Zadatak 441 (Katarina, maturantica)

Odredite sva rješenja jednadžbe $tg\,x + \frac{4}{tg\,x} = 4.$

Rješenje 441

Ponovimo!

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2 \quad , \quad a^2 = 0 \;\Rightarrow\; a = 0.$$

Trigonometrijska jednadžba tg x = a

Skup rješenja jednadžbe $tg\,x = a,\ a \in R,$ je $\{\, x_\circ + k \cdot \pi : k \in Z \,\},$ gdje je $x_\circ \in R$ jedno rješenje te jednadžbe.

$$tg\,x = a \;\Rightarrow\; tg\,x = tg\,x_\circ \;\Rightarrow\; x = x_\circ + k \cdot \pi \;,\; k \in Z.$$

Skup cijelih brojeva označavamo slovom Z, a zapisujemo

$$Z = \{\, ..., -3, -2, -1, 0, 1, 2, 3, ... \,\}.$$

$$tg\,x + \frac{4}{tg\,x} = 4 \;\Rightarrow\; tg\,x + \frac{4}{tg\,x} = 4 \;/\cdot tg\,x \;\Rightarrow\; tg^2 x + 4 = 4 \cdot tg\,x \;\Rightarrow\; tg^2 x - 4 \cdot tg\,x + 4 = 0 \;\Rightarrow$$

$$\Rightarrow\; (tg\,x - 2)^2 = 0 \;\Rightarrow\; tg\,x - 2 = 0 \;\Rightarrow\; tg\,x = 2 \;\Rightarrow\; x = tg^{-1}(2) \;\Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{džepno računalo} \\ \text{Mode: RAD} \end{bmatrix} \Rightarrow\; x = 1.107... + k \cdot \pi,\ k \in Z.$$

Vježba 441

Odredite sva rješenja jednadžbe $\frac{1}{4} \cdot tg\,x + \frac{1}{tg\,x} = 1.$

Rezultat: $x = 1.107... + k \cdot \pi,\ k \in Z.$

Zadatak 442 (Kristijan, gimnazija)

Ako je $\alpha + \beta = \frac{\pi}{2}\ (\alpha \neq \beta),$ onda je $\frac{tg(\alpha - \beta)}{tg\,\alpha - tg\,\beta}$ jednako:

$$A.\ \frac{1}{3} \qquad B.\ \frac{1}{2} \qquad C.\ 2 \qquad D.\ 3 \qquad E.\ \frac{2}{3}$$

Rješenje 442

Ponovimo!

$$tg(x-y) = \frac{tg\,x - tg\,y}{1 + tg\,x \cdot tg\,y} \quad , \quad tg\left(\frac{\pi}{2} - x\right) = ctg\,x \quad , \quad tg\,x \cdot ctg\,x = 1.$$

$$tg(\alpha - \beta) = \frac{tg\,\alpha - tg\,\beta}{1 + tg\,\alpha \cdot tg\,\beta} \;\Rightarrow \begin{bmatrix} \alpha + \beta = \frac{\pi}{2} \\ \beta = \frac{\pi}{2} - \alpha \end{bmatrix} \Rightarrow\; tg(\alpha - \beta) = \frac{tg\,\alpha - tg\,\beta}{1 + tg\,\alpha \cdot tg\left(\frac{\pi}{2} - \alpha\right)} \;\Rightarrow$$

$$\Rightarrow\; tg(\alpha - \beta) = \frac{tg\,\alpha - tg\,\beta}{1 + tg\,\alpha \cdot ctg\,\alpha} \;\Rightarrow\; tg(\alpha - \beta) = \frac{tg\,\alpha - tg\,\beta}{1 + 1} \;\Rightarrow\; tg(\alpha - \beta) = \frac{tg\,\alpha - tg\,\beta}{2} \;\Rightarrow$$

$$\Rightarrow\; tg(\alpha - \beta) = \frac{tg\,\alpha - tg\,\beta}{2} \;/\cdot \frac{1}{tg\,\alpha - tg\,\beta} \;\Rightarrow\; \frac{tg(\alpha - \beta)}{tg\,\alpha - tg\,\beta} = \frac{1}{2}.$$

Odgovor je pod B.

Vježba 442

Ako je $\alpha+\beta=\frac{\pi}{2}\;(\alpha\neq\beta)$, onda je $\frac{tg\,\alpha-tg\,\beta}{tg(\alpha-\beta)}$ jednako:

$$A.\ \frac{1}{3}\qquad B.\ \frac{1}{2}\qquad C.\ 2\qquad D.\ 3\qquad E.\ \frac{2}{3}$$

Rezultat: C.

Zadatak 443 (Vox, gimnazija)

Neka su α i β kutovi u pravokutnom trokutu (α ≠ 90º, β ≠ 90º). Ako je $tg\,\alpha=\frac{7}{24}$, onda je sin β jednako:

$$A.\ \frac{7}{25}\qquad B.\ \frac{1}{2}\qquad C.\ \frac{3}{2}\qquad D.\ \frac{24}{27}\qquad E.\ \frac{24}{25}$$

Rješenje 443

Ponovimo!

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$ctg\left(90^\circ-\alpha\right)=tg\,\alpha\quad,\quad \sin\alpha=\frac{1}{\sqrt{1+ctg^2\alpha}}.$$

Trokut je dio ravnine omeđen s tri dužine. Te dužine zovemo stranice trokuta.
Pravokutni trokuti imaju jedan pravi kut (kut od 90º). Stranice koje zatvaraju pravi kut zovu se katete, a najdulja stranica je hipotenuza pravokutnog trokuta.
Za šiljaste kutove α i β pravokutnog trokuta vrijedi:

$$\alpha+\beta=90^\circ.$$

$$\alpha+\beta=90^\circ\;\Rightarrow\;\beta=90^\circ-\alpha\;\Rightarrow\;ctg\,\beta=ctg\left(90^\circ-\alpha\right)\;\Rightarrow\;ctg\,\beta=tg\,\alpha\;\Rightarrow$$

$$\Rightarrow\left[tg\,\alpha=\frac{7}{24}\right]\Rightarrow\;ctg\,\beta=\frac{7}{24}.$$

Sada je:

$$\sin\beta=\frac{1}{\sqrt{1+ctg^2\beta}}\;\Rightarrow\;\sin\beta=\frac{1}{\sqrt{1+\left(\frac{7}{24}\right)^2}}\;\Rightarrow\;\sin\beta=\frac{1}{\sqrt{1+\frac{49}{576}}}\;\Rightarrow\;\sin\beta=\frac{1}{\sqrt{\frac{1}{1}+\frac{49}{576}}}\;\Rightarrow$$

$$\Rightarrow\;\sin\beta=\frac{1}{\sqrt{\frac{576+49}{576}}}\;\Rightarrow\;\sin\beta=\frac{1}{\sqrt{\frac{625}{576}}}\;\Rightarrow\;\sin\beta=\frac{1}{\frac{25}{24}}\;\Rightarrow\;\sin\beta=\frac{\frac{1}{1}}{\frac{25}{24}}\;\Rightarrow\;\sin\beta=\frac{24}{25}.$$

Odgovor je pod E.

Vježba 443

Odmor!

Rezultat: …

Zadatak 444 (Mladen, srednja škola)

Ako je $\alpha+\beta+\gamma=\pi$, onda je $\cos^2\alpha+\cos^2\beta+\cos^2\gamma$ jednako:

$$A.\ 1-2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma \qquad B.\ 1+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma$$

$$C.\ 4\cdot\sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\sin\frac{\gamma}{2} \qquad D.\ 4\cdot\cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2} \qquad E.\ 2+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma$$

Rješenje 444

Ponovimo!

$$\cos(\pi-x)=-\cos x \quad , \quad \cos(x+y)=\cos x\cdot\cos y-\sin x\cdot\sin y \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$(a\cdot b)^n=a^n\cdot b^n \quad , \quad \cos^2 x+\sin^2 x=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\alpha+\beta+\gamma=\pi \Rightarrow \alpha+\beta=\pi-\gamma \ \cos(\alpha+\beta)=\cos(\pi-\gamma) \Rightarrow$$

$$\Rightarrow \cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta=-\cos\gamma \Rightarrow \cos\alpha\cdot\cos\beta+\cos\gamma=\sin\alpha\cdot\sin\beta \Rightarrow$$

$$\Rightarrow \cos\alpha\cdot\cos\beta+\cos\gamma=\sin\alpha\cdot\sin\beta \ /^2 \Rightarrow (\cos\alpha\cdot\cos\beta+\cos\gamma)^2=(\sin\alpha\cdot\sin\beta)^2 \Rightarrow$$

$$\Rightarrow (\cos\alpha\cdot\cos\beta)^2+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma+\cos^2\gamma=\sin^2\alpha\cdot\sin^2\beta \Rightarrow$$

$$\Rightarrow \cos^2\alpha\cdot\cos^2\beta+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma+\cos^2\gamma=\left(1-\cos^2\alpha\right)\cdot\sin^2\beta \Rightarrow$$

$$\Rightarrow \cos^2\alpha\cdot\left(1-\sin^2\beta\right)+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma+\cos^2\gamma=\sin^2\beta-\cos^2\alpha\cdot\sin^2\beta \Rightarrow$$

$$\Rightarrow \cos^2\alpha-\cos^2\alpha\cdot\sin^2\beta+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma+\cos^2\gamma=\sin^2\beta-\cos^2\alpha\cdot\sin^2\beta \Rightarrow$$

$$\Rightarrow \cos^2\alpha-\cos^2\alpha\cdot\sin^2\beta+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma+\cos^2\gamma=\sin^2\beta-\cos^2\alpha\cdot\sin^2\beta \Rightarrow$$

$$\Rightarrow \cos^2\alpha+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma+\cos^2\gamma=\sin^2\beta \Rightarrow$$

$$\Rightarrow \cos^2\alpha+2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma+\cos^2\gamma=1-\cos^2\beta \Rightarrow$$

$$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma=1-2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma.$$

Odgovor je pod A.

Vježba 444

Ako je $\alpha+\beta+\gamma=\pi$, onda je $1-\cos^2\alpha-\cos^2\beta-\cos^2\gamma$ jednako:

$$A.\ 2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma \qquad B.\ -2\cdot\cos\alpha\cdot\cos\beta\cdot\cos\gamma$$

$$C.\ \sin\frac{\alpha}{2}\cdot\sin\frac{\beta}{2}\cdot\sin\frac{\gamma}{2} \qquad D.\ \cos\frac{\alpha}{2}\cdot\cos\frac{\beta}{2}\cdot\cos\frac{\gamma}{2} \qquad E.\ \cos\alpha\cdot\cos\beta\cdot\cos\gamma$$

Rezultat: A.

Zadatak 445 (Tea, srednja škola)

Koji je od navedenih brojeva najveći?

$$A.\ \cos 47 \qquad B.\ \sin 92 \qquad C.\ \cos 47^\circ \qquad D.\ \sin 92^\circ$$

Rješenje 445

Ponovimo!

Najvažnije jedinice mjere kuta su: stupanj (znak ^{0}) i radijan (znak rad).

Mjera punog kuta = 360°.
Mjera punog kuta = $2 \cdot \pi \ rad$.

$$\left.\begin{array}{l}\left.\begin{array}{l}\cos 47\\ \sin 92\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{džepno računalo}\\ \text{stanje RAD}\end{array}\right] \Rightarrow \left.\begin{array}{l}\cos 47 = -0.9923354692\\ \sin 92 = -0.7794660696\end{array}\right\}\\ \\ \left.\begin{array}{l}\cos 47^\circ\\ \sin 92^\circ\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{džepno računalo}\\ \text{stanje DEG}\end{array}\right] \Rightarrow \left.\begin{array}{l}\cos 47^\circ = 0.6819983601\\ \sin 92^\circ = 0.999390827\end{array}\right\}\end{array}\right\}.$$

Usporedimo vrijednosti svih zadanih izraza.
Odgovor je pod D.

Vježba 445

Koji je od navedenih brojeva najmanji?

$A.\ \cos 47 \qquad B.\ \sin 92 \qquad C.\ \cos 47^\circ \qquad D.\ \sin 92^\circ$

Rezultat: A.

Zadatak 446 (Marta, srednja škola)

Dokaži da je $\dfrac{1-\cos(2\cdot x)+\sin(2\cdot x)}{1+\cos(2\cdot x)+\sin(2\cdot x)} = tg\ x \ , \ tg\ x \neq -1.$

Rješenje 446

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad \sin(2\cdot x) = 2\cdot \sin x \cdot \cos x.$$

$$1-\cos(2\cdot x) = 2\cdot sin^2 x \quad , \quad 1+\cos(2\cdot x) = 2\cdot \cos^2 x \quad , \quad \frac{\sin x}{\cos x} = tg\ x.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

$$\frac{1-\cos(2\cdot x)+\sin(2\cdot x)}{1+\cos(2\cdot x)+\sin(2\cdot x)} = \frac{2\cdot \sin^2 x + 2\cdot \sin x\cdot \cos x}{2\cdot \cos^2 x + 2\cdot \sin x \cdot \cos x} = \frac{2\cdot \sin x\cdot(\sin x+\cos x)}{2\cdot \cos x\cdot(\cos x + \sin x)} =$$

$$= \frac{2\cdot \sin x\cdot(\sin x+\cos x)}{2\cdot \cos x\cdot(\sin x+\cos x)} = \frac{\sin x}{\cos x} = tg\ x.$$

Vježba 446

Dokaži da je $\dfrac{\sin^2 x + \sin x\cdot \cos x}{\cos^2 x + \sin x \cdot \cos x} = tg\ x \ , \ tg\ x \neq -1.$

Rezultat: Dokaz analogan.