

**Zadatak 141 (Kranjc, pomorska škola)**

Pojednostavnite:  $\left(\frac{1}{a} - \frac{1}{b}\right)^{-1} : \left(\frac{1}{a^{-1}} - \frac{1}{b^{-1}}\right)^{-1}$ .

**Rješenje 141**

Ponovimo!

$$a^{-n} = \frac{1}{a^n}, \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n.$$

$$\left(\frac{1}{a} - \frac{1}{b}\right)^{-1} : \left(\frac{1}{a^{-1}} - \frac{1}{b^{-1}}\right)^{-1} = \left(\frac{b-a}{a \cdot b}\right)^{-1} : (a-b)^{-1} = \frac{a \cdot b}{b-a} : \frac{1}{a-b} = \frac{a \cdot b}{b-a} \cdot \frac{a-b}{1} = \frac{a \cdot b}{b-a} \cdot \frac{-(b-a)}{1} = -a \cdot b.$$

**Vježba 141**

Pojednostavnite:  $\left(\frac{1}{a} - \frac{1}{b}\right)^{-1} \cdot \left(\frac{1}{a^{-1}} - \frac{1}{b^{-1}}\right)$ .

**Rezultat:**  $-a \cdot b$ .**Zadatak 142 (Los-Habanos, gimnazija)**

Pojednostavnite izraz:  $\frac{(b^{-2} - a^{-2})^{-1}}{(a^{-2} + b^{-2}) \cdot (a \cdot b^{-3} - a^{-3} \cdot b)^{-1}}$ .

**Rješenje 142**

Ponovimo!

$$\text{Razlika kvadrata: } (x+y) \cdot (x-y) = x^2 - y^2, \quad a^{-n} = \frac{1}{a^n}, \quad \frac{1}{a^{-n}} = a^n.$$

1. inačica

$$\begin{aligned} & \frac{(b^{-2} - a^{-2})^{-1}}{(a^{-2} + b^{-2}) \cdot (a \cdot b^{-3} - a^{-3} \cdot b)^{-1}} = \frac{\frac{a}{b^3} - \frac{b}{a^3}}{(a^{-2} + b^{-2}) \cdot (b^{-2} - a^{-2})} = \frac{\frac{a^4 - b^4}{a^3 \cdot b^3}}{\left(\frac{1}{a^2} + \frac{1}{b^2}\right) \cdot \left(\frac{1}{b^2} - \frac{1}{a^2}\right)} \\ & = \frac{\frac{a^4 - b^4}{a^3 \cdot b^3}}{\frac{b^2 + a^2}{a^2 \cdot b^2} \cdot \frac{a^2 - b^2}{a^2 \cdot b^2}} = \frac{\frac{a^4 - b^4}{a^3 \cdot b^3}}{\frac{(b^2 + a^2) \cdot (a^2 - b^2)}{a^4 \cdot b^4}} = \frac{\frac{a^4 - b^4}{a^3 \cdot b^3}}{\frac{(a^2 + b^2) \cdot (a^2 - b^2)}{a^4 \cdot b^4}} = \frac{\frac{a^4 - b^4}{a^3 \cdot b^3}}{\frac{a^4 - b^4}{a^4 \cdot b^4}} = \frac{a^4 \cdot b^4}{a^3 \cdot b^3} = a \cdot b. \end{aligned}$$

2. inačica

$$\begin{aligned} & \frac{(b^{-2} - a^{-2})^{-1}}{(a^{-2} + b^{-2}) \cdot (a \cdot b^{-3} - a^{-3} \cdot b)^{-1}} = \frac{a \cdot b^{-3} - a^{-3} \cdot b}{(a^{-2} + b^{-2}) \cdot (b^{-2} - a^{-2})} = \frac{a \cdot b \cdot (b^{-4} - a^{-4})}{(b^{-2} + a^{-2}) \cdot (b^{-2} - a^{-2})} \\ & = \frac{a \cdot b \cdot (b^{-4} - a^{-4})}{b^{-4} - a^{-4}} = a \cdot b. \end{aligned}$$

**Vježba 142**

Pojednostavnite izraz:  $\frac{(a \cdot b^{-3} - a^{-3} \cdot b) \cdot (a^{-2} + b^{-2})^{-1}}{b^{-2} - a^{-2}}$ .

**Rezultat:**  $a \cdot b$ .

**Zadatak 143 (Los-Habanos, gimnazija)**

Pojednostavnite izraz: 
$$\frac{\frac{1}{x} + \frac{1}{y} - \frac{a}{x \cdot y}}{\frac{1}{x^2} + \frac{1}{y^2} + \frac{2}{x \cdot y} - \frac{a^2}{x^2 \cdot y^2}} \cdot (a + x + y).$$

**Rješenje 143**

Ponovimo!

Kvadrat zbroja:  $(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2$  , Razlika kvadrata:  $(x+y) \cdot (x-y) = x^2 - y^2$ .

$$\begin{aligned} \frac{\frac{1}{x} + \frac{1}{y} - \frac{a}{x \cdot y}}{\frac{1}{x^2} + \frac{1}{y^2} + \frac{2}{x \cdot y} - \frac{a^2}{x^2 \cdot y^2}} \cdot (a + x + y) &= \frac{\frac{y+x-a}{x \cdot y}}{\frac{y^2 + x^2 + 2 \cdot x \cdot y - a^2}{x^2 \cdot y^2}} \cdot (a + x + y) = \\ &= \frac{y+x-a}{\frac{1}{y^2 + x^2 + 2 \cdot x \cdot y - a^2}} \cdot (a + x + y) = \frac{y+x-a}{\frac{1}{(y+x)^2 - a^2}} \cdot (a + x + y) = \frac{x \cdot y \cdot (y+x-a)}{(y+x)^2 - a^2} \cdot (a + x + y) = \\ &= \frac{x \cdot y \cdot (y+x-a)}{x \cdot y} \cdot (a + x + y) = \frac{y+x-a}{(y+x-a) \cdot (y+x+a)} \cdot (a + x + y) = x \cdot y. \end{aligned}$$

**Vježba 143**

Pojednostavnite izraz: 
$$(a + x + y) : \frac{\frac{1}{x^2} + \frac{1}{y^2} + \frac{2}{x \cdot y} - \frac{a^2}{x^2 \cdot y^2}}{\frac{1}{x} + \frac{1}{y} - \frac{a}{x \cdot y}}.$$

**Rezultat:**  $x \cdot y.$ **Zadatak 144 (Sanela, maturantica gimnazije)**

Reducirajte izraz:  $(x^3 - 1) \cdot (x+1)^{-1} \cdot (x^3 + 1) \cdot (x^4 + x^2 + 1)^{-1}.$

**Rješenje 144**

Ponovimo!

$$a^{-n} = \frac{1}{a^n} \quad , \quad a^3 - b^3 = (a-b) \cdot (a^2 + a \cdot b + b^2) \quad , \quad a^3 + b^3 = (a+b) \cdot (a^2 - a \cdot b + b^2)$$

$$a^2 - b^2 = (a-b) \cdot (a+b) \quad , \quad a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2.$$

Najprije rastavimo trinom  $x^4 + x^2 + 1$  na faktore:

$$\begin{aligned} x^4 + x^2 + 1 &= x^4 + 2 \cdot x^2 - x^2 + 1 = (x^4 + 2 \cdot x^2 + 1) - x^2 = (x^2 + 1)^2 - x^2 = \\ &= (x^2 + 1 - x) \cdot (x^2 + 1 + x) = (x^2 - x + 1) \cdot (x^2 + x + 1). \end{aligned}$$

Sada reduciramo algebarski izraz:

$$(x^3 - 1) \cdot (x+1)^{-1} \cdot (x^3 + 1) \cdot (x^4 + x^2 + 1)^{-1} = (x^3 - 1) \cdot \frac{1}{x+1} \cdot (x^3 + 1) \cdot \frac{1}{x^4 + x^2 + 1} =$$

$$\begin{aligned}
&= \frac{x^3-1}{1} \cdot \frac{1}{x+1} \cdot \frac{x^3+1}{1} \cdot \frac{1}{x^4+x^2+1} = \frac{(x^3-1) \cdot (x^3+1)}{(x+1) \cdot (x^4+x^2+1)} = \frac{(x-1) \cdot (x^2+x+1) \cdot (x+1) \cdot (x^2-x+1)}{(x+1) \cdot (x^2+x+1) \cdot (x^2-x+1)} = \\
&= \frac{(x-1) \cdot (x^2+x+1) \cdot (x+1) \cdot (x^2-x+1)}{(x+1) \cdot (x^2+x+1) \cdot (x^2-x+1)} = x-1.
\end{aligned}$$

### Vježba 144

Reducirajte izraz:  $(x^3-1) \cdot (x^2-1)^{-1} \cdot (x^3+1) \cdot (x^4+x^2+1)^{-1}$ .

**Rezultat:** 1.

### Zadatak 145 (Sanela, maturantica gimnazije)

Reducirajte izraz:  $\frac{x^3+y^3}{x+y} : (x^2-y^2) + \frac{2 \cdot y}{x+y} - \frac{x \cdot y}{x^2-y^2}$ .

### Rješenje 145

Ponovimo!

$$a^3+b^3=(a+b) \cdot (a^2-a \cdot b+b^2), \quad a^2-b^2=(a-b) \cdot (a+b), \quad \frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}, \quad \frac{n}{1} = n.$$

Pojednostavljujemo algebarski izraz:

$$\begin{aligned}
&\frac{x^3+y^3}{x+y} : (x^2-y^2) + \frac{2 \cdot y}{x+y} - \frac{x \cdot y}{x^2-y^2} = \frac{(x+y) \cdot (x^2-x \cdot y+y^2)}{x+y} \cdot \frac{1}{x^2-y^2} + \frac{2 \cdot y}{x+y} - \frac{x \cdot y}{(x-y) \cdot (x+y)} = \\
&= \frac{x^2-x \cdot y+y^2}{1} \cdot \frac{1}{(x-y) \cdot (x+y)} + \frac{2 \cdot y}{x+y} - \frac{x \cdot y}{(x-y) \cdot (x+y)} = \frac{x^2-x \cdot y+y^2}{(x-y) \cdot (x+y)} + \frac{2 \cdot y}{x+y} - \frac{x \cdot y}{(x-y) \cdot (x+y)} = \\
&= \frac{x^2-x \cdot y+y^2+2 \cdot y \cdot (x-y)-x \cdot y}{(x-y) \cdot (x+y)} = \frac{x^2-x \cdot y+y^2+2 \cdot x \cdot y-2 \cdot y^2-x \cdot y}{(x-y) \cdot (x+y)} = \frac{x^2-y^2}{x^2-y^2} = 1.
\end{aligned}$$

### Vježba 145

Reducirajte izraz:  $\frac{x^3+y^3}{x+y} : \frac{x^2-x \cdot y+y^2}{x}$ .

**Rezultat:** x.

### Zadatak 146 (Tajanstvena, ekonomska škola)

Pojednostavnite:  $\sqrt[3]{a^2 \cdot b} \cdot \sqrt[4]{a^3 \cdot b^5} \cdot \sqrt[6]{a^{-1} \cdot b^{-2}}$ .

### Rješenje 146

Ponovimo!

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}, \quad (a^n)^m = a^{n \cdot m}, \quad a^n \cdot a^m = a^{n+m}, \quad a^n \cdot b^n = (a \cdot b)^n.$$

Budući da je 12 najmanji zajednički višekratnik brojeva 3, 4 i 6 sve korijene svedemo na 12 – ti korijen:

$$\begin{aligned}
&\sqrt[3]{a^2 \cdot b} \cdot \sqrt[4]{a^3 \cdot b^5} \cdot \sqrt[6]{a^{-1} \cdot b^{-2}} = \sqrt[12]{(a^2 \cdot b)^4} \cdot \sqrt[12]{(a^3 \cdot b^5)^3} \cdot \sqrt[12]{(a^{-1} \cdot b^{-2})^2} = \\
&= \sqrt[12]{a^8 \cdot b^4} \cdot \sqrt[12]{a^9 \cdot b^{15}} \cdot \sqrt[12]{a^{-2} \cdot b^{-4}} = \sqrt[12]{a^8 \cdot b^4 \cdot a^9 \cdot b^{15} \cdot a^{-2} \cdot b^{-4}} = \sqrt[12]{a^{15} \cdot b^{15}} = \\
&= \sqrt[12]{(a^5 \cdot b^5)^3} = \sqrt[4]{(a^5 \cdot b^5)^3} = \sqrt[4]{a^5 \cdot b^5} = \sqrt[4]{(a \cdot b)^5} = \sqrt[4]{(a \cdot b)^4 \cdot (a \cdot b)} =
\end{aligned}$$

$$= \sqrt[4]{(a \cdot b)^4} \cdot \sqrt[4]{a \cdot b} = \sqrt[4]{(a \cdot b)^4} \cdot \sqrt[4]{a \cdot b} = a \cdot b \cdot \sqrt[4]{a \cdot b}.$$

### Vježba 146

Pojednostavnite:  $\sqrt[3]{a^2 \cdot b} \cdot \sqrt[6]{a^{-1} \cdot b^{-2}}$ .

**Rezultat:**  $\sqrt{a}$ .

### Zadatak 147 (Tajanstvena, ekonomska škola)

Izračunajte:  $\sqrt{125} - 5 \cdot \sqrt{45} + 27 \cdot \sqrt{20}$ .

#### Rješenje 147

Ponovimo!

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \quad , \quad \sqrt{a^2} = a, a \geq 0.$$

Svaki korijen djelomično korjenujemo:

- $\sqrt{125} = \sqrt{25 \cdot 5} = \sqrt{25} \cdot \sqrt{5} = \sqrt{5^2} \cdot \sqrt{5} = 5 \cdot \sqrt{5}$
- $\sqrt{45} = \sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = \sqrt{3^2} \cdot \sqrt{5} = 3 \cdot \sqrt{5}$
- $\sqrt{20} = \sqrt{4 \cdot 5} = \sqrt{4} \cdot \sqrt{5} = \sqrt{2^2} \cdot \sqrt{5} = 2 \cdot \sqrt{5}$ .

Sada je:

$$\begin{aligned} \sqrt{125} - 5 \cdot \sqrt{45} + 27 \cdot \sqrt{20} &= \sqrt{25 \cdot 5} - 5 \cdot \sqrt{9 \cdot 5} + 27 \cdot \sqrt{4 \cdot 5} = 5 \cdot \sqrt{5} - 5 \cdot 3 \cdot \sqrt{5} + 27 \cdot 2 \cdot \sqrt{5} = \\ &= 5 \cdot \sqrt{5} - 15 \cdot \sqrt{5} + 54 \cdot \sqrt{5} = 44 \cdot \sqrt{5}. \end{aligned}$$

### Vježba 147

Izračunajte:  $\sqrt{125} - 5 \cdot \sqrt{45}$ .

**Rezultat:**  $-10 \cdot \sqrt{5}$ .

### Zadatak 148 (Matea, Dajana, Anamarija, hotelijerska škola)

Pojednostavnite:  $4^{2.5} \cdot 64^{-\frac{7}{6}} \cdot \left(\frac{1}{8}\right)^{-\frac{2}{3}}$ .

#### Rješenje 148

Ponovimo!

$$(a^n)^m = a^{n \cdot m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^0 = 1.$$

$$4^{2.5} \cdot 64^{-\frac{7}{6}} \cdot \left(\frac{1}{8}\right)^{-\frac{2}{3}} = (2^2)^{2.5} \cdot (2^6)^{-\frac{7}{6}} \cdot \left(\frac{1}{2^3}\right)^{-\frac{2}{3}} = 2^5 \cdot 2^{-7} \cdot (2^{-3})^{-\frac{2}{3}} = 2^5 \cdot 2^{-7} \cdot 2^2 = 2^0 = 1.$$

### Vježba 148

Pojednostavnite:  $4^{2.5} \cdot 64^{\frac{7}{6}}$ .

**Rezultat:**  $2^{12}$ .

### Zadatak 149 (Ana, ekomomska škola)

Izračunajte:  $\left(2 \cdot a - \frac{4 \cdot a - 1}{2 \cdot a}\right) \cdot \frac{4 \cdot a^2}{1 - 4 \cdot a^2} : \left(\frac{1}{a} - \frac{4}{1 + 2 \cdot a}\right)$ .

#### Rješenje 149

Ponovimo!

$$x^2 - y^2 = (x-y) \cdot (x+y) \quad , \quad x^2 - 2 \cdot x \cdot y + y^2 = (x-y)^2 .$$

$$\begin{aligned} & \left( 2 \cdot a - \frac{4 \cdot a - 1}{2 \cdot a} \right) \cdot \frac{4 \cdot a^2}{1 - 4 \cdot a^2} : \left( \frac{1}{a} - \frac{4}{1 + 2 \cdot a} \right) = \frac{(2 \cdot a)^2 - (4 \cdot a - 1)}{2 \cdot a} \cdot \frac{4 \cdot a^2}{1 - 4 \cdot a^2} : \frac{1 + 2 \cdot a - 4 \cdot a}{a \cdot (1 + 2 \cdot a)} = \\ & = \frac{4 \cdot a^2 - 4 \cdot a + 1}{2 \cdot a} \cdot \frac{4 \cdot a^2}{(1 - 2 \cdot a) \cdot (1 + 2 \cdot a)} : \frac{1 - 2 \cdot a}{a \cdot (1 + 2 \cdot a)} = \frac{1 - 4 \cdot a + 4 \cdot a^2}{2 \cdot a} \cdot \frac{4 \cdot a^2}{(1 - 2 \cdot a) \cdot (1 + 2 \cdot a)} \cdot \frac{a \cdot (1 + 2 \cdot a)}{1 - 2 \cdot a} = \\ & = \frac{(1 - 2 \cdot a)^2}{2} \cdot \frac{4 \cdot a^2}{1 - 2 \cdot a} \cdot \frac{1}{1 - 2 \cdot a} = \frac{(1 - 2 \cdot a)^2}{2} \cdot \frac{4 \cdot a^2}{(1 - 2 \cdot a)^2} = \frac{4 \cdot a^2}{2} = 2 \cdot a^2 . \end{aligned}$$

### Vježba 149

Izračunajte:  $\left( 2 \cdot a - \frac{4 \cdot a - 1}{2 \cdot a} \right) : \frac{1 - 4 \cdot a^2}{4 \cdot a^2} : \left( \frac{1}{a} - \frac{4}{1 + 2 \cdot a} \right)$ .

**Rezultat:**  $2 \cdot a^2$ .

### Zadatak 150 (Ana, ekomomska škola)

Izračunajte:  $\left\{ a^4 - (a^2 - 1) \cdot [(a-1)^2 + 2 \cdot a] \right\}^2$ .

### Rješenje 150

Ponovimo!

$$(x-y)^2 = x^2 - 2 \cdot x \cdot y + y^2 \quad , \quad (x-y) \cdot (x+y) = x^2 - y^2 \quad , \quad (a^n)^m = a^{n \cdot m} .$$

$$\begin{aligned} \left\{ a^4 - (a^2 - 1) \cdot [(a-1)^2 + 2 \cdot a] \right\}^2 &= \left\{ a^4 - (a^2 - 1) \cdot [a^2 - 2 \cdot a + 1 + 2 \cdot a] \right\}^2 = \left\{ a^4 - (a^2 - 1) \cdot [a^2 + 1] \right\}^2 = \\ &= \left\{ a^4 - \left( (a^2)^2 - 1 \right) \right\}^2 = \left\{ a^4 - (a^4 - 1) \right\}^2 = \left\{ a^4 - a^4 + 1 \right\}^2 = 1^2 = 1 . \end{aligned}$$

### Vježba 150

Izračunajte:  $\left\{ a^4 - (a^2 - 1) \cdot [(a+1)^2 - 2 \cdot a] \right\}^2$ .

**Rezultat:** 1.

### Zadatak 151 (Rea, gimnazija)

Skrati razlomak:  $\frac{a^2 + 2 \cdot a \cdot b + b^2 - c^2}{a^2 + c^2 + a \cdot b + 2 \cdot a \cdot c + b \cdot c}$ .

### Rješenje 151

Ponovimo!

$$x^2 + 2 \cdot x \cdot y + y^2 = (x+y)^2 \quad , \quad x \cdot y + x \cdot z = x \cdot (y+z) \quad , \quad x^2 - y^2 = (x-y) \cdot (x+y) .$$

$$\begin{aligned} \frac{a^2 + 2 \cdot a \cdot b + b^2 - c^2}{a^2 + c^2 + a \cdot b + 2 \cdot a \cdot c + b \cdot c} &= [2 \cdot a \cdot c = a \cdot c + a \cdot c] = \frac{(a+b)^2 - c^2}{a^2 + a \cdot b + a \cdot c + a \cdot c + b \cdot c + c^2} = \\ &= \frac{(a+b-c) \cdot (a+b+c)}{a \cdot (a+b+c) + c \cdot (a+b+c)} = \frac{(a+b-c) \cdot (a+b+c)}{(a+b+c) \cdot (a+c)} = \frac{a+b-c}{a+c} . \end{aligned}$$

### Vježba 151

Skrati razlomak:  $\frac{a^2 + c^2 + a \cdot b + 2 \cdot a \cdot c + b \cdot c}{a^2 + 2 \cdot a \cdot b + b^2 - c^2}$ .

**Rezultat:**  $\frac{a+c}{a+b-c}$ .

**Zadatak 152 (Jan, Zoran, Luka, gimnazija)**

Pojednostavni:  $\left[ \frac{1}{\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)^{-2}} - \left(\frac{\sqrt{a}-\sqrt{b}}{a^{\frac{3}{2}} - b^{\frac{3}{2}}}\right)^{-1} \right] \cdot (a \cdot b)^{-\frac{1}{2}}$ .

**Rješenje 152**

Ponovimo!

$a^{-n} = \frac{1}{a^n}$  ,  $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$  ,  $n\sqrt{a^m} = a^{\frac{m}{n}}$  ,  $(x+y)^2 = x^2 + 2 \cdot x \cdot y + y^2$

$x^3 - y^3 = (x-y) \cdot (x^2 + x \cdot y + y^2)$  ,  $\sqrt{a^2} = a$  ,  $a \geq 0$  ,  $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$ .

$$\begin{aligned} & \left[ \frac{1}{\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)^{-2}} - \left(\frac{\sqrt{a}-\sqrt{b}}{a^{\frac{3}{2}} - b^{\frac{3}{2}}}\right)^{-1} \right] \cdot (a \cdot b)^{-\frac{1}{2}} = \left[ \left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)^2 - \frac{a^{\frac{3}{2}} - b^{\frac{3}{2}}}{\sqrt{a}-\sqrt{b}} \right] \cdot \frac{1}{(a \cdot b)^{\frac{1}{2}}} = \\ & = \left[ (\sqrt{a} + \sqrt{b})^2 - \frac{(\sqrt{a})^3 - (\sqrt{b})^3}{\sqrt{a}-\sqrt{b}} \right] \cdot \frac{1}{\sqrt{a \cdot b}} = \left[ a + 2 \cdot \sqrt{a \cdot b} + b - \frac{(\sqrt{a}-\sqrt{b}) \cdot (a + \sqrt{a \cdot b} + b)}{\sqrt{a}-\sqrt{b}} \right] \cdot \frac{1}{\sqrt{a \cdot b}} = \\ & = \left[ a + 2 \cdot \sqrt{a \cdot b} + b - (a + \sqrt{a \cdot b} + b) \right] \cdot \frac{1}{\sqrt{a \cdot b}} = \left[ a + 2 \cdot \sqrt{a \cdot b} + b - a - \sqrt{a \cdot b} - b \right] \cdot \frac{1}{\sqrt{a \cdot b}} = \sqrt{a \cdot b} \cdot \frac{1}{\sqrt{a \cdot b}} = 1. \end{aligned}$$

**Vježba 152**

Pojednostavni:  $\left[ \frac{1}{\left(a^{\frac{1}{2}} + b^{\frac{1}{2}}\right)^{-2}} - \left(\frac{\sqrt{a}-\sqrt{b}}{a^{\frac{3}{2}} - b^{\frac{3}{2}}}\right)^{-1} \right]$ .

**Rezultat:**  $\sqrt{a \cdot b}$ .

**Zadatak 153 (Hrvoje, gimnazija)**

Neka je  $x = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}$  i  $y = \frac{(a-b+c) \cdot (a+b-c)}{(a+b+c) \cdot (b+c-a)}$ . Izračunajte  $(x+1) \cdot (y+1)$ .

**Rješenje 153**

Ponovimo!

Kvadrat zbroja:  $(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2$  , razlika kvadrata:  $(x+y) \cdot (x-y) = x^2 - y^2$

kvadrat razlike:  $(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2$ .

Računamo:

$$\begin{aligned}
 \bullet \quad x+1 &= \frac{b^2+c^2-a^2}{2 \cdot b \cdot c} + 1 = \frac{b^2+c^2-a^2+2 \cdot b \cdot c}{2 \cdot b \cdot c} = \frac{-a^2+b^2+c^2+2 \cdot b \cdot c}{2 \cdot b \cdot c}. \\
 \bullet \quad y+1 &= \frac{(a-b+c) \cdot (a+b-c)}{(a+b+c) \cdot (b+c-a)} + 1 = \frac{(a-b+c) \cdot (a+b-c) + (a+b+c) \cdot (b+c-a)}{(a+b+c) \cdot (b+c-a)} = \\
 &= \frac{(a-(b-c)) \cdot (a+(b-c)) + ((b+c)+a) \cdot ((b+c)-a)}{((b+c)+a) \cdot ((b+c)-a)} = \frac{a^2 - (b-c)^2 + (b+c)^2 - a^2}{(b+c)^2 - a^2} = \\
 &= \frac{-(b-c)^2 + (b+c)^2}{(b+c)^2 - a^2} = \frac{-(b^2 - 2 \cdot b \cdot c + c^2) + b^2 + 2 \cdot b \cdot c + c^2}{b^2 + 2 \cdot b \cdot c + c^2 - a^2} = \frac{-b^2 + 2 \cdot b \cdot c - c^2 + b^2 + 2 \cdot b \cdot c + c^2}{-a^2 + b^2 + c^2 + 2 \cdot b \cdot c} = \\
 &= \frac{4 \cdot b \cdot c}{-a^2 + b^2 + c^2 + 2 \cdot b \cdot c}.
 \end{aligned}$$

Sada izravno slijedi:

$$(x+1) \cdot (y+1) = \frac{-a^2+b^2+c^2+2 \cdot b \cdot c}{2 \cdot b \cdot c} \cdot \frac{4 \cdot b \cdot c}{-a^2+b^2+c^2+2 \cdot b \cdot c} = 2.$$

### Vježba 153

Neka je  $x = \frac{b^2+c^2-a^2}{2 \cdot b \cdot c}$  i  $y = \frac{(a-b+c) \cdot (a+b-c)}{(a+b+c) \cdot (b+c-a)}$ . Izračunajte  $(2 \cdot x + 2) \cdot (y + 1)$ .

**Rezultat:** 4.

### Zadatak 154 (Marijana, maturantica)

Ako je  $x^{-x} - x = 1$ , pojednostavnite  $\left(1 + \frac{1}{x}\right)^{\frac{1}{x}} \cdot \left(\frac{1}{x}\right)^{-\frac{1}{x}}$ .

### Rješenje 154

Ponovimo!

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n, \quad a^n \cdot b^n = (a \cdot b)^n, \quad (a^n)^m = a^{n \cdot m}, \quad a^{-n} = \frac{1}{a^n}.$$

$$x^{-x} - x = 1 \Rightarrow x^{-x} = 1 + x.$$

Sada je:

$$\left(1 + \frac{1}{x}\right)^{\frac{1}{x}} \cdot \left(\frac{1}{x}\right)^{-\frac{1}{x}} = \left(1 + \frac{1}{x}\right)^{\frac{1}{x}} \cdot x^{\frac{1}{x}} = \left(x \cdot \left(1 + \frac{1}{x}\right)\right)^{\frac{1}{x}} = (x+1)^{\frac{1}{x}} = (x^{-x})^{\frac{1}{x}} = x^{-1} = \frac{1}{x}.$$

### Vježba 154

Ako je  $x^x - x = 1$ , pojednostavnite  $\left(1 + \frac{1}{x}\right)^{\frac{1}{x}} \cdot \left(\frac{1}{x}\right)^{-\frac{1}{x}}$ .

**Rezultat:** x.

### Zadatak 155 (Vedran, gimnazija)

Kolika je vrijednost izraza  $x^4 + \frac{1}{x^4}$ , ako je  $x^2 - 3 \cdot x + 1 = 0$ ?

### Rješenje 155

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad (a^n)^m = a^{n \cdot m}.$$

$$\begin{aligned}
x^2 - 3 \cdot x + 1 = 0 \quad /: x &\Rightarrow \frac{x^2}{x} - \frac{3 \cdot x}{x} + \frac{1}{x} = 0 \Rightarrow x - 3 + \frac{1}{x} = 0 \Rightarrow x + \frac{1}{x} = 3 \quad /: 2 \Rightarrow \left(x + \frac{1}{x}\right)^2 = 3^2 \Rightarrow \\
&\Rightarrow x^2 + 2 \cdot x \cdot \frac{1}{x} + \left(\frac{1}{x}\right)^2 = 9 \Rightarrow x^2 + 2 + \frac{1}{x^2} = 9 \Rightarrow x^2 + \frac{1}{x^2} = 9 - 2 \Rightarrow x^2 + \frac{1}{x^2} = 7 \quad /: 2 \Rightarrow \\
&\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 7^2 \Rightarrow \left(x^2\right)^2 + 2 \cdot x^2 \cdot \frac{1}{x^2} + \left(\frac{1}{x^2}\right)^2 = 7^2 \Rightarrow x^4 + 2 + \frac{1}{x^4} = 49 \Rightarrow \\
&\Rightarrow x^4 + \frac{1}{x^4} = 49 - 2 \Rightarrow x^4 + \frac{1}{x^4} = 47.
\end{aligned}$$

### Vježba 155

Kolika je vrijednost izraza  $x^4 + \frac{1}{x^4}$ , ako je  $x^2 - 4 \cdot x + 1 = 0$ ?

**Rezultat:** 194.

### Zadatak 156 (Iva, gimnazija)

Kolika je vrijednost izraza  $x^3 + y^3$ , ako je  $x + y = 4$  i  $x^2 + y^2 = 10$ ?

### Rješenje 156

Ponovimo!

$$\begin{aligned}
(a+b)^2 &= a^2 + 2 \cdot a \cdot b + b^2, & a^3 + b^3 &= (a+b) \cdot (a^2 - a \cdot b + b^2) \\
(a+b)^3 &= a^3 + 3 \cdot a^2 \cdot b + 3 \cdot a \cdot b^2 + b^3.
\end{aligned}$$

1. inačica

$$\begin{aligned}
\left. \begin{array}{l} x + y = 4 \\ x^2 + y^2 = 10 \end{array} \right\} &\Rightarrow \left. \begin{array}{l} x + y = 4 \quad /: 2 \\ x^2 + y^2 = 10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x^2 + 2 \cdot x \cdot y + y^2 = 16 \\ x^2 + y^2 = 10 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{oduzmemo} \\ \text{jednadžbe} \end{array} \right] \Rightarrow \\
&\Rightarrow x^2 + 2 \cdot x \cdot y + y^2 - x^2 - y^2 = 16 - 10 \Rightarrow 2 \cdot x \cdot y = 6 \quad /: 2 \Rightarrow x \cdot y = 3.
\end{aligned}$$

Vrijednost zadanog izraza iznosi:

$$\begin{aligned}
\left. \begin{array}{l} x + y = 4, \quad x^2 + y^2 = 10, \quad x \cdot y = 3 \\ x^3 + y^3 = (x+y) \cdot (x^2 - x \cdot y + y^2) \end{array} \right\} &\Rightarrow \left. \begin{array}{l} x + y = 4, \quad x^2 + y^2 = 10, \quad x \cdot y = 3 \\ x^3 + y^3 = (x+y) \cdot (x^2 + y^2 - x \cdot y) \end{array} \right\} \Rightarrow \\
&\Rightarrow x^3 + y^3 = 4 \cdot (10 - 3) \Rightarrow x^3 + y^3 = 4 \cdot 7 \Rightarrow x^3 + y^3 = 28.
\end{aligned}$$

2. inačica

Zbog identiteta

$$x \cdot y = \frac{1}{2} \cdot \left[ (x+y)^2 - (x^2 + y^2) \right],$$

vrijedi

$$x \cdot y = \frac{1}{2} \cdot [4^2 - 10] \Rightarrow x \cdot y = \frac{1}{2} \cdot [16 - 10] \Rightarrow x \cdot y = \frac{1}{2} \cdot 6 \Rightarrow x \cdot y = 3.$$

Vrijednost zadanog izraza iznosi:

$$\begin{aligned}
\left. \begin{array}{l} x + y = 4, \quad x^2 + y^2 = 10, \quad x \cdot y = 3 \\ x^3 + y^3 = (x+y) \cdot (x^2 - x \cdot y + y^2) \end{array} \right\} &\Rightarrow \left. \begin{array}{l} x + y = 4, \quad x^2 + y^2 = 10, \quad x \cdot y = 3 \\ x^3 + y^3 = (x+y) \cdot (x^2 + y^2 - x \cdot y) \end{array} \right\} \Rightarrow \\
&\Rightarrow x^3 + y^3 = 4 \cdot (10 - 3) \Rightarrow x^3 + y^3 = 4 \cdot 7 \Rightarrow x^3 + y^3 = 28.
\end{aligned}$$

3. inačica

$$\left. \begin{array}{l} x+y=4 \\ x^2+y^2=10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x+y=4 \text{ / } 2 \\ x^2+y^2=10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x^2+2 \cdot x \cdot y+y^2=16 \\ x^2+y^2=10 \end{array} \right\} \Rightarrow \left. \begin{array}{l} 2 \cdot x \cdot y=16-x^2-y^2 \\ x^2+y^2=10 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} 2 \cdot x \cdot y=16-(x^2+y^2) \\ x^2+y^2=10 \end{array} \right\} \Rightarrow 2 \cdot x \cdot y=16-10 \Rightarrow 2 \cdot x \cdot y=6 \text{ / } 2 \Rightarrow x \cdot y=3.$$

Vrijednost zadanog izraza iznosi:

$$\left. \begin{array}{l} x+y=4, x^2+y^2=10, x \cdot y=3 \\ x^3+y^3=(x+y) \cdot (x^2-x \cdot y+y^2) \end{array} \right\} \Rightarrow \left. \begin{array}{l} x+y=4, x^2+y^2=10, x \cdot y=3 \\ x^3+y^3=(x+y) \cdot (x^2+y^2-x \cdot y) \end{array} \right\} \Rightarrow$$

$$\Rightarrow x^3+y^3=4 \cdot (10-3) \Rightarrow x^3+y^3=4 \cdot 7 \Rightarrow x^3+y^3=28.$$

### Vježba 156

Kolika je vrijednost izraza  $x^3+y^3$ , ako je  $x+y=5$  i  $x^2+y^2=5$ ?

**Rezultat:** - 25.

### Zadatak 157 (Elena, gimnazija)

Riješite jednadžbu:  $\frac{a}{b} \cdot \left(1 - \frac{a}{x}\right) + \frac{b}{a} \cdot \left(1 - \frac{b}{x}\right) = 1$ .

### Rješenje 157

Ponovimo!

$$a^3+b^3=(a+b) \cdot (a^2-a \cdot b+b^2), \quad \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c}.$$

1. inačica

$$\frac{a}{b} \cdot \left(1 - \frac{a}{x}\right) + \frac{b}{a} \cdot \left(1 - \frac{b}{x}\right) = 1 \Rightarrow \frac{a}{b} - \frac{a^2}{b \cdot x} + \frac{b}{a} - \frac{b^2}{a \cdot x} = 1 \text{ / } \cdot a \cdot b \cdot x \Rightarrow a^2 \cdot x - a^3 + b^2 \cdot x - b^3 = a \cdot b \cdot x \Rightarrow$$

$$\Rightarrow a^2 \cdot x + b^2 \cdot x - a \cdot b \cdot x = a^3 + b^3 \Rightarrow (a^2 + b^2 - a \cdot b) \cdot x = a^3 + b^3 \text{ / } \cdot \frac{1}{a^2 + b^2 - a \cdot b} \Rightarrow$$

$$\Rightarrow x = \frac{a^3 + b^3}{a^2 + b^2 - a \cdot b} \Rightarrow x = \frac{(a+b) \cdot (a^2 - a \cdot b + b^2)}{a^2 - a \cdot b + b^2} \Rightarrow x = a + b.$$

2. inačica

$$\frac{a}{b} \cdot \left(1 - \frac{a}{x}\right) + \frac{b}{a} \cdot \left(1 - \frac{b}{x}\right) = 1 \Rightarrow \frac{a}{b} - \frac{a^2}{b \cdot x} + \frac{b}{a} - \frac{b^2}{a \cdot x} = 1 \Rightarrow \frac{a}{b} + \frac{b}{a} - 1 = \frac{a^2}{b \cdot x} + \frac{b^2}{a \cdot x} \Rightarrow$$

$$\Rightarrow \frac{a^2 + b^2 - a \cdot b}{a \cdot b} = \frac{1}{x} \cdot \left(\frac{a^2}{b} + \frac{b^2}{a}\right) \Rightarrow \frac{a^2 + b^2 - a \cdot b}{a \cdot b} = \frac{1}{x} \cdot \frac{a^3 + b^3}{a \cdot b} \text{ / } \cdot \frac{a \cdot b \cdot x}{a^2 + b^2 - a \cdot b} \Rightarrow$$

$$\Rightarrow x = \frac{a^3 + b^3}{a^2 + b^2 - a \cdot b} \Rightarrow x = \frac{(a+b) \cdot (a^2 - a \cdot b + b^2)}{a^2 - a \cdot b + b^2} \Rightarrow x = a + b.$$

### Vježba 157

Riješite jednadžbu:  $\frac{a}{b} \cdot \left(1 - \frac{a}{x}\right) + \frac{b}{a} \cdot \left(1 - \frac{b}{x}\right) = 0$ .

**Rezultat:**  $x = \frac{a^3 + b^3}{a^2 + b^2}$ .

**Zadatak 158 (Ana, gimnazija)**

Kolika je vrijednost broja  $x + y$  ako je  $(x + \sqrt{x^2 + 1}) \cdot (y + \sqrt{y^2 + 1}) = 1$ ?

**Rješenje 158**

Ponovimo!

$$(a-b) \cdot (a+b) = a^2 - b^2.$$

Zadanu jednakost pomnožimo sa  $x - \sqrt{x^2 + 1}$  i sa  $y - \sqrt{y^2 + 1}$ :

$$\left. \begin{aligned} (x + \sqrt{x^2 + 1}) \cdot (y + \sqrt{y^2 + 1}) &= 1 \cdot (x - \sqrt{x^2 + 1}) \\ (x + \sqrt{x^2 + 1}) \cdot (y + \sqrt{y^2 + 1}) &= 1 \cdot (y - \sqrt{y^2 + 1}) \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} (x + \sqrt{x^2 + 1}) \cdot (x - \sqrt{x^2 + 1}) \cdot (y + \sqrt{y^2 + 1}) &= x - \sqrt{x^2 + 1} \\ (x + \sqrt{x^2 + 1}) \cdot (y + \sqrt{y^2 + 1}) \cdot (y - \sqrt{y^2 + 1}) &= y - \sqrt{y^2 + 1} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} (x^2 - (x^2 + 1)) \cdot (y + \sqrt{y^2 + 1}) &= x - \sqrt{x^2 + 1} \\ (x + \sqrt{x^2 + 1}) \cdot (y^2 - (y^2 + 1)) &= y - \sqrt{y^2 + 1} \end{aligned} \right\} \Rightarrow \left. \begin{aligned} (x^2 - x^2 - 1) \cdot (y + \sqrt{y^2 + 1}) &= x - \sqrt{x^2 + 1} \\ (x + \sqrt{x^2 + 1}) \cdot (y^2 - y^2 - 1) &= y - \sqrt{y^2 + 1} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} -1 \cdot (y + \sqrt{y^2 + 1}) &= x - \sqrt{x^2 + 1} \\ -1 \cdot (x + \sqrt{x^2 + 1}) &= y - \sqrt{y^2 + 1} \end{aligned} \right\} \Rightarrow \left. \begin{aligned} -y - \sqrt{y^2 + 1} &= x - \sqrt{x^2 + 1} \\ -x - \sqrt{x^2 + 1} &= y - \sqrt{y^2 + 1} \end{aligned} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{aligned} -y - x &= -\sqrt{x^2 + 1} + \sqrt{y^2 + 1} \cdot (-1) \\ -x - y &= -\sqrt{y^2 + 1} + \sqrt{x^2 + 1} \cdot (-1) \end{aligned} \right\} \Rightarrow \left. \begin{aligned} x + y &= \sqrt{x^2 + 1} - \sqrt{y^2 + 1} \\ x + y &= \sqrt{y^2 + 1} - \sqrt{x^2 + 1} \end{aligned} \right\} \Rightarrow \left[ \begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array} \right] \Rightarrow$$

$$\Rightarrow 2 \cdot (x + y) = \sqrt{x^2 + 1} - \sqrt{y^2 + 1} + \sqrt{y^2 + 1} - \sqrt{x^2 + 1} \Rightarrow 2 \cdot (x + y) = 0 \quad /: 2 \Rightarrow x + y = 0.$$

**Vježba 158**

Kolika je vrijednost broja  $x + y$  ako je  $(x - \sqrt{x^2 + 1}) \cdot (y - \sqrt{y^2 + 1}) = 1$ ?

**Rezultat:** 0.

**Zadatak 159 (Tina, gimnazija)**

Dokažite da za svaki realni broj  $x$  vrijedi nejednakost:  $x^2 - x + 2 > 0$ .

**Rješenje 159**

Ponovimo!

$$a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2, \quad a^2 \geq 0, \quad a \in \mathbb{R}.$$

$$x^2 - x + 2 = x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 2 = x^2 - x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 2 = x^2 - x + \frac{1}{4} - \frac{1}{4} + 2 =$$

$$= \left(x^2 - x + \frac{1}{4}\right) - \frac{1}{4} + 2 = \left(x - \frac{1}{2}\right)^2 + \frac{7}{4} \geq \frac{7}{4} > 0.$$

Dakle, za svaki realni broj  $x$  vrijedi:

$$\left(x - \frac{1}{2}\right)^2 + \frac{7}{4} > 0 \Rightarrow x^2 - x + 2 > 0.$$

### Vježba 159

Dokažite da za svaki realni broj  $x$  vrijedi nejednakost:  $x^2 - x + 1 > 0$ .

**Rezultat:**  $\left(x - \frac{1}{2}\right)^2 + \frac{3}{4} \geq \frac{3}{4} > 0.$

### Zadatak 160 (Tina, gimnazija)

Dokažite da za sve realne brojeve  $x$  i  $y$  vrijedi nejednakost:  $x^2 + y^2 + 1 \geq x \cdot y + x + y$ .

### Rješenje 160

Ponovimo!

$$a^2 - 2 \cdot a \cdot b + b^2 = (a - b)^2, \quad a^2 \geq 0, \quad a \in \mathbb{R}.$$

$$\begin{aligned} x^2 + y^2 + 1 \geq x \cdot y + x + y &\Rightarrow x^2 + y^2 + 1 - x \cdot y - x - y \geq 0 \quad / \cdot 2 \Rightarrow 2 \cdot x^2 + 2 \cdot y^2 + 2 - 2 \cdot x \cdot y - 2 \cdot x - 2 \cdot y \geq 0 \Rightarrow \\ &\Rightarrow x^2 - 2 \cdot x \cdot y + y^2 + x^2 - 2 \cdot x + 1 + y^2 - 2 \cdot y + 1 \geq 0 \Rightarrow (x^2 - 2 \cdot x \cdot y + y^2) + (x^2 - 2 \cdot x + 1) + (y^2 - 2 \cdot y + 1) \geq 0 \Rightarrow \\ &\Rightarrow (x - y)^2 + (x - 1)^2 + (y - 1)^2 \geq 0. \end{aligned}$$

Dobivena nejednakost istinita je za sve realne brojeve  $x$  i  $y$  pa je i polazna nejednakost istinita za sve  $x$  i  $y$ .  
Jednakost vrijedi ako i samo ako je  $x = y = 1$ .

### Vježba 160

Dokažite da za sve realne brojeve  $x$  i  $y$  vrijedi nejednakost:  $x^2 + y^2 \geq 2 \cdot (x + y - 1)$ .

**Rezultat:**  $(x - 1)^2 + (y - 1)^2 \geq 0.$