Zadatak 081 (Ivan, strojarska škola)

Koliko iznosi $\sqrt{2}\,\%$ od $\left(\sqrt{5}-\sqrt{3}\right)\cdot\sqrt{4+\sqrt{15}}$?

Rješenje 081

$$\sqrt{2}\,\%\cdot\left(\sqrt{5}-\sqrt{3}\right)\cdot\sqrt{4+\sqrt{15}}=\frac{\sqrt{2}}{100}\cdot\sqrt{\left(\sqrt{5}-\sqrt{3}\right)^2\cdot\left(4+\sqrt{15}\right)}=\frac{\sqrt{2}}{100}\cdot\sqrt{\left(5-2\cdot\sqrt{15}+3\right)\cdot\left(4+\sqrt{15}\right)}=$$

$$=\frac{\sqrt{2}}{100}\cdot\sqrt{\left(8-2\cdot\sqrt{15}\right)\cdot\left(4+\sqrt{15}\right)}=\frac{\sqrt{2}}{100}\cdot\sqrt{2\cdot\underbrace{\left(4-\sqrt{15}\right)\cdot\left(4+\sqrt{15}\right)}_{\text{razlika kvadrata}}}=\frac{\sqrt{2}}{100}\cdot\sqrt{2\cdot(16-15)}=\frac{\sqrt{2}}{100}\cdot\sqrt{2\cdot1}=$$

$$=\frac{\sqrt{2}}{100}\cdot\sqrt{2}=\frac{\left(\sqrt{2}\right)^2}{100}=\frac{2}{100}=0.02.$$

Vježba 081

Koliko iznosi $\sqrt{8}\,\%$ od $\left(\sqrt{5}-\sqrt{3}\right)\cdot\sqrt{4+\sqrt{15}}$?

Rezultat: 0.04.

Zadatak 082 (Viki, komercijalna škola)

Pojednostavnite: $\dfrac{a^4+1-2\cdot a^2}{1-a-a^2+a^3}$.

Rješenje 082

$$\frac{a^4+1-2\cdot a^2}{1-a-a^2+a^3}=\frac{a^4-2\cdot a^2+1}{a^3-a^2-a+1}=\frac{\left(a^2-1\right)^2}{a^2\cdot(a-1)-(a-1)}=\frac{\left(a^2-1\right)^2}{(a-1)\cdot\left(a^2-1\right)}=\frac{\left(a^2-1\right)\cdot\left(a^2-1\right)}{(a-1)\cdot\left(a^2-1\right)}=$$

$$=\frac{a^2-1}{a-1}=\frac{(a-1)\cdot(a+1)}{a-1}=a+1.$$

Vježba 082

Pojednostavnite: $\dfrac{a^4+1-2\cdot a^2}{a^3+a^2-a-1}$.

Rezultat: a – 1.

Zadatak 083 (Had, hotelijerska škola)

Ako je $\dfrac{a}{b}+\dfrac{b}{a}=3$, koliko je $\dfrac{a^3}{b^3}+\dfrac{b^3}{a^3}$?

Rješenje 083

Ponovimo!

$$(x+y)^3=x^3+3\cdot x^2\cdot y+3\cdot x\cdot y^2+y^3\qquad,\qquad\left(\frac{x}{y}\right)^n=\frac{x^n}{y^n}$$

$$\frac{a}{b}+\frac{b}{a}=3\;\Rightarrow\;\frac{a}{b}+\frac{b}{a}=3\;/^3\;\Rightarrow\;\left(\frac{a}{b}+\frac{b}{a}\right)^3=3^3\;\Rightarrow\;\left(\frac{a}{b}\right)^3+3\cdot\left(\frac{a}{b}\right)^2\cdot\frac{b}{a}+3\cdot\frac{a}{b}\cdot\left(\frac{b}{a}\right)^2+\left(\frac{b}{a}\right)^3=27\;\Rightarrow$$

$$\Rightarrow\;\frac{a^3}{b^3}+3\cdot\frac{a^2}{b^2}\cdot\frac{b}{a}+3\cdot\frac{a}{b}\cdot\frac{b^2}{a^2}+\frac{b^3}{a^3}=27\;\Rightarrow\;\frac{a^3}{b^3}+3\cdot\frac{a}{b}+3\cdot\frac{b}{a}+\frac{b^3}{a^3}=27\;\Rightarrow\;\frac{a^3}{b^3}+\frac{b^3}{a^3}+3\cdot\left(\frac{a}{b}+\frac{b}{a}\right)=27\;\Rightarrow$$

$$\Rightarrow\;\frac{a^3}{b^3}+\frac{b^3}{a^3}+3\cdot3=27\;\Rightarrow\;\frac{a^3}{b^3}+\frac{b^3}{a^3}=18.$$

Vježba 083

Ako je $\frac{a}{b}+\frac{b}{a}=4$, koliko je $\frac{a^3}{b^3}+\frac{b^3}{a^3}$?

Rezultat: 15.

Zadatak 084 (Had, hotelijerska škola)

Izračunaj: $\sqrt[4]{17+12\cdot\sqrt{2}}$.

Rješenje 084

Ponovimo!

$$a^2+2\cdot a\cdot b+b^2=(a+b)^2$$

$$\sqrt[4]{17+12\cdot\sqrt{2}}=\sqrt[4]{9+12\cdot\sqrt{2}+8}=\sqrt[4]{3^2+2\cdot3\cdot2\cdot\sqrt{2}+\left(2\cdot\sqrt{2}\right)^2}=\sqrt[4]{\left(3+2\cdot\sqrt{2}\right)^2}=\sqrt{3+2\cdot\sqrt{2}}=$$

$$=\sqrt{2+2\cdot\sqrt{2}\cdot1+1}=\sqrt{\left(\sqrt{2}\right)^2+2\cdot\sqrt{2}\cdot1+1^2}=\sqrt{\left(\sqrt{2}+1\right)^2}=\sqrt{2}+1.$$

Vježba 084

Izračunaj: $\sqrt{4+2\cdot\sqrt{3}}$.

Rezultat: $\sqrt{3}+1$.

Zadatak 085 (1A, hotelijerska škola)

Izračunaj $\frac{(-2)^{-3}}{(-0.2)^3}-\left(\frac{2}{5}\right)^{-3}\cdot(-2)^{-2}\cdot0.5^{-2}$.

Rješenje 085

$$\frac{(-2)^{-3}}{(-0.2)^3}-\left(\frac{2}{5}\right)^{-3}\cdot(-2)^{-2}\cdot0.5^{-2}=\frac{\left(-\frac{1}{2}\right)^3}{\left(-\frac{2}{10}\right)^3}-\left(\frac{5}{2}\right)^3\cdot\left(-\frac{1}{2}\right)^2\cdot\left(\frac{5}{10}\right)^{-2}=\frac{\left(-\frac{1}{2}\right)^3}{\left(-\frac{1}{5}\right)^3}-\left(\frac{5}{2}\right)^3\cdot\left(-\frac{1}{2}\right)^2\cdot\left(\frac{1}{2}\right)^{-2}=$$

$$=\frac{-\frac{1}{8}}{-\frac{1}{125}}-\frac{125}{8}\cdot\frac{1}{4}\cdot2^2=\frac{125}{8}-\frac{125}{8}\cdot\frac{1}{4}\cdot4=\frac{125}{8}-\frac{125}{8}=0.$$

Vježba 085

Izračunaj $\frac{(-2)^{-3}}{(-0.2)^3}+\left(\frac{2}{5}\right)^{-3}\cdot(-2)^{-2}\cdot0.5^{-2}$.

Rezultat: 31.25.

Zadatak 086 (1A, hotelijerska škola)

Pojednostavni: $\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(a^{-1}+b^{-1}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)$.

Rješenje 086

$$\left(\sqrt{a}+\sqrt{b}\right)^{-2}\cdot\left(a^{-1}+b^{-1}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^2}\cdot\left(\frac{1}{a}+\frac{1}{b}\right)+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^3}\cdot\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)=$$

$$=\frac{1}{\left(\sqrt{a}+\sqrt{b}\right)^{2}}\cdot\frac{b+a}{a\cdot b}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^{3}}\cdot\frac{\sqrt{b}+\sqrt{a}}{\sqrt{a}\cdot\sqrt{b}}=\frac{a+b}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^{3}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a\cdot b}}=$$

$$=\frac{a+b}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}+\frac{2}{\left(\sqrt{a}+\sqrt{b}\right)^{2}}\cdot\frac{1}{\sqrt{a\cdot b}}=\frac{a+b}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}+\frac{2}{\sqrt{a\cdot b}\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=\left[a\cdot b=\left(\sqrt{a\cdot b}\right)^{2}\right]=$$

$$=\frac{a+b+2\cdot\sqrt{a\cdot b}}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=\frac{\left(\sqrt{a}\right)^{2}+2\cdot\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^{2}}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=\frac{\left(\sqrt{a}+\sqrt{b}\right)^{2}}{a\cdot b\cdot\left(\sqrt{a}+\sqrt{b}\right)^{2}}=\frac{1}{a\cdot b}.$$

Vježba 086

Pojednostavni: $\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}\right)\cdot\frac{a\cdot b}{\sqrt{a}+\sqrt{b}}.$

Rezultat: $\sqrt{a\cdot b}.$

Zadatak 087 (Viky, gimnazija)

Pojednostavni izraz: $\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}.$

Rješenje 087

Budimo kreativni, budimo maštoviti! Riješimo zadatak na 5 načina!

1.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=\frac{(a-b)\cdot\left(\sqrt{a}+\sqrt{b}\right)-(a-b)\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}=$$

$$=\frac{a\cdot\sqrt{a}+a\cdot\sqrt{b}-b\cdot\sqrt{a}-b\cdot\sqrt{b}-\left(a\cdot\sqrt{a}-a\cdot\sqrt{b}-b\cdot\sqrt{a}+b\cdot\sqrt{b}\right)}{\left(\sqrt{a}\right)^{2}-\left(\sqrt{b}\right)^{2}}=$$

$$=\frac{a\cdot\sqrt{a}+a\cdot\sqrt{b}-b\cdot\sqrt{a}-b\cdot\sqrt{b}-a\cdot\sqrt{a}+a\cdot\sqrt{b}+b\cdot\sqrt{a}-b\cdot\sqrt{b}}{a-b}=\frac{2\cdot a\cdot\sqrt{b}-2\cdot b\cdot\sqrt{b}}{a-b}=\frac{2\cdot\sqrt{b}\cdot(a-b)}{a-b}=2\cdot\sqrt{b}.$$

2.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=\frac{(a-b)\cdot\left(\sqrt{a}+\sqrt{b}\right)-(a-b)\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}=\frac{(a-b)\cdot\left[\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}\right]}{\left(\sqrt{a}\right)^{2}-\left(\sqrt{b}\right)^{2}}=$$

$$=\frac{(a-b)\cdot 2\cdot\sqrt{b}}{a-b}=2\cdot\sqrt{b}.$$

3.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=(a-b)\cdot\left[\frac{1}{\sqrt{a}-\sqrt{b}}-\frac{1}{\sqrt{a}+\sqrt{b}}\right]=(a-b)\cdot\frac{\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}=$$

$$=(a-b)\cdot\frac{2\cdot\sqrt{b}}{\left(\sqrt{a}\right)^{2}-\left(\sqrt{b}\right)^{2}}=(a-b)\cdot\frac{2\cdot\sqrt{b}}{a-b}=2\cdot\sqrt{b}.$$

4.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=\frac{\left(\sqrt{a}\right)^{2}-\left(\sqrt{b}\right)^{2}}{\sqrt{a}-\sqrt{b}}-\frac{\left(\sqrt{a}\right)^{2}-\left(\sqrt{b}\right)^{2}}{\sqrt{a}+\sqrt{b}}=$$

$$=\frac{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}-\sqrt{b}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}+\sqrt{b}}=\sqrt{a}+\sqrt{b}-\left(\sqrt{a}-\sqrt{b}\right)=\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}=2\cdot\sqrt{b}.$$

5.inačica

$$\frac{a-b}{\sqrt{a}-\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}=\frac{a-b}{\sqrt{a}-\sqrt{b}}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}+\sqrt{b}}-\frac{a-b}{\sqrt{a}+\sqrt{b}}\cdot\frac{\sqrt{a}-\sqrt{b}}{\sqrt{a}-\sqrt{b}}=\frac{(a-b)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}-\frac{(a-b)\cdot\left(\sqrt{a}-\sqrt{b}\right)}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}=$$

$$=\frac{(a-b)\cdot\left(\sqrt{a}+\sqrt{b}\right)}{a-b}-\frac{(a-b)\cdot\left(\sqrt{a}-\sqrt{b}\right)}{a-b}=\sqrt{a}+\sqrt{b}-\left(\sqrt{a}-\sqrt{b}\right)=\sqrt{a}+\sqrt{b}-\sqrt{a}+\sqrt{b}=2\cdot\sqrt{b}.$$

Vježba 087

Pojednostavni izraz: $\frac{a-b}{\sqrt{a}-\sqrt{b}}+\frac{a-b}{\sqrt{a}+\sqrt{b}}.$

Rezultat: $2\cdot\sqrt{a}.$

Zadatak 088 (Mira, gimnazija)

Pojednostavni izraz: $\frac{a^3\cdot b^{-1}-a^{-1}\cdot b^3}{a\cdot b^{-1}+a^{-1}\cdot b}\cdot\left(\frac{a^2-b^2}{a\cdot b}\right)^{-1}$, za $a\cdot b\neq 0, |a|\neq|b|.$

Rješenje 088

1.inačica

$$\frac{a^3\cdot b^{-1}-a^{-1}\cdot b^3}{a\cdot b^{-1}+a^{-1}\cdot b}\cdot\left(\frac{a^2-b^2}{a\cdot b}\right)^{-1}=\frac{\frac{a^3}{b}-\frac{b^3}{a}}{\frac{a}{b}+\frac{b}{a}}\cdot\frac{a\cdot b}{a^2-b^2}=\frac{\frac{a^4-b^4}{a\cdot b}}{\frac{a^2+b^2}{a\cdot b}}\cdot\frac{a\cdot b}{a^2-b^2}=$$

$$=\frac{a^4-b^4}{a^2+b^2}\cdot\frac{a\cdot b}{a^2-b^2}=\frac{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)}{a^2+b^2}\cdot\frac{a\cdot b}{a^2-b^2}=a\cdot b.$$

2.inačica

$$\frac{a^3\cdot b^{-1}-a^{-1}\cdot b^3}{a\cdot b^{-1}+a^{-1}\cdot b}\cdot\left(\frac{a^2-b^2}{a\cdot b}\right)^{-1}=\left[\text{prvi razlomak proširujemo s } a\cdot b\right]=$$

$$=\frac{\left(a^3\cdot b^{-1}-a^{-1}\cdot b^3\right)\cdot a\cdot b}{\left(a\cdot b^{-1}+a^{-1}\cdot b\right)\cdot a\cdot b}\cdot\left(\frac{a^2-b^2}{a\cdot b}\right)^{-1}=\frac{\left(a^3\cdot b^{-1}-a^{-1}\cdot b^3\right)\cdot a\cdot b}{\left(a\cdot b^{-1}+a^{-1}\cdot b\right)\cdot a\cdot b}\cdot\frac{a\cdot b}{a^2-b^2}=\frac{a^4-b^4}{a^2+b^2}\cdot\frac{a\cdot b}{a^2-b^2}=$$

$$=\frac{\left(a^2-b^2\right)\cdot\left(a^2+b^2\right)}{a^2+b^2}\cdot\frac{a\cdot b}{a^2-b^2}=a\cdot b.$$

Vježba 088

Pojednostavni izraz: $\frac{a^3\cdot b^{-1}-a^{-1}\cdot b^3}{a\cdot b^{-1}+a^{-1}\cdot b}\cdot\frac{1}{a^2-b^2}$, za $a\cdot b\neq 0, |a|\neq|b|.$

Rezultat: 1.

Zadatak 089 (Mirna, komercijalna škola)

Napiši u obliku potencije s bazom 2: $\frac{8^{n+2}\cdot 4^{n-2}}{16^{n-1}\cdot 2^{n+5}}.$

Rješenje 089

Ponovimo!

$$\left(a^n\right)^m = a^{n\cdot m} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \frac{a^n}{a^m} = a^n : a^m = a^{n-m}$$

$$\frac{8^{n+2}\cdot 4^{n-2}}{16^{n-1}\cdot 2^{n+5}} = \frac{\left(2^3\right)^{n+2}\cdot\left(2^2\right)^{n-2}}{\left(2^4\right)^{n-1}\cdot 2^{n+5}} = \frac{2^{3\cdot n+6}\cdot 2^{2\cdot n-4}}{2^{4\cdot n-4}\cdot 2^{n+5}} = \frac{2^{3\cdot n+6+2\cdot n-4}}{2^{4\cdot n-4+n+5}} = \frac{2^{5\cdot n+2}}{2^{5\cdot n+1}} =$$

$$= 2^{5\cdot n+2} : 2^{5\cdot n+1} = 2^{5\cdot n+2-(5\cdot n+1)} = 2^{5\cdot n+2-5\cdot n-1} = 2^1 = 2.$$

Vježba 089

Napiši u obliku potencije s bazom 2: $\frac{16^{n-1}\cdot 2^{n+5}}{8^{n+2}\cdot 4^{n-2}}$.

Rezultat: 2^{-1}.

Zadatak 090 (Vedrana, gimnazija)

Dokaži ako je $x = \frac{a-b}{a+b}$, $y = \frac{b-c}{b+c}$, $z = \frac{c-a}{c+a}$ onda je $(1+x)\cdot(1+y)\cdot(1+z) = (1-x)\cdot(1-y)\cdot(1-z)$.

Rješenje 090

1.inačica

Izračunat ćemo posebno lijevu stranu, a posebno desnu stranu jednakosti i usporediti ih:

$$\left.\begin{array}{l} L = (1+x)\cdot(1+y)\cdot(1+z) \\ D = (1-x)\cdot(1-y)\cdot(1-z) \end{array}\right\} \Rightarrow \left.\begin{array}{l} L = \left(1+\frac{a-b}{a+b}\right)\cdot\left(1+\frac{b-c}{b+c}\right)\cdot\left(1+\frac{c-a}{c+a}\right) \\ D = \left(1-\frac{a-b}{a+b}\right)\cdot\left(1-\frac{b-c}{b+c}\right)\cdot\left(1-\frac{c-a}{c+a}\right) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} L = \frac{a+b+a-b}{a+b}\cdot\frac{b+c+b-c}{b+c}\cdot\frac{c+a+c-a}{c+a} \\ D = \frac{a+b-a+b}{a+b}\cdot\frac{b+c-b+c}{b+c}\cdot\frac{c+a-c+a}{c+a} \end{array}\right\} \Rightarrow \left.\begin{array}{l} L = \frac{2\cdot a}{a+b}\cdot\frac{2\cdot b}{b+c}\cdot\frac{2\cdot c}{c+a} \\ D = \frac{2\cdot b}{a+b}\cdot\frac{2\cdot c}{b+c}\cdot\frac{2\cdot a}{c+a} \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} L = \frac{8\cdot a\cdot b\cdot c}{(a+b)\cdot(b+c)\cdot(c+a)} \\ D = \frac{8\cdot a\cdot b\cdot c}{(a+b)\cdot(b+c)\cdot(c+a)} \end{array}\right\} \Rightarrow L = D.$$

2.inačica

Neka je L = (1 + x) · (1 + y) · (1 + z) i D = (1 – x) · (1 – y) · (1 – z). Računamo omjer $\frac{L}{D}$:

$$\frac{L}{D} = \frac{(1+x)\cdot(1+y)\cdot(1+z)}{(1-x)\cdot(1-y)\cdot(1-z)} \Rightarrow \frac{L}{D} = \frac{\left(1+\frac{a-b}{a+b}\right)\cdot\left(1+\frac{b-c}{b+c}\right)\cdot\left(1+\frac{c-a}{c+a}\right)}{\left(1-\frac{a-b}{a+b}\right)\cdot\left(1-\frac{b-c}{b+c}\right)\cdot\left(1-\frac{c-a}{c+a}\right)} \Rightarrow$$

$$\Rightarrow \frac{L}{D} = \frac{\frac{a+b+a-b}{a+b}\cdot\frac{b+c+b-c}{b+c}\cdot\frac{c+a+c-a}{c+a}}{\frac{a+b-a+b}{a+b}\cdot\frac{b+c-b+c}{b+c}\cdot\frac{c+a-c+a}{c+a}} \Rightarrow \frac{L}{D} = \frac{\frac{2\cdot a}{a+b}\cdot\frac{2\cdot b}{b+c}\cdot\frac{2\cdot c}{c+a}}{\frac{2\cdot b}{a+b}\cdot\frac{2\cdot c}{b+c}\cdot\frac{2\cdot a}{c+a}} \Rightarrow$$

$$\frac{L}{D}=\frac{\frac{8\cdot a\cdot b\cdot c}{(a+b)\cdot(b+c)\cdot(c+a)}}{\frac{8\cdot a\cdot b\cdot c}{(a+b)\cdot(b+c)\cdot(c+a)}}\Rightarrow \frac{L}{D}=1\Rightarrow L=D.$$

Vježba 090

Dokaži ako je $x=a,\ y=\frac{1}{a}$ onda je $(1-x)\cdot(1+y)=y-x.$

Rezultat: Jednakost je točna.

Zadatak 091 (Vedrana, gimnazija)

Izračunaj: $\left[a\cdot(1-a)^{-\frac{2}{3}}+\frac{a^2}{(1-a)^{\frac{5}{3}}}\right]:\left[(1-a)^{\frac{1}{3}}\cdot(1-a)^{-2}\right].$

Rješenje 091

Ponovimo!

$$a^n\cdot a^m=a^{n+m}\quad,\quad \frac{1}{a^n}=a^{-n}\quad,\quad a:b^{-1}=a\cdot b\quad,\quad a^0=1$$

$$\left[a\cdot(1-a)^{-\frac{2}{3}}+\frac{a^2}{(1-a)^{\frac{5}{3}}}\right]:\left[(1-a)^{\frac{1}{3}}\cdot(1-a)^{-2}\right]=\left[a\cdot(1-a)^{-\frac{2}{3}}+a^2\cdot(1-a)^{-\frac{5}{3}}\right]:(1-a)^{-\frac{5}{3}}=$$

$$=\left[a\cdot(1-a)^{-\frac{2}{3}}+a^2\cdot(1-a)^{-\frac{5}{3}}\right]\cdot(1-a)^{\frac{5}{3}}=a\cdot(1-a)^{-\frac{2}{3}}\cdot(1-a)^{\frac{5}{3}}+a^2\cdot(1-a)^{-\frac{5}{3}}\cdot(1-a)^{\frac{5}{3}}=$$

$$=a\cdot(1-a)^1+a^2\cdot(1-a)^0=a\cdot(1-a)+a^2\cdot 1=a-a^2+a^2=a.$$

Vježba 091

Izračunaj: $\left[a\cdot(1-a)^2\right]\cdot\left[(1-a)^3:(1-a)^5\right].$

Rezultat: a.

Zadatak 092 (Gregor, gimnazija)

Koliko iznosi izraz: $\frac{a+\sqrt{a\cdot b}+b}{\sqrt{a^3}-\sqrt{b^3}}\cdot\frac{\sqrt{a^3}-\sqrt{a\cdot b^2}}{a+\sqrt{a\cdot b}}$?

Rješenje 092

Ponovimo!

$$\text{razlika kubova}\quad x^3-y^3=(x-y)\cdot\left(x^2+x\cdot y+y^2\right)$$

$$\text{razlika kvadrata}\quad x^2-y^2=(x-y)\cdot(x+y)$$

Računamo vrijednost izraza:

$$\frac{a+\sqrt{a\cdot b}+b}{\sqrt{a^3}-\sqrt{b^3}}\cdot\frac{\sqrt{a^3}-\sqrt{a\cdot b^2}}{a+\sqrt{a\cdot b}}=\frac{a+\sqrt{a\cdot b}+b}{\left(\sqrt{a}\right)^3-\left(\sqrt{b}\right)^3}\cdot\frac{\sqrt{a^2\cdot a}-\sqrt{a\cdot b^2}}{\left(\sqrt{a}\right)^2+\sqrt{a\cdot b}}=$$

$$=\frac{a+\sqrt{a\cdot b}+b}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left[\left(\sqrt{a}\right)^2+\sqrt{a}\cdot\sqrt{b}+\left(\sqrt{b}\right)^2\right]}\cdot\frac{a\cdot\sqrt{a}-b\cdot\sqrt{a}}{\left(\sqrt{a}\right)^2+\sqrt{a}\cdot\sqrt{b}}=$$

$$=\frac{a+\sqrt{a\cdot b}+b}{\left(\sqrt{a}-\sqrt{b}\right)\cdot\left[a+\sqrt{a\cdot b}+b\right]}\cdot\frac{\sqrt{a}\cdot(a-b)}{\sqrt{a}\cdot\left(\sqrt{a}+\sqrt{b}\right)}=\frac{1}{\sqrt{a}-\sqrt{b}}\cdot\frac{a-b}{\sqrt{a}+\sqrt{b}}=\frac{a-b}{\left(\sqrt{a}\right)^2-\left(\sqrt{b}\right)^2}=\frac{a-b}{a-b}=1.$$

Vježba 092

Koliko iznosi izraz: $\frac{a-\sqrt{a\cdot b}}{a-b}\cdot\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}}$?

Rezultat: 1.

Zadatak 093 (Brucoš, gimnazija)

Neka je $x^2 = 1 + x$. Ako je $x^{10} = a + b \cdot x$, koliko iznosi a + b?

Rješenje 093

Najprije riješimo kvadratnu jednadžbu:

$$x^2=1+x \Rightarrow x^2-x-1=0 \Rightarrow x_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+4}}{2} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{5}}{2}.$$

Budući da vrijednosti x_1 i x_2 treba uvrstiti u drugu jednadžbu $x^{10} = a + b \cdot x$, prvo izračunajmo $\left(\frac{1\pm\sqrt{5}}{2}\right)^{10}$:

- $\left(1\pm\sqrt{5}\right)^2=1\pm2\cdot\sqrt{5}+5=6\pm2\cdot\sqrt{5},$
- $\left(1\pm\sqrt{5}\right)^4=\left(\left(1\pm\sqrt{5}\right)^2\right)^2=\left(6\pm2\cdot\sqrt{5}\right)^2=36\pm24\cdot\sqrt{5}+20=56\pm24\cdot\sqrt{5},$
- $\left(1\pm\sqrt{5}\right)^8=\left(\left(1\pm\sqrt{5}\right)^4\right)^2=\left(56\pm24\cdot\sqrt{5}\right)^2=3136\pm2688\cdot\sqrt{5}+2880=6016\pm2688\cdot\sqrt{5},$
- $\left(1\pm\sqrt{5}\right)^{10}=\left(1\pm\sqrt{5}\right)^8\cdot\left(1\pm\sqrt{5}\right)^2=\left(6016\pm2688\cdot\sqrt{5}\right)\cdot\left(6\pm2\cdot\sqrt{5}\right)=62976\pm28160\cdot\sqrt{5}.$

Sada je:

$$\left(\frac{1\pm\sqrt{5}}{2}\right)^{10}=\frac{\left(1\pm\sqrt{5}\right)^{10}}{2^{10}}=\frac{62976\pm28160\cdot\sqrt{5}}{1024}=\frac{62976}{1024}\pm\frac{28160\cdot\sqrt{5}}{1024}=\frac{123}{2}\pm\frac{55\cdot\sqrt{5}}{2}.$$

Postavimo sustav jednadžbi s nepoznanicama a i b:

$$\left.\begin{array}{l} x=\frac{1+\sqrt{5}}{2} \Rightarrow x^{10}=\left(\frac{1+\sqrt{5}}{2}\right)^{10}=\frac{123}{2}+\frac{55\cdot\sqrt{5}}{2} \\ x^{10}=a+b\cdot x \end{array}\right\} \Rightarrow \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2} \quad (1).$$

$$\left.\begin{array}{l} x=\frac{1-\sqrt{5}}{2} \Rightarrow x^{10}=\left(\frac{1-\sqrt{5}}{2}\right)^{10}=\frac{123}{2}-\frac{55\cdot\sqrt{5}}{2} \\ x^{10}=a+b\cdot x \end{array}\right\} \Rightarrow \frac{123}{2}-\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \quad (2).$$

Oduzmemo jednadžbe (1) i (2):

$$\left.\begin{array}{l} \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2} \\ \frac{123}{2}-\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1+\sqrt{5}}{2} \\ \frac{123}{2}-\frac{55\cdot\sqrt{5}}{2}=a+b\cdot\frac{1-\sqrt{5}}{2} \quad /\cdot(-1) \end{array}\right\} \Rightarrow$$

$$\left.\begin{matrix}\dfrac{123}{2}+\dfrac{55\cdot\sqrt{5}}{2}=a+b\cdot\dfrac{1+\sqrt{5}}{2}\\ -\dfrac{123}{2}+\dfrac{55\cdot\sqrt{5}}{2}=-a-b\cdot\dfrac{1-\sqrt{5}}{2}\end{matrix}\right\}\Rightarrow 55\cdot\sqrt{5}=b\cdot\sqrt{5}\ /:\sqrt{5}\Rightarrow b=55.$$

Uvrstimo b = 55 natrag u prvu jednadžbu (1):

$$\frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+55\cdot\frac{1+\sqrt{5}}{2}\Rightarrow\frac{123}{2}+\frac{55\cdot\sqrt{5}}{2}=a+\frac{55}{2}+\frac{55\cdot\sqrt{5}}{2}\Rightarrow a=\frac{123}{2}-\frac{55}{2}\Rightarrow a=\frac{68}{2}\Rightarrow a=34.$$

Zbroj a i b iznosi:

$$a+b=34+55=89.$$

Vježba 093

Neka je $x^2 = 1 + x$. Ako je $x^{10} = a + b\cdot x$, koliko iznosi $b - a$?

Rezultat: 21.

Zadatak 094 (Ivan, gimnazija)

Pojednostavnite izraz: $\left(\frac{x+y}{x}-\frac{2\cdot x}{x-y}\right)\cdot\frac{x-y}{x^2+y^2}$.

Rješenje 094

Budući da se pojavljuje okrugla zagrada, najprije ćemo izračunati izraz u njoj, tj. izvršiti operaciju oduzimanja danih algebarskih razlomaka. Zatim dobiveni algebarski razlomak pomnožimo razlomkom $\frac{x-y}{x^2+y^2}$ i skratimo jednake faktore. Pišemo:

$$\left(\frac{x+y}{x}-\frac{2\cdot x}{x-y}\right)\cdot\frac{x-y}{x^2+y^2}=\frac{(x+y)\cdot(x-y)-2\cdot x^2}{x\cdot(x-y)}\cdot\frac{x-y}{x^2+y^2}=\left[\begin{matrix}\text{razlika kvadrata}\\(a+b)\cdot(a-b)=a^2-b^2\end{matrix}\right]=\frac{x^2-y^2-2\cdot x^2}{x\cdot(x-y)}\cdot\frac{x-y}{x^2+y^2}=$$

$$=\frac{-x^2-y^2}{x\cdot\underbrace{(x-y)}_{1}}\cdot\frac{\overbrace{x-y}^{1}}{x^2+y^2}=\frac{-x^2-y^2}{x}\cdot\frac{1}{x^2+y^2}=\frac{-\left(x^2+y^2\right)}{x}\cdot\frac{1}{x^2+y^2}=\frac{-\overbrace{\left(x^2+y^2\right)}^{1}}{x}\cdot\frac{1}{\underbrace{x^2+y^2}_{1}}=-\frac{1}{x}.$$

Vježba 094

Pojednostavnite izraz: $\left(\frac{x+y}{x}-\frac{2\cdot x}{x-y}\right)\cdot\frac{y-x}{x^2+y^2}$.

Rezultat: $\frac{1}{x}$.

Zadatak 095 (Antonija ♥ Vedran, hotelijerska škola, tehnička škola)

Izračunajte: $\dfrac{\frac{13}{21}+\left(\frac{1}{2}+\frac{1}{7}\right):\frac{18}{13}-0.15}{\left(\frac{1}{3}+0.5\right):\frac{5}{2}+0.2}$.

Rješenje 095

Najprije decimalne brojeve napišemo u obliku razlomaka i skratimo ako je moguće:

- $0.15=\frac{15}{100}=\frac{15:5}{100:5}=\frac{3}{20}$,
- $0.5=\frac{5}{10}=\frac{5:5}{10:5}=\frac{1}{2}$,
- $0.2=\frac{2}{10}=\frac{2:2}{10:2}=\frac{1}{5}$.

$$\frac{\frac{13}{21}+\left(\frac{1}{2}+\frac{1}{7}\right):\frac{18}{13}-0.15}{\left(\frac{1}{3}+0.5\right):\frac{5}{2}+0.2}=\frac{\frac{13}{21}+\left(\frac{1}{2}+\frac{1}{7}\right):\frac{18}{13}-\frac{3}{20}}{\left(\frac{1}{3}+\frac{1}{2}\right):\frac{5}{2}+\frac{1}{5}}=\left[\begin{matrix}\text{računamo u}\\ \text{zagradama}\end{matrix}\right]=\frac{\frac{13}{21}+\frac{7+2}{14}:\frac{18}{13}-\frac{3}{20}}{\frac{2+3}{6}:\frac{5}{2}+\frac{1}{5}}=\frac{\frac{13}{21}+\frac{9}{14}:\frac{18}{13}-\frac{3}{20}}{\frac{5}{6}:\frac{5}{2}+\frac{1}{5}}=$$

$$=\left[\begin{matrix}\text{dijelimo}\\ \text{razlomke}\end{matrix}\right]=\frac{\frac{13}{21}+\frac{9}{14}\cdot\frac{13}{18}-\frac{3}{20}}{\frac{5}{6}\cdot\frac{2}{5}+\frac{1}{5}}=\frac{\frac{13}{21}+\frac{1}{14}\cdot\frac{13}{2}-\frac{3}{20}}{\frac{1}{3}\cdot\frac{1}{1}+\frac{1}{5}}=\frac{\frac{13}{21}+\frac{13}{28}-\frac{3}{20}}{\frac{1}{3}+\frac{1}{5}}=\frac{\frac{260+195-63}{420}}{\frac{5+3}{15}}=\frac{\frac{392}{420}}{\frac{8}{15}}=\frac{392\cdot 15}{420\cdot 8}=\frac{49\cdot 15}{420\cdot 1}=$$

$$=\frac{49\cdot 15}{420\cdot 1}=\frac{49\cdot 15}{420}=\frac{7\cdot 15}{60}=\frac{7\cdot 15}{60}=\frac{7\cdot 1}{4}=\frac{7}{4}=1\frac{3}{4}.$$

Vježba 095

Izračunajte: $\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)\cdot 24.$

Rezultat: 26.

Zadatak 096 (Atila, gimnazija)

Pojednostavnite: $\frac{(a+3)^6-(a+3)^4}{(a+2)^4-(a+2)^2}\cdot\frac{(a+1)^3-(a+1)}{(a+3)^5-(a+3)^3}.$

Rješenje 096

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b)$$

U brojniku prvog razlomka izlučimo $(a + 3)^4$, a u nazivniku $(a + 2)^2$.
U brojniku drugog razlomka izlučimo (a + 1), a u nazivniku $(a + 3)^3$.

$$\frac{(a+3)^6-(a+3)^4}{(a+2)^4-(a+2)^2}\cdot\frac{(a+1)^3-(a+1)}{(a+3)^5-(a+3)^3}=\frac{(a+3)^4\cdot\left[(a+3)^2-1\right]}{(a+2)^2\cdot\left[(a+2)^2-1\right]}\cdot\frac{(a+1)\cdot\left[(a+1)^2-1\right]}{(a+3)^3\cdot\left[(a+3)^2-1\right]}=$$

$$=\frac{(a+3)^4}{(a+2)^2\cdot\left[(a+2)^2-1\right]}\cdot\frac{(a+1)\cdot\left[(a+1)^2-1\right]}{(a+3)^3}=\frac{(a+3)^4}{(a+2)^2\cdot\left[(a+2)^2-1\right]}\cdot\frac{(a+1)\cdot\left[(a+1)^2-1\right]}{(a+3)^3}=$$

$$=\frac{a+3}{(a+2)^2\cdot\left[(a+2)^2-1\right]}\cdot\frac{(a+1)\cdot\left[(a+1)^2-1\right]}{1}=\frac{a+3}{(a+2)^2\cdot(a+2-1)\cdot(a+2+1)}\cdot\frac{(a+1)\cdot(a+1-1)\cdot(a+1+1)}{1}=$$

$$=\frac{a+3}{(a+2)^2\cdot(a+1)\cdot(a+3)}\cdot\frac{(a+1)\cdot a\cdot(a+2)}{1}=\frac{a+3}{(a+2)^2\cdot(a+1)\cdot(a+3)}\cdot\frac{(a+1)\cdot a\cdot(a+2)}{1}=\frac{1}{(a+2)^2}\cdot\frac{a\cdot(a+2)}{1}=$$

$$=\frac{1}{(a+2)^2}\cdot\frac{a\cdot(a+2)}{1}=\frac{a}{a+2}.$$

Vježba 096

Pojednostavnite: $\frac{(a+3)^6-(a+3)^4}{(a+2)^4-(a+2)^2}\cdot\frac{(a+1)^3-(a+1)}{(a+3)^5-(a+3)^3}\cdot\frac{a+2}{a}.$

Rezultat: 1.

Zadatak 097 (1A, hotelijerska škola)

Izračunajte: $1-\frac{1+\frac{1}{1+2}}{2+\frac{1}{1+3}}+\frac{2+\frac{1}{3+4}}{3+\frac{1}{3+6}}.$

Rješenje 097

Ponovimo!

Zbrajanje razlomaka: $\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}$, oduzimanje razlomaka: $\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}$, $n=\frac{n}{1}$

dvojni razlomak: $\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$

$$1-\frac{1+\frac{1}{1+2}}{2+\frac{1}{1+3}}+\frac{2+\frac{1}{3+4}}{3+\frac{1}{3+6}}=1-\frac{1+\frac{1}{3}}{2+\frac{1}{4}}+\frac{2+\frac{1}{7}}{3+\frac{1}{9}}=1-\frac{\frac{1}{1}+\frac{1}{3}}{\frac{2}{1}+\frac{1}{4}}+\frac{\frac{2}{1}+\frac{1}{7}}{\frac{3}{1}+\frac{1}{9}}=1-\frac{\frac{3+1}{3}}{\frac{8+1}{4}}+\frac{\frac{14+1}{7}}{\frac{27+1}{9}}=1-\frac{\frac{4}{3}}{\frac{9}{4}}+\frac{\frac{15}{7}}{\frac{28}{9}}=1-\frac{4\cdot 4}{3\cdot 9}+\frac{15\cdot 9}{7\cdot 28}=$$

$$=1-\frac{4\cdot 4}{3\cdot 9}+\frac{15\cdot 9}{7\cdot 28}=1-\frac{16}{27}+\frac{135}{196}=\frac{1}{1}-\frac{16}{27}+\frac{135}{196}=\left[\begin{array}{c}\text{višekratnik}\\ v(27,196)=27\cdot 196=5292\end{array}\right]=$$

$$=\frac{5292-16\cdot 196+135\cdot 27}{5292}=\frac{5292-3136+3645}{5292}=\frac{5801}{5292}.$$

Vježba 097

Izračunajte: $1+\frac{1+\frac{1}{1+2}}{2+\frac{1}{1+3}}-\frac{2+\frac{1}{3+4}}{3+\frac{1}{3+6}}.$

Rezultat: $\frac{4783}{5292}.$

Zadatak 098 (Medena, gimnazija)

Napišite u obliku potencije: $2^{11}\cdot 9^5+4^5\cdot 3^{11}+36^5.$

Rješenje 098

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^n\cdot b^n=(a\cdot b)^n$$

1.inačica

$$2^{11}\cdot 9^5+4^5\cdot 3^{11}+36^5=2^{11}\cdot\left(3^2\right)^5+\left(2^2\right)^5\cdot 3^{11}+\left(6^2\right)^5=2^{11}\cdot 3^{10}+2^{10}\cdot 3^{11}+6^{10}=$$

$$=2^{11}\cdot 3^{10}+2^{10}\cdot 3^{11}+(2\cdot 3)^{10}=2^{11}\cdot 3^{10}+2^{10}\cdot 3^{11}+2^{10}\cdot 3^{10}=\left[\text{izlučujemo } 2^{10}\cdot 3^{10}\right]=$$

$$=2^{10}\cdot 3^{10}\cdot[2+3+1]=(2\cdot 3)^{10}\cdot 6=6^{10}\cdot 6=6^{11}.$$

2.inačica

$$2^{11}\cdot 9^5+4^5\cdot 3^{11}+36^5=2^{11}\cdot\left(3^2\right)^5+\left(2^2\right)^5\cdot 3^{11}+\left(6^2\right)^5=2^{11}\cdot 3^{10}+2^{10}\cdot 3^{11}+6^{10}=$$

$$=2\cdot 2^{10}\cdot 3^{10}+2^{10}\cdot 3\cdot 3^{10}+6^{10}=2\cdot(2\cdot 3)^{10}+3\cdot(2\cdot 3)^{10}+6^{10}=2\cdot 6^{10}+3\cdot 6^{10}+6^{10}=\left[\text{izlučujemo } 6^{10}\right]=$$

$$=6^{10}\cdot[2+3+1]=6^{10}\cdot 6=6^{11}.$$

Vježba 098

Napišite u obliku potencije: $3^9+6\cdot 9^4$.

Rezultat: 3^{10}.

Zadatak 099 (Medena, gimnazija)

Napišite u obliku potencije: $20\cdot 4^5+3\cdot 2^{13}+5\cdot 8^4$.

Rješenje 099

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^n\cdot a^m=a^{n+m}$$

1.inačica

$$20\cdot 4^5+3\cdot 2^{13}+5\cdot 8^4=20\cdot\left(2^2\right)^5+3\cdot 2^{13}+5\cdot\left(2^3\right)^4=20\cdot 2^{10}+3\cdot 2^{13}+5\cdot 2^{12}=\left[\begin{array}{l}\text{izlučuje se potencija s}\\ \text{najmanjim eksponentom } 2^{10}\end{array}\right]=$$

$$=2^{10}\cdot\left[20+3\cdot 2^3+5\cdot 2^2\right]=2^{10}\cdot[20+3\cdot 8+5\cdot 4]=2^{10}\cdot[20+24+20]=2^{10}\cdot 64=2^{10}\cdot 2^6=2^{16}.$$

2.inačica

$$20\cdot 4^5+3\cdot 2^{13}+5\cdot 8^4=\left[\begin{array}{l}\text{rastavimo broj 20}\\ \text{na proste faktore}\end{array}\right]=2\cdot 2\cdot 5\cdot 4^5+3\cdot 2^{13}+5\cdot 8^4=2^2\cdot 5\cdot\left(2^2\right)^5+3\cdot 2^{13}+5\cdot\left(2^3\right)^4=$$

$$=2^2\cdot 5\cdot 2^{10}+3\cdot 2^{13}+5\cdot 2^{12}=5\cdot 2^{12}+3\cdot 2^{13}+5\cdot 2^{12}=\left[\begin{array}{l}\text{izlučujemo potenciju s}\\ \text{manjim eksponentom } 2^{12}\end{array}\right]=2^{12}\cdot[5+3\cdot 2+5]=$$

$$=2^{12}\cdot[5+6+5]=2^{12}\cdot 16=2^{12}\cdot 2^4=2^{16}.$$

Vježba 099

Napišite u obliku potencije: $2\cdot 16^3-3\cdot 4^6+5\cdot 8^4$.

Rezultat: 2^{14}.

Zadatak 100 (Ivana, gimnazija)

Ako je $\frac{a}{b}=5$, koliko je $\frac{a+b}{a-b}$?

Rješenje 100

1.inačica

$$\frac{a+b}{a-b}=\left[\text{proširimo razlomak s } \frac{1}{b}\right]=\frac{\frac{a+b}{b}}{\frac{a-b}{b}}=\frac{\frac{a}{b}+\frac{b}{b}}{\frac{a}{b}-\frac{b}{b}}=\frac{\frac{a}{b}+1}{\frac{a}{b}-1}=\frac{5+1}{5-1}=\frac{6}{4}=\frac{3}{2}.$$

2.inačica

$$\frac{a+b}{a-b}=\left[\begin{array}{c}\text{supstitucija}\\ \frac{a}{b}=5\ \Rightarrow\ a=5\cdot b\end{array}\right]=\frac{5\cdot b+b}{5\cdot b-b}=\frac{6\cdot b}{4\cdot b}=\frac{6}{4}=\frac{3}{2}.$$

3.inačica

$$\frac{a+b}{a-b}=\left[\begin{array}{l}\text{u brojniku i}\\ \text{nazivniku izlučimo } b\end{array}\right]=\frac{b\cdot\left(\frac{a}{b}+1\right)}{b\cdot\left(\frac{a}{b}-1\right)}=\frac{b\cdot\left(\frac{a}{b}+1\right)}{b\cdot\left(\frac{a}{b}-1\right)}=\frac{\frac{a}{b}+1}{\frac{a}{b}-1}=\frac{5+1}{5-1}=\frac{6}{4}=\frac{3}{2}.$$

Vježba 100

Ako je $\frac{a}{b}=2$, koliko je $\frac{a+b}{a-b}$?

Rezultat: 3.