Zadatak 501 (ET, gimnazija)

Ako je $\sin(\alpha)+\cos(\beta)=m$ i $\cos(\alpha)-\cos(\beta)=n,$ koliko je $\sin(2\cdot\alpha)$?

$A.\ m+n-1 \qquad B.\ m^2+n^2+1 \qquad C.\ (m+n)^2-1 \qquad D.\ (m-n)^2-1$

Rješenje 501

Ponovimo!

$$\left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a+c=b+d.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \cos^2(x)+\sin^2(x)=1 \quad , \quad \sin(2\cdot x)=2\cdot\sin(x)\cdot\cos(x).$$

$$\left.\begin{array}{l} \sin(\alpha)+\cos(\beta)=m \\ \cos(\alpha)-\cos(\beta)=n \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow \sin(\alpha)+\cos(\beta)+\cos(\alpha)-\cos(\beta)=m+n \Rightarrow$$

$$\Rightarrow \sin(\alpha)+\cos(\beta)+\cos(\alpha)-\cos(\beta)=m+n \Rightarrow \sin(\alpha)+\cos(\alpha)=m+n \Rightarrow$$

$$\Rightarrow \sin(\alpha)+\cos(\alpha)=m+n \ /^2 \Rightarrow (\sin(\alpha)+\cos(\alpha))^2=(m+n)^2 \Rightarrow$$

$$\Rightarrow \sin^2(\alpha)+2\cdot\sin(\alpha)\cdot\cos(\alpha)+\cos^2(\alpha)=(m+n)^2 \Rightarrow 1+\sin(2\cdot\alpha)=(m+n)^2 \Rightarrow$$

$$\Rightarrow \sin(2\cdot\alpha)=(m+n)^2-1.$$

Odgovor je pod C.

Vježba 501

Ako je $\sin(\alpha)-\cos(\beta)=m$ i $\cos(\alpha)+\cos(\beta)=n,$ koliko je $\sin(2\cdot\alpha)$?

$A.\ m+n-1 \qquad B.\ m^2+n^2+1 \qquad C.\ (m+n)^2-1 \qquad D.\ (m-n)^2-1$

Rezultat: C.

Zadatak 502 (Bug, maturant)

Čemu je nakon pojednostavljivanja jednak izraz $3\cdot\sin(4\cdot\pi+x)+\cos\left(\frac{\pi}{2}+x\right)$ za svaki x?

$A.\ -4\cdot\sin(x) \qquad B.\ -2\cdot\sin(x) \qquad C.\ 2\cdot\sin(x) \qquad D.\ 4\cdot\sin(x)$

Rješenje 502

Ponovimo!

$$\sin(4\cdot\pi)=0 \quad , \quad \cos(4\cdot\pi)=1 \quad , \quad \sin\left(\frac{\pi}{2}\right)=1 \quad , \quad \cos\left(\frac{\pi}{2}\right)=0.$$

Funkcija sinus je periodična funkcija s periodom $k\cdot 2\cdot\pi \ , \ k\in Z.$
Za sinus najmanji od tih perioda je $2\cdot\pi$ pa je njezin temeljni period $2\cdot\pi.$

$$\sin(x+k\cdot 2\cdot\pi)=\sin(x).$$

Formula redukcije

$$\cos\left(\frac{\pi}{2}+x\right)=-\sin(x).$$

Funkcije zbroja

$$\sin(x+y)=\sin(x)\cdot\cos(y)+\cos(x)\cdot\sin(y) \quad , \quad \cos(x+y)=\cos(x)\cdot\cos(y)-\sin(x)\cdot\sin(y).$$

1.inačica

$$3\cdot\sin(4\cdot\pi+x)+\cos\left(\frac{\pi}{2}+x\right)=3\cdot\sin(x+2\cdot 2\cdot\pi)+\cos\left(\frac{\pi}{2}+x\right)=3\cdot\sin(x)-\sin(x)=2\cdot\sin(x).$$

Odgovor je pod C.

2.inačica

$$3\cdot\sin(4\cdot\pi+x)+\cos\left(\frac{\pi}{2}+x\right)=$$

$$=3\cdot\left(\sin(4\cdot\pi)\cdot\cos(x)+\cos(4\cdot\pi)\cdot\sin(x)\right)+\cos\left(\frac{\pi}{2}\right)\cdot\cos(x)-\sin\left(\frac{\pi}{2}\right)\cdot\sin(x)=$$

$$=3\cdot\left(0\cdot\cos(x)+1\cdot\sin(x)\right)+0\cdot\cos(x)-1\cdot\sin(x)=3\cdot\sin(x)-\sin(x)=2\cdot\sin(x).$$

Odgovor je pod C.

Vježba 502

Odmor!

Rezultat: …

Zadatak 503 (Jelena, gimnazija)

Za koji α jednadžba $2\cdot x^2-4\cdot x\cdot\cos(\alpha)+3\cdot\sin(\alpha)=0$ ima dvostruko rješenje?

Rješenje 503

Ponovimo!

$$\cos^2(\alpha)+\sin^2(\alpha)=1 \quad , \quad (a\cdot b)^n=a^n\cdot b^2 \quad , \quad a-\frac{b}{c}=\frac{a\cdot c-b}{c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \quad , \; n\neq 0 \; , \; n\neq 1.$$

Trigonometrijska jednadžba $\sin x=a \; , \; |a|\le 1.$

Skup rješenja jednadžbe $\sin x=a$, $|a|\le 1$, je $\{x_0+k\cdot 2\cdot\pi : k\in Z\}\cup\{\pi-x_0+k\cdot 2\cdot\pi : k\in Z\}$

gdje je $x_0\in R$ jedno rješenje te jednadžbe.

Diskriminanta kvadratne jednadžbe

$$a\cdot x^2+b\cdot x+c=0$$

je broj

$$D=b^2-4\cdot a\cdot c.$$

- Ako je D > 0, jednadžba ima dva realna rješenja.
- **Ako je D = 0, jednadžba ima jedno dvostruko realno rješenje.**
- Ako je D < 0, jednadžba ima kompleksno – konjugirana rješenja.

$$2\cdot x^2-4\cdot x\cdot\cos(\alpha)+3\cdot\sin(\alpha)=0 \Rightarrow \left.\begin{array}{l} 2\cdot x^2-4\cdot x\cdot\cos(\alpha)+3\cdot\sin(\alpha)=0 \\ a=2 \; , \; b=-4\cdot\cos(\alpha) \; , \; c=3\cdot\sin(\alpha) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[b^2-4\cdot a\cdot c=0\right] \Rightarrow (-4\cdot\cos(\alpha))^2-4\cdot 2\cdot 3\cdot\sin(\alpha)=0 \Rightarrow 16\cdot\cos^2(\alpha)-24\cdot\sin(\alpha)=0 \Rightarrow$$

$$\Rightarrow 16\cdot\cos^2(\alpha)-24\cdot\sin(\alpha)=0 \;/:8 \Rightarrow 2\cdot\cos^2(\alpha)-3\cdot\sin(\alpha)=0 \Rightarrow$$

$$\Rightarrow 2\cdot\left(1-\sin^2(\alpha)\right)-3\cdot\sin(\alpha)=0 \Rightarrow 2-2\cdot\sin^2(\alpha)-3\cdot\sin(\alpha)=0 \Rightarrow$$

$$\Rightarrow -2\cdot\sin^2(\alpha)-3\cdot\sin(\alpha)+2=0 \Rightarrow -2\cdot\sin^2(\alpha)-3\cdot\sin(\alpha)+2=0 \;/\cdot(-1) \Rightarrow$$

$$\Rightarrow 2\cdot\sin^2(\alpha)+3\cdot\sin(\alpha)-2=0 \Rightarrow \left.\begin{matrix} 2\cdot\sin^2(\alpha)+3\cdot\sin(\alpha)-2=0 \\ a=2\ ,\ b=3\ ,\ c=-2 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left[(\sin(\alpha))_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \right] \Rightarrow (\sin(\alpha))_{1,2}=\frac{-3\pm\sqrt{3^2-4\cdot 2\cdot(-2)}}{2\cdot 2} \Rightarrow$$

$$\Rightarrow (\sin(\alpha))_{1,2}=\frac{-3\pm\sqrt{9+16}}{4} \Rightarrow (\sin(\alpha))_{1,2}=\frac{-3\pm\sqrt{25}}{4} \Rightarrow (\sin(\alpha))_{1,2}=\frac{-3\pm 5}{4} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} (\sin(\alpha))_1=\frac{-3-5}{4} \\ (\sin(\alpha))_2=\frac{-3+5}{4} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} (\sin(\alpha))_1=\frac{-8}{4} \\ (\sin(\alpha))_2=\frac{2}{4} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} (\sin(\alpha))_1=\frac{-8}{4} \\ (\sin(\alpha))_2=\frac{2}{4} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} (\sin(\alpha))_1=-2 \text{ nema smisla} \\ (\sin(\alpha))_2=\frac{1}{2} \end{matrix}\right\} \Rightarrow \sin(\alpha)=\frac{1}{2} \Rightarrow \alpha=\sin^{-1}\left(\frac{1}{2}\right) \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \alpha_1=\frac{\pi}{6}+k\cdot 2\cdot\pi\ ,\ k\in Z \\ \alpha_2=\pi-\frac{\pi}{6}+k\cdot 2\cdot\pi\ ,\ k\in Z \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \alpha_1=\frac{\pi}{6}+k\cdot 2\cdot\pi\ ,\ k\in Z \\ \alpha_2=\frac{5\cdot\pi}{6}+k\cdot 2\cdot\pi\ ,\ k\in Z \end{matrix}\right\}.$$

Vježba 503

Odmor!

Rezultat: …

Zadatak 504 (Tom, maturant)

Odredite sve vrijednosti x iz zadanoga sustava jednadžba.

$$\begin{cases} 2\cdot x=y+\frac{\pi}{3} \\ \sin(y-x)=0.5 \end{cases}.$$

Rješenje 504

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\ ,\quad a-\frac{b}{c}=\frac{a\cdot c-b}{c}\ ,\quad \left.\begin{matrix} a=b \\ c=d \end{matrix}\right\} \Rightarrow a+c=b+d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Trigonometrijska jednadžba $\sin x=a\ ,\ |a|\leq 1$

Skup rješenja jednadžbe $\sin x=a,\ |a|\leq 1,$ je $\{x_\circ+k\cdot 2\cdot\pi : k\in Z\}\cup\{\pi-x_\circ+k\cdot 2\cdot\pi : k\in Z\}$

gdje je $x_\circ\in R$ jedno rješenje te jednadžbe.

Trigonometrijska jednadžba dat će dva skupa rješenja.

$$\sin(y-x)=0.5 \Rightarrow y-x=\sin^{-1}(0.5) \Rightarrow \left.\begin{array}{l} y-x=\frac{\pi}{6}+k\cdot 2\cdot\pi \ , \ k\in Z \\ y-x=\pi-\frac{\pi}{6}+k\cdot 2\cdot\pi \ , \ k\in Z \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} y-x=\frac{\pi}{6}+k\cdot 2\cdot\pi \ , \ k\in Z \text{ prvo rješenje} \\ y-x=\frac{5\cdot\pi}{6}+k\cdot 2\cdot\pi \ , \ k\in Z \text{ drugo rješenje} \end{array}\right\}.$$

Zato moramo riješiti dva sustava jednadžba.

Prvi sustav

$$\left.\begin{array}{l} 2\cdot x=y+\frac{\pi}{3} \\ y-x=\frac{\pi}{6}+k\cdot 2\cdot\pi \end{array}\right\} \Rightarrow \left.\begin{array}{l} 2\cdot x-y=\frac{\pi}{3} \\ -x+y=\frac{\pi}{6}+k\cdot 2\cdot\pi \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow$$

$$\Rightarrow x=\frac{\pi}{3}+\frac{\pi}{6}+k\cdot 2\cdot\pi \Rightarrow x=\frac{2\cdot\pi+\pi}{6}+k\cdot 2\cdot\pi \Rightarrow x=\frac{3\cdot\pi}{6}+k\cdot 2\cdot\pi \Rightarrow x=\frac{3\cdot\pi}{6}+k\cdot 2\cdot\pi \Rightarrow$$

$$\Rightarrow x=\frac{\pi}{2}+k\cdot 2\cdot\pi \ , \ k\in Z.$$

Drugi sustav

$$\left.\begin{array}{l} 2\cdot x=y+\frac{\pi}{3} \\ y-x=\frac{5\cdot\pi}{6}+k\cdot 2\cdot\pi \end{array}\right\} \Rightarrow \left.\begin{array}{l} 2\cdot x-y=\frac{\pi}{3} \\ -x+y=\frac{5\cdot\pi}{6}+k\cdot 2\cdot\pi \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow$$

$$\Rightarrow x=\frac{\pi}{3}+\frac{5\cdot\pi}{6}+k\cdot 2\cdot\pi \Rightarrow x=\frac{2\cdot\pi+5\cdot\pi}{6}+k\cdot 2\cdot\pi \Rightarrow$$

$$\Rightarrow x=\frac{7\cdot\pi}{6}+k\cdot 2\cdot\pi \ , \ k\in Z.$$

Vježba 504

Odmor!

Rezultat: …

Zadatak 505 (Viki, gimnazija)

Zadana je jednadžba $\sin^6(x)+\cos^6(x)=a$, gdje je a realan parametar. Za koje vrijednosti parametra a zadana jednadžba ima realna rješenja?

Rješenje 505

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right) \quad , \quad \sin^2(x)+\cos^2(x)=1.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \sin(2\cdot x)=2\cdot\sin(x)\cdot\cos(x).$$

$$\sin^2(x)=\frac{1-\cos(2\cdot x)}{2} \quad , \quad a\le b \ , \ c>0 \Rightarrow a\cdot c\le b\cdot c \quad , \quad a\le b \ , \ c>0 \Rightarrow \frac{a}{c}\le\frac{b}{c}.$$

$$a\cdot\frac{b}{c}=\frac{a\cdot b}{c} \quad , \quad \frac{a}{b}-c=\frac{a-b\cdot c}{b} \quad , \quad a\le b \ , \ c\in R \Rightarrow a+c\le b+c.$$

$$-1 \le \cos(x) \le 1 \text{ , za svaki } x.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \text{ , } n \ne 0 \text{ , } n \ne 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n} \text{ , } n \ne 0 \text{ , } n \ne 1.$$

$$\sin^6(x) + \cos^6(x) = a \Rightarrow \left(\sin^2(x)\right)^3 + \left(\cos^2(x)\right)^3 = a \Rightarrow$$

$$\Rightarrow \left(\sin^2(x) + \cos^2(x)\right) \cdot \left(\left(\sin^2(x)\right)^2 - \sin^2(x) \cdot \cos^2(x) + \left(\cos^2(x)\right)^2\right) = a \Rightarrow$$

$$\Rightarrow 1 \cdot \left(\left(\sin^2(x)\right)^2 - \sin^2(x) \cdot \cos^2(x) + \left(\cos^2(x)\right)^2\right) = a \Rightarrow$$

$$\Rightarrow \left(\sin^2(x)\right)^2 - \sin^2(x) \cdot \cos^2(x) + \left(\cos^2(x)\right)^2 = a \Rightarrow$$

$$\Rightarrow \left(\sin^2(x)\right)^2 + 2 \cdot \sin^2(x) \cdot \cos^2(x) + \left(\cos^2(x)\right)^2 - 3 \cdot \sin^2(x) \cdot \cos^2(x) = a \Rightarrow$$

$$\Rightarrow \left(\sin^2(x) + \cos^2(x)\right)^2 - 3 \cdot \sin^2(x) \cdot \cos^2(x) = a \Rightarrow 1^2 - 3 \cdot \sin^2(x) \cdot \cos^2(x) = a \Rightarrow$$

$$\Rightarrow 1 - 3 \cdot \sin^2(x) \cdot \cos^2(x) = a \Rightarrow -3 \cdot \sin^2(x) \cdot \cos^2(x) = a - 1 \Rightarrow$$

$$\Rightarrow -3 \cdot \sin^2(x) \cdot \cos^2(x) = a - 1 \; / \cdot \left(-\frac{4}{3}\right) \Rightarrow 4 \cdot \sin^2(x) \cdot \cos^2(x) = \frac{4 - 4 \cdot a}{3} \Rightarrow$$

$$\Rightarrow \left(2 \cdot \sin(x) \cdot \cos(x)\right)^2 = \frac{4 - 4 \cdot a}{3} \Rightarrow \sin^2(2 \cdot x) = \frac{4 - 4 \cdot a}{3} \Rightarrow \frac{1 - \cos(4 \cdot x)}{2} = \frac{4 - 4 \cdot a}{3} \Rightarrow$$

$$\Rightarrow \frac{1 - \cos(4 \cdot x)}{2} = \frac{4 - 4 \cdot a}{3} \; / \cdot 2 \Rightarrow 1 - \cos(4 \cdot x) = \frac{8 - 8 \cdot a}{3} \Rightarrow -\cos(4 \cdot x) = \frac{8 - 8 \cdot a}{3} - 1 \Rightarrow$$

$$\Rightarrow -\cos(4 \cdot x) = \frac{8 - 8 \cdot a - 3}{3} \Rightarrow -\cos(4 \cdot x) = \frac{5 - 8 \cdot a}{3} \Rightarrow -\cos(4 \cdot x) = \frac{5 - 8 \cdot a}{3} \; / \cdot (-1) \Rightarrow$$

$$\Rightarrow \cos(4 \cdot x) = \frac{8 \cdot a - 5}{3}.$$

Budući da za trigonometrijsku funkciju kosinus vrijedi

$$-1 \le \cos(x) \le 1,$$

nužan i dovoljan uvjet za egzistenciju realnih rješenja zadane jednadžbe je

$$-1 \le \frac{8 \cdot a - 5}{3} \le 1 \Rightarrow -1 \le \frac{8 \cdot a - 5}{3} \le 1 \; / \cdot 3 \Rightarrow -3 \le 8 \cdot a - 5 \le 3 \Rightarrow -3 \le 8 \cdot a - 5 \le 3 \; / + 5 \Rightarrow$$

$$\Rightarrow -3 + 5 \le 8 \cdot a - 5 + 5 \le 3 + 5 \Rightarrow 2 \le 8 \cdot a - 5 + 5 \le 8 \Rightarrow 2 \le 8 \cdot a \le 8 \Rightarrow$$

$$\Rightarrow 2 \le 8 \cdot a \le 8 \; / \cdot \frac{1}{8} \Rightarrow \frac{2}{8} \le a \le 1 \Rightarrow \frac{2}{8} \le a \le 1 \Rightarrow \frac{1}{4} \le a \le 1 \Rightarrow a \in \left[\frac{1}{4}, 1\right].$$

Vježba 505

Odmor!

Rezultat: …

Zadatak 506 (Dragan, maturant)

Zbroj svih rješenja jednadžbe $\frac{2}{1+ctg^2(x)} = 5 \cdot \sin(x) - 2$ u intervalu $\langle 0, \pi \rangle$ iznosi:

$A.\ 1.5 \cdot \pi \qquad B.\ 2 \cdot \pi \qquad C.\ \pi \qquad D.\ 0.75 \cdot \pi$

Rješenje 506

Ponovimo!

$$ctg^2(x) = \frac{\cos^2(x)}{\sin^2(x)} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \sin^2(x) + \cos^2(x) = 1.$$

$$-1 \le \sin(x) \le 1 \quad , \quad \frac{a}{\frac{b}{c}} = \frac{a \cdot c}{b} \quad , \quad a - \frac{b}{c} = \frac{a \cdot c - b}{c} \quad , \quad \frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \ne 0 \quad , \quad n \ne 1.$$

Trigonometrijska jednadžba $\sin x = a \; , \; |a| \le 1$

Skup rješenja jednadžbe $\sin x = a, \; |a| \le 1$, je $\{x_o + k \cdot 2 \cdot \pi : k \in Z\} \cup \{\pi - x_o + k \cdot 2 \cdot \pi : k \in Z\}$

gdje je $x_o \in R$ jedno rješenje te jednadžbe.

$$\frac{2}{1+ctg^2(x)} = 5 \cdot \sin(x) - 2 \Rightarrow \frac{2}{1+\frac{\cos^2(x)}{\sin^2(x)}} = 5 \cdot \sin(x) - 2 \Rightarrow \frac{2}{\frac{1}{1}+\frac{\cos^2(x)}{\sin^2(x)}} = 5 \cdot \sin(x) - 2 \Rightarrow$$

$$\Rightarrow \frac{2}{\frac{\sin^2(x)+\cos^2(x)}{\sin^2(x)}} = 5 \cdot \sin(x) - 2 \Rightarrow \frac{2}{\frac{1}{\sin^2(x)}} = 5 \cdot \sin(x) - 2 \Rightarrow 2 \cdot \sin^2(x) = 5 \cdot \sin(x) - 2 \Rightarrow$$

$$\Rightarrow 2 \cdot \sin^2(x) - 5 \cdot \sin(x) + 2 = 0 \Rightarrow \left[\begin{matrix} \text{zamjena} \\ \sin(x) = t \end{matrix}\right] \Rightarrow 2 \cdot t^2 - 5 \cdot t + 2 = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} 2 \cdot t^2 - 5 \cdot t + 2 = 0 \\ a = 2 \; , \; b = -5 \; , \; c = 2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} a = 2 \; , \; b = -5 \; , \; c = 2 \\ t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{matrix}\right\} \Rightarrow t_{1,2} = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 2 \cdot 2}}{2 \cdot 2} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{5 \pm \sqrt{25-16}}{4} \Rightarrow t_{1,2} = \frac{5 \pm \sqrt{9}}{4} \Rightarrow t_{1,2} = \frac{5 \pm 3}{4} \Rightarrow \left.\begin{matrix} t_1 = \frac{5+3}{4} \\ t_2 = \frac{5-3}{4} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = \frac{8}{4} \\ t_2 = \frac{2}{4} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t_1 = \frac{8}{4} \\ t_2 = \frac{2}{4} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = 2 \\ t_2 = \frac{1}{2} \end{array}\right\}.$$

Vraćamo se na zamjenu.

- $\left.\begin{array}{l} t = 2 \\ \sin(x) = t \end{array}\right\} \Rightarrow \sin(x) = 2$ nema smisla.

- $\left.\begin{array}{l} t = \frac{1}{2} \\ \sin(x) = t \end{array}\right\} \Rightarrow \sin(x) = \frac{1}{2} \Rightarrow x = \sin^{-1}\left(\frac{1}{2}\right) \Rightarrow \left[\begin{array}{c} \text{na intervalu} \\ \langle 0, \pi \rangle \end{array}\right] \Rightarrow \left.\begin{array}{l} x_1 = \frac{\pi}{6} \\ x_2 = \pi - \frac{\pi}{6} \end{array}\right\} \Rightarrow$

$$\Rightarrow \left.\begin{array}{l} x_1 = \frac{\pi}{6} \\ x_2 = \frac{5 \cdot \pi}{6} \end{array}\right\}.$$

Zbroj rješenja je

$$x_1 + x_2 = \frac{\pi}{6} + \frac{5 \cdot \pi}{6} \Rightarrow x_1 + x_2 = \frac{\pi + 5 \cdot \pi}{6} \Rightarrow x_1 + x_2 = \frac{6 \cdot \pi}{6} \Rightarrow x_1 + x_2 = \frac{6 \cdot \pi}{6} \Rightarrow x_1 + x_2 = \pi.$$

Odgovor je pod C.

Vježba 506

Odmor!

Rezultat: …

Zadatak 507 (Laura, srednja škola)

Ako je $tg(x) + ctg(x) = m$, onda je $tg^3(x) + ctg^3(x)$ jednako:

$A.\ m \cdot (m - 3) \qquad B.\ m^2 - 3 \qquad C.\ m^3 - 1 \qquad D.\ m^3 - 3 \cdot m$

Rješenje 507

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad tg(x) \cdot ctg(x) = 1.$$

$$a^3 + b^3 = (a+b) \cdot \left(a^2 - a \cdot b + b^2\right) \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Zadanu jednadžbu kvadriramo.

$$tg(x) + ctg(x) = m \Rightarrow tg(x) + ctg(x) = m \ /^2 \Rightarrow (tg(x) + ctg(x))^2 = m^2 \Rightarrow$$

$$\Rightarrow tg^2(x) + 2 \cdot tg(x) \cdot ctg(x) + ctg^2(x) = m^2 \Rightarrow tg^2(x) + 2 \cdot 1 + ctg^2(x) = m^2 \Rightarrow$$

$$\Rightarrow tg^2(x) + 2 + ctg^2(x) = m^2 \Rightarrow tg^2(x) + ctg^2(x) = m^2 - 2.$$

Konačno imamo:

$$tg^3(x) + ctg^3(x) = (tg(x) + ctg(x)) \cdot \left(tg^2(x) - tg(x) \cdot ctg(x) + ctg^2(x)\right) =$$

$$=\left(tg(x)+ctg(x)\right)\cdot\left(tg^2(x)-1+ctg^2(x)\right)=\left(tg(x)+ctg(x)\right)\cdot\left(tg^2(x)+ctg^2(x)-1\right)=$$

$$=\begin{bmatrix} tg(x)+ctg(x)=m \\ tg^2(x)+ctg^2(x)=m^2-2 \end{bmatrix}=m\cdot\left(m^2-2-1\right)=m\cdot\left(m^2-3\right)=m^3-3\cdot m.$$

Odgovor je pod D.

Vježba 507

Odmor!

Rezultat: …

Zadatak 508 (Luka, srednja škola)

Ako je $x+y=\frac{3\cdot\pi}{4}$, koliko je $\left(1+ctg(x)\right)\cdot\left(1+ctg(y)\right)$?

$A.\ 2 \qquad B.\ 1 \qquad C.\ 0 \qquad D.\ -1$

Rješenje 508

Ponovimo!

$$a\cdot\frac{b}{a}=a \quad , \quad ctg\left(\frac{3\cdot\pi}{4}\right)=-1.$$

$$ctg(x+y)=\frac{ctg(x)\cdot ctg(y)-1}{ctg(y)+ctg(x)} \quad , \quad ctg(x-y)=\frac{ctg(x)\cdot ctg(y)+1}{ctg(y)-ctg(x)}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

1.inačica

$$ctg(x+y)=\frac{ctg(x)\cdot ctg(y)-1}{ctg(y)+ctg(x)} \Rightarrow \left[x+y=\frac{3\cdot\pi}{4}\right] \Rightarrow ctg\left(\frac{3\cdot\pi}{4}\right)=\frac{ctg(x)\cdot ctg(y)-1}{ctg(y)+ctg(x)} \Rightarrow$$

$$\Rightarrow -1=\frac{ctg(x)\cdot ctg(y)-1}{ctg(y)+ctg(x)} \Rightarrow -1=\frac{ctg(x)\cdot ctg(y)-1}{ctg(y)+ctg(x)} \ /\cdot\left(ctg(y)+ctg(x)\right) \Rightarrow$$

$$\Rightarrow -1\cdot\left(ctg(y)+ctg(x)\right)=ctg(x)\cdot ctg(y)-1 \Rightarrow -ctg(y)-ctg(x)=ctg(x)\cdot ctg(y)-1 \Rightarrow$$

$$\Rightarrow -ctg(y)-ctg(x)-ctg(x)\cdot ctg(y)=-1 \Rightarrow -ctg(y)-ctg(x)-ctg(x)\cdot ctg(y)=-1 \ /\cdot(-1) \Rightarrow$$

$$\Rightarrow ctg(y)+ctg(x)+ctg(x)\cdot ctg(y)=1.$$

Sada imamo:

$$\left(1+ctg(x)\right)\cdot\left(1+ctg(y)\right)=1+ctg(y)+ctg(x)+ctg(x)\cdot ctg(y)=$$

$$=1+\left(ctg(y)+ctg(x)+ctg(x)\cdot ctg(y)\right)=1+1=2.$$

Odgovor je pod A.

2.inačica

$$x+y=\frac{3\cdot\pi}{4} \Rightarrow y=\frac{3\cdot\pi}{4}-x.$$

Dalje slijedi:

$$\left(1+ctg(x)\right)\cdot\left(1+ctg(y)\right)=\left(1+ctg(x)\right)\cdot\left(1+ctg\left(\frac{3\cdot\pi}{4}-x\right)\right)=$$

$$=\left(1+ctg\left(x\right)\right)\cdot\left(1+\frac{ctg\left(\frac{3\cdot\pi}{4}\right)\cdot ctg\left(x\right)+1}{ctg\left(x\right)-ctg\left(\frac{3\cdot\pi}{4}\right)}\right)=\left(1+ctg\left(x\right)\right)\cdot\left(1+\frac{-1\cdot ctg\left(x\right)+1}{ctg\left(x\right)-\left(-1\right)}\right)=$$

$$=\left(1+ctg\left(x\right)\right)\cdot\left(1+\frac{-ctg\left(x\right)+1}{ctg\left(x\right)+1}\right)=1+ctg\left(x\right)-ctg\left(x\right)+1=1+ctg\left(x\right)-ctg\left(x\right)+1=2.$$

Odgovor je pod A.

Vježba 508

Odmor!

Rezultat: …

Zadatak 509 (Štrumf, maturant)

Zbroj najmanje i najveće vrijednosti funkcije $f\left(x\right)=3\cdot\sin^2\left(x\right)+5\cdot\cos^2\left(x\right)$ iznosi:

$A.\ 3 \qquad B.\ 5 \qquad C.\ 8 \qquad D.\ 7$

Rješenje 509

Ponovimo!

$$\sin^2\left(x\right)+\cos^2\left(x\right)=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

Za funkcije $\sin\left(x\right)$ i $\cos\left(x\right)$ vrijedi:

$$-1\le\sin\left(x\right)\le1 \quad , \quad -1\le\cos\left(x\right)\le1 \ , \ x\in R.$$

Za funkcije $\sin^2\left(x\right)$ i $\cos^2\left(x\right)$ vrijedi:

$$0\le\sin^2\left(x\right)\le1 \quad , \quad 0\le\cos^2\left(x\right)\le1 \ , \ x\in R.$$

1.inačica

Funkcija $f\left(x\right)=3\cdot\sin^2\left(x\right)+5\cdot\cos^2\left(x\right)$ ima:

- najmanju vrijednost

$$3\cdot0+5\cdot0=0+0=0$$

- najveću vrijednost

$$3\cdot1+5\cdot1=3+5=8.$$

Zbroj najmanje i najveće vrijednosti funkcije f je

$$0+8=8.$$

Odgovor je pod C.

2.inačica

Preoblikujemo zadanu funkciju.

$$f\left(x\right)=3\cdot\sin^2\left(x\right)+5\cdot\cos^2\left(x\right)\ \Rightarrow\ f\left(x\right)=3\cdot\left(1-\cos^2\left(x\right)\right)+5\cdot\cos^2\left(x\right)\ \Rightarrow$$

$$\Rightarrow\ f\left(x\right)=3-3\cdot\cos^2\left(x\right)+5\cdot\cos^2\left(x\right)\ \Rightarrow\ f\left(x\right)=3+2\cdot\cos^2\left(x\right).$$

Funkcija $f\left(x\right)=3+2\cdot\cos^2\left(x\right)$ ima:

- najmanju vrijednost za $\cos^2\left(x\right)=0$ i ona iznosi

$$3+2\cdot0=3+0=3$$

- najveću vrijednost za $\cos^2\left(x\right)=1$ i ona iznosi

$$3+2\cdot 1=3+2=5.$$

Zbroj najmanje i najveće vrijednosti funkcije f je

$$3+5=8.$$

Odgovor je pod C.

3.inačica

Preoblikujemo zadanu funkciju.

$$f(x)=3\cdot\sin^2(x)+5\cdot\cos^2(x)\ \Rightarrow\ f(x)=3\cdot\sin^2(x)+5\cdot\left(1-\sin^2(x)\right)\ \Rightarrow$$

$$\Rightarrow\ f(x)=3\cdot\sin^2(x)+5-5\cdot\sin^2(x)\ \Rightarrow\ f(x)=5-2\cdot\sin^2(x).$$

Funkcija $f(x)=5-2\cdot\sin^2(x)$ ima:

- najmanju vrijednost za $\sin^2(x)=1$ i ona iznosi
$$5-2\cdot 1=5-2=3$$
- najveću vrijednost za $\sin^2(x)=0$ i ona iznosi
$$5-2\cdot 0=5-0=5.$$

Zbroj najmanje i najveće vrijednosti funkcije f je

$$3+5=8.$$

Odgovor je pod C.

Vježba 509

Zbroj najmanje i najveće vrijednosti funkcije $f(x)=2\cdot\sin^2(x)+5\cdot\cos^2(x)$ iznosi:

$A.\ 3 \qquad B.\ 5 \qquad C.\ 8 \qquad D.\ 7$

Rezultat: D.

Zadatak 510 (Helena, ekonomska škola)

Riješite jednadžbu $\left(\sin(x)-\cos(x)\right)^2=\sin(2\cdot x)$.

Rješenje 510

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad \sin^2(x)+\cos^2(x)=1\quad,\quad \sin(2\cdot x)=2\cdot\sin(x)\cdot\cos(x).$$

$$a-\frac{b}{c}=\frac{a\cdot c-b}{c}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Trigonometrijska jednadžba $\sin x=a\ ,\ |a|\le 1$

Skup rješenja jednadžbe $\sin x=a,\ |a|\le 1,$ je $\left\{x_o+k\cdot 2\cdot\pi : k\in Z\right\}\cup\left\{\pi-x_o+k\cdot 2\cdot\pi : k\in Z\right\}$

gdje je $x_o\in R$ jedno rješenje te jednadžbe.

$$\left(\sin(x)-\cos(x)\right)^2=\sin(2\cdot x)\ \Rightarrow\ \sin^2(x)-2\cdot\sin(x)\cdot\cos(x)+\cos^2(x)=\sin(2\cdot x)\ \Rightarrow$$

$$\Rightarrow\ 1-2\cdot\sin(x)\cdot\cos(x)=\sin(2\cdot x)\ \Rightarrow\ 1-\sin(2\cdot x)=\sin(2\cdot x)\ \Rightarrow\ 1=\sin(2\cdot x)+\sin(2\cdot x)\ \Rightarrow$$

$$\Rightarrow\ 1=2\cdot\sin(2\cdot x)\ \Rightarrow\ 2\cdot\sin(2\cdot x)=1\ \Rightarrow\ 2\cdot\sin(2\cdot x)=1\ /\cdot\frac{1}{2}\ \Rightarrow\ \sin(2\cdot x)=\frac{1}{2}\ \Rightarrow$$

$$\Rightarrow\ \begin{bmatrix}\text{zamjena}\\ 2\cdot x=t\end{bmatrix}\ \Rightarrow\ \sin(t)=\frac{1}{2}\ \Rightarrow\ t=\sin^{-1}\left(\frac{1}{2}\right)\ \Rightarrow\ \text{RAD}\ \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t_1 = \frac{\pi}{6} + k \cdot 2 \cdot \pi \\ t_2 = \pi - \frac{\pi}{6} + k \cdot 2 \cdot \pi \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = \frac{\pi}{6} + k \cdot 2 \cdot \pi \\ t_2 = \frac{5 \cdot \pi}{6} + k \cdot 2 \cdot \pi \end{array}\right\}.$$

Vraćamo se zamjeni.

- $$\left.\begin{array}{l} t = \frac{\pi}{6} + k \cdot 2 \cdot \pi \\ 2 \cdot x = t \end{array}\right\} \Rightarrow 2 \cdot x = \frac{\pi}{6} + k \cdot 2 \cdot \pi \Rightarrow 2 \cdot x = \frac{\pi}{6} + k \cdot 2 \cdot \pi \; / \cdot \frac{1}{2} \Rightarrow$$

$$\Rightarrow x_1 = \frac{\pi}{12} + k \cdot \pi \; , \; k \in Z$$

- $$\left.\begin{array}{l} t = \frac{5 \cdot \pi}{6} + k \cdot 2 \cdot \pi \\ 2 \cdot x = t \end{array}\right\} \Rightarrow 2 \cdot x = \frac{5 \cdot \pi}{6} + k \cdot 2 \cdot \pi \Rightarrow 2 \cdot x = \frac{5 \cdot \pi}{6} + k \cdot 2 \cdot \pi \; / \cdot \frac{1}{2} \Rightarrow$$

$$\Rightarrow x_2 = \frac{5 \cdot \pi}{6} + k \cdot \pi \; , \; k \in Z.$$

Vježba 510

Odmor!

Rezultat: …