Zadatak 421 (Marija, gimnazija)

Ako je $\cos^2\alpha+\cos^2\beta=a$, onda je $\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)$ jednako:

$A.\ a-1 \qquad B.\ 1-a \qquad C.\ a+1 \qquad D.\ a^2-1$

Rješenje 421

Ponovimo!

$$\cos x\cdot\cos y=\frac{1}{2}\cdot(\cos(x-y)+\cos(x+y)) \quad , \quad \cos(2\cdot x)=2\cdot\cos^2 x-1.$$

$$\frac{a}{b}\cdot b=a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Preoblikujemo zadani izraz:

$$\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)=\frac{1}{2}\cdot(\cos(\alpha+\beta-(\alpha-\beta))+\cos(\alpha+\beta+\alpha-\beta))=$$

$$=\frac{1}{2}\cdot(\cos(\alpha+\beta-\alpha+\beta)+\cos(\alpha+\beta+\alpha-\beta))=\frac{1}{2}\cdot(\cos(\alpha+\beta-\alpha+\beta)+\cos(\alpha+\beta+\alpha-\beta))=$$

$$=\frac{1}{2}\cdot(\cos(2\cdot\beta)+\cos(2\cdot\alpha))=\frac{1}{2}\cdot(\cos(2\cdot\alpha)+\cos(2\cdot\beta))=\left[\cos(2\cdot x)=2\cdot\cos^2 x-1\right]=$$

$$=\frac{1}{2}\cdot\left(2\cdot\cos^2\alpha-1+2\cdot\cos^2\beta-1\right)=\frac{1}{2}\cdot\left(2\cdot\cos^2\alpha+2\cdot\cos^2\beta-2\right)=$$

$$=\cos^2\alpha+\cos^2\beta-1=\left[\begin{array}{c}\text{uvjet}\\ \cos^2\alpha+\cos^2\beta=a\end{array}\right]=a-1.$$

Odgovor je pod A.

Vježba 421

Ako je $\cos^2\alpha+\cos^2\beta=a+2$, onda je $\cos(\alpha+\beta)\cdot\cos(\alpha-\beta)$ jednako:

$A.\ a-1 \qquad B.\ 1-a \qquad C.\ a+1 \qquad D.\ a^2-1$

Rezultat: C.

Zadatak 422 (Patrik, gimnazija)

Izračunajte bez uporabe džepnog računala $\sin 75^\circ\cdot\cos 75^\circ$.

Rješenje 422

Ponovimo!

$$n=\frac{n}{1} \quad , \quad 2\cdot\sin x\cdot\cos x=\sin(2\cdot x) \quad , \quad \sin\left(180^\circ-x\right)=\sin x \quad , \quad \sin 0^\circ=1.$$

$$\sin 30^\circ=\frac{1}{2} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$\sin x\cdot\cos y=\frac{1}{2}\cdot(\sin(x+y)+\sin(x-y)).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

1.inačica

$$\sin 75^\circ \cdot \cos 75^\circ = \left[n = \frac{n}{1} \right] = \frac{\sin 75^\circ \cdot \cos 75^\circ}{1} = \left[\frac{a}{b} = \frac{a \cdot n}{b \cdot n} \right] = \frac{2 \cdot \sin 75^\circ \cdot \cos 75^\circ}{2 \cdot 1} =$$

$$= \frac{2 \cdot \sin 75^\circ \cdot \cos 75^\circ}{2} = \left[2 \cdot \sin x \cdot \cos x = \sin(2 \cdot x) \right] = \frac{\sin(2 \cdot 75^\circ)}{2} = \frac{\sin 150^\circ}{2} =$$

$$= \left[\sin(180^\circ - x) = \sin x \right] = \frac{\sin(180^\circ - 30^\circ)}{2} = \frac{\sin 30^\circ}{2} = \frac{\frac{1}{2}}{2} = \frac{\frac{1}{2}}{\frac{2}{1}} = \frac{1}{4}.$$

2.inačica

$$\sin 75^\circ \cdot \cos 75^\circ = \left[\sin x \cdot \cos y = \frac{1}{2} \cdot \left(\sin(x+y) + \sin(x-y) \right) \right] =$$

$$= \frac{1}{2} \cdot \left(\sin(75^\circ + 75^\circ) + \sin(75^\circ - 75^\circ) \right) = \frac{1}{2} \cdot \left(\sin 150^\circ + \sin 0^\circ \right) = \frac{1}{2} \cdot \left(\sin 150^\circ + 0 \right) =$$

$$= \frac{1}{2} \cdot \sin 150^\circ = \left[\sin(180^\circ - x) = \sin x \right] = \frac{1}{2} \cdot \sin(180^\circ - 30^\circ) = \frac{1}{2} \cdot \sin 30^\circ =$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}.$$

Vježba 422

Izračunajte bez uporabe džepnog računala $4 \cdot \sin 75^\circ \cdot \cos 75^\circ$.

Rezultat: 1.

Zadatak 423 (Patrik, gimnazija)

Izračunajte bez uporabe džepnog računala $\cos 15^\circ \cdot \sin 75^\circ$.

Rješenje 423

Ponovimo!

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad , \quad \cos 0^\circ = 1 \quad , \quad \sin 60^\circ = \frac{\sqrt{3}}{2} \quad , \quad \sin 90^\circ = 1.$$

$$\sin(90^\circ - x) = \cos x \quad , \quad \sin(-x) = -\sin x \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

$$\cos x \cdot \cos y = \frac{1}{2} \cdot \left(\cos(x+y) + \cos(x-y) \right).$$

$$\cos x \cdot \sin y = \frac{1}{2} \cdot \left(\sin(x+y) - \sin(x-y) \right).$$

$$\cos^2 x = \frac{1 + \cos(2 \cdot x)}{2}.$$

1.inačica

$$\cos 15^\circ \cdot \sin 75^\circ = \left[\sin(90^\circ - x) = \cos x \right] = \cos 15^\circ \cdot \sin(90^\circ - 15^\circ) = \cos 15^\circ \cdot \cos 15^\circ =$$

$$=\left[\cos x\cdot\cos y=\frac{1}{2}\cdot\left(\cos\left(x+y\right)+\cos\left(x-y\right)\right)\right]=$$

$$=\frac{1}{2}\cdot\left(\cos\left(15^\circ+15^\circ\right)+\cos\left(15^\circ-15^\circ\right)\right)=\frac{1}{2}\cdot\left(\cos 30^\circ+\cos 0^\circ\right)=\frac{1}{2}\cdot\left(\frac{\sqrt{3}}{2}+1\right)=\frac{1}{2}\cdot\left(1+\frac{\sqrt{3}}{2}\right).$$

2.inačica

$$\cos 15^\circ\cdot\sin 75^\circ=\left[\cos x\cdot\sin y=\frac{1}{2}\cdot\left(\sin\left(x+y\right)-\sin\left(x-y\right)\right)\right]=$$

$$=\frac{1}{2}\cdot\left(\sin\left(15^\circ+75^\circ\right)-\sin\left(15^\circ-75^\circ\right)\right)=\frac{1}{2}\cdot\left(\sin 90^\circ-\sin\left(-60^\circ\right)\right)=\left[\sin\left(-x\right)=-\sin x\right]=$$

$$=\frac{1}{2}\cdot\left(\sin 90^\circ+\sin 60^\circ\right)=\frac{1}{2}\cdot\left(1+\frac{\sqrt{3}}{2}\right).$$

3.inačica

$$\cos 15^\circ\cdot\sin 75^\circ=\left[\sin\left(90^\circ-x\right)=\cos x\right]=\cos 15^\circ\cdot\sin\left(90^\circ-15^\circ\right)=\cos 15^\circ\cdot\cos 15^\circ=$$

$$=\cos^2 15^\circ=\left[\cos^2 x=\frac{1+\cos\left(2\cdot x\right)}{2}\right]=\frac{1+\cos\left(2\cdot 15^\circ\right)}{2}=\frac{1+\cos 30^\circ}{2}=$$

$$=\frac{1}{2}\cdot\left(1+\cos 30^\circ\right)=\frac{1}{2}\cdot\left(1+\frac{\sqrt{3}}{2}\right).$$

Vježba 423

Izračunajte bez uporabe džepnog računala $2\cdot\cos 15^\circ\cdot\sin 75^\circ$.

Rezultat: $1+\frac{\sqrt{3}}{2}.$

Zadatak 424 (Branka, gimnazija)

Dokaži identitet $\frac{\sin^4\alpha+\cos^4\alpha-1}{\sin^6\alpha+\cos^6\alpha-1}=\frac{2}{3}.$

Rješenje 424

Ponovimo!

$$a^1=a\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^n:a^m=a^{n-m}\quad,\quad\cos^2\alpha+\sin^2\alpha=1.$$

$$a^2+2\cdot a\cdot b+b^2=\left(a+b\right)^2\quad,\quad a^3+b^3=\left(a+b\right)\cdot\left(a^2-a\cdot b+b^2\right).$$

$$\frac{-a}{-b}=\frac{a}{b}\quad,\quad a^2-b^2=\left(a-b\right)\cdot\left(a+b\right).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

Lijevu stranu postupno preoblikujemo.

$$\frac{\sin^4\alpha+\cos^4\alpha-1}{\sin^6\alpha+\cos^6\alpha-1}=\frac{\sin^4\alpha+\cos^4\alpha-\left(\sin^2\alpha+\cos^2\alpha\right)}{\sin^6\alpha+\cos^6\alpha-\left(\sin^2\alpha+\cos^2\alpha\right)}=$$

$$=\frac{\sin^4\alpha+\cos^4\alpha-\sin^2\alpha-\cos^2\alpha}{\sin^6\alpha+\cos^6\alpha-\sin^2\alpha-\cos^2\alpha}=\frac{-\left(-\sin^4\alpha-\cos^4\alpha+\sin^2\alpha+\cos^2\alpha\right)}{-\left(-\sin^6\alpha-\cos^6\alpha+\sin^2\alpha+\cos^2\alpha\right)}=$$

$$=\frac{-\sin^4\alpha-\cos^4\alpha+\sin^2\alpha+\cos^2\alpha}{-\sin^6\alpha-\cos^6\alpha+\sin^2\alpha+\cos^2\alpha}=\frac{\sin^2\alpha-\sin^4\alpha+\cos^2\alpha-\cos^4\alpha}{\sin^2\alpha-\sin^6\alpha+\cos^2\alpha-\cos^6\alpha}=$$

$$=\frac{\sin^2\alpha\cdot\left(1-\sin^2\alpha\right)+\cos^2\alpha\cdot\left(1-\cos^2\alpha\right)}{\sin^2\alpha\cdot\left(1-\sin^4\alpha\right)+\cos^2\alpha\cdot\left(1-\cos^4\alpha\right)}=\frac{\sin^2\alpha\cdot\cos^2\alpha+\cos^2\alpha\cdot\sin^2\alpha}{\sin^2\alpha\cdot\left(1^2-\left(\sin^2\alpha\right)^2\right)+\cos^2\alpha\cdot\left(1^2-\left(\cos^2\alpha\right)^2\right)}=$$

$$=\frac{\sin^2\alpha\cdot\cos^2\alpha+\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\left(1-\sin^2\alpha\right)\cdot\left(1+\sin^2\alpha\right)+\cos^2\alpha\cdot\left(1-\cos^2\alpha\right)\cdot\left(1+\cos^2\alpha\right)}=$$

$$=\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha\cdot\left(1+\sin^2\alpha\right)+\cos^2\alpha\cdot\sin^2\alpha\cdot\left(1+\cos^2\alpha\right)}=$$

$$=\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha\cdot\left(1+\sin^2\alpha\right)+\sin^2\alpha\cdot\cos^2\alpha\cdot\left(1+\cos^2\alpha\right)}=$$

$$=\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha\cdot\left(1+\sin^2\alpha+1+\cos^2\alpha\right)}=\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha\cdot\left(1+\sin^2\alpha+1+\cos^2\alpha\right)}=$$

$$=\frac{2}{1+\sin^2\alpha+1+\cos^2\alpha}=\frac{2}{2+\sin^2\alpha+\cos^2\alpha}=\frac{2}{2+1}=\frac{2}{3}.$$

2.inačica

Lijevu stranu postupno preoblikujemo.

$$\frac{\sin^4\alpha+\cos^4\alpha-1}{\sin^6\alpha+\cos^6\alpha-1}=\frac{\sin^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha-2\cdot\sin^2\alpha\cdot\cos^2\alpha-1}{\left(\sin^2\alpha\right)^3+\left(\cos^2\alpha\right)^3}=$$

$$=\frac{\left(\sin^2\alpha\right)^2+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\left(\cos^2\alpha\right)^2-2\cdot\sin^2\alpha\cdot\cos^2\alpha-1}{\left(\sin^2\alpha+\cos^2\alpha\right)\cdot\left(\left(\sin^2\alpha\right)^2-\sin^2\alpha\cdot\cos^2\alpha+\left(\cos^2\alpha\right)^2\right)-1}=$$

$$=\frac{\left(\sin^2\alpha+\cos^2\alpha\right)^2-2\cdot\sin^2\alpha\cdot\cos^2\alpha-1}{1\cdot\left(\sin^4\alpha-\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha\right)-1}=\frac{1^2-2\cdot\sin^2\alpha\cdot\cos^2\alpha-1}{\sin^4\alpha-\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha-1}=$$

$$=\frac{1-2\cdot\sin^2\alpha\cdot\cos^2\alpha-1}{\sin^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha-3\cdot\sin^2\alpha\cdot\cos^2\alpha-1}=$$

$$=\frac{1-2\cdot\sin^2\alpha\cdot\cos^2\alpha-1}{\left(\sin^2\alpha\right)^2+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\left(\cos^2\alpha\right)^2-3\cdot\sin^2\alpha\cdot\cos^2\alpha-1}=$$

$$=\frac{-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\left(\sin^2\alpha+\cos^2\alpha\right)^2-3\cdot\sin^2\alpha\cdot\cos^2\alpha-1}=\frac{-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{1^2-3\cdot\sin^2\alpha\cdot\cos^2\alpha-1}=$$

$$=\frac{-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{1-3\cdot\sin^2\alpha\cdot\cos^2\alpha-1}=\frac{-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{1-3\cdot\sin^2\alpha\cdot\cos^2\alpha-1}=\frac{-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{-3\cdot\sin^2\alpha\cdot\cos^2\alpha}=$$

$$=\frac{-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{-3\cdot\sin^2\alpha\cdot\cos^2\alpha}=\frac{2}{3}.$$

Vježba 424

Dokaži identitet $\dfrac{\sin^6\alpha+\cos^6\alpha-1}{\sin^4\alpha+\cos^4\alpha-1}=\dfrac{3}{2}.$

Rezultat: Dokaz analogan.

Zadatak 425 ( **, gimnazija)**

Riješi jednadžbu: $\sqrt{3}\cdot\sin x-\cos x=0.$

Rješenje 425

Ponovimo!

$$tg\,x=\frac{\sin x}{\cos x}\quad,\quad tg\frac{\pi}{6}=\frac{\sqrt{3}}{3}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}.$$

$$\left(\sqrt{a}\right)^2=a.$$

Trigonometrijska jednadžba tg x = a

Skup rješenja jednadžbe $tg\,x=a,\ a\in R,$ je $\left\{x_\circ+k\cdot\pi : k\in Z\right\},$ gdje je $x_\circ\in R$ jedno rješenje te jednadžbe.

$$tg\,x=a\ \Rightarrow\ tg\,x=tg\,x_\circ\ \Rightarrow\ x=x_\circ+k\cdot\pi\ ,\ k\in Z.$$

Skup cijelih brojeva označavamo slovom Z, a zapisujemo

$$Z=\left\{...,-3,-2,-1,0,1,2,3,...\right\}.$$

$$\sqrt{3}\cdot\sin x-\cos x=0\ \Rightarrow\ \sqrt{3}\cdot\sin x=\cos x\ \Rightarrow\ \sqrt{3}\cdot\sin x=\cos x\ /\cdot\frac{1}{\sqrt{3}\cdot\cos x}\ \Rightarrow$$

$$\Rightarrow \frac{\sin x}{\cos x}=\frac{1}{\sqrt{3}} \Rightarrow tg\, x=\frac{1}{\sqrt{3}} \Rightarrow tg\, x=\frac{1}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \Rightarrow tg\, x=\frac{\sqrt{3}}{\left(\sqrt{3}\right)^2} \Rightarrow tg\, x=\frac{\sqrt{3}}{3} \Rightarrow$$

$$\Rightarrow x=tg^{-1}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow x=\frac{\pi}{6}+k\cdot\pi,\ k\in Z.$$

Vježba 425

Riješi jednadžbu: $\cos x-\sqrt{3}\cdot\sin x=0.$

Rezultat: $x=\frac{\pi}{6}+k\cdot\pi,\ k\in Z.$

Zadatak 426 (Martin, gimnazija)

Dokažite jednakost: $tg\, 9^\circ-tg\, 27^\circ-tg\, 63^\circ+tg\, 81^\circ=4.$

Rješenje 426

Ponovimo!

$$tg\,\alpha+tg\,\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta} \quad , \quad \sin 90^\circ=1 \quad , \quad \cos 90^\circ=0 \quad , \quad n=\frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$\cos\alpha\cdot\cos\beta=\frac{1}{2}\cdot\left(\cos(\alpha+\beta)+\cos(\alpha-\beta)\right), \quad \cos(-\alpha)=\cos\alpha.$$

$$\cos\left(90^\circ-\alpha\right)=\sin\alpha \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}, \quad \sin\alpha-\sin\beta=2\cdot\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$tg\, 9^\circ-tg\, 27^\circ-tg\, 63^\circ+tg\, 81^\circ=\left(tg\, 9^\circ+tg\, 81^\circ\right)-\left(tg\, 27^\circ+tg\, 63^\circ\right)=$$

$$=\left[tg\,\alpha+tg\,\beta=\frac{\sin(\alpha+\beta)}{\cos\alpha\cdot\cos\beta}\right]=\frac{\sin\left(9^\circ+81^\circ\right)}{\cos 9^\circ\cdot\cos 81^\circ}-\frac{\sin\left(27^\circ+63^\circ\right)}{\cos 27^\circ\cdot\cos 63^\circ}=$$

$$=\frac{\sin 90^\circ}{\cos 9^\circ\cdot\cos 81^\circ}-\frac{\sin 90^\circ}{\cos 27^\circ\cdot\cos 63^\circ}=\left[\sin 90^\circ=1\right]=\frac{1}{\cos 9^\circ\cdot\cos 81^\circ}-\frac{1}{\cos 27^\circ\cdot\cos 63^\circ}=$$

$$=\left[\cos\alpha\cdot\cos\beta=\frac{1}{2}\cdot\left(\cos(\alpha+\beta)+\cos(\alpha-\beta)\right)\right]=$$

$$=\frac{1}{\frac{1}{2}\cdot\left(\cos\left(9^\circ+81^\circ\right)+\cos\left(9^\circ-81^\circ\right)\right)}-\frac{1}{\frac{1}{2}\cdot\left(\cos\left(27^\circ+63^\circ\right)+\cos\left(27^\circ-63^\circ\right)\right)}=$$

$$=\frac{\frac{1}{1}}{\frac{1}{2}\cdot\left(\cos 90^\circ+\cos\left(-72^\circ\right)\right)}-\frac{\frac{1}{1}}{\frac{1}{2}\cdot\left(\cos 90^\circ+\cos\left(-36^\circ\right)\right)}=\left[\cos(-\alpha)=\cos\alpha\right]=$$

$$=\frac{2}{\cos 90^\circ+\cos 72^\circ}-\frac{2}{\cos 90^\circ+\cos 36^\circ}=\left[\cos 90^\circ=0\right]=\frac{2}{0+\cos 72^\circ}-\frac{2}{0+\cos 36^\circ}=$$

$$=\frac{2}{\cos 72^\circ}-\frac{2}{\cos 36^\circ}=\left[\cos\left(90^\circ-\alpha\right)=\sin\alpha\right]=\frac{2}{\cos\left(90^\circ-18^\circ\right)}-\frac{2}{\cos\left(90^\circ-54^\circ\right)}=$$

$$=\frac{2}{\sin 18^\circ}-\frac{2}{\sin 54^\circ}=2\cdot\left(\frac{1}{\sin 18^\circ}-\frac{1}{\sin 54^\circ}\right)=2\cdot\frac{\sin 54^\circ-\sin 18^\circ}{\sin 18^\circ\cdot\sin 54^\circ}=$$

$$=\left[\sin\alpha-\sin\beta=2\cdot\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}\right]=2\cdot\frac{2\cdot\cos\frac{54^\circ+18^\circ}{2}\cdot\sin\frac{54^\circ-18^\circ}{2}}{\sin 18^\circ\cdot\sin 54^\circ}=$$

$$=2\cdot\frac{2\cdot\cos\frac{72^\circ}{2}\cdot\sin\frac{36^\circ}{2}}{\sin 18^\circ\cdot\sin 54^\circ}=4\cdot\frac{\cos\frac{72^\circ}{2}\cdot\sin\frac{36^\circ}{2}}{\sin 18^\circ\cdot\sin 54^\circ}=4\cdot\frac{\cos 36^\circ\cdot\sin 18^\circ}{\sin 18^\circ\cdot\sin 54^\circ}=4\cdot\frac{\cos 36^\circ\cdot\sin 18^\circ}{\sin 18^\circ\cdot\sin 54^\circ}=$$

$$=4\cdot\frac{\cos 36^\circ}{\sin 54^\circ}=\left[\cos\left(90^\circ-\alpha\right)=\sin\alpha\right]=4\cdot\frac{\cos\left(90^\circ-54^\circ\right)}{\sin 54^\circ}=4\cdot\frac{\sin 54^\circ}{\sin 54^\circ}=4\cdot\frac{\sin 54^\circ}{\sin 54^\circ}=4.$$

Vježba 426

Dokažite jednakost: $tg\,9^\circ-tg\,27^\circ-tg\,63^\circ+tg\,81^\circ-4=0.$

Rezultat: Dokaz analogan.

Zadatak 427 (Lea, srednja škola)

Pojednostavnite izraz: $\frac{2\cdot\cos 48^\circ+\sin 42^\circ}{3\cdot\cos 48^\circ}.$

Rješenje 427

Ponovimo!

$$\sin\left(90^\circ-\alpha\right)=\cos\alpha.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{2\cdot\cos 48^\circ+\sin 42^\circ}{3\cdot\cos 48^\circ}=\left[\sin\left(90^\circ-\alpha\right)=\cos\alpha\right]=\frac{2\cdot\cos 48^\circ+\sin\left(90^\circ-48^\circ\right)}{3\cdot\cos 48^\circ}=$$

$$=\frac{2\cdot\cos 48^\circ+\cos 48^\circ}{3\cdot\cos 48^\circ}=\frac{3\cdot\cos 48^\circ}{3\cdot\cos 48^\circ}=\frac{3\cdot\cos 48^\circ}{3\cdot\cos 48^\circ}=1.$$

Vježba 427

Pojednostavnite izraz: $\frac{3\cdot\cos 48^\circ+\sin 42^\circ}{4\cdot\cos 48^\circ}.$

Rezultat: 1.

Zadatak 428 (Luka, gimnazija)

Preoblikuj u umnožak: $tg\,40^\circ + ctg\,40^\circ$.

Rješenje 428

Ponovimo!

$$ctg\left(90^\circ - \alpha\right) = tg\,\alpha \quad , \quad tg\,\alpha + tg\,\beta = \frac{\sin\left(\alpha+\beta\right)}{\cos\alpha\cdot\cos\beta} \quad , \quad \sin 90^\circ = 1 \quad , \quad \cos\left(90^\circ - \alpha\right) = \sin\alpha.$$

$$\sin\left(90^\circ - \alpha\right) = \cos\alpha \quad , \quad 2\cdot\sin\alpha\cdot\cos\alpha = \sin\left(2\cdot\alpha\right).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a\cdot n}{b\cdot n} \ , \ n \ne 0 \ , \ n \ne 1.$$

$$tg\,40^\circ + ctg\,40^\circ = \left[\,ctg\left(90^\circ - \alpha\right) = tg\,\alpha\,\right] = tg\,40^\circ + ctg\left(90^\circ - 50^\circ\right) = tg\,40^\circ + tg\,50^\circ =$$

$$= \left[\,tg\,\alpha + tg\,\beta = \frac{\sin\left(\alpha+\beta\right)}{\cos\alpha\cdot\cos\beta}\,\right] = \frac{\sin\left(40^\circ + 50^\circ\right)}{\cos 40^\circ\cdot\cos 50^\circ} = \frac{\sin 90^\circ}{\cos 40^\circ\cdot\cos 50^\circ} = \left[\,\sin 90^\circ = 1\,\right] =$$

$$= \frac{1}{\cos 40^\circ\cdot\cos 50^\circ} = \left[\,\cos\left(90^\circ - \alpha\right) = \sin\alpha\,\right] = \frac{1}{\cos 40^\circ\cdot\cos\left(90^\circ - 40^\circ\right)} = \frac{1}{\cos 40^\circ\cdot\sin 40^\circ} =$$

$$= \frac{2\cdot 1}{2\cdot\cos 40^\circ\cdot\sin 40^\circ} = \frac{2}{\sin\left(2\cdot 40^\circ\right)} = \frac{2}{\sin 80^\circ} = \left[\,\sin\left(90^\circ - \alpha\right) = \cos\alpha\,\right] =$$

$$= \frac{2}{\sin\left(90^\circ - 10^\circ\right)} = \frac{2}{\cos 10^\circ}.$$

Vježba 428

Preoblikuj u umnožak: $\frac{1}{2}\cdot\left(tg\,40^\circ + ctg\,40^\circ\right)$.

Rezultat: $\frac{1}{\cos 10^\circ}$.

Zadatak 429 (Help, gimnazija)

Dokazati identitet: $\frac{\sin\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$.

Rješenje 429

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d} \quad , \quad \left(a-b\right)\cdot\left(a+b\right) = a^2 - b^2 \quad , \quad \cos^2\alpha + \sin^2\alpha = 1 \quad , \quad a^1 = a.$$

$$\frac{a^n}{a^m} = a^{n-m} \quad , \quad a^n\cdot a^m = a^{n+m}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

1.inačica

$$\frac{\sin\alpha}{1-\cos\alpha}=\left[\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\right]=\frac{\sin\alpha}{1-\cos\alpha}\cdot\frac{1+\cos\alpha}{1+\cos\alpha}=\frac{\sin\alpha\cdot(1+\cos\alpha)}{(1-\cos\alpha)\cdot(1+\cos\alpha)}=$$

$$=\frac{\sin\alpha\cdot(1+\cos\alpha)}{1-\cos^2\alpha}=\frac{\sin\alpha\cdot(1+\cos\alpha)}{\sin^2\alpha}=\frac{\sin\alpha\cdot(1+\cos\alpha)}{\sin^2\alpha}=\frac{1+\cos\alpha}{\sin\alpha}.$$

2.inačica

$$\frac{1+\cos\alpha}{\sin\alpha}=\left[\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\right]=\frac{1+\cos\alpha}{\sin\alpha}\cdot\frac{1-\cos\alpha}{1-\cos\alpha}=\frac{(1+\cos\alpha)\cdot(1-\cos\alpha)}{\sin\alpha\cdot(1-\cos\alpha)}=\frac{1-\cos^2\alpha}{\sin\alpha\cdot(1-\cos\alpha)}=$$

$$=\frac{\sin^2\alpha}{\sin\alpha\cdot(1-\cos\alpha)}=\frac{\sin^2\alpha}{\sin\alpha\cdot(1-\cos\alpha)}=\frac{\sin\alpha}{1-\cos\alpha}.$$

3.inačica

$$\frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}\ \Rightarrow\ \frac{\sin\alpha}{1-\cos\alpha}=\frac{1+\cos\alpha}{\sin\alpha}\ /\cdot(1-\cos\alpha)\cdot\sin\alpha\ \Rightarrow$$

$$\Rightarrow\ \sin^2\alpha=(1+\cos\alpha)\cdot(1-\cos\alpha)\ \Rightarrow\ \sin^2\alpha=1-\cos^2\alpha\ \Rightarrow\ \sin^2\alpha=\sin^2\alpha.$$

Vježba 429

Dokazati identitet: $\frac{1+\cos\alpha}{\sin\alpha}=\frac{\sin\alpha}{1-\cos\alpha}.$

Rezultat: Dokaz analogan.

Zadatak 430 (Domagoj, gimnazija)

Dokazati nejednakost: $|\sin\alpha|+|\cos\alpha|\geq 1.$

Rješenje 430

Ponovimo!

$$a\geq b\ ,\ c>0\ \Rightarrow\ a\cdot c\geq b\cdot c\quad,\quad \cos^2\alpha+\sin^2\alpha=1\quad,\quad \left.\begin{matrix}a\geq 0\\ b\geq 0\end{matrix}\right\}\ \Rightarrow\ a\cdot b\geq 0.$$

$$\left.\begin{matrix}a\geq b\\ c=d\end{matrix}\right\}\ \Rightarrow\ a+c\geq b+d\quad,\quad a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

$$a^2\geq b^2\ ,\ a>b>0\ \Rightarrow\ a\geq b.$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x|=\begin{cases}x,\ x\geq 0\\ -x,\ x<0.\end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x \geq 0$, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, $x < 0$, je $|x| = -x$.

$$\boxed{|x| \ge 0 \quad , \quad |x|^2 = x^2.}$$

Budući da je

$$|\sin\alpha| \ge 0 \quad , \quad |\cos\alpha| \ge 0,$$

slijedi

$$|\sin\alpha| \cdot |\cos\alpha| \ge 0 \Rightarrow |\sin\alpha| \cdot |\cos\alpha| \ge 0 \ / \cdot 2 \Rightarrow 2 \cdot |\sin\alpha| \cdot |\cos\alpha| \ge 0.$$

Sada je:

$$\left.\begin{array}{l} 2 \cdot |\sin\alpha| \cdot |\cos\alpha| \ge 0 \\ |\sin\alpha|^2 + |\cos\alpha|^2 = 1 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{nejednakost} \\ \text{i jednakost} \end{array}\right] \Rightarrow 2 \cdot |\sin\alpha| \cdot |\cos\alpha| + |\sin\alpha|^2 + |\cos\alpha|^2 \ge 0 + 1 \Rightarrow$$

$$\Rightarrow |\sin\alpha|^2 + 2 \cdot |\sin\alpha| \cdot |\cos\alpha| + |\cos\alpha|^2 \ge 1 \Rightarrow \left(|\sin\alpha| + |\cos\alpha|\right)^2 \ge 1 \Rightarrow$$

$$\Rightarrow \left(|\sin\alpha| + |\cos\alpha|\right)^2 \ge 1 \ / \sqrt{\ } \Rightarrow |\sin\alpha| + |\cos\alpha| \ge 1.$$

Vježba 430

Dokazati nejednakost: $|\sin\alpha| + |\cos\alpha| - 1 \ge 0.$

Rezultat: Dokaz analogan.

Zadatak 431 (Max, gimnazija)

Pokaži da se kao rezultat eliminacije x i y iz jednadžbi sin x + sin y = 2 · a, cos x + cos y = 2 · b, cos (x – y) = – 4 · a · b dobije $(a+b)^2 = \frac{1}{2}.$

Rješenje 431

Ponovimo!

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad (a \cdot b)^n = a^n \cdot b^n \quad , \quad \left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow a + c = b + d.$$

$$\cos^2 x + \sin^2 x = 1 \quad , \quad \cos(x-y) = \cos x \cdot \cos y + \sin x \cdot \sin y.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \ne 0 \ , \ n \ne 1.$$

$$\left.\begin{array}{l} \sin x + \sin y = 2 \cdot a \\ \cos x + \cos y = 2 \cdot b \\ \cos(x-y) = -4 \cdot a \cdot b \end{array}\right\} \Rightarrow \left.\begin{array}{l} \sin x + \sin y = 2 \cdot a \ /^2 \\ \cos x + \cos y = 2 \cdot b \ /^2 \\ \cos(x-y) = -4 \cdot a \cdot b \ / \cdot (-2) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} (\sin x + \sin y)^2 = (2 \cdot a)^2 \\ (\cos x + \cos y)^2 = (2 \cdot b)^2 \\ -2 \cdot \cos(x-y) = 8 \cdot a \cdot b \end{array}\right\} \Rightarrow \left.\begin{array}{l} \sin^2 x + 2 \cdot \sin x \cdot \sin y + \sin^2 y = 4 \cdot a^2 \\ \cos^2 x + 2 \cdot \cos x \cdot \cos y + \cos^2 y = 4 \cdot b^2 \\ -2 \cdot \cos(x-y) = 8 \cdot a \cdot b \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow$$

$$\Rightarrow \sin^2 x + 2\cdot\sin x\cdot\sin y + \sin^2 y + \cos^2 x + 2\cdot\cos x\cdot\cos y + \cos^2 y - 2\cdot\cos(x-y) =$$

$$= 4\cdot a^2 + 4\cdot b^2 + 8\cdot a\cdot b \Rightarrow$$

$$\Rightarrow \sin^2 x + \cos^2 x + \sin^2 y + \cos^2 y + 2\cdot\cos x\cdot\cos y + 2\cdot\sin x\cdot\sin y - 2\cdot\cos(x-y) =$$

$$= 4\cdot\left(a^2 + b^2 + 2\cdot a\cdot b\right) \Rightarrow$$

$$\Rightarrow \left(\sin^2 x + \cos^2 x\right) + \left(\sin^2 y + \cos^2 y\right) + 2\cdot(\cos x\cdot\cos y + \sin x\cdot\sin y) - 2\cdot\cos(x-y) =$$

$$= 4\cdot\left(a^2 + 2\cdot a\cdot b + b^2\right) \Rightarrow$$

$$\Rightarrow 1 + 1 + 2\cdot\cos(x-y) - 2\cdot\cos(x-y) = 4\cdot(a+b)^2 \Rightarrow$$

$$\Rightarrow 2 + 2\cdot\cos(x-y) - 2\cdot\cos(x-y) = 4\cdot(a+b)^2 \Rightarrow 2 = 4\cdot(a+b)^2 \Rightarrow$$

$$\Rightarrow 4\cdot(a+b)^2 = 2 \Rightarrow 4\cdot(a+b)^2 = 2 \ /\cdot\frac{1}{4} \Rightarrow (a+b)^2 = \frac{2}{4} \Rightarrow (a+b)^2 = \frac{2}{4} \Rightarrow (a+b)^2 = \frac{1}{2}.$$

Vježba 431

Pokaži da se kao rezultat eliminacije x i y iz jednadžbi sin x + sin y – 2 · a = 0, cos x + cos y – 2 · b = 0, cos (x – y) + 4 · a · b = 0 dobije $(a+b)^2 = \frac{1}{2}$.

Rezultat: Dokaz analogan.

Zadatak 432 (Max, gimnazija)

Ako je $ctg\,\alpha\cdot ctg\,\beta + ctg\,\alpha\cdot ctg\,\gamma + ctg\,\beta\cdot ctg\,\gamma = 1$ (α, β, γ šiljasti kutovi), pokazati da je $\alpha + \beta + \gamma = \pi$.

Rješenje 432

Ponovimo!

$$ctg\,(x+y) = \frac{ctg\,x\cdot ctg\,y - 1}{ctg\,x + ctg\,y} \quad , \quad ctg\,(\pi - x) = -ctg\,x.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Postupno preoblikujemo zadanu jednakost.

$$ctg\,\alpha\cdot ctg\,\beta + ctg\,\alpha\cdot ctg\,\gamma + ctg\,\beta\cdot ctg\,\gamma = 1 \Rightarrow ctg\,\alpha\cdot ctg\,\beta + ctg\,\alpha\cdot ctg\,\gamma = 1 - ctg\,\beta\cdot ctg\,\gamma \Rightarrow$$

$$\Rightarrow ctg\,\alpha\cdot(ctg\,\beta + ctg\,\gamma) = -(ctg\,\beta\cdot ctg\,\gamma - 1) \Rightarrow$$

$$\Rightarrow ctg\,\alpha\cdot(ctg\,\beta + ctg\,\gamma) = -(ctg\,\beta\cdot ctg\,\gamma - 1) \ /\cdot\frac{1}{ctg\,\beta + ctg\,\gamma} \Rightarrow$$

$$\Rightarrow ctg\,\alpha = -\frac{ctg\,\beta\cdot ctg\,\gamma - 1}{ctg\,\beta + ctg\,\gamma} \Rightarrow \left[ctg\,(x-y) = \frac{ctg\,x\cdot ctg\,y - 1}{ctg\,x + ctg\,y}\right] \Rightarrow ctg\,\alpha = -ctg\,(\beta+\gamma) \Rightarrow$$

$$\Rightarrow \left[ctg\,(\pi - x) = -ctg\,x\right] \Rightarrow ctg\,\alpha = ctg\,(\pi - (\beta+\gamma)) \Rightarrow \left[\begin{array}{l}\alpha,\ \beta,\ \gamma \\ \text{šiljasti kutovi}\end{array}\right] \Rightarrow$$

$$\Rightarrow \alpha = \pi - \beta - \gamma \Rightarrow \alpha + \beta + \gamma = \pi.$$

Vježba 432

Ako je $ctg\,\alpha \cdot ctg\,\beta + ctg\,\alpha \cdot ctg\,\gamma + ctg\,\beta \cdot ctg\,\gamma - 1 = 0$ (α, β, γ šiljasti kutovi), pokazati da je $\alpha + \beta + \gamma = \pi$.

Rezultat: Dokaz analogan.

Zadatak 433 (Miroslav, srednja škola)

Riješite jednadžbu $\sin^{1988} x + \cos^{1000} x = 1$.

Rješenje 433

Ponovimo!

$$-1 \le \sin x \le 1 \quad , \quad -1 \le \cos x \le 1 \quad , \quad a^{2 \cdot n} \ge 0 \ , \ a \in R.$$

Skup cijelih brojeva označavamo slovom Z, a zapisujemo

$$Z = \{ \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}.$$

Parni brojevi su oni brojevi koji su djeljivi s 2, a neparni su oni koji nisu djeljivi s 2.
Da je neki cijeli broj m neparan znači da se može napisati u obliku

$$m = 2 \cdot (\text{neki cijeli broj}) + 1 \quad , \quad m = 2 \cdot k + 1 \ , \ k \in Z.$$

Neparni brojevi mogu se prikazati općom formulom

$$2 \cdot n + 1,$$

gdje n pripada skupu cijelih brojeva.
Eksponenti 1988 i 1000 parni su brojevi pa je

$$\sin^{1988} x \ge 0 \text{ i } \cos^{1000} x \ge 0.$$

Da bi jednadžba

$$\sin^{1988} x + \cos^{1000} x = 1$$

imala realna rješenja mora vrijediti:

- $\sin x = 0$, a $\cos x = -1$ ili $\cos x = 1$
- $\cos x = 0$, a $\sin x = -1$ ili $\sin x = 1$.

Ako je

$$\sin x = 0 \Rightarrow x = \sin^{-1}(0) \Rightarrow x = k \cdot \pi, \ k \in Z.$$

Sami se uvjerimo da je za $x = k \cdot \pi$ ili cos x = – 1 ili cos x = 1.
Ako je

$$\cos x = 0 \Rightarrow x = \cos^{-1}(0) \Rightarrow x = (2 \cdot k + 1) \cdot \frac{\pi}{2}, \ k \in Z.$$

Sami se uvjerimo da je za $x = (2 \cdot k + 1) \cdot \frac{\pi}{2}$ ili sin x = – 1 ili sin x = 1.
Opće rješenje zadane jednadžbe tada glasi:

$$x = k \cdot \frac{\pi}{2}, \ k \in Z.$$

Vježba 433

Riješite jednadžbu $\sin^{988} x + \cos^{100} x = 1$.

Rezultat: $x = k \cdot \frac{\pi}{2}, \ k \in Z.$

Zadatak 434 (Frenky, gimnazija)

Izračunajte zbroj $\frac{\sin 1}{\cos 0 \cdot \cos 1}+\frac{\sin 1}{\cos 1 \cdot \cos 2}+\frac{\sin 1}{\cos 2 \cdot \cos 3}+\ \ldots\ \frac{\sin 1}{\cos (n-1) \cdot \cos n}$.

Rješenje 434

Ponovimo!

$$\sin (\alpha-\beta)=\sin \alpha \cdot \cos \beta-\cos \alpha \cdot \sin \beta .$$

$$\frac{a-b}{n}=\frac{a}{n}-\frac{b}{n} \quad , \quad tg\, \alpha=\frac{\sin \alpha}{\cos \alpha} \quad , \quad tg\, 0=0 .$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot(b+c)=a \cdot b+a \cdot c \quad , \quad a \cdot b+a \cdot c=a \cdot(b+c) .$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n}=\frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1 .$$

Uočimo da vrijedi:

$$\frac{\sin 1}{\cos (k-1) \cdot \cos k}=\frac{\sin (k-(k-1))}{\cos (k-1) \cdot \cos k} \Rightarrow \frac{\sin 1}{\cos (k-1) \cdot \cos k}=\frac{\sin k \cdot \cos (k-1)-\cos k \cdot \sin (k-1)}{\cos (k-1) \cdot \cos k} \Rightarrow$$

$$\Rightarrow \frac{\sin 1}{\cos (k-1) \cdot \cos k}=\frac{\sin k \cdot \cos (k-1)}{\cos (k-1) \cdot \cos k}-\frac{\cos k \cdot \sin (k-1)}{\cos (k-1) \cdot \cos k} \Rightarrow$$

$$\Rightarrow \frac{\sin 1}{\cos (k-1) \cdot \cos k}=\frac{\sin k \cdot \cos (k-1)}{\cos (k-1) \cdot \cos k}-\frac{\cos k \cdot \sin (k-1)}{\cos (k-1) \cdot \cos k} \Rightarrow \frac{\sin 1}{\cos (k-1) \cdot \cos k}=\frac{\sin k}{\cos k}-\frac{\sin (k-1)}{\cos (k-1)} \Rightarrow$$

$$\Rightarrow \frac{\sin 1}{\cos (k-1) \cdot \cos k}=tg\, k-tg\,(k-1) .$$

Preoblikujemo zadani zbroj pa dobijemo:

$$\frac{\sin 1}{\cos 0 \cdot \cos 1}+\frac{\sin 1}{\cos 1 \cdot \cos 2}+\frac{\sin 1}{\cos 2 \cdot \cos 3}+\ \ldots\ \frac{\sin 1}{\cos (n-1) \cdot \cos n}=$$

$$=(tg\, 1-tg\, 0)+(tg\, 2-tg\, 1)+(tg\, 3-tg\, 2)+\ \ldots\ +(tg\, n-tg\,(n-1))=$$

$$=tg\, 1-tg\, 0+tg\, 2-tg\, 1+tg\, 3-tg\, 2+\ \ldots\ +tg\, n-tg\,(n-1)=$$

$$=tg\, 1-0+tg\, 2-tg\, 1+tg\, 3-tg\, 2+\ \ldots\ +tg\, n-tg\,(n-1)=$$

$$=tg\, 1+tg\, 2-tg\, 1+tg\, 3-tg\, 2+\ \ldots\ +tg\, n-tg\,(n-1)=$$

$$=tg\, 1+tg\, 2-tg\, 1+tg\, 3-tg\, 2+\ \ldots\ +tg\, n-tg\,(n-1)=tg\, n .$$

Vježba 434

Izračunajte zbroj $\frac{\sin 1}{\cos 0 \cdot \cos 1}+\frac{\sin 1}{\cos 1 \cdot \cos 2}+\frac{\sin 1}{\cos 2 \cdot \cos 3}$.

Rezultat: tg 3.

Zadatak 435 (Josip, gimnazija)

Ako je $x=\dfrac{1}{\cos\alpha\cdot\cos\beta}$, $y=\dfrac{tg\,\alpha}{\cos\beta}$, $z=tg\,\beta$ izračunajte $x^2-y^2-z^2$.

$A.\ 0 \qquad B.\ 1 \qquad C.\ -1 \qquad D.\ \sin\alpha$

Rješenje 435

Ponovimo!

$$\frac{1}{\cos^2 x}=1+tg^2 x \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$\left.\begin{array}{l} x=\dfrac{1}{\cos\alpha\cdot\cos\beta} \\ y=\dfrac{tg\,\alpha}{\cos\beta} \\ z=tg\,\beta \end{array}\right\} \Rightarrow \left[\begin{array}{c}\text{kvadriramo}\\ \text{jednakosti}\end{array}\right] \Rightarrow \left.\begin{array}{l} x=\dfrac{1}{\cos\alpha\cdot\cos\beta}\ /^2 \\ y=\dfrac{tg\,\alpha}{\cos\beta}\ /^2 \\ z=tg\,\beta\ /^2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2=\left(\dfrac{1}{\cos\alpha\cdot\cos\beta}\right)^2 \\ y^2=\left(\dfrac{tg\,\alpha}{\cos\beta}\right)^2 \\ z^2=tg^2\beta \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x^2=\dfrac{1}{\cos^2\alpha\cdot\cos^2\beta} \\ y^2=\dfrac{tg^2\alpha}{\cos^2\beta} \\ z^2=tg^2\beta \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2=\dfrac{1}{\cos^2\alpha}\cdot\dfrac{1}{\cos^2\beta} \\ y^2=tg^2\alpha\cdot\dfrac{1}{\cos^2\beta} \\ z^2=tg^2\beta \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2=\left(1+tg^2\alpha\right)\cdot\left(1+tg^2\beta\right) \\ y^2=tg^2\alpha\cdot\left(1+tg^2\beta\right) \\ z^2=tg^2\beta \end{array}\right\}.$$

Sada je

$$x^2-y^2-z^2=\left(1+tg^2\alpha\right)\cdot\left(1+tg^2\beta\right)-tg^2\alpha\cdot\left(1+tg^2\beta\right)-tg^2\beta \Rightarrow$$

$$\Rightarrow x^2-y^2-z^2=1+tg^2\beta+tg^2\alpha+tg^2\alpha\cdot tg^2\beta-tg^2\alpha-tg^2\alpha\cdot tg^2\beta-tg^2\beta \Rightarrow$$

$$\Rightarrow x^2-y^2-z^2=1+tg^2\beta+tg^2\alpha+tg^2\alpha\cdot tg^2\beta-tg^2\alpha-tg^2\alpha\cdot tg^2\beta-tg^2\beta \Rightarrow$$

$$\Rightarrow x^2-y^2-z^2=1.$$

Odgovor je pod B.

Vježba 435

Ako je $x=\dfrac{1}{\cos\alpha\cdot\cos\beta}$, $y=\dfrac{\sin\alpha}{\cos\alpha\cdot\cos\beta}$, $z=tg\,\beta$ izračunajte $x^2-y^2-z^2$.

$A.\ 0 \qquad B.\ 1 \qquad C.\ -1 \qquad D.\ \sin\alpha$

Rezultat: B.

Zadatak 436 (Josip, gimnazija)

Ako je $x=\frac{a}{\cos\alpha}+b\cdot tg\,\beta,\ y=a\cdot tg\,\alpha+\frac{b}{\cos\alpha}$ izračunajte x^2-y^2.

$$A.\ 1\qquad B.\ a^2\qquad C.\ a^2-b^2\qquad D.\ a^2+b^2$$

Rješenje 436

Ponovimo!

$$\frac{1}{\cos^2 x}=1+tg^2 x\quad,\quad\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad,\quad(a\cdot b)^n=a^n\cdot b^n\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left.\begin{array}{l}x=\frac{a}{\cos\alpha}+b\cdot tg\,\alpha\\ y=a\cdot tg\,\alpha+\frac{b}{\cos\alpha}\end{array}\right\}\Rightarrow\begin{bmatrix}\text{kvadriramo}\\ \text{jednakosti}\end{bmatrix}\Rightarrow\left.\begin{array}{l}x=\frac{a}{\cos\alpha}+b\cdot tg\,\alpha\ /^2\\ y=a\cdot tg\,\alpha+\frac{b}{\cos\alpha}\ /^2\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}x^2=\left(\frac{a}{\cos\alpha}+b\cdot tg\,\alpha\right)^2\\ y^2=\left(a\cdot tg\,\alpha+\frac{b}{\cos\alpha}\right)^2\end{array}\right\}\Rightarrow\left.\begin{array}{l}x^2=\left(\frac{a}{\cos\alpha}\right)^2+2\cdot\frac{a}{\cos\alpha}\cdot b\cdot tg\,\alpha+(b\cdot tg\,\alpha)^2\\ y^2=(a\cdot tg\,\alpha)^2+2\cdot a\cdot tg\,\alpha\cdot\frac{b}{\cos\alpha}+\left(\frac{b}{\cos\alpha}\right)^2\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}x^2=\frac{a^2}{\cos^2\alpha}+2\cdot a\cdot b\cdot\frac{tg\,\alpha}{\cos\alpha}+b^2\cdot tg^2\,\alpha\\ y^2=a^2\cdot tg^2\,\alpha+2\cdot a\cdot b\cdot\frac{tg\,\alpha}{\cos\alpha}+\frac{b^2}{\cos^2\alpha}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}x^2=a^2\cdot\frac{1}{\cos^2\alpha}+2\cdot a\cdot b\cdot\frac{tg\,\alpha}{\cos\alpha}+b^2\cdot tg^2\,\alpha\\ y^2=a^2\cdot tg^2\,\alpha+2\cdot a\cdot b\cdot\frac{tg\,\alpha}{\cos\alpha}+b^2\cdot\frac{1}{\cos^2\alpha}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}x^2=a^2\cdot\left(1+tg^2\,\alpha\right)+2\cdot a\cdot b\cdot\frac{tg\,\alpha}{\cos\alpha}+b^2\cdot tg^2\,\alpha\\ y^2=a^2\cdot tg^2\,\alpha+2\cdot a\cdot b\cdot\frac{tg\,\alpha}{\cos\alpha}+b^2\cdot\left(1+tg^2\,\alpha\right)\end{array}\right\}.$$

Sada je:

$$x^2-y^2=$$

$$=a^2\cdot\left(1+tg^2\,\alpha\right)+2\cdot a\cdot b\cdot\frac{tg\,\alpha}{\cos\alpha}+b^2\cdot tg^2\,\alpha-\left(a^2\cdot tg^2\,\alpha+2\cdot a\cdot b\cdot\frac{tg\,\alpha}{\cos\alpha}+b^2\cdot\left(1+tg^2\,\alpha\right)\right)\Rightarrow$$

$$\Rightarrow x^2-y^2=$$

$$= a^2 \cdot \left(1 + tg^2 \alpha\right) + 2 \cdot a \cdot b \cdot \frac{tg\,\alpha}{\cos\alpha} + b^2 \cdot tg^2 \alpha - a^2 \cdot tg^2 \alpha - 2 \cdot a \cdot b \cdot \frac{tg\,\alpha}{\cos\alpha} - b^2 \cdot \left(1 + tg^2 \alpha\right) \Rightarrow$$

$$\Rightarrow x^2 - y^2 =$$

$$= a^2 \cdot \left(1 + tg^2 \alpha\right) + 2 \cdot a \cdot b \cdot \frac{tg\,\alpha}{\cos\alpha} + b^2 \cdot tg^2 \alpha - a^2 \cdot tg^2 \alpha - 2 \cdot a \cdot b \cdot \frac{tg\,\alpha}{\cos\alpha} - b^2 \cdot \left(1 + tg^2 \alpha\right) \Rightarrow$$

$$\Rightarrow x^2 - y^2 = a^2 \cdot \left(1 + tg^2 \alpha\right) + b^2 \cdot tg^2 \alpha - a^2 \cdot tg^2 \alpha - b^2 \cdot \left(1 + tg^2 \alpha\right) \Rightarrow$$

$$\Rightarrow x^2 - y^2 = a^2 + a^2 \cdot tg^2 \alpha + b^2 \cdot tg^2 \alpha - a^2 \cdot tg^2 \alpha - b^2 - b^2 \cdot tg^2 \alpha \Rightarrow$$

$$\Rightarrow x^2 - y^2 = a^2 + a^2 \cdot tg^2 \alpha + b^2 \cdot tg^2 \alpha - a^2 \cdot tg^2 \alpha - b^2 - b^2 \cdot tg^2 \alpha \Rightarrow$$

$$\Rightarrow x^2 - y^2 = a^2 - b^2.$$

Odgovor je pod C.

Vježba 436

Ako je $x = \frac{a}{\cos\alpha} + b \cdot tg\,\beta,\ y = a \cdot tg\,\alpha + \frac{b}{\cos\alpha}$ izračunajte $y^2 - x^2$.

$A.\ 1 \qquad B.\ b^2 \qquad C.\ b^2 - a^2 \qquad D.\ a^2 + b^2$

Rezultat: C.

Zadatak 437 (Ivica, srednja škola)

Ako je $ctg\,\alpha = \frac{3}{4},\ ctg\,\beta = \frac{1}{7},\ 0 < \alpha < \frac{\pi}{2},\ 0 < \beta < \frac{\pi}{2}$, izračunati $\alpha + \beta$.

Rješenje 437

Ponovimo!

$$ctg\left(\alpha + \beta\right) = \frac{ctg\,\alpha \cdot ctg\,\beta - 1}{ctg\,\alpha + ctg\,\beta} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad ctg\left(\frac{3 \cdot \pi}{4}\right) = -1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ ,\ n \neq 0 \ ,\ n \neq 1.$$

$$ctg\left(\alpha + \beta\right) = \frac{ctg\,\alpha \cdot ctg\,\beta - 1}{ctg\,\alpha + ctg\,\beta} \Rightarrow \left[\begin{array}{l} ctg\,\alpha = \frac{3}{4} \\ ctg\,\beta = \frac{1}{7} \end{array}\right] \Rightarrow ctg\left(\alpha + \beta\right) = \frac{\frac{3}{4} \cdot \frac{1}{7} - 1}{\frac{3}{4} + \frac{1}{7}} \Rightarrow$$

$$\Rightarrow ctg\left(\alpha + \beta\right) = \frac{\frac{3}{28} - 1}{\frac{3}{4} + \frac{1}{7}} \Rightarrow ctg\left(\alpha + \beta\right) = \frac{\frac{3}{28} - \frac{1}{1}}{\frac{3}{4} + \frac{1}{7}} \Rightarrow ctg\left(\alpha + \beta\right) = \frac{\frac{3 - 28}{28}}{\frac{21 + 4}{28}} \Rightarrow ctg\left(\alpha + \beta\right) = \frac{-\frac{25}{28}}{\frac{25}{28}} \Rightarrow$$

$$\Rightarrow ctg(\alpha+\beta)=\frac{-\frac{25}{28}}{\frac{25}{28}} \Rightarrow ctg(\alpha+\beta)=-1 \Rightarrow \alpha+\beta=ctg^{-1}(-1) \Rightarrow \alpha+\beta=\frac{3\cdot\pi}{4}.$$

Vježba 437

Ako je $ctg\,\alpha=\frac{3}{4},\ ctg\,\beta=\frac{1}{7},\ 0<\alpha<\frac{\pi}{2},\ 0<\beta<\frac{\pi}{2}$, izračunati $\frac{1}{3}\cdot(\alpha+\beta)$.

Rezultat: $\frac{\pi}{4}$.

Zadatak 438 (Tomislav, gimnazija)

Za koji realan broj t vrijedi $\frac{\sin x-\sin^3 x}{1+\cos(2\cdot x)}=t\cdot\sin x$ za svaki $x\neq\frac{k\cdot\pi}{2},\ k\in Z$?

Rješenje 438

Ponovimo!

$$a^1=a \quad , \quad a^n:a^m=a^{n-m} \quad , \quad \cos^2 x+\sin^2 x=1 \quad , \quad \cos(2\cdot x)=\cos^2 x-\sin^2 x.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ ,\ n\neq 0\ ,\ n\neq 1.$$

Skup cijelih brojeva označavamo slovom Z, a zapisujemo

$$Z=\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}.$$

$$\frac{\sin x-\sin^3 x}{1+\cos(2\cdot x)}=t\cdot\sin x \Rightarrow \frac{\sin x\cdot(1-\sin^2 x)}{\cos^2 x+\sin^2 x+\cos^2 x-\sin^2 x}=t\cdot\sin x \Rightarrow$$

$$\Rightarrow \frac{\sin x\cdot\cos^2 x}{\cos^2 x+\sin^2 x+\cos^2 x-\sin^2 x}=t\cdot\sin x \Rightarrow \frac{\sin x\cdot\cos^2 x}{\cos^2 x+\cos^2 x}=t\cdot\sin x \Rightarrow$$

$$\Rightarrow \frac{\sin x\cdot\cos^2 x}{2\cdot\cos^2 x}=t\cdot\sin x \Rightarrow \frac{\sin x\cdot\cos^2 x}{2\cdot\cos^2 x}=t\cdot\sin x \Rightarrow \frac{\sin x}{2}=t\cdot\sin x \Rightarrow$$

$$\Rightarrow \frac{\sin x}{2}=t\cdot\sin x \Rightarrow t\cdot\sin x=\frac{\sin x}{2} \Rightarrow t\cdot\sin x=\frac{\sin x}{2}\ /:\sin x \Rightarrow t=\frac{1}{2}.$$

Vježba 438

Za koji realan broj t vrijedi $\frac{\sin x-\sin^3 x}{1+\cos(2\cdot x)}-t\cdot\sin x=0$ za svaki $x\neq\frac{k\cdot\pi}{2},\ k\in Z$?

Rezultat: $\frac{1}{2}$.

Zadatak 439 (Marija, gimnazija)

Ako su a, b, c tri pozitivna broja, $0<\alpha<\pi$ i $a^2=b^2+c^2-2\cdot b\cdot c\cdot\cos\alpha$, pokažite da je $|b-c|<a<b+c$.

Rješenje 439

Ponovimo!

$$\cos 0 = 1 \quad , \quad \cos \pi = -1 \quad , \quad a < b \ , \ c > 0 \ \Rightarrow \ a \cdot c < b \cdot c \quad , \quad a^2 - 2 \cdot a \cdot b + b^2 = (a - b)^2 .$$

$$\sqrt{a^2} = |a| \quad , \quad \sqrt{a^2} = a \ , \ a \geq 0 \quad , \quad a^2 + 2 \cdot a \cdot b + b^2 = (a + b)^2 .$$

$$a > b \ , \ c > 0 \ \Rightarrow \ a \cdot c > b \cdot c .$$

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x| = \begin{cases} x, \ x \geq 0 \\ -x, \ x < 0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x \geq 0$, vrijedi $|x| = x$.
Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, $x < 0$, je $|x| = -x$.
Budući da je

$$0 < \alpha < \pi,$$

slijedi

$$-1 < \cos \alpha < 1.$$

Preoblikujemo zadanu jednakost.

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha \ \Rightarrow \ 2 \cdot b \cdot c \cdot \cos \alpha = b^2 + c^2 - a^2 \ \Rightarrow$$

$$\Rightarrow \ 2 \cdot b \cdot c \cdot \cos \alpha = b^2 + c^2 - a^2 \ / \cdot \frac{1}{2 \cdot b \cdot c} \ \Rightarrow \ \cos \alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}.$$

Prvi slučaj

$$\cos \alpha < 1 \ \Rightarrow \ \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c} < 1 \ \Rightarrow \ [b \cdot c > 0] \ \Rightarrow \ \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c} < 1 \ / \cdot 2 \cdot b \cdot c \ \Rightarrow$$

$$\Rightarrow \ b^2 + c^2 - a^2 < 2 \cdot b \cdot c \ \Rightarrow \ b^2 - 2 \cdot b \cdot c + c^2 < a^2 \ \Rightarrow \ (b - c)^2 < a^2 \ \Rightarrow$$

$$\Rightarrow \ (b - c)^2 < a^2 \ / \sqrt{\ } \ \Rightarrow \ \sqrt{(b - c)^2} < \sqrt{a^2} \ \Rightarrow \ |b - c| < a.$$

Drugi slučaj

$$\cos \alpha > -1 \ \Rightarrow \ \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c} > -1 \ \Rightarrow \ [b \cdot c > 0] \ \Rightarrow \ \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c} > -1 \ / \cdot 2 \cdot b \cdot c \ \Rightarrow$$

$$\Rightarrow \ b^2 + c^2 - a^2 > -2 \cdot b \cdot c \ \Rightarrow \ b^2 + 2 \cdot b \cdot c + c^2 > a^2 \ \Rightarrow \ (b + c)^2 > a^2 \ \Rightarrow$$

$$\Rightarrow \ (b + c)^2 > a^2 \ / \sqrt{\ } \ \Rightarrow \ \sqrt{(b + c)^2} > \sqrt{a^2} \ \Rightarrow \ b + c > a \ \Rightarrow \ a < b + c.$$

Konačno je:

$$\left. \begin{array}{l} |b - c| < a \\ a < b + c \end{array} \right\} \ \Rightarrow \ |b - c| < a < b + c.$$

Vježba 439

Ako su a, b, c tri pozitivna broja, $0 < \beta < \pi$ i $b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta$, pokažite da je $|a - c| < b < a + c$.

Rezultat: Dokaz analogan.

Zadatak 440 (Robert, maturant)

Bez uporabe računala dokažite: $\cos 40^\circ + \cos 80^\circ = \cos 20^\circ$.

Rješenje 440

Ponovimo!

$$\cos\alpha + \cos\beta = 2\cdot\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2} \quad , \quad \cos(-\alpha) = \cos\alpha \quad , \quad \cos 60^\circ = \frac{1}{2}.$$

$$\cos 120^\circ = -\frac{1}{2} \quad , \quad \cos\left(180^\circ - \alpha\right) = -\cos\alpha.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \;\; , \;\; n \neq 0 \;\; , \;\; n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

1.inačica

$$\cos 40^\circ + \cos 80^\circ = 2\cdot\cos\frac{40^\circ + 80^\circ}{2}\cdot\cos\frac{40^\circ - 80^\circ}{2} = 2\cdot\cos\frac{120^\circ}{2}\cdot\cos\left(-\frac{40^\circ}{2}\right) =$$

$$= 2\cdot\cos\frac{120^\circ}{2}\cdot\cos\frac{40^\circ}{2} = 2\cdot\cos\frac{120^\circ}{2}\cdot\cos\frac{40^\circ}{2} = 2\cdot\cos 60^\circ\cdot\cos 20^\circ = 2\cdot\frac{1}{2}\cdot\cos 20^\circ =$$

$$= 2\cdot\frac{1}{2}\cdot\cos 20^\circ = \cos 20^\circ.$$

2.inačica

$$\cos 40^\circ + \cos 80^\circ = \cos 20^\circ \;\Rightarrow\; \cos 40^\circ + \cos 80^\circ - \cos 20^\circ = 0 \;\Rightarrow$$

$$\Rightarrow\; \cos 40^\circ + \cos 80^\circ + \cos\left(180^\circ - 20^\circ\right) = 0 \;\Rightarrow\; \cos 40^\circ + \cos 80^\circ + \cos 160^\circ = 0 \;\Rightarrow$$

$$\Rightarrow\; \cos 40^\circ + \left(\cos 80^\circ + \cos 160^\circ\right) = 0 \;\Rightarrow\; \cos 40^\circ + 2\cdot\cos\frac{80^\circ + 160^\circ}{2}\cdot\cos\frac{80^\circ - 160^\circ}{2} = 0 \;\Rightarrow$$

$$\Rightarrow\; \cos 40^\circ + 2\cdot\cos\frac{240^\circ}{2}\cdot\cos\frac{80^\circ}{2} = 0 \;\Rightarrow\; \cos 40^\circ + 2\cdot\cos 120^\circ\cdot\cos 40^\circ = 0 \;\Rightarrow$$

$$\Rightarrow\; \cos 40^\circ + 2\cdot\left(-\frac{1}{2}\right)\cdot\cos 40^\circ = 0 \;\Rightarrow\; \cos 40^\circ + 2\cdot\left(-\frac{1}{2}\right)\cdot\cos 40^\circ = 0 \;\Rightarrow$$

$$\Rightarrow\; \cos 40^\circ - \cos 40^\circ = 0 \;\Rightarrow\; \cos 40^\circ - \cos 40^\circ = 0 \;\Rightarrow\; 0 = 0.$$

3.inačica

$$\cos 40^\circ + \cos 80^\circ = \cos 20^\circ \;\Rightarrow\; \cos 40^\circ + \cos 80^\circ - \cos 20^\circ = 0 \;\Rightarrow$$

$$\Rightarrow\; \left(\cos 40^\circ + \cos 80^\circ\right) - \cos 20^\circ = 0 \;\Rightarrow\; 2\cdot\cos\frac{40^\circ + 80^\circ}{2}\cdot\cos\frac{40^\circ - 80^\circ}{2} - \cos 20^\circ = 0 \;\Rightarrow$$

$$\Rightarrow\; 2\cdot\cos\frac{120^\circ}{2}\cdot\cos\left(-\frac{40^\circ}{2}\right) - \cos 20^\circ = 0 \;\Rightarrow\; 2\cdot\cos\frac{120^\circ}{2}\cdot\cos\frac{40^\circ}{2} - \cos 20^\circ = 0 \;\Rightarrow$$

$$\Rightarrow\ 2\cdot\cos 60^\circ\cdot\cos 20^\circ-\cos 20^\circ=0\ \Rightarrow\ 2\cdot\frac{1}{2}\cdot\cos 20^\circ-\cos 20^\circ=0\ \Rightarrow$$

$$\Rightarrow\ 2\cdot\frac{1}{2}\cdot\cos 20^\circ-\cos 20^\circ=0\ \Rightarrow\ \cos 20^\circ-\cos 20^\circ=0\ \Rightarrow\ \cos 20^\circ-\cos 20^\circ=0\ \Rightarrow\ 0=0.$$

Vježba 440

Odmor!

Rezultat: …