[8]{7^3}$.

Zadatak 180 (Robertina kumica ☺, ekonomska škola)

Izračunaj: $\sqrt[3]{\sqrt[3]{a^{-4}}\cdot\sqrt{a}}$.

Rješenje 180

Ponovimo!

$$\sqrt[n]{a^m}=a^{\frac{m}{n}} \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^1=a \quad , \quad a\cdot\sqrt{b}=\sqrt{a^2\cdot b}.$$

$$\sqrt[n]{\sqrt[m]{a}}=\sqrt[n\cdot m]{a} \quad , \quad \sqrt[n]{a^m}=\sqrt[n\cdot p]{a^{m\cdot p}}.$$

1.inačica

Redom pretvaramo korijene u racionalni eksponent, te primjenom svojstava potencije pojednostavimo rezultat:

$$\sqrt[3]{\sqrt[3]{a^{-4}}\cdot\sqrt{a}}=\left(\sqrt[3]{a^{-4}}\cdot\sqrt{a}\right)^{\frac{1}{3}}=\left(\left(a^{-4}\cdot\sqrt{a}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}=\left(\left(a^{-4}\cdot a^{\frac{1}{2}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}=\left(\left(a^{-4+\frac{1}{2}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}=$$

$$=\left(\left(a^{-\frac{7}{2}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}=\left(a^{-\frac{7}{6}}\right)^{\frac{1}{3}}=a^{-\frac{7}{18}}=\sqrt[18]{a^{-7}}.$$

2.inačica

Korijene prvo svedemo na jedan korijen:

$$\sqrt[3]{\sqrt[3]{a^{-4}}\cdot\sqrt{a}}=\sqrt[3]{\sqrt[3]{\sqrt{\left(a^{-4}\right)^2}\cdot a}}=\sqrt[3]{\sqrt[3]{\sqrt{a^{-8}\cdot a}}}=\sqrt[3]{\sqrt[3]{\sqrt{a^{-8}\cdot a^1}}}=\sqrt[3]{\sqrt[3]{\sqrt{a^{-7}}}}=$$

$$=\sqrt[9]{\sqrt{a^{-7}}}=\sqrt[18]{a^{-7}}.$$

Vježba 180

Izračunaj: $\sqrt[5]{\sqrt[3]{a^{-4}}\cdot\sqrt{a}}$.

Rezultat: $\sqrt[30]{a^{-7}}$.