

Zadatak 121 (Anamarija, gimnazija)

Pojednostavnite: $\frac{1}{\sqrt[4]{3}-\sqrt[4]{2}}$.

Rješenje 121

Ponovimo!

Razlika kvadrata:

$$(a-b) \cdot (a+b) = a^2 - b^2.$$

Množenje korijena:

$$\sqrt[n]{a} \cdot \sqrt[m]{b} = \sqrt[n \cdot m]{a^m \cdot b^n}.$$

$$\begin{aligned} \frac{1}{\sqrt[4]{3}-\sqrt[4]{2}} &= \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \frac{1}{\sqrt[4]{3}-\sqrt[4]{2}} \cdot \frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt[4]{3}+\sqrt[4]{2}} = \frac{\sqrt[4]{3}+\sqrt[4]{2}}{(\sqrt[4]{3})^2 - (\sqrt[4]{2})^2} = \frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt{3}-\sqrt{2}} = \\ &= \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \frac{\sqrt[4]{3}+\sqrt[4]{2}}{\sqrt{3}-\sqrt{2}} \cdot \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = \frac{(\sqrt[4]{3}+\sqrt[4]{2}) \cdot (\sqrt{3}+\sqrt{2})}{(\sqrt{3})^2 - (\sqrt{2})^2} = \frac{(\sqrt[4]{3}+\sqrt[4]{2}) \cdot (\sqrt{3}+\sqrt{2})}{3-2} = \\ &= \frac{(\sqrt[4]{3}+\sqrt[4]{2}) \cdot (\sqrt{3}+\sqrt{2})}{1} = (\sqrt[4]{3}+\sqrt[4]{2}) \cdot (\sqrt{3}+\sqrt{2}) = \sqrt[4]{3} \cdot \sqrt{3} + \sqrt[4]{3} \cdot \sqrt{2} + \sqrt[4]{2} \cdot \sqrt{3} + \sqrt[4]{2} \cdot \sqrt{2} = \\ &= \sqrt[4]{3 \cdot 3^2} + \sqrt[4]{3 \cdot 2^2} + \sqrt[4]{2 \cdot 3^2} + \sqrt[4]{2 \cdot 2^2} = \sqrt[4]{27} + \sqrt[4]{12} + \sqrt[4]{18} + \sqrt[4]{8}. \end{aligned}$$

Vježba 121

Pojednostavnite: $\frac{1}{\sqrt[4]{3}+\sqrt[4]{2}}$.

Rezultat: $\sqrt[4]{27} + \sqrt[4]{12} + \sqrt[4]{18} + \sqrt[4]{8}$.

Zadatak 122 (Iva, gimnazija)

Pojednostavnite: $\frac{a^2-4}{2 \cdot a-4}$.

Rješenje 122

Ponovimo!

Razlika kvadrata:

$$(a-b) \cdot (a+b) = a^2 - b^2.$$

$$\frac{a^2-4}{2 \cdot a-4} = \left[\begin{array}{l} \text{u brojniku je razlika kvadrata,} \\ \text{u nazivniku izlučimo broj 2} \end{array} \right] = \frac{(a-2) \cdot (a+2)}{2 \cdot (a-2)} = \left[\begin{array}{l} \text{kratimo faktore} \\ a-2 \end{array} \right] = \frac{(a-2) \cdot (a+2)}{2 \cdot (a-2)} = \frac{a+2}{2}.$$

Vježba 122

Pojednostavnite: $\frac{a^2-9}{2 \cdot a-6}$.

Rezultat: $\frac{a+3}{2}$.

Zadatak 123 (Anamarija, gimnazija)

Izračunajte: $\left(\frac{x-1}{x+1}-1\right) : \left(\frac{x+1}{x-1}-1\right)$.

Rješenje 123

U obje zagrade je računski operacija oduzimanje pa zato moramo naći najmanji zajednički nazivnik za svaku zagradu:

$$\left(\frac{x-1}{x+1}-1\right) : \left(\frac{x+1}{x-1}-1\right) = \frac{x-1-1 \cdot (x+1)}{x+1} : \frac{x+1-1 \cdot (x-1)}{x-1} = \frac{x-1-x-1}{x+1} : \frac{x+1-x+1}{x-1} =$$

$$= \frac{x-1-x-1}{x+1} : \frac{x+1-x+1}{x-1} = \frac{-2}{x+1} : \frac{2}{x-1} = \frac{-2}{x+1} \cdot \frac{x-1}{2} = \frac{-2}{x+1} \cdot \frac{x-1}{2} = \frac{-1}{x+1} \cdot \frac{x-1}{1} = \frac{-(x-1)}{x+1} = \frac{-x+1}{x+1} = \frac{1-x}{1+x}$$

Vježba 123

Izračunajte: $\left(\frac{x-1}{x+1}-1\right) : \frac{2}{x+1}$.

Rezultat: -1.

Zadatak 124 (Nena, Sarah, hotelijerska škola - THK)

Rastavite na faktore: $a \cdot (a+b) - c \cdot (b+c)$.

Rješenje 124

Ponovimo!

Zakon distribucije množenja prema zbrajanju: $a \cdot (b+c) = a \cdot b + a \cdot c$, $a \cdot b + a \cdot c = a \cdot (b+c)$.

Razlika kvadrata: $a^2 - b^2 = (a-b) \cdot (a+b)$, $(a-b) \cdot (a+b) = a^2 - b^2$.

$$a \cdot (a+b) - c \cdot (b+c) = a^2 + a \cdot b - b \cdot c - c^2 = \left[\begin{array}{l} \text{grupiramo prvi i četvrti član, dobijemo razliku kvadrata} \\ \text{grupiramo drugi i treći član, izlučimo b} \end{array} \right] =$$

$$= a^2 - c^2 + a \cdot b - b \cdot c = (a-c) \cdot (a+c) + b \cdot (a-c) = \left[\text{izlučimo zagradu } (a-c) \right] = (a-c) \cdot (a+c+b).$$

Vježba 124

Rastavite na faktore: $a \cdot (a+b) + c \cdot (b-c)$.

Rezultat: $(a+c) \cdot (a+b-c)$.

Zadatak 125 (Nena, Sarah, hotelijerska škola - THK)

Rastavite na faktore: $a^2 - b^2 + 2 \cdot b \cdot c - c^2$.

Rješenje 125

Ponovimo!

Razlika kvadrata: $a^2 - b^2 = (a-b) \cdot (a+b)$, $(a-b) \cdot (a+b) = a^2 - b^2$.

Kvadrat razlike: $(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2$, $a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2$.

$$a^2 - b^2 + 2 \cdot b \cdot c - c^2 = \left[\begin{array}{l} \text{iz zadnja tri člana izlučimo predznak minus,} \\ \text{dobije se kvadrata razlike} \end{array} \right] = a^2 - (b^2 - 2 \cdot b \cdot c + c^2) =$$

$$= a^2 - (b-c)^2 = \left[\begin{array}{l} \text{razlika kvadrata} \\ I = a, II = b-c \end{array} \right] = (a - (b-c)) \cdot (a + (b-c)) = (a-b+c) \cdot (a+b-c).$$

Vježba 125

Rastavite na faktore: $b^2 - 2 \cdot b \cdot c + c^2 - a^2$.

Rezultat: $(b-c-a) \cdot (b-c+a)$.

Zadatak 126 (Nena, Sarah, hotelijerska škola - THK)

Rastavite na faktore: $2 \cdot (x+1)^2 - 18 \cdot (2-x)^2$.

Rješenje 126

Ponovimo!

Zakon distribucije množenja prema zbrajanju: $a \cdot (b+c) = a \cdot b + a \cdot c$, $a \cdot b + a \cdot c = a \cdot (b+c)$.

Razlika kvadrata: $a^2 - b^2 = (a-b) \cdot (a+b)$, $(a-b) \cdot (a+b) = a^2 - b^2$.

$$2 \cdot (x+1)^2 - 18 \cdot (2-x)^2 = \left[\text{izlučimo broj 2} \right] = 2 \cdot \left[(x+1)^2 - 9 \cdot (2-x)^2 \right] =$$

$$= \left[\begin{array}{l} \text{razlika kvadrata} \\ I = x+1, II = 3 \cdot (2-x) \end{array} \right] = 2 \cdot [(x+1) - 3 \cdot (2-x)] \cdot [(x+1) + 3 \cdot (2-x)] = 2 \cdot [x+1-6+3 \cdot x] \cdot [x+1+6-3 \cdot x] = \\ = 2 \cdot [4 \cdot x - 5] \cdot [7 - 2 \cdot x].$$

Vježba 126

Rastavite na faktore: $2 \cdot (x+1)^2 - 2 \cdot (2-x)^2$.

Rezultat: $6 \cdot (2 \cdot x - 1)$.

Zadatak 127 (4A, hotelijerska škola)

Rastavite na faktore algebarski izraz: $6 \cdot x^2 - 5 \cdot x \cdot y - 4 \cdot y^2$.

Rješenje 127

$$6 \cdot x^2 - 5 \cdot x \cdot y - 4 \cdot y^2 = 6 \cdot x^2 - 5 \cdot x \cdot y - 4 \cdot y^2 = 6 \cdot x^2 + 3 \cdot x \cdot y - 8 \cdot x \cdot y - 4 \cdot y^2 = \\ = 3 \cdot x \cdot (2 \cdot x + y) - 4 \cdot y \cdot (2 \cdot x + y) = (2 \cdot x + y)(3 \cdot x - 4 \cdot y).$$

Vježba 127

Rastavite na faktore algebarski izraz: $4 \cdot a^2 - 12 \cdot a \cdot b + 9 \cdot b^2 - 2 \cdot a + 3 \cdot b$.

Rezultat: $4 \cdot a^2 - 12 \cdot a \cdot b + 9 \cdot b^2 - 2 \cdot a + 3 \cdot b = 4 \cdot a^2 - 6 \cdot a \cdot b - 6 \cdot a \cdot b + 9 \cdot b^2 - 2 \cdot a + 3 \cdot b = \\ = 2 \cdot a \cdot (2 \cdot a - 3 \cdot b) - 3 \cdot b \cdot (2 \cdot a - 3 \cdot b) - (2 \cdot a - 3 \cdot b) = (2 \cdot a - 3 \cdot b) \cdot (2 \cdot a - 3 \cdot b - 1)$.

Zadatak 128 (4A, hotelijerska škola)

Rastavite na faktore algebarski izraz: $x^3 + 5 \cdot x^2 + 3 \cdot x - 9$.

Rješenje 128

Ponovimo!

Razlika kubova: $a^3 - b^3 = (a-b) \cdot (a^2 + a \cdot b + b^2)$. Razlika kvadrata: $a^2 - b^2 = (a-b) \cdot (a+b)$.

$$x^3 + 5 \cdot x^2 + 3 \cdot x - 9 = x^3 + 5 \cdot x^2 + 3 \cdot x - 9 = x^3 + 5 \cdot x^2 + 3 \cdot x - 1 - 5 - 3 = x^3 - 1 + 5 \cdot x^2 - 5 + 3 \cdot x - 3 = \\ = (x-1) \cdot (x^2 + x + 1) + 5 \cdot (x^2 - 1) + 3 \cdot (x-1) = (x-1) \cdot (x^2 + x + 1) + 5 \cdot (x-1) \cdot (x+1) + 3 \cdot (x-1) = \\ = (x-1) \cdot (x^2 + x + 1 + 5 \cdot (x+1) + 3) = (x-1) \cdot (x^2 + x + 1 + 5 \cdot x + 5 + 3) = (x-1) \cdot (x^2 + 6 \cdot x + 9) = (x-1) \cdot (x+3)^2.$$

Vježba 128

Rastavite na faktore algebarski izraz: $x^3 + x^2 + 4$.

Rezultat: $x^3 + x^2 + 4 = x^3 + x^2 + 8 - 4 = x^3 + 8 + x^2 - 4 = (x+2) \cdot (x^2 - 2 \cdot x + 4) + (x-2) \cdot (x+2) = \\ = (x+2) \cdot (x^2 - 2 \cdot x + 4 + x - 2) = (x+2) \cdot (x^2 - x + 2)$.

Zadatak 129 (Anamarija, gimnazija)

Dokažite da vrijednost razlomka ne ovisi o vrijednosti broja n: $\frac{16^{n-1}}{8^{n-2} \cdot 2^{n+2}}$.

Rješenje 129

Ponovimo!

$$(a^n)^m = a^{n \cdot m}, \quad a^n \cdot a^m = a^{n+m}, \quad a^n \cdot b^n = (a \cdot b)^n.$$

1. inačica

$$\frac{16^{n-1}}{8^{n-2} \cdot 2^{n+2}} = \frac{(2^4)^{n-1}}{(2^3)^{n-2} \cdot 2^{n+2}} = \frac{2^{4n-4}}{2^{3n-6} \cdot 2^{n+2}} = \frac{2^{4n-4}}{2^{3n-6+n+2}} = \frac{2^{4n-4}}{2^{4n-4}} = 1.$$

2. inačica

$$\begin{aligned}\frac{16^{n-1}}{8^{n-2} \cdot 2^{n+2}} &= \frac{16^n \cdot 16^{-1}}{8^n \cdot 8^{-2} \cdot 2^n \cdot 2^2} = \frac{16^n \cdot 16^{-1}}{(8 \cdot 2)^n \cdot 8^{-2} \cdot 2^2} = \frac{16^n \cdot 16^{-1}}{16^n \cdot 8^{-2} \cdot 2^2} = \frac{16^n \cdot 16^{-1}}{16^n \cdot 8^{-2} \cdot 2^2} = \frac{16^{-1}}{8^{-2} \cdot 2^2} = \\ &= \frac{8^2}{16 \cdot 2^2} = \frac{64}{16 \cdot 4} = \frac{64}{64} = 1.\end{aligned}$$

Vježba 129

Dokažite da vrijednost razlomka ne ovisi o vrijednosti broja n : $\frac{25^{n+1} \cdot 5^{n+2}}{125^{n-1}}$.

Rezultat: 5^7 .

Zadatak 130 (Deny, strojarska škola)

Ako je $a > 1$, izračunajte $\frac{a-1}{a+1} \cdot \frac{\sqrt{a^4 - 2 \cdot a^2 + 1}}{a^2 - 2 \cdot a + 1} + 5$.

Rješenje 130

Ponovimo!

$$a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2, \quad \sqrt{a^2} = |a|, \quad |a| = a \text{ za } a \geq 0, \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

$$\begin{aligned}\frac{a-1}{a+1} \cdot \frac{\sqrt{a^4 - 2 \cdot a^2 + 1}}{a^2 - 2 \cdot a + 1} + 5 &= \frac{a-1}{a+1} \cdot \frac{\sqrt{(a^2-1)^2}}{(a-1)^2} + 5 = \frac{a-1}{a+1} \cdot \frac{|a^2-1|}{(a-1)^2} + 5 = \left[\begin{array}{l} a > 1 \Rightarrow a^2 - 1 > 0 \\ a^2 - 1 = a^2 - 1 \end{array} \right] = \\ &= \frac{a-1}{a+1} \cdot \frac{a^2-1}{(a-1)^2} + 5 = \frac{a-1}{a+1} \cdot \frac{(a-1) \cdot (a+1)}{(a-1) \cdot (a-1)} + 5 = 1 + 5 = 6.\end{aligned}$$

Vježba 130

Ako je $a > 1$, izračunajte $(a+1) \cdot \frac{\sqrt{a^2 - 2 \cdot a + 1}}{a^2 - 1} + 5$.

Rezultat: 6.

Zadatak 131 (Jasenska, srednja škola)

Izračunajte $\sqrt{(1-\sqrt{2})^2} + \sqrt{(\sqrt{2}-\sqrt{3})^2} + \sqrt{(\sqrt{3}-2)^2}$.

Rješenje 131

Ponovimo!

$$\sqrt{a^2} = |a|, \quad |a| = a \text{ za } a \geq 0, \quad |a| = -a \text{ za } a < 0.$$

$$\sqrt{(1-\sqrt{2})^2} + \sqrt{(\sqrt{2}-\sqrt{3})^2} + \sqrt{(\sqrt{3}-2)^2} = |1-\sqrt{2}| + |\sqrt{2}-\sqrt{3}| + |\sqrt{3}-2|.$$

Pogledajmo kakvi su po predznacima (pozitivni ili negativni) izrazi pod znakom apsolutne vrijednosti:

$$\left. \begin{array}{l} 1-\sqrt{2} \approx 1-1.41 = -0.41 < 0 \text{ negativno} \\ \sqrt{2}-\sqrt{3} \approx 1.41-1.73 = -0.32 < 0 \text{ negativno} \\ \sqrt{3}-2 \approx 1.73-2 = -0.27 < 0 \text{ negativno} \end{array} \right\}.$$

Po definiciji apsolutne vrijednosti je:

$$|a| = -a \text{ za } a < 0.$$

Zato je:

$$\begin{aligned} & \sqrt{(1-\sqrt{2})^2} + \sqrt{(\sqrt{2}-\sqrt{3})^2} + \sqrt{(\sqrt{3}-2)^2} = |1-\sqrt{2}| + |\sqrt{2}-\sqrt{3}| + |\sqrt{3}-2| = \\ & = -(1-\sqrt{2}) - (\sqrt{2}-\sqrt{3}) - (\sqrt{3}-2) = -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + 2 = -1 + \sqrt{2} - \sqrt{2} + \sqrt{3} - \sqrt{3} + 2 = -1 + 2 = 1. \end{aligned}$$

Vježba 131

Izračunajte $\sqrt{(1-\sqrt{2})^2}$

Rezultat: $\sqrt{2}-1$.

Zadatak 132 (Jasenska, srednja škola)

Izračunajte $\frac{(\sqrt{8}-3)+(1-\sqrt{2})}{(\sqrt{2}-2)+\sqrt{18}}$.

Rješenje 132

Ponovimo!

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \quad , \quad (\sqrt{a})^2 = a \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$\begin{aligned} \frac{(\sqrt{8}-3)+(1-\sqrt{2})}{(\sqrt{2}-2)+\sqrt{18}} &= \left[\begin{array}{l} \text{riješimo} \\ \text{se zagrada} \end{array} \right] = \frac{\sqrt{8}-3+1-\sqrt{2}}{\sqrt{2}-2+\sqrt{18}} = \left[\begin{array}{l} \text{djelomično} \\ \text{korjenujemo} \end{array} \right] = \frac{\sqrt{4 \cdot 2}-2-\sqrt{2}}{\sqrt{2}-2+\sqrt{9 \cdot 2}} = \frac{\sqrt{4} \cdot \sqrt{2}-2-\sqrt{2}}{\sqrt{2}-2+\sqrt{9} \cdot \sqrt{2}} = \\ &= \frac{2 \cdot \sqrt{2}-2-\sqrt{2}}{\sqrt{2}-2+3 \cdot \sqrt{2}} = \frac{\sqrt{2}-2}{4 \cdot \sqrt{2}-2} = \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \frac{\sqrt{2}-2}{4 \cdot \sqrt{2}-2} \cdot \frac{4 \cdot \sqrt{2}+2}{4 \cdot \sqrt{2}+2} = \frac{4 \cdot (\sqrt{2})^2 + 2 \cdot \sqrt{2} - 8 \cdot \sqrt{2} - 4}{(4 \cdot \sqrt{2})^2 - 2^2} = \\ &= \frac{4 \cdot 2 + 2 \cdot \sqrt{2} - 8 \cdot \sqrt{2} - 4}{16 \cdot 2 - 4} = \frac{8 - 6 \cdot \sqrt{2} - 4}{32 - 4} = \frac{4 - 6 \cdot \sqrt{2}}{28} = \frac{2 \cdot (2 - 3 \cdot \sqrt{2})}{28} = \frac{2 - 3 \cdot \sqrt{2}}{14}. \end{aligned}$$

Vježba 132

Izračunajte $\frac{1}{\sqrt{2}-1}$.

Rezultat: $\sqrt{2}+1$.

Zadatak 133 (Jasenska, srednja škola)

Izračunajte $(\sqrt{3}-1)^2 \cdot (4+2 \cdot \sqrt{3})$.

Rješenje 133

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \quad , \quad (\sqrt{a})^2 = a \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$\begin{aligned} (\sqrt{3}-1)^2 \cdot (4+2 \cdot \sqrt{3}) &= \left((\sqrt{3})^2 - 2 \cdot \sqrt{3} \cdot 1 + 1^2 \right) \cdot (4+2 \cdot \sqrt{3}) = (3-2 \cdot \sqrt{3}+1) \cdot (4+2 \cdot \sqrt{3}) = \\ &= (4-2 \cdot \sqrt{3}) \cdot (4+2 \cdot \sqrt{3}) = \left[\begin{array}{l} \text{razlika} \\ \text{kvadrata} \end{array} \right] = 4^2 - (2 \cdot \sqrt{3})^2 = 16 - 4 \cdot 3 = 16 - 12 = 4. \end{aligned}$$

Vježba 133

Izračunajte $(\sqrt{2}-1)^2 \cdot (3+2 \cdot \sqrt{2})$.

Rezultat: 1.

Zadatak 134 (Jasenska, srednja škola)

Izračunajte $\frac{\sqrt{3}}{2 \cdot \sqrt{3} - 3}$.

Rješenje 134

Ponovimo!

$$(a-b) \cdot (a+b) = a^2 - b^2 \quad , \quad (\sqrt{a})^2 = a.$$

1. inačica

$$\begin{aligned} \frac{\sqrt{3}}{2 \cdot \sqrt{3} - 3} &= \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \frac{\sqrt{3}}{2 \cdot \sqrt{3} - 3} \cdot \frac{2 \cdot \sqrt{3} + 3}{2 \cdot \sqrt{3} + 3} = \frac{\sqrt{3} \cdot (2 \cdot \sqrt{3} + 3)}{(2 \cdot \sqrt{3})^2 - 3^2} = \frac{2 \cdot (\sqrt{3})^2 + 3 \cdot \sqrt{3}}{4 \cdot 3 - 9} = \frac{2 \cdot 3 + 3 \cdot \sqrt{3}}{12 - 9} = \\ &= \frac{6 + 3 \cdot \sqrt{3}}{3} = \frac{3 \cdot (2 + \sqrt{3})}{3} = 2 + \sqrt{3}. \end{aligned}$$

2. inačica

$$\begin{aligned} \frac{\sqrt{3}}{2 \cdot \sqrt{3} - 3} &= \frac{\sqrt{3}}{2 \cdot \sqrt{3} - (\sqrt{3})^2} = \frac{\sqrt{3}}{\sqrt{3} \cdot (2 - \sqrt{3})} = \frac{\sqrt{3}}{\sqrt{3} \cdot (2 - \sqrt{3})} = \frac{1}{2 - \sqrt{3}} = \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \\ &= \frac{1}{2 - \sqrt{3}} \cdot \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{2^2 - (\sqrt{3})^2} = \frac{2 + \sqrt{3}}{4 - 3} = \frac{2 + \sqrt{3}}{1} = 2 + \sqrt{3}. \end{aligned}$$

Vježba 134

Izračunajte $\frac{\sqrt{2}}{\sqrt{2} - 2}$.

Rezultat: $-(1 + \sqrt{2})$.

Zadatak 135 (Jasenska, srednja škola)

Izračunajte $(\sqrt{2 + \sqrt{3}} + \sqrt{2 - \sqrt{3}})^2$.

Rješenje 135

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad (\sqrt{a})^2 = a \quad , \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b} \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$\begin{aligned} (\sqrt{2 + \sqrt{3}} + \sqrt{2 - \sqrt{3}})^2 &= (\sqrt{2 + \sqrt{3}})^2 + 2 \cdot \sqrt{2 + \sqrt{3}} \cdot \sqrt{2 - \sqrt{3}} + (\sqrt{2 - \sqrt{3}})^2 = \\ &= 2 + \sqrt{3} + 2 \cdot \sqrt{(2 + \sqrt{3}) \cdot (2 - \sqrt{3})} + 2 - \sqrt{3} = 2 + \sqrt{3} + 2 \cdot \sqrt{(2 + \sqrt{3}) \cdot (2 - \sqrt{3})} + 2 - \sqrt{3} = \\ &= 4 + 2 \cdot \sqrt{(2 + \sqrt{3}) \cdot (2 - \sqrt{3})} = 4 + 2 \cdot \sqrt{2^2 - (\sqrt{3})^2} = 4 + 2 \cdot \sqrt{4 - 3} = 4 + 2 \cdot \sqrt{1} = 4 + 2 \cdot 1 = 6. \end{aligned}$$

Vježba 135

Izračunajte $(\sqrt{3 + \sqrt{5}} - \sqrt{3 - \sqrt{5}})^2$.

Rezultat: 2.

Zadatak 136 (Jasenska, srednja škola)

Izračunajte $\frac{1}{2 - \sqrt{2} + \sqrt{3} - \sqrt{6}}$.

Rješenje 136

Ponovimo!

$$(\sqrt{a})^2 = a \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$\begin{aligned} \frac{1}{2-\sqrt{2}+\sqrt{3}-\sqrt{6}} &= \frac{1}{(\sqrt{2})^2 - \sqrt{2} + \sqrt{3} - \sqrt{6}} = \frac{1}{(\sqrt{2})^2 - \sqrt{2} + \sqrt{3} - \sqrt{3} \cdot 2} = \frac{1}{\sqrt{2} \cdot (\sqrt{2}-1) - \sqrt{3} \cdot (\sqrt{2}-1)} = \\ &= \frac{1}{(\sqrt{2}-1) \cdot (\sqrt{2}-\sqrt{3})} = \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \frac{1}{(\sqrt{2}-1) \cdot (\sqrt{2}-\sqrt{3})} \cdot \frac{(\sqrt{2}+1) \cdot (\sqrt{2}+\sqrt{3})}{(\sqrt{2}+1) \cdot (\sqrt{2}+\sqrt{3})} = \\ &= \frac{(\sqrt{2}+1) \cdot (\sqrt{2}+\sqrt{3})}{\left((\sqrt{2})^2 - 1 \right) \cdot \left((\sqrt{2})^2 - (\sqrt{3})^2 \right)} = \frac{(\sqrt{2}+1) \cdot (\sqrt{2}+\sqrt{3})}{(2-1) \cdot (2-3)} = \frac{(\sqrt{2}+1) \cdot (\sqrt{2}+\sqrt{3})}{1 \cdot (-1)} = -(\sqrt{2}+1) \cdot (\sqrt{2}+\sqrt{3}). \end{aligned}$$

Vježba 136

Izračunajte $\frac{1}{\sqrt{6}-\sqrt{3}}$.

Rezultat: $\frac{\sqrt{6}+\sqrt{3}}{3}$.

Zadatak 137 (Jasenka, srednja škola)

Izračunajte $\frac{\sqrt{x}+1}{1+\sqrt{x}+x} : \frac{1}{x^2-\sqrt{x}}$.

Rješenje 137

Ponovimo!

$$a^3 - b^3 = (a-b) \cdot (a^2 + a \cdot b + b^2) \quad , \quad a^2 = (\sqrt{a})^4 \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

$$\begin{aligned} \frac{\sqrt{x}+1}{1+\sqrt{x}+x} : \frac{1}{x^2-\sqrt{x}} &= \frac{\sqrt{x}+1}{1+\sqrt{x}+x} \cdot \frac{x^2-\sqrt{x}}{1} = \frac{\sqrt{x}+1}{1+\sqrt{x}+x} \cdot \frac{(\sqrt{x})^4 - \sqrt{x}}{1} = \frac{\sqrt{x}+1}{1+\sqrt{x}+x} \cdot \frac{\sqrt{x} \cdot \left((\sqrt{x})^3 - 1 \right)}{1} = \\ &= \frac{\sqrt{x}+1}{1+\sqrt{x}+x} \cdot \frac{\sqrt{x} \cdot (\sqrt{x}-1) \left((\sqrt{x})^2 + \sqrt{x} \cdot 1 + 1^2 \right)}{1} = \frac{\sqrt{x}+1}{1+\sqrt{x}+x} \cdot \frac{\sqrt{x} \cdot (\sqrt{x}-1) (x + \sqrt{x} + 1)}{1} = \\ &= \frac{\sqrt{x}+1}{1+\sqrt{x}+x} \cdot \frac{\sqrt{x} \cdot (\sqrt{x}-1) (x + \sqrt{x} + 1)}{1} = \sqrt{x} \cdot (\sqrt{x}+1) \cdot (\sqrt{x}-1) = \sqrt{x} \cdot \left((\sqrt{x})^2 - 1 \right) = \sqrt{x} \cdot (x-1). \end{aligned}$$

Vježba 137

Izračunajte $(x^2 - \sqrt{x}) : \frac{1 + \sqrt{x} + x}{\sqrt{x} + 1}$.

Rezultat: $\sqrt{x} \cdot (x-1)$.

Zadatak 138 (Jasenka, srednja škola)

Izračunajte $\frac{36}{(3 \cdot \sqrt{2} - 2 \cdot \sqrt{3})^2}$.

Rješenje 138

Ponovimo!

$$\begin{aligned}
 (a-b)^2 &= a^2 - 2 \cdot a \cdot b + b^2, \quad (\sqrt{a})^2 = a, \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}, \quad (a-b) \cdot (a+b) = a^2 - b^2. \\
 \frac{36}{(3 \cdot \sqrt{2} - 2 \cdot \sqrt{3})^2} &= \frac{36}{(3 \cdot \sqrt{2})^2 - 2 \cdot 3 \cdot \sqrt{2} \cdot 2 \cdot \sqrt{3} + (2 \cdot \sqrt{3})^2} = \frac{36}{9 \cdot 2 - 12 \cdot \sqrt{6} + 4 \cdot 3} = \frac{36}{18 - 12 \cdot \sqrt{6} + 12} = \\
 &= \frac{36}{30 - 12 \cdot \sqrt{6}} = \frac{36}{6 \cdot (5 - 2 \cdot \sqrt{6})} = \frac{36}{6 \cdot (5 - 2 \cdot \sqrt{6})} = \frac{6}{5 - 2 \cdot \sqrt{6}} \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] = \frac{6}{5 - 2 \cdot \sqrt{6}} \cdot \frac{5 + 2 \cdot \sqrt{6}}{5 + 2 \cdot \sqrt{6}} = \\
 &= \frac{6 \cdot (5 + 2 \cdot \sqrt{6})}{5^2 - (2 \cdot \sqrt{6})^2} = \frac{6 \cdot (5 + 2 \cdot \sqrt{6})}{25 - 4 \cdot 6} = \frac{6 \cdot (5 + 2 \cdot \sqrt{6})}{25 - 24} = \frac{6 \cdot (5 + 2 \cdot \sqrt{6})}{1} = 6 \cdot (5 + 2 \cdot \sqrt{6}).
 \end{aligned}$$

Vježba 138

Izračunajte $\frac{6}{(3 \cdot \sqrt{2} - 2 \cdot \sqrt{3})^2}$.

Rezultat: $5 + 2 \cdot \sqrt{6}$.

Zadatak 139 (Jasenka, srednja škola)

Koliko je $x^2 - x \cdot y + y^2$ za $x = \frac{1}{\sqrt{3} - \sqrt{2}}$, $y = \frac{1}{\sqrt{3} + \sqrt{2}}$.

Rješenje 139

Ponovimo!

$$(a \pm b)^2 = a^2 \pm 2 \cdot a \cdot b + b^2, \quad (\sqrt{a})^2 = a, \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}, \quad (a-b) \cdot (a+b) = a^2 - b^2.$$

1. inačica

Posebno izračunamo x i y, a zatim njihove vrijednosti uvrstimo u zadani izraz:

$$\begin{aligned}
 \left. \begin{array}{l} x = \frac{1}{\sqrt{3} - \sqrt{2}} \\ y = \frac{1}{\sqrt{3} + \sqrt{2}} \end{array} \right\} &\Rightarrow \left[\begin{array}{l} \text{racionalizacija} \\ \text{nazivnika} \end{array} \right] \Rightarrow \left. \begin{array}{l} x = \frac{1}{\sqrt{3} - \sqrt{2}} \cdot \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} \\ y = \frac{1}{\sqrt{3} + \sqrt{2}} \cdot \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \frac{\sqrt{3} + \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} \\ y = \frac{\sqrt{3} - \sqrt{2}}{(\sqrt{3})^2 - (\sqrt{2})^2} \end{array} \right\} \Rightarrow \\
 &\Rightarrow \left. \begin{array}{l} x = \frac{\sqrt{3} + \sqrt{2}}{3 - 2} \\ y = \frac{\sqrt{3} - \sqrt{2}}{3 - 2} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \frac{\sqrt{3} + \sqrt{2}}{1} \\ y = \frac{\sqrt{3} - \sqrt{2}}{1} \end{array} \right\} \Rightarrow \left. \begin{array}{l} x = \sqrt{3} + \sqrt{2} \\ y = \sqrt{3} - \sqrt{2} \end{array} \right\}.
 \end{aligned}$$

Vrijednost izraza iznosi:

$$\begin{aligned}
 x^2 - x \cdot y + y^2 &= (\sqrt{3} + \sqrt{2})^2 - (\sqrt{3} + \sqrt{2}) \cdot (\sqrt{3} - \sqrt{2}) + (\sqrt{3} - \sqrt{2})^2 = \\
 &= (\sqrt{3})^2 + 2 \cdot \sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^2 - \left((\sqrt{3})^2 - (\sqrt{2})^2 \right) + (\sqrt{3})^2 - 2 \cdot \sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^2 = \\
 &= 3 + 2 \cdot \sqrt{6} + 2 - (3 - 2) + 3 - 2 \cdot \sqrt{6} + 2 = 3 + 2 \cdot \sqrt{6} + 2 - (3 - 2) + 3 - 2 \cdot \sqrt{6} + 2 = 3 + 2 - 1 + 3 + 2 = 9.
 \end{aligned}$$

2. inačica

Odmah uvrstimo vrijednosti za x i y u zadani izraz:

$$\left. \begin{array}{l} x = \frac{1}{\sqrt{3} - \sqrt{2}}, \quad y = \frac{1}{\sqrt{3} + \sqrt{2}} \\ x^2 - x \cdot y + y^2 = ? \end{array} \right\} \Rightarrow \left(\frac{1}{\sqrt{3} - \sqrt{2}} \right)^2 - \frac{1}{\sqrt{3} - \sqrt{2}} \cdot \frac{1}{\sqrt{3} + \sqrt{2}} + \left(\frac{1}{\sqrt{3} + \sqrt{2}} \right)^2 =$$

$$\begin{aligned}
&= \frac{1}{(\sqrt{3}-\sqrt{2})^2} - \frac{1}{(\sqrt{3}-\sqrt{2}) \cdot (\sqrt{3}+\sqrt{2})} + \frac{1}{(\sqrt{3}+\sqrt{2})^2} = \\
&= \frac{1}{(\sqrt{3})^2 - 2 \cdot \sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^2} - \frac{1}{(\sqrt{3})^2 - (\sqrt{2})^2} + \frac{1}{(\sqrt{3})^2 + 2 \cdot \sqrt{3} \cdot \sqrt{2} + (\sqrt{2})^2} = \\
&= \frac{1}{3-2\sqrt{6}+2} - \frac{1}{3-2} + \frac{1}{3+2\sqrt{6}+2} = \frac{1}{5-2\sqrt{6}} - \frac{1}{1} + \frac{1}{5+2\sqrt{6}} \left[\begin{array}{l} \text{zbrajanje} \\ \text{razlomaka} \end{array} \right] = \\
&= \frac{5+2\sqrt{6} - (5-2\sqrt{6}) \cdot (5+2\sqrt{6}) + 5-2\sqrt{6}}{(5-2\sqrt{6}) \cdot (5+2\sqrt{6})} = \frac{5+2\sqrt{6} - (5^2 - (2\sqrt{6})^2) + 5-2\sqrt{6}}{5^2 - (2\sqrt{6})^2} = \\
&= \frac{5 - (25 - 4 \cdot 6) + 5}{25 - 4 \cdot 6} = \frac{5 - (25 - 24) + 5}{25 - 24} = \frac{5 - 1 + 5}{1} = 9.
\end{aligned}$$

Vježba 139

Koliko je $x^2 + y^2$ za $x = \frac{1}{\sqrt{2}-1}$, $y = \frac{1}{\sqrt{2}+1}$.

Rezultat: 6.

Zadatak 140 (Maturant, gimnazija)

Nadite vrijednost algebarskog izraza $\left(x + \frac{x \cdot y + y^2}{x + y}\right) \cdot \left(\frac{a^3 + 8}{x^2 - y^2} : \frac{a^2 - 2 \cdot a + 4}{x - y}\right)$ za $|x| \neq |y|$.

Rješenje 140

Ponovimo!

$$a^3 + b^3 = (a+b) \cdot (a^2 - a \cdot b + b^2), \quad a^2 - b^2 = (a-b) \cdot (a+b), \quad a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2.$$

$$\begin{aligned}
&\left(x + \frac{x \cdot y + y^2}{x + y}\right) \cdot \left(\frac{a^3 + 8}{x^2 - y^2} : \frac{a^2 - 2 \cdot a + 4}{x - y}\right) = \frac{x \cdot (x + y) + x \cdot y + y^2}{x + y} \cdot \left(\frac{(a+2) \cdot (x^2 - 2 \cdot a + 4)}{(x-y) \cdot (x+y)} : \frac{a^2 - 2 \cdot a + 4}{x - y}\right) = \\
&= \frac{x \cdot (x + y) + x \cdot y + y^2}{x + y} \cdot \frac{(a+2) \cdot (x^2 - 2 \cdot a + 4)}{(x-y) \cdot (x+y)} \cdot \frac{x-y}{a^2 - 2 \cdot a + 4} = \frac{x^2 + x \cdot y + x \cdot y + y^2}{x + y} \cdot \frac{a+2}{x+y} \cdot 1 = \\
&= \frac{x^2 + 2 \cdot x \cdot y + y^2}{x + y} \cdot \frac{a+2}{x+y} = \frac{(x+y)^2}{(x+y)^2} \cdot (a+2) = a+2.
\end{aligned}$$

Vježba 140

Nadite vrijednost algebarskog izraza $\frac{a^3 + 8}{x^2 - y^2} : \frac{a^2 - 2 \cdot a + 4}{x - y}$ za $|x| \neq |y|$.

Rezultat: $\frac{a+2}{x+y}$.