

**Zadatak 441 (Katarina, maturantica)**

Odredite sva rješenja jednadžbe  $\operatorname{tg} x + \frac{4}{\operatorname{tg} x} = 4$ .

**Rješenje 441**

Ponovimo!

$$a^1 = a, \quad a^n \cdot a^m = a^{n+m}, \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2, \quad a^2 = 0 \Rightarrow a = 0.$$

**Trigonometrijska jednadžba  $\operatorname{tg} x = a$** 

Skup rješenja jednadžbe  $\operatorname{tg} x = a$ ,  $a \in \mathbb{R}$ , je  $\{x_0 + k \cdot \pi : k \in \mathbb{Z}\}$ , gdje je  $x_0 \in \mathbb{R}$  jedno rješenje te jednadžbe.

$$\operatorname{tg} x = a \Rightarrow \operatorname{tg} x = \operatorname{tg} x_0 \Rightarrow x = x_0 + k \cdot \pi, \quad k \in \mathbb{Z}.$$

Skup cijelih brojeva označavamo slovom  $\mathbb{Z}$ , a zapisujemo

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}.$$

$$\begin{aligned} \operatorname{tg} x + \frac{4}{\operatorname{tg} x} = 4 &\Rightarrow \operatorname{tg} x + \frac{4}{\operatorname{tg} x} = 4 \quad / \cdot \operatorname{tg} x \Rightarrow \operatorname{tg}^2 x + 4 = 4 \cdot \operatorname{tg} x \Rightarrow \operatorname{tg}^2 x - 4 \cdot \operatorname{tg} x + 4 = 0 \Rightarrow \\ &\Rightarrow (\operatorname{tg} x - 2)^2 = 0 \Rightarrow \operatorname{tg} x - 2 = 0 \Rightarrow \operatorname{tg} x = 2 \Rightarrow x = \operatorname{tg}^{-1}(2) \Rightarrow \\ &\Rightarrow \left[ \begin{array}{l} \text{džepno računalo} \\ \text{Mode: RAD} \end{array} \right] \Rightarrow x = 1.107\dots + k \cdot \pi, \quad k \in \mathbb{Z}. \end{aligned}$$

**Vježba 441**

Odredite sva rješenja jednadžbe  $\frac{1}{4} \cdot \operatorname{tg} x + \frac{1}{\operatorname{tg} x} = 1$ .

**Rezultat:**  $x = 1.107\dots + k \cdot \pi$ ,  $k \in \mathbb{Z}$ .

**Zadatak 442 (Kristijan, gimnazija)**

Ako je  $\alpha + \beta = \frac{\pi}{2}$  ( $\alpha \neq \beta$ ), onda je  $\frac{\operatorname{tg}(\alpha - \beta)}{\operatorname{tg} \alpha - \operatorname{tg} \beta}$  jednako:

A.  $\frac{1}{3}$       B.  $\frac{1}{2}$       C. 2      D. 3      E.  $\frac{2}{3}$

**Rješenje 442**

Ponovimo!

$$\operatorname{tg}(x - y) = \frac{\operatorname{tg} x - \operatorname{tg} y}{1 + \operatorname{tg} x \cdot \operatorname{tg} y}, \quad \operatorname{tg}\left(\frac{\pi}{2} - x\right) = \operatorname{ctg} x, \quad \operatorname{tg} x \cdot \operatorname{ctg} x = 1.$$

$$\begin{aligned} \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \cdot \operatorname{tg} \beta} &\Rightarrow \left[ \begin{array}{l} \alpha + \beta = \frac{\pi}{2} \\ \beta = \frac{\pi}{2} - \alpha \end{array} \right] \Rightarrow \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \cdot \operatorname{tg}\left(\frac{\pi}{2} - \alpha\right)} \Rightarrow \\ \Rightarrow \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + \operatorname{tg} \alpha \cdot \operatorname{ctg} \alpha} &\Rightarrow \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{1 + 1} \Rightarrow \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{2} \Rightarrow \\ \Rightarrow \operatorname{tg}(\alpha - \beta) = \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{2} &/ \cdot \frac{1}{\operatorname{tg} \alpha - \operatorname{tg} \beta} \Rightarrow \frac{\operatorname{tg}(\alpha - \beta)}{\operatorname{tg} \alpha - \operatorname{tg} \beta} = \frac{1}{2}. \end{aligned}$$

Odgovor je pod B.

### Vježba 442

Ako je  $\alpha + \beta = \frac{\pi}{2}$  ( $\alpha \neq \beta$ ), onda je  $\frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{\operatorname{tg}(\alpha - \beta)}$  jednako:

A.  $\frac{1}{3}$       B.  $\frac{1}{2}$       C. 2      D. 3      E.  $\frac{2}{3}$

**Rezultat:** C.

### Zadatak 443 (Vox, gimnazija)

Neka su  $\alpha$  i  $\beta$  kutovi u pravokutnom trokutu ( $\alpha \neq 90^\circ$ ,  $\beta \neq 90^\circ$ ). Ako je  $\operatorname{tg} \alpha = \frac{7}{24}$ , onda je  $\sin \beta$  jednako:

A.  $\frac{7}{25}$       B.  $\frac{1}{2}$       C.  $\frac{3}{2}$       D.  $\frac{24}{27}$       E.  $\frac{24}{25}$

### Rješenje 443

Ponovimo!

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad n = \frac{n}{1}, \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

$$\operatorname{ctg}(90^\circ - \alpha) = \operatorname{tg} \alpha, \quad \sin \alpha = \frac{1}{\sqrt{1 + \operatorname{ctg}^2 \alpha}}.$$

Trokut je dio ravnine omeđen s tri dužine. Te dužine zovemo stranice trokuta.

Pravokutni trokuti imaju jedan pravi kut (kut od  $90^\circ$ ). Stranice koje zatvaraju pravi kut zovu se katete, a najdulja stranica je hipotenuza pravokutnog trokuta.

Za šiljaste kutove  $\alpha$  i  $\beta$  pravokutnog trokuta vrijedi:

$$\alpha + \beta = 90^\circ.$$

$$\alpha + \beta = 90^\circ \Rightarrow \beta = 90^\circ - \alpha \Rightarrow \operatorname{ctg} \beta = \operatorname{ctg}(90^\circ - \alpha) \Rightarrow \operatorname{ctg} \beta = \operatorname{tg} \alpha \Rightarrow$$

$$\Rightarrow \left[ \operatorname{tg} \alpha = \frac{7}{24} \right] \Rightarrow \operatorname{ctg} \beta = \frac{7}{24}.$$

Sada je:

$$\sin \beta = \frac{1}{\sqrt{1 + \operatorname{ctg}^2 \beta}} \Rightarrow \sin \beta = \frac{1}{\sqrt{1 + \left(\frac{7}{24}\right)^2}} \Rightarrow \sin \beta = \frac{1}{\sqrt{1 + \frac{49}{576}}} \Rightarrow \sin \beta = \frac{1}{\sqrt{1 + \frac{49}{576}}} \Rightarrow$$

$$\Rightarrow \sin \beta = \frac{1}{\sqrt{\frac{576 + 49}{576}}} \Rightarrow \sin \beta = \frac{1}{\sqrt{\frac{625}{576}}} \Rightarrow \sin \beta = \frac{1}{\frac{25}{24}} \Rightarrow \sin \beta = \frac{1}{\frac{25}{24}} \Rightarrow \sin \beta = \frac{24}{25}.$$

Odgovor je pod E.

### Vježba 443

Odmor!

**Rezultat:** ...

**Zadatak 444 (Mladen, srednja škola)**

Ako je  $\alpha + \beta + \gamma = \pi$ , onda je  $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma$  jednako:

- A.  $1 - 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma$       B.  $1 + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma$   
 C.  $4 \cdot \sin \frac{\alpha}{2} \cdot \sin \frac{\beta}{2} \cdot \sin \frac{\gamma}{2}$       D.  $4 \cdot \cos \frac{\alpha}{2} \cdot \cos \frac{\beta}{2} \cdot \cos \frac{\gamma}{2}$       E.  $2 + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma$

**Rješenje 444**

Ponovimo!

$$\cos(\pi - x) = -\cos x \quad , \quad \cos(x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y \quad , \quad (a + b)^2 = a^2 + 2 \cdot a \cdot b + b^2 .$$

$$(a \cdot b)^n = a^n \cdot b^n \quad , \quad \cos^2 x + \sin^2 x = 1 .$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c) .$$

$$\begin{aligned} \alpha + \beta + \gamma = \pi &\Rightarrow \alpha + \beta = \pi - \gamma \quad \cos(\alpha + \beta) = \cos(\pi - \gamma) \Rightarrow \\ &\Rightarrow \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta = -\cos \gamma \Rightarrow \cos \alpha \cdot \cos \beta + \cos \gamma = \sin \alpha \cdot \sin \beta \Rightarrow \\ &\Rightarrow \cos \alpha \cdot \cos \beta + \cos \gamma = \sin \alpha \cdot \sin \beta \quad |^2 \Rightarrow (\cos \alpha \cdot \cos \beta + \cos \gamma)^2 = (\sin \alpha \cdot \sin \beta)^2 \Rightarrow \\ &\Rightarrow (\cos \alpha \cdot \cos \beta)^2 + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma + \cos^2 \gamma = \sin^2 \alpha \cdot \sin^2 \beta \Rightarrow \\ &\Rightarrow \cos^2 \alpha \cdot \cos^2 \beta + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma + \cos^2 \gamma = (1 - \cos^2 \alpha) \cdot \sin^2 \beta \Rightarrow \\ &\Rightarrow \cos^2 \alpha \cdot (1 - \sin^2 \beta) + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma + \cos^2 \gamma = \sin^2 \beta - \cos^2 \alpha \cdot \sin^2 \beta \Rightarrow \\ &\Rightarrow \cos^2 \alpha - \cos^2 \alpha \cdot \sin^2 \beta + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma + \cos^2 \gamma = \sin^2 \beta - \cos^2 \alpha \cdot \sin^2 \beta \Rightarrow \\ &\Rightarrow \cos^2 \alpha - \cos^2 \alpha \cdot \sin^2 \beta + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma + \cos^2 \gamma = \sin^2 \beta - \cos^2 \alpha \cdot \sin^2 \beta \Rightarrow \\ &\Rightarrow \cos^2 \alpha + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma + \cos^2 \gamma = \sin^2 \beta \Rightarrow \\ &\Rightarrow \cos^2 \alpha + 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma + \cos^2 \gamma = 1 - \cos^2 \beta \Rightarrow \\ &\Rightarrow \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 - 2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma . \end{aligned}$$

Odgovor je pod A.

**Vježba 444**

Ako je  $\alpha + \beta + \gamma = \pi$ , onda je  $1 - \cos^2 \alpha - \cos^2 \beta - \cos^2 \gamma$  jednako:

- A.  $2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma$       B.  $-2 \cdot \cos \alpha \cdot \cos \beta \cdot \cos \gamma$   
 C.  $\sin \frac{\alpha}{2} \cdot \sin \frac{\beta}{2} \cdot \sin \frac{\gamma}{2}$       D.  $\cos \frac{\alpha}{2} \cdot \cos \frac{\beta}{2} \cdot \cos \frac{\gamma}{2}$       E.  $\cos \alpha \cdot \cos \beta \cdot \cos \gamma$

**Rezultat:**      A.

**Zadatak 445 (Tea, srednja škola)**

Koji je od navedenih brojeva najveći?

- A.  $\cos 47$       B.  $\sin 92$       C.  $\cos 47^\circ$       D.  $\sin 92^\circ$

**Rješenje 445**

Ponovimo!

Najvažnije jedinice mjere kuta su: stupanj (znak  $^\circ$ ) i radijan (znak rad).

Mjera punog kuta =  $360^\circ$ .

Mjera punog kuta =  $2 \cdot \pi \text{ rad}$ .

$$\left. \begin{array}{l} \cos 47 \\ \sin 92 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{džepno računalo} \\ \text{stanje RAD} \end{array} \right] \Rightarrow \left. \begin{array}{l} \cos 47 = -0.9923354692 \\ \sin 92 = -0.7794660696 \end{array} \right\}$$
$$\left. \begin{array}{l} \cos 47^\circ \\ \sin 92^\circ \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{džepno računalo} \\ \text{stanje DEG} \end{array} \right] \Rightarrow \left. \begin{array}{l} \cos 47^\circ = 0.6819983601 \\ \sin 92^\circ = 0.999390827 \end{array} \right\}$$

Usporedimo vrijednosti svih zadanih izraza.

Odgovor je pod D.

### Vježba 445

Koji je od navedenih brojeva najmanji?

- A.  $\cos 47$       B.  $\sin 92$       C.  $\cos 47^\circ$       D.  $\sin 92^\circ$

**Rezultat:** A.

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