

**Zadatak 421 (Marija, gimnazija)**

Ako je  $\cos^2 \alpha + \cos^2 \beta = a$ , onda je  $\cos(\alpha + \beta) \cdot \cos(\alpha - \beta)$  jednako:

- A.  $a-1$       B.  $1-a$       C.  $a+1$       D.  $a^2-1$

**Rješenje 421**

Ponovimo!

$$\cos x \cdot \cos y = \frac{1}{2} \cdot (\cos(x-y) + \cos(x+y)) \quad , \quad \cos(2 \cdot x) = 2 \cdot \cos^2 x - 1.$$

$$\frac{a}{b} \cdot b = a.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Preoblikujemo zadani izraz:

$$\begin{aligned} \cos(\alpha + \beta) \cdot \cos(\alpha - \beta) &= \frac{1}{2} \cdot (\cos(\alpha + \beta - (\alpha - \beta)) + \cos(\alpha + \beta + \alpha - \beta)) = \\ &= \frac{1}{2} \cdot (\cos(\alpha + \beta - \alpha + \beta) + \cos(\alpha + \beta + \alpha - \beta)) = \frac{1}{2} \cdot (\cos(\alpha + \beta - \alpha + \beta) + \cos(\alpha + \beta + \alpha - \beta)) = \\ &= \frac{1}{2} \cdot (\cos(2 \cdot \beta) + \cos(2 \cdot \alpha)) = \frac{1}{2} \cdot (\cos(2 \cdot \alpha) + \cos(2 \cdot \beta)) = \left[ \cos(2 \cdot x) = 2 \cdot \cos^2 x - 1 \right] = \\ &= \frac{1}{2} \cdot (2 \cdot \cos^2 \alpha - 1 + 2 \cdot \cos^2 \beta - 1) = \frac{1}{2} \cdot (2 \cdot \cos^2 \alpha + 2 \cdot \cos^2 \beta - 2) = \\ &= \cos^2 \alpha + \cos^2 \beta - 1 = \left[ \begin{array}{l} \text{uvjet} \\ \cos^2 \alpha + \cos^2 \beta = a \end{array} \right] = a - 1. \end{aligned}$$

Odgovor je pod A.

**Vježba 421**

Ako je  $\cos^2 \alpha + \cos^2 \beta = a + 2$ , onda je  $\cos(\alpha + \beta) \cdot \cos(\alpha - \beta)$  jednako:

- A.  $a-1$       B.  $1-a$       C.  $a+1$       D.  $a^2-1$

**Rezultat:**      C.

**Zadatak 422 (Patrik, gimnazija)**

Izračunajte bez uporabe džepnog računala  $\sin 75^\circ \cdot \cos 75^\circ$ .

**Rješenje 422**

Ponovimo!

$$n = \frac{n}{1} \quad , \quad 2 \cdot \sin x \cdot \cos x = \sin(2 \cdot x) \quad , \quad \sin(180^\circ - x) = \sin x \quad , \quad \sin 0^\circ = 1.$$

$$\sin 30^\circ = \frac{1}{2} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \quad , \quad \frac{\frac{a}{b} \cdot \frac{c}{d}}{\frac{a \cdot c}{b \cdot d}}.$$

$$\sin x \cdot \cos y = \frac{1}{2} \cdot (\sin(x+y) + \sin(x-y)).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

1. inačica

$$\begin{aligned} \sin 75^\circ \cdot \cos 75^\circ &= \left[ n = \frac{n}{1} \right] = \frac{\sin 75^\circ \cdot \cos 75^\circ}{1} = \left[ \frac{a}{b} = \frac{a \cdot n}{b \cdot n} \right] = \frac{2 \cdot \sin 75^\circ \cdot \cos 75^\circ}{2 \cdot 1} = \\ &= \frac{2 \cdot \sin 75^\circ \cdot \cos 75^\circ}{2} = \left[ 2 \cdot \sin x \cdot \cos x = \sin(2 \cdot x) \right] = \frac{\sin(2 \cdot 75^\circ)}{2} = \frac{\sin 150^\circ}{2} = \\ &= \left[ \sin(180^\circ - x) = \sin x \right] = \frac{\sin(180^\circ - 30^\circ)}{2} = \frac{\sin 30^\circ}{2} = \frac{\frac{1}{2}}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}. \end{aligned}$$

2. inačica

$$\begin{aligned} \sin 75^\circ \cdot \cos 75^\circ &= \left[ \sin x \cdot \cos y = \frac{1}{2} \cdot (\sin(x+y) + \sin(x-y)) \right] = \\ &= \frac{1}{2} \cdot (\sin(75^\circ + 75^\circ) + \sin(75^\circ - 75^\circ)) = \frac{1}{2} \cdot (\sin 150^\circ + \sin 0^\circ) = \frac{1}{2} \cdot (\sin 150^\circ + 0) = \\ &= \frac{1}{2} \cdot \sin 150^\circ = \left[ \sin(180^\circ - x) = \sin x \right] = \frac{1}{2} \cdot \sin(180^\circ - 30^\circ) = \frac{1}{2} \cdot \sin 30^\circ = \\ &= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}. \end{aligned}$$

### Vježba 422

Izračunajte bez uporabe džepnog računala  $4 \cdot \sin 75^\circ \cdot \cos 75^\circ$ .

**Rezultat:** 1.

### Zadatak 423 (Patrik, gimnazija)

Izračunajte bez uporabe džepnog računala  $\cos 15^\circ \cdot \sin 75^\circ$ .

### Rješenje 423

Ponovimo!

$$\begin{aligned} \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \cos 0^\circ = 1, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \sin 90^\circ = 1. \\ \sin(90^\circ - x) = \cos x, \quad \sin(-x) = -\sin x, \quad a^1 = a, \quad a^n \cdot a^m = a^{n+m}. \end{aligned}$$

$$\cos x \cdot \cos y = \frac{1}{2} \cdot (\cos(x+y) + \cos(x-y)).$$

$$\cos x \cdot \sin y = \frac{1}{2} \cdot (\sin(x+y) - \sin(x-y)).$$

$$\cos^2 x = \frac{1 + \cos(2 \cdot x)}{2}.$$

1. inačica

$$\cos 15^\circ \cdot \sin 75^\circ = \left[ \sin(90^\circ - x) = \cos x \right] = \cos 15^\circ \cdot \sin(90^\circ - 15^\circ) = \cos 15^\circ \cdot \cos 15^\circ =$$

$$= \left[ \cos x \cdot \cos y = \frac{1}{2} \cdot (\cos(x+y) + \cos(x-y)) \right] =$$

$$= \frac{1}{2} \cdot (\cos(15^\circ + 15^\circ) + \cos(15^\circ - 15^\circ)) = \frac{1}{2} \cdot (\cos 30^\circ + \cos 0^\circ) = \frac{1}{2} \cdot \left( \frac{\sqrt{3}}{2} + 1 \right) = \frac{1}{2} \cdot \left( 1 + \frac{\sqrt{3}}{2} \right).$$

2. inačica

$$\cos 15^\circ \cdot \sin 75^\circ = \left[ \cos x \cdot \sin y = \frac{1}{2} \cdot (\sin(x+y) - \sin(x-y)) \right] =$$

$$= \frac{1}{2} \cdot (\sin(15^\circ + 75^\circ) - \sin(15^\circ - 75^\circ)) = \frac{1}{2} \cdot (\sin 90^\circ - \sin(-60^\circ)) = [\sin(-x) = -\sin x] =$$

$$= \frac{1}{2} \cdot (\sin 90^\circ + \sin 60^\circ) = \frac{1}{2} \cdot \left( 1 + \frac{\sqrt{3}}{2} \right).$$

3. inačica

$$\cos 15^\circ \cdot \sin 75^\circ = [\sin(90^\circ - x) = \cos x] = \cos 15^\circ \cdot \sin(90^\circ - 15^\circ) = \cos 15^\circ \cdot \cos 15^\circ =$$

$$= \cos^2 15^\circ = \left[ \cos^2 x = \frac{1 + \cos(2 \cdot x)}{2} \right] = \frac{1 + \cos(2 \cdot 15^\circ)}{2} = \frac{1 + \cos 30^\circ}{2} =$$

$$= \frac{1}{2} \cdot (1 + \cos 30^\circ) = \frac{1}{2} \cdot \left( 1 + \frac{\sqrt{3}}{2} \right).$$

### Vježba 423

Izračunajte bez uporabe džepnog računala  $2 \cdot \cos 15^\circ \cdot \sin 75^\circ$ .

**Rezultat:**  $1 + \frac{\sqrt{3}}{2}$ .

### Zadatak 424 (Branka, gimnazija)

Dokaži identitet  $\frac{\sin^4 \alpha + \cos^4 \alpha - 1}{\sin^6 \alpha + \cos^6 \alpha - 1} = \frac{2}{3}$ .

### Rješenje 424

Ponovimo!

$$a^1 = a \quad , \quad (a^n)^m = a^{n \cdot m} \quad , \quad a^n : a^m = a^{n-m} \quad , \quad \cos^2 \alpha + \sin^2 \alpha = 1.$$

$$a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2 \quad , \quad a^3 + b^3 = (a+b) \cdot (a^2 - a \cdot b + b^2).$$

$$\frac{-a}{-b} = \frac{a}{b} \quad , \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

1. inačica

Lijevu stranu postupno preoblikujemo.

$$\begin{aligned}
 \frac{\sin^4 \alpha + \cos^4 \alpha - 1}{\sin^6 \alpha + \cos^6 \alpha - 1} &= \frac{\sin^4 \alpha + \cos^4 \alpha - (\sin^2 \alpha + \cos^2 \alpha)}{\sin^6 \alpha + \cos^6 \alpha - (\sin^2 \alpha + \cos^2 \alpha)} = \\
 &= \frac{\sin^4 \alpha + \cos^4 \alpha - \sin^2 \alpha - \cos^2 \alpha}{\sin^6 \alpha + \cos^6 \alpha - \sin^2 \alpha - \cos^2 \alpha} = \frac{-(-\sin^4 \alpha - \cos^4 \alpha + \sin^2 \alpha + \cos^2 \alpha)}{-(-\sin^6 \alpha - \cos^6 \alpha + \sin^2 \alpha + \cos^2 \alpha)} = \\
 &= \frac{-\sin^4 \alpha - \cos^4 \alpha + \sin^2 \alpha + \cos^2 \alpha}{-\sin^6 \alpha - \cos^6 \alpha + \sin^2 \alpha + \cos^2 \alpha} = \frac{\sin^2 \alpha - \sin^4 \alpha + \cos^2 \alpha - \cos^4 \alpha}{\sin^2 \alpha - \sin^6 \alpha + \cos^2 \alpha - \cos^6 \alpha} = \\
 &= \frac{\sin^2 \alpha \cdot (1 - \sin^2 \alpha) + \cos^2 \alpha \cdot (1 - \cos^2 \alpha)}{\sin^2 \alpha \cdot (1 - \sin^4 \alpha) + \cos^2 \alpha \cdot (1 - \cos^4 \alpha)} = \frac{\sin^2 \alpha \cdot \cos^2 \alpha + \cos^2 \alpha \cdot \sin^2 \alpha}{\sin^2 \alpha \cdot (1^2 - (\sin^2 \alpha)^2) + \cos^2 \alpha \cdot (1^2 - (\cos^2 \alpha)^2)} = \\
 &= \frac{\sin^2 \alpha \cdot \cos^2 \alpha + \sin^2 \alpha \cdot \cos^2 \alpha}{\sin^2 \alpha \cdot (1 - \sin^2 \alpha) \cdot (1 + \sin^2 \alpha) + \cos^2 \alpha \cdot (1 - \cos^2 \alpha) \cdot (1 + \cos^2 \alpha)} = \\
 &= \frac{2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{\sin^2 \alpha \cdot \cos^2 \alpha \cdot (1 + \sin^2 \alpha) + \cos^2 \alpha \cdot \sin^2 \alpha \cdot (1 + \cos^2 \alpha)} = \\
 &= \frac{2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{\sin^2 \alpha \cdot \cos^2 \alpha \cdot (1 + \sin^2 \alpha) + \sin^2 \alpha \cdot \cos^2 \alpha \cdot (1 + \cos^2 \alpha)} = \\
 &= \frac{2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{\sin^2 \alpha \cdot \cos^2 \alpha \cdot (1 + \sin^2 \alpha + 1 + \cos^2 \alpha)} = \frac{2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{\sin^2 \alpha \cdot \cos^2 \alpha \cdot (1 + \sin^2 \alpha + 1 + \cos^2 \alpha)} = \\
 &= \frac{2}{1 + \sin^2 \alpha + 1 + \cos^2 \alpha} = \frac{2}{2 + \sin^2 \alpha + \cos^2 \alpha} = \frac{2}{2 + 1} = \frac{2}{3}.
 \end{aligned}$$

2. inačica

Lijevu stranu postupno preoblikujemo.

$$\begin{aligned}
 \frac{\sin^4 \alpha + \cos^4 \alpha - 1}{\sin^6 \alpha + \cos^6 \alpha - 1} &= \frac{\sin^4 \alpha + 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha + \cos^4 \alpha - 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1}{(\sin^2 \alpha)^3 + (\cos^2 \alpha)^3} = \\
 &= \frac{(\sin^2 \alpha)^2 + 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha + (\cos^2 \alpha)^2 - 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1}{(\sin^2 \alpha + \cos^2 \alpha) \cdot ((\sin^2 \alpha)^2 - \sin^2 \alpha \cdot \cos^2 \alpha + (\cos^2 \alpha)^2) - 1} =
 \end{aligned}$$

$$\begin{aligned}
&= \frac{(\sin^2 \alpha + \cos^2 \alpha)^2 - 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1}{1 \cdot (\sin^4 \alpha - \sin^2 \alpha \cdot \cos^2 \alpha + \cos^4 \alpha) - 1} = \frac{1^2 - 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1}{\sin^4 \alpha - \sin^2 \alpha \cdot \cos^2 \alpha + \cos^4 \alpha - 1} = \\
&= \frac{1 - 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1}{\sin^4 \alpha + 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha + \cos^4 \alpha - 3 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1} = \\
&= \frac{1 - 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1}{(\sin^2 \alpha)^2 + 2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha + (\cos^2 \alpha)^2 - 3 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1} = \\
&= \frac{-2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{(\sin^2 \alpha + \cos^2 \alpha)^2 - 3 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1} = \frac{-2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{1^2 - 3 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1} = \\
&= \frac{-2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{1 - 3 \cdot \sin^2 \alpha \cdot \cos^2 \alpha - 1} = \frac{-2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{-3 \cdot \sin^2 \alpha \cdot \cos^2 \alpha} = \\
&= \frac{-2 \cdot \sin^2 \alpha \cdot \cos^2 \alpha}{-3 \cdot \sin^2 \alpha \cdot \cos^2 \alpha} = \frac{2}{3}.
\end{aligned}$$

### Vježba 424

Dokaži identitet  $\frac{\sin^6 \alpha + \cos^6 \alpha - 1}{\sin^4 \alpha + \cos^4 \alpha - 1} = \frac{3}{2}$ .

**Rezultat:** Dokaz analogan.



### Zadatak 425 ( , gimnazija)

Riješi jednačinu:  $\sqrt{3} \cdot \sin x - \cos x = 0$ .

### Rješenje 425

Ponovimo!

$$\operatorname{tg} x = \frac{\sin x}{\cos x}, \quad \operatorname{tg} \frac{\pi}{6} = \frac{\sqrt{3}}{3}, \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \quad a^1 = a, \quad a^n \cdot a^m = a^{n+m}.$$

$$(\sqrt{a})^2 = a.$$

**Trigonometrijska jednačina  $\operatorname{tg} x = a$**

Skup rješenja jednačine  $\operatorname{tg} x = a$ ,  $a \in \mathbb{R}$ , je  $\{x_0 + k \cdot \pi : k \in \mathbb{Z}\}$ , gdje je  $x_0 \in \mathbb{R}$  jedno rješenje te jednačine.

$$\operatorname{tg} x = a \Rightarrow \operatorname{tg} x = \operatorname{tg} x_0 \Rightarrow x = x_0 + k \cdot \pi, \quad k \in \mathbb{Z}.$$

Skup cijelih brojeva označavamo slovom  $\mathbb{Z}$ , a zapisujemo

$$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}.$$

$$\sqrt{3} \cdot \sin x - \cos x = 0 \Rightarrow \sqrt{3} \cdot \sin x = \cos x \Rightarrow \sqrt{3} \cdot \sin x = \cos x \cdot \frac{1}{\sqrt{3} \cdot \cos x} \Rightarrow$$

$$\Rightarrow \frac{\sin x}{\cos x} = \frac{1}{\sqrt{3}} \Rightarrow \operatorname{tg} x = \frac{1}{\sqrt{3}} \Rightarrow \operatorname{tg} x = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \operatorname{tg} x = \frac{\sqrt{3}}{(\sqrt{3})^2} \Rightarrow \operatorname{tg} x = \frac{\sqrt{3}}{3} \Rightarrow$$

$$\Rightarrow x = \operatorname{tg}^{-1}\left(\frac{\sqrt{3}}{3}\right) \Rightarrow x = \frac{\pi}{6} + k \cdot \pi, \quad k \in \mathbb{Z}.$$

### Vježba 425

Riješi jednačbu:  $\cos x - \sqrt{3} \cdot \sin x = 0$ .

**Rezultat:**  $x = \frac{\pi}{6} + k \cdot \pi, \quad k \in \mathbb{Z}.$

### Zadatak 426 (Martin, gimnazija)

Dokažite jednakost:  $\operatorname{tg} 9^\circ - \operatorname{tg} 27^\circ - \operatorname{tg} 63^\circ + \operatorname{tg} 81^\circ = 4$ .

### Rješenje 426

Ponovimo!

$$\operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cdot \cos \beta}, \quad \sin 90^\circ = 1, \quad \cos 90^\circ = 0, \quad n = \frac{n}{1}, \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

$$\cos \alpha \cdot \cos \beta = \frac{1}{2} \cdot (\cos(\alpha + \beta) + \cos(\alpha - \beta)), \quad \cos(-\alpha) = \cos \alpha.$$

$$\cos(90^\circ - \alpha) = \sin \alpha, \quad \frac{\frac{a}{b} - \frac{c}{d}}{\frac{a}{b} \cdot \frac{c}{d}} = \frac{a \cdot d - b \cdot c}{b \cdot d}, \quad \sin \alpha - \sin \beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned} \operatorname{tg} 9^\circ - \operatorname{tg} 27^\circ - \operatorname{tg} 63^\circ + \operatorname{tg} 81^\circ &= (\operatorname{tg} 9^\circ + \operatorname{tg} 81^\circ) - (\operatorname{tg} 27^\circ + \operatorname{tg} 63^\circ) = \\ &= \left[ \operatorname{tg} \alpha + \operatorname{tg} \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cdot \cos \beta} \right] = \frac{\sin(9^\circ + 81^\circ)}{\cos 9^\circ \cdot \cos 81^\circ} - \frac{\sin(27^\circ + 63^\circ)}{\cos 27^\circ \cdot \cos 63^\circ} = \\ &= \frac{\sin 90^\circ}{\cos 9^\circ \cdot \cos 81^\circ} - \frac{\sin 90^\circ}{\cos 27^\circ \cdot \cos 63^\circ} = \left[ \sin 90^\circ = 1 \right] = \frac{1}{\cos 9^\circ \cdot \cos 81^\circ} - \frac{1}{\cos 27^\circ \cdot \cos 63^\circ} = \\ &= \left[ \cos \alpha \cdot \cos \beta = \frac{1}{2} \cdot (\cos(\alpha + \beta) + \cos(\alpha - \beta)) \right] = \\ &= \frac{1}{\frac{1}{2} \cdot (\cos(9^\circ + 81^\circ) + \cos(9^\circ - 81^\circ))} - \frac{1}{\frac{1}{2} \cdot (\cos(27^\circ + 63^\circ) + \cos(27^\circ - 63^\circ))} = \end{aligned}$$

$$\begin{aligned}
&= \frac{\frac{1}{2}}{\frac{1}{2} \cdot (\cos 90^\circ + \cos(-72^\circ))} - \frac{\frac{1}{2}}{\frac{1}{2} \cdot (\cos 90^\circ + \cos(-36^\circ))} = [\cos(-\alpha) = \cos \alpha] = \\
&= \frac{2}{\cos 90^\circ + \cos 72^\circ} - \frac{2}{\cos 90^\circ + \cos 36^\circ} = [\cos 90^\circ = 0] = \frac{2}{0 + \cos 72^\circ} - \frac{2}{0 + \cos 36^\circ} = \\
&= \frac{2}{\cos 72^\circ} - \frac{2}{\cos 36^\circ} = [\cos(90^\circ - \alpha) = \sin \alpha] = \frac{2}{\cos(90^\circ - 18^\circ)} - \frac{2}{\cos(90^\circ - 54^\circ)} = \\
&= \frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = 2 \cdot \left( \frac{1}{\sin 18^\circ} - \frac{1}{\sin 54^\circ} \right) = 2 \cdot \frac{\sin 54^\circ - \sin 18^\circ}{\sin 18^\circ \cdot \sin 54^\circ} = \\
&= \left[ \sin \alpha - \sin \beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \sin \frac{\alpha - \beta}{2} \right] = 2 \cdot \frac{2 \cdot \cos \frac{54^\circ + 18^\circ}{2} \cdot \sin \frac{54^\circ - 18^\circ}{2}}{\sin 18^\circ \cdot \sin 54^\circ} = \\
&= 2 \cdot \frac{2 \cdot \cos \frac{72^\circ}{2} \cdot \sin \frac{36^\circ}{2}}{\sin 18^\circ \cdot \sin 54^\circ} = 4 \cdot \frac{\cos \frac{72^\circ}{2} \cdot \sin \frac{36^\circ}{2}}{\sin 18^\circ \cdot \sin 54^\circ} = 4 \cdot \frac{\cos 36^\circ \cdot \sin 18^\circ}{\sin 18^\circ \cdot \sin 54^\circ} = 4 \cdot \frac{\cos 36^\circ \cdot \sin 18^\circ}{\sin 18^\circ \cdot \sin 54^\circ} = \\
&= 4 \cdot \frac{\cos 36^\circ}{\sin 54^\circ} = [\cos(90^\circ - \alpha) = \sin \alpha] = 4 \cdot \frac{\cos(90^\circ - 54^\circ)}{\sin 54^\circ} = 4 \cdot \frac{\sin 54^\circ}{\sin 54^\circ} = 4 \cdot \frac{\sin 54^\circ}{\sin 54^\circ} = 4.
\end{aligned}$$

### Vježba 426

Dokažite jednakost:  $\operatorname{tg} 9^\circ - \operatorname{tg} 27^\circ - \operatorname{tg} 63^\circ + \operatorname{tg} 81^\circ - 4 = 0$ .

**Rezultat:** Dokaz analogan.

### Zadatak 427 (Lea, srednja škola)

Pojednostavnite izraz:  $\frac{2 \cdot \cos 48^\circ + \sin 42^\circ}{3 \cdot \cos 48^\circ}$ .

### Rješenje 427

Ponovimo!

$$\sin(90^\circ - \alpha) = \cos \alpha.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\begin{aligned}
\frac{2 \cdot \cos 48^\circ + \sin 42^\circ}{3 \cdot \cos 48^\circ} &= \left[ \sin(90^\circ - \alpha) = \cos \alpha \right] = \frac{2 \cdot \cos 48^\circ + \sin(90^\circ - 48^\circ)}{3 \cdot \cos 48^\circ} = \\
&= \frac{2 \cdot \cos 48^\circ + \cos 48^\circ}{3 \cdot \cos 48^\circ} = \frac{3 \cdot \cos 48^\circ}{3 \cdot \cos 48^\circ} = \frac{3 \cdot \cos 48^\circ}{3 \cdot \cos 48^\circ} = 1.
\end{aligned}$$

### Vježba 427

Pojednostavnite izraz:  $\frac{3 \cdot \cos 48^\circ + \sin 42^\circ}{4 \cdot \cos 48^\circ}$ .

**Rezultat:** 1.

**Zadatak 428 (Luka, gimnazija)**

Preoblikuj u umnožak:  $tg 40^\circ + ctg 40^\circ$ .

**Rješenje 428**

Ponovimo!

$$ctg(90^\circ - \alpha) = tg \alpha, \quad tg \alpha + tg \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cdot \cos \beta}, \quad \sin 90^\circ = 1, \quad \cos(90^\circ - \alpha) = \sin \alpha.$$

$$\sin(90^\circ - \alpha) = \cos \alpha, \quad 2 \cdot \sin \alpha \cdot \cos \alpha = \sin(2 \cdot \alpha).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\begin{aligned} \frac{a}{b} &= \frac{a \cdot n}{b \cdot n}, \quad n \neq 0, \quad n \neq 1. \\ tg 40^\circ + ctg 40^\circ &= [ctg(90^\circ - \alpha) = tg \alpha] = tg 40^\circ + ctg(90^\circ - 50^\circ) = tg 40^\circ + tg 50^\circ = \\ &= \left[ tg \alpha + tg \beta = \frac{\sin(\alpha + \beta)}{\cos \alpha \cdot \cos \beta} \right] = \frac{\sin(40^\circ + 50^\circ)}{\cos 40^\circ \cdot \cos 50^\circ} = \frac{\sin 90^\circ}{\cos 40^\circ \cdot \cos 50^\circ} = \left[ \sin 90^\circ = 1 \right] = \\ &= \frac{1}{\cos 40^\circ \cdot \cos 50^\circ} = \left[ \cos(90^\circ - \alpha) = \sin \alpha \right] = \frac{1}{\cos 40^\circ \cdot \cos(90^\circ - 40^\circ)} = \frac{1}{\cos 40^\circ \cdot \sin 40^\circ} = \\ &= \frac{2 \cdot 1}{2 \cdot \cos 40^\circ \cdot \sin 40^\circ} = \frac{2}{\sin(2 \cdot 40^\circ)} = \frac{2}{\sin 80^\circ} = \left[ \sin(90^\circ - \alpha) = \cos \alpha \right] = \\ &= \frac{2}{\sin(90^\circ - 10^\circ)} = \frac{2}{\cos 10^\circ}. \end{aligned}$$

**Vježba 428**

Preoblikuj u umnožak:  $\frac{1}{2} \cdot (tg 40^\circ + ctg 40^\circ)$ .

**Rezultat:**  $\frac{1}{\cos 10^\circ}$ .

**Zadatak 429 (Help, gimnazija)**

Dokazati identitet:  $\frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha}$ .

**Rješenje 429**

Ponovimo!

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \quad (a-b) \cdot (a+b) = a^2 - b^2, \quad \cos^2 \alpha + \sin^2 \alpha = 1, \quad a^1 = a.$$

$$\frac{a^n}{a^m} = a^{n-m}, \quad a^n \cdot a^m = a^{n+m}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice



$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n}, \quad n \neq 0, \quad n \neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

1. inačica

$$\begin{aligned} \frac{\sin \alpha}{1 - \cos \alpha} &= \left[ \frac{a}{b} = \frac{a \cdot n}{b \cdot n} \right] = \frac{\sin \alpha}{1 - \cos \alpha} \cdot \frac{1 + \cos \alpha}{1 + \cos \alpha} = \frac{\sin \alpha \cdot (1 + \cos \alpha)}{(1 - \cos \alpha) \cdot (1 + \cos \alpha)} = \\ &= \frac{\sin \alpha \cdot (1 + \cos \alpha)}{1 - \cos^2 \alpha} = \frac{\sin \alpha \cdot (1 + \cos \alpha)}{\sin^2 \alpha} = \frac{\sin \alpha \cdot (1 + \cos \alpha)}{\sin^2 \alpha} = \frac{1 + \cos \alpha}{\sin \alpha}. \end{aligned}$$

2. inačica

$$\begin{aligned} \frac{1 + \cos \alpha}{\sin \alpha} &= \left[ \frac{a}{b} = \frac{a \cdot n}{b \cdot n} \right] = \frac{1 + \cos \alpha}{\sin \alpha} \cdot \frac{1 - \cos \alpha}{1 - \cos \alpha} = \frac{(1 + \cos \alpha) \cdot (1 - \cos \alpha)}{\sin \alpha \cdot (1 - \cos \alpha)} = \frac{1 - \cos^2 \alpha}{\sin \alpha \cdot (1 - \cos \alpha)} = \\ &= \frac{\sin^2 \alpha}{\sin \alpha \cdot (1 - \cos \alpha)} = \frac{\sin^2 \alpha}{\sin \alpha \cdot (1 - \cos \alpha)} = \frac{\sin \alpha}{1 - \cos \alpha}. \end{aligned}$$

3. inačica

$$\begin{aligned} \frac{\sin \alpha}{1 - \cos \alpha} &= \frac{1 + \cos \alpha}{\sin \alpha} \Rightarrow \frac{\sin \alpha}{1 - \cos \alpha} = \frac{1 + \cos \alpha}{\sin \alpha} \cdot (1 - \cos \alpha) \cdot \sin \alpha \Rightarrow \\ \Rightarrow \sin^2 \alpha &= (1 + \cos \alpha) \cdot (1 - \cos \alpha) \Rightarrow \sin^2 \alpha = 1 - \cos^2 \alpha \Rightarrow \sin^2 \alpha = \sin^2 \alpha. \end{aligned}$$

### Vježba 429

Dokazati identitet:  $\frac{1 + \cos \alpha}{\sin \alpha} = \frac{\sin \alpha}{1 - \cos \alpha}$ .

**Rezultat:** Dokaz analogan.

### Zadatak 430 (Domagoj, gimnazija)

Dokazati nejednakost:  $|\sin \alpha| + |\cos \alpha| \geq 1$ .

### Rješenje 430

Ponovimo!

$$a \geq b, \quad c > 0 \Rightarrow a \cdot c \geq b \cdot c, \quad \left. \begin{array}{l} \cos^2 \alpha + \sin^2 \alpha = 1 \\ a \geq 0 \\ b \geq 0 \end{array} \right\} \Rightarrow a \cdot b \geq 0.$$

$$\left. \begin{array}{l} a \geq b \\ c = d \end{array} \right\} \Rightarrow a + c \geq b + d, \quad a^2 + 2 \cdot a \cdot b + b^2 = (a + b)^2.$$

$$a^2 \geq b^2, \quad a > b > 0 \Rightarrow a \geq b.$$

Za realni broj  $x$  njegova je apsolutna vrijednost (modul) broj  $|x|$  koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj  $x$  pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki  $x$ ,  $x \geq 0$ , vrijedi  $|x| = x$ .

Ako je  $x$  negativan broj, njegova apsolutna vrijednost je suprotan broj  $-x$  koji je pozitivan. Za svaki  $x$ ,  $x < 0$ , je  $|x| = -x$ .

$$|x| \geq 0, \quad |x|^2 = x^2.$$

Budući da je

$$|\sin \alpha| \geq 0, \quad |\cos \alpha| \geq 0,$$

slijedi

$$|\sin \alpha| \cdot |\cos \alpha| \geq 0 \Rightarrow |\sin \alpha| \cdot |\cos \alpha| \geq 0 \cdot 2 \Rightarrow 2 \cdot |\sin \alpha| \cdot |\cos \alpha| \geq 0.$$

Sada je:

$$\left. \begin{array}{l} 2 \cdot |\sin \alpha| \cdot |\cos \alpha| \geq 0 \\ |\sin \alpha|^2 + |\cos \alpha|^2 = 1 \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{zbrojimo} \\ \text{nejednakost} \\ \text{i jednakost} \end{array} \right] \Rightarrow 2 \cdot |\sin \alpha| \cdot |\cos \alpha| + |\sin \alpha|^2 + |\cos \alpha|^2 \geq 0 + 1 \Rightarrow$$

$$\Rightarrow |\sin \alpha|^2 + 2 \cdot |\sin \alpha| \cdot |\cos \alpha| + |\cos \alpha|^2 \geq 1 \Rightarrow (|\sin \alpha| + |\cos \alpha|)^2 \geq 1 \Rightarrow$$

$$\Rightarrow (|\sin \alpha| + |\cos \alpha|)^2 \geq 1 \cdot \sqrt{\quad} \Rightarrow |\sin \alpha| + |\cos \alpha| \geq 1.$$

### Vježba 430

Dokazati nejednakost:  $|\sin \alpha| + |\cos \alpha| - 1 \geq 0$ .

**Rezultat:** Dokaz analogan.

### Zadatak 431 (Max, gimnazija)

Pokaži da se kao rezultat eliminacije  $x$  i  $y$  iz jednadžbi  $\sin x + \sin y = 2 \cdot a$ ,  
 $\cos x + \cos y = 2 \cdot b$ ,  $\cos(x - y) = -4 \cdot a \cdot b$  dobije  $(a + b)^2 = \frac{1}{2}$ .

### Rješenje 431

Ponovimo!

$$\left. \begin{array}{l} (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \\ (a \cdot b)^n = a^n \cdot b^n, \quad \left. \begin{array}{l} a = b \\ c = d \end{array} \right\} \Rightarrow a + c = b + d. \end{array} \right\}$$

$$\cos^2 x + \sin^2 x = 1, \quad \cos(x - y) = \cos x \cdot \cos y + \sin x \cdot \sin y.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\left. \begin{array}{l} \sin x + \sin y = 2 \cdot a \\ \cos x + \cos y = 2 \cdot b \\ \cos(x - y) = -4 \cdot a \cdot b \end{array} \right\} \Rightarrow \left. \begin{array}{l} \sin x + \sin y = 2 \cdot a \cdot 1^2 \\ \cos x + \cos y = 2 \cdot b \cdot 1^2 \\ \cos(x - y) = -4 \cdot a \cdot b \cdot (-2) \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} (\sin x + \sin y)^2 = (2 \cdot a)^2 \\ (\cos x + \cos y)^2 = (2 \cdot b)^2 \\ -2 \cdot \cos(x - y) = 8 \cdot a \cdot b \end{array} \right\} \Rightarrow \left. \begin{array}{l} \sin^2 x + 2 \cdot \sin x \cdot \sin y + \sin^2 y = 4 \cdot a^2 \\ \cos^2 x + 2 \cdot \cos x \cdot \cos y + \cos^2 y = 4 \cdot b^2 \\ -2 \cdot \cos(x - y) = 8 \cdot a \cdot b \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array} \right] \Rightarrow$$

$$\begin{aligned}
&\Rightarrow \sin^2 x + 2 \cdot \sin x \cdot \sin y + \sin^2 y + \cos^2 x + 2 \cdot \cos x \cdot \cos y + \cos^2 y - 2 \cdot \cos(x-y) = \\
&\quad = 4 \cdot a^2 + 4 \cdot b^2 + 8 \cdot a \cdot b \Rightarrow \\
&\Rightarrow \sin^2 x + \cos^2 x + \sin^2 y + \cos^2 y + 2 \cdot \cos x \cdot \cos y + 2 \cdot \sin x \cdot \sin y - 2 \cdot \cos(x-y) = \\
&\quad = 4 \cdot (a^2 + b^2 + 2 \cdot a \cdot b) \Rightarrow \\
&\Rightarrow (\sin^2 x + \cos^2 x) + (\sin^2 y + \cos^2 y) + 2 \cdot (\cos x \cdot \cos y + \sin x \cdot \sin y) - 2 \cdot \cos(x-y) = \\
&\quad = 4 \cdot (a^2 + 2 \cdot a \cdot b + b^2) \Rightarrow \\
&\quad \Rightarrow 1+1+2 \cdot \cos(x-y) - 2 \cdot \cos(x-y) = 4 \cdot (a+b)^2 \Rightarrow \\
&\quad \Rightarrow 2+2 \cdot \cos(x-y) - 2 \cdot \cos(x-y) = 4 \cdot (a+b)^2 \Rightarrow 2 = 4 \cdot (a+b)^2 \Rightarrow \\
&\Rightarrow 4 \cdot (a+b)^2 = 2 \Rightarrow 4 \cdot (a+b)^2 = 2 \cdot \frac{1}{4} \Rightarrow (a+b)^2 = \frac{2}{4} \Rightarrow (a+b)^2 = \frac{2}{4} \Rightarrow (a+b)^2 = \frac{1}{2}.
\end{aligned}$$

### Vježba 431

Pokaži da se kao rezultat eliminacije  $x$  i  $y$  iz jednadžbi  $\sin x + \sin y - 2 \cdot a = 0$ ,  
 $\cos x + \cos y - 2 \cdot b = 0$ ,  $\cos(x-y) + 4 \cdot a \cdot b = 0$  dobije  $(a+b)^2 = \frac{1}{2}$ .

**Rezultat:** Dokaz analogan.

### Zadatak 432 (Max, gimnazija)

Ako je  $\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta + \operatorname{ctg} \alpha \cdot \operatorname{ctg} \gamma + \operatorname{ctg} \beta \cdot \operatorname{ctg} \gamma = 1$  ( $\alpha, \beta, \gamma$  šiljasti kutovi), pokazati da je  
 $\alpha + \beta + \gamma = \pi$ .

### Rješenje 432

Ponovimo!

$$\operatorname{ctg}(x+y) = \frac{\operatorname{ctg} x \cdot \operatorname{ctg} y - 1}{\operatorname{ctg} x + \operatorname{ctg} y}, \quad \operatorname{ctg}(\pi - x) = -\operatorname{ctg} x.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Postupno preoblikujemo zadanu jednakost.

$$\begin{aligned}
&\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta + \operatorname{ctg} \alpha \cdot \operatorname{ctg} \gamma + \operatorname{ctg} \beta \cdot \operatorname{ctg} \gamma = 1 \Rightarrow \operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta + \operatorname{ctg} \alpha \cdot \operatorname{ctg} \gamma = 1 - \operatorname{ctg} \beta \cdot \operatorname{ctg} \gamma \Rightarrow \\
&\quad \Rightarrow \operatorname{ctg} \alpha \cdot (\operatorname{ctg} \beta + \operatorname{ctg} \gamma) = -( \operatorname{ctg} \beta \cdot \operatorname{ctg} \gamma - 1 ) \Rightarrow \\
&\quad \Rightarrow \operatorname{ctg} \alpha \cdot (\operatorname{ctg} \beta + \operatorname{ctg} \gamma) = -( \operatorname{ctg} \beta \cdot \operatorname{ctg} \gamma - 1 ) \cdot \frac{1}{\operatorname{ctg} \beta + \operatorname{ctg} \gamma} \Rightarrow \\
&\Rightarrow \operatorname{ctg} \alpha = -\frac{\operatorname{ctg} \beta \cdot \operatorname{ctg} \gamma - 1}{\operatorname{ctg} \beta + \operatorname{ctg} \gamma} \Rightarrow \left[ \operatorname{ctg}(x-y) = \frac{\operatorname{ctg} x \cdot \operatorname{ctg} y - 1}{\operatorname{ctg} x + \operatorname{ctg} y} \right] \Rightarrow \operatorname{ctg} \alpha = -\operatorname{ctg}(\beta + \gamma) \Rightarrow \\
&\Rightarrow \left[ \operatorname{ctg}(\pi - x) = -\operatorname{ctg} x \right] \Rightarrow \operatorname{ctg} \alpha = \operatorname{ctg}(\pi - (\beta + \gamma)) \Rightarrow \left[ \begin{array}{l} \alpha, \beta, \gamma \\ \text{šiljasti kutovi} \end{array} \right] \Rightarrow \\
&\quad \Rightarrow \alpha = \pi - \beta - \gamma \Rightarrow \alpha + \beta + \gamma = \pi.
\end{aligned}$$

### Vježba 432

Ako je  $\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta + \operatorname{ctg} \alpha \cdot \operatorname{ctg} \gamma + \operatorname{ctg} \beta \cdot \operatorname{ctg} \gamma - 1 = 0$  ( $\alpha, \beta, \gamma$  šiljasti kutovi), pokazati da je  $\alpha + \beta + \gamma = \pi$ .

**Rezultat:** Dokaz analogan.

### Zadatak 433 (Miroslav, srednja škola)

Riješite jednadžbu  $\sin^{1988} x + \cos^{1000} x = 1$ .

### Rješenje 433

Ponovimo!

$$-1 \leq \sin x \leq 1, \quad -1 \leq \cos x \leq 1, \quad a^{2 \cdot n} \geq 0, \quad a \in \mathbb{R}.$$

Skup cijelih brojeva označavamo slovom  $\mathbb{Z}$ , a zapisujemo

$$\mathbb{Z} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}.$$

Parni brojevi su oni brojevi koji su djeljivi s 2, a neparni su oni koji nisu djeljivi s 2. Da je neki cijeli broj  $m$  neparan znači da se može napisati u obliku

$$m = 2 \cdot (\text{neki cijeli broj}) + 1, \quad m = 2 \cdot k + 1, \quad k \in \mathbb{Z}.$$

Neparni brojevi mogu se prikazati općom formulom

$$2 \cdot n + 1,$$

gdje  $n$  pripada skupu cijelih brojeva.

Eksponenti 1988 i 1000 parni su brojevi pa je

$$\sin^{1988} x \geq 0 \text{ i } \cos^{1000} x \geq 0.$$

Da bi jednadžba

$$\sin^{1988} x + \cos^{1000} x = 1$$

imala realna rješenja mora vrijediti:

- $\sin x = 0$ , a  $\cos x = -1$  ili  $\cos x = 1$
- $\cos x = 0$ , a  $\sin x = -1$  ili  $\sin x = 1$ .

Ako je

$$\sin x = 0 \Rightarrow x = \sin^{-1}(0) \Rightarrow x = k \cdot \pi, \quad k \in \mathbb{Z}.$$

Sami se uvjerimo da je za  $x = k \cdot \pi$  ili  $\cos x = -1$  ili  $\cos x = 1$ .

Ako je

$$\cos x = 0 \Rightarrow x = \cos^{-1}(0) \Rightarrow x = (2 \cdot k + 1) \cdot \frac{\pi}{2}, \quad k \in \mathbb{Z}.$$

Sami se uvjerimo da je za  $x = (2 \cdot k + 1) \cdot \frac{\pi}{2}$  ili  $\sin x = -1$  ili  $\sin x = 1$ .

Opće rješenje zadane jednadžbe tada glasi:

$$x = k \cdot \frac{\pi}{2}, \quad k \in \mathbb{Z}.$$

### Vježba 433

Riješite jednadžbu  $\sin^{988} x + \cos^{100} x = 1$ .

**Rezultat:**  $x = k \cdot \frac{\pi}{2}, \quad k \in \mathbb{Z}.$

**Zadatak 434 (Frenky, gimnazija)**

Izračunajte zbroj  $\frac{\sin 1}{\cos 0 \cdot \cos 1} + \frac{\sin 1}{\cos 1 \cdot \cos 2} + \frac{\sin 1}{\cos 2 \cdot \cos 3} + \dots + \frac{\sin 1}{\cos(n-1) \cdot \cos n}$ .

**Rješenje 434**

Ponovimo!

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta.$$

$$\frac{a-b}{n} = \frac{a}{n} - \frac{b}{n}, \quad \operatorname{tg} \alpha = \frac{\sin \alpha}{\cos \alpha}, \quad \operatorname{tg} 0 = 0.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Uočimo da vrijedi:

$$\begin{aligned} \frac{\sin 1}{\cos(k-1) \cdot \cos k} &= \frac{\sin(k-(k-1))}{\cos(k-1) \cdot \cos k} \Rightarrow \frac{\sin 1}{\cos(k-1) \cdot \cos k} = \frac{\sin k \cdot \cos(k-1) - \cos k \cdot \sin(k-1)}{\cos(k-1) \cdot \cos k} \Rightarrow \\ &\Rightarrow \frac{\sin 1}{\cos(k-1) \cdot \cos k} = \frac{\sin k \cdot \cos(k-1)}{\cos(k-1) \cdot \cos k} - \frac{\cos k \cdot \sin(k-1)}{\cos(k-1) \cdot \cos k} \Rightarrow \\ \Rightarrow \frac{\sin 1}{\cos(k-1) \cdot \cos k} &= \frac{\sin k \cdot \cos(k-1)}{\cos(k-1) \cdot \cos k} - \frac{\cos k \cdot \sin(k-1)}{\cos(k-1) \cdot \cos k} \Rightarrow \frac{\sin 1}{\cos(k-1) \cdot \cos k} = \frac{\sin k}{\cos k} - \frac{\sin(k-1)}{\cos(k-1)} \Rightarrow \\ &\Rightarrow \frac{\sin 1}{\cos(k-1) \cdot \cos k} = \operatorname{tg} k - \operatorname{tg}(k-1). \end{aligned}$$

Preoblikujemo zadani zbroj pa dobijemo:

$$\begin{aligned} &\frac{\sin 1}{\cos 0 \cdot \cos 1} + \frac{\sin 1}{\cos 1 \cdot \cos 2} + \frac{\sin 1}{\cos 2 \cdot \cos 3} + \dots + \frac{\sin 1}{\cos(n-1) \cdot \cos n} = \\ &= (\operatorname{tg} 1 - \operatorname{tg} 0) + (\operatorname{tg} 2 - \operatorname{tg} 1) + (\operatorname{tg} 3 - \operatorname{tg} 2) + \dots + (\operatorname{tg} n - \operatorname{tg}(n-1)) = \\ &= \operatorname{tg} 1 - \operatorname{tg} 0 + \operatorname{tg} 2 - \operatorname{tg} 1 + \operatorname{tg} 3 - \operatorname{tg} 2 + \dots + \operatorname{tg} n - \operatorname{tg}(n-1) = \\ &= \operatorname{tg} 1 - 0 + \operatorname{tg} 2 - \operatorname{tg} 1 + \operatorname{tg} 3 - \operatorname{tg} 2 + \dots + \operatorname{tg} n - \operatorname{tg}(n-1) = \\ &= \operatorname{tg} 1 + \operatorname{tg} 2 - \operatorname{tg} 1 + \operatorname{tg} 3 - \operatorname{tg} 2 + \dots + \operatorname{tg} n - \operatorname{tg}(n-1) = \\ &= \operatorname{tg} 1 + \operatorname{tg} 2 - \operatorname{tg} 1 + \operatorname{tg} 3 - \operatorname{tg} 2 + \dots + \operatorname{tg} n - \operatorname{tg}(n-1) = \operatorname{tg} n. \end{aligned}$$

**Vježba 434**

Izračunajte zbroj  $\frac{\sin 1}{\cos 0 \cdot \cos 1} + \frac{\sin 1}{\cos 1 \cdot \cos 2} + \frac{\sin 1}{\cos 2 \cdot \cos 3}$ .

**Rezultat:**  $\operatorname{tg} 3$ .

**Zadatak 435 (Josip, gimnazija)**

Ako je  $x = \frac{1}{\cos \alpha \cdot \cos \beta}$ ,  $y = \frac{\operatorname{tg} \alpha}{\cos \beta}$ ,  $z = \operatorname{tg} \beta$  izračunajte  $x^2 - y^2 - z^2$ .

- A. 0      B. 1      C. -1      D.  $\sin \alpha$

**Rješenje 435**

Ponovimo!

$$\frac{1}{\cos^2 x} = 1 + \operatorname{tg}^2 x, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad (a \cdot b)^n = a^n \cdot b^n, \quad \frac{a \cdot c}{b \cdot d} = \frac{a \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Množenje zagrada

$$(a + b) \cdot (c + d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

$$\left. \begin{array}{l} x = \frac{1}{\cos \alpha \cdot \cos \beta} \\ y = \frac{\operatorname{tg} \alpha}{\cos \beta} \\ z = \operatorname{tg} \beta \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{kvadriramo} \\ \text{jednakosti} \end{array} \right] \Rightarrow \left. \begin{array}{l} x = \frac{1}{\cos \alpha \cdot \cos \beta} / 2 \\ y = \frac{\operatorname{tg} \alpha}{\cos \beta} / 2 \\ z = \operatorname{tg} \beta / 2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x^2 = \left(\frac{1}{\cos \alpha \cdot \cos \beta}\right)^2 \\ y^2 = \left(\frac{\operatorname{tg} \alpha}{\cos \beta}\right)^2 \\ z^2 = \operatorname{tg}^2 \beta \end{array} \right\} \Rightarrow$$

$$\left. \begin{array}{l} x^2 = \frac{1}{\cos^2 \alpha \cdot \cos^2 \beta} \\ \Rightarrow y^2 = \frac{\operatorname{tg}^2 \alpha}{\cos^2 \beta} \\ z^2 = \operatorname{tg}^2 \beta \end{array} \right\} \Rightarrow \left. \begin{array}{l} x^2 = \frac{1}{\cos^2 \alpha} \cdot \frac{1}{\cos^2 \beta} \\ y^2 = \operatorname{tg}^2 \alpha \cdot \frac{1}{\cos^2 \beta} \\ z^2 = \operatorname{tg}^2 \beta \end{array} \right\} \Rightarrow \left. \begin{array}{l} x^2 = (1 + \operatorname{tg}^2 \alpha) \cdot (1 + \operatorname{tg}^2 \beta) \\ y^2 = \operatorname{tg}^2 \alpha \cdot (1 + \operatorname{tg}^2 \beta) \\ z^2 = \operatorname{tg}^2 \beta \end{array} \right\}.$$

Sada je

$$\begin{aligned} x^2 - y^2 - z^2 &= (1 + \operatorname{tg}^2 \alpha) \cdot (1 + \operatorname{tg}^2 \beta) - \operatorname{tg}^2 \alpha \cdot (1 + \operatorname{tg}^2 \beta) - \operatorname{tg}^2 \beta \Rightarrow \\ &\Rightarrow x^2 - y^2 - z^2 = 1 + \operatorname{tg}^2 \beta + \operatorname{tg}^2 \alpha + \operatorname{tg}^2 \alpha \cdot \operatorname{tg}^2 \beta - \operatorname{tg}^2 \alpha - \operatorname{tg}^2 \alpha \cdot \operatorname{tg}^2 \beta - \operatorname{tg}^2 \beta \Rightarrow \\ &\Rightarrow x^2 - y^2 - z^2 = 1 + \operatorname{tg}^2 \beta + \operatorname{tg}^2 \alpha + \operatorname{tg}^2 \alpha \cdot \operatorname{tg}^2 \beta - \operatorname{tg}^2 \alpha - \operatorname{tg}^2 \alpha \cdot \operatorname{tg}^2 \beta - \operatorname{tg}^2 \beta \Rightarrow \\ &\Rightarrow x^2 - y^2 - z^2 = 1. \end{aligned}$$

Odgovor je pod B.

**Vježba 435**

Ako je  $x = \frac{1}{\cos \alpha \cdot \cos \beta}$ ,  $y = \frac{\sin \alpha}{\cos \alpha \cdot \cos \beta}$ ,  $z = \operatorname{tg} \beta$  izračunajte  $x^2 - y^2 - z^2$ .

- A. 0      B. 1      C. -1      D.  $\sin \alpha$

**Rezultat:** B.

**Zadatak 436 (Josip, gimnazija)**

Ako je  $x = \frac{a}{\cos \alpha} + b \cdot \operatorname{tg} \alpha$ ,  $y = a \cdot \operatorname{tg} \alpha + \frac{b}{\cos \alpha}$  izračunajte  $x^2 - y^2$ .

- A. 1      B.  $a^2$       C.  $a^2 - b^2$       D.  $a^2 + b^2$

**Rješenje 436**

Ponovimo!

$$\frac{1}{\cos^2 x} = 1 + \operatorname{tg}^2 x, \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad (a \cdot b)^n = a^n \cdot b^n, \quad \frac{a \cdot c}{b \cdot d} = \frac{a \cdot c}{b \cdot d}.$$

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$\left. \begin{array}{l} x = \frac{a}{\cos \alpha} + b \cdot \operatorname{tg} \alpha \\ y = a \cdot \operatorname{tg} \alpha + \frac{b}{\cos \alpha} \end{array} \right\} \Rightarrow \left[ \begin{array}{l} \text{kvadriramo} \\ \text{jednakosti} \end{array} \right] \Rightarrow \left. \begin{array}{l} x = \frac{a}{\cos \alpha} + b \cdot \operatorname{tg} \alpha \quad / \quad ^2 \\ y = a \cdot \operatorname{tg} \alpha + \frac{b}{\cos \alpha} \quad / \quad ^2 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} x^2 = \left( \frac{a}{\cos \alpha} + b \cdot \operatorname{tg} \alpha \right)^2 \\ y^2 = \left( a \cdot \operatorname{tg} \alpha + \frac{b}{\cos \alpha} \right)^2 \end{array} \right\} \Rightarrow \left. \begin{array}{l} x^2 = \left( \frac{a}{\cos \alpha} \right)^2 + 2 \cdot \frac{a}{\cos \alpha} \cdot b \cdot \operatorname{tg} \alpha + (b \cdot \operatorname{tg} \alpha)^2 \\ y^2 = (a \cdot \operatorname{tg} \alpha)^2 + 2 \cdot a \cdot \operatorname{tg} \alpha \cdot \frac{b}{\cos \alpha} + \left( \frac{b}{\cos \alpha} \right)^2 \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} x^2 = \frac{a^2}{\cos^2 \alpha} + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot \operatorname{tg}^2 \alpha \\ y^2 = a^2 \cdot \operatorname{tg}^2 \alpha + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + \frac{b^2}{\cos^2 \alpha} \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} x^2 = a^2 \cdot \frac{1}{\cos^2 \alpha} + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot \operatorname{tg}^2 \alpha \\ y^2 = a^2 \cdot \operatorname{tg}^2 \alpha + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot \frac{1}{\cos^2 \alpha} \end{array} \right\} \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} x^2 = a^2 \cdot (1 + \operatorname{tg}^2 \alpha) + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot \operatorname{tg}^2 \alpha \\ y^2 = a^2 \cdot \operatorname{tg}^2 \alpha + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot (1 + \operatorname{tg}^2 \alpha) \end{array} \right\}.$$

Sada je:

$$x^2 - y^2 =$$

$$= a^2 \cdot (1 + \operatorname{tg}^2 \alpha) + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot \operatorname{tg}^2 \alpha - \left( a^2 \cdot \operatorname{tg}^2 \alpha + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot (1 + \operatorname{tg}^2 \alpha) \right) \Rightarrow$$

$$\Rightarrow x^2 - y^2 =$$

$$\begin{aligned}
&= a^2 \cdot (1 + \operatorname{tg}^2 \alpha) + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot \operatorname{tg}^2 \alpha - a^2 \cdot \operatorname{tg}^2 \alpha - 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} - b^2 \cdot (1 + \operatorname{tg}^2 \alpha) \Rightarrow \\
&\quad \Rightarrow x^2 - y^2 = \\
&= a^2 \cdot (1 + \operatorname{tg}^2 \alpha) + 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} + b^2 \cdot \operatorname{tg}^2 \alpha - a^2 \cdot \operatorname{tg}^2 \alpha - 2 \cdot a \cdot b \cdot \frac{\operatorname{tg} \alpha}{\cos \alpha} - b^2 \cdot (1 + \operatorname{tg}^2 \alpha) \Rightarrow \\
&\quad \Rightarrow x^2 - y^2 = a^2 \cdot (1 + \operatorname{tg}^2 \alpha) + b^2 \cdot \operatorname{tg}^2 \alpha - a^2 \cdot \operatorname{tg}^2 \alpha - b^2 \cdot (1 + \operatorname{tg}^2 \alpha) \Rightarrow \\
&\quad \Rightarrow x^2 - y^2 = a^2 + a^2 \cdot \operatorname{tg}^2 \alpha + b^2 \cdot \operatorname{tg}^2 \alpha - a^2 \cdot \operatorname{tg}^2 \alpha - b^2 - b^2 \cdot \operatorname{tg}^2 \alpha \Rightarrow \\
&\quad \Rightarrow x^2 - y^2 = a^2 + a^2 \cdot \operatorname{tg}^2 \alpha + b^2 \cdot \operatorname{tg}^2 \alpha - a^2 \cdot \operatorname{tg}^2 \alpha - b^2 - b^2 \cdot \operatorname{tg}^2 \alpha \Rightarrow \\
&\quad \Rightarrow x^2 - y^2 = a^2 - b^2.
\end{aligned}$$

Odgovor je pod C.

### Vježba 436

Ako je  $x = \frac{a}{\cos \alpha} + b \cdot \operatorname{tg} \beta$ ,  $y = a \cdot \operatorname{tg} \alpha + \frac{b}{\cos \alpha}$  izračunajte  $y^2 - x^2$ .

- A. 1      B.  $b^2$       C.  $b^2 - a^2$       D.  $a^2 + b^2$

**Rezultat:** C.

### Zadatak 437 (Ivica, srednja škola)

Ako je  $\operatorname{ctg} \alpha = \frac{3}{4}$ ,  $\operatorname{ctg} \beta = \frac{1}{7}$ ,  $0 < \alpha < \frac{\pi}{2}$ ,  $0 < \beta < \frac{\pi}{2}$ , izračunati  $\alpha + \beta$ .

### Rješenje 437

Ponovimo!

$$\operatorname{ctg}(\alpha + \beta) = \frac{\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta} = \frac{\frac{a}{b} \cdot \frac{c}{d} - 1}{\frac{a}{b} + \frac{c}{d}} = \frac{\frac{a \cdot c}{b \cdot d} - 1}{\frac{a \cdot d + b \cdot c}{b \cdot d}}, \quad n = \frac{n}{1}, \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}, \quad \operatorname{ctg}\left(\frac{3 \cdot \pi}{4}\right) = -1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

$$\operatorname{ctg}(\alpha + \beta) = \frac{\operatorname{ctg} \alpha \cdot \operatorname{ctg} \beta - 1}{\operatorname{ctg} \alpha + \operatorname{ctg} \beta} \Rightarrow \left[ \begin{array}{l} \operatorname{ctg} \alpha = \frac{3}{4} \\ \operatorname{ctg} \beta = \frac{1}{7} \end{array} \right] \Rightarrow \operatorname{ctg}(\alpha + \beta) = \frac{\frac{3}{4} \cdot \frac{1}{7} - 1}{\frac{3}{4} + \frac{1}{7}} \Rightarrow$$

$$\Rightarrow \operatorname{ctg}(\alpha + \beta) = \frac{\frac{3}{4} - 1}{\frac{3}{4} + \frac{1}{7}} \Rightarrow \operatorname{ctg}(\alpha + \beta) = \frac{\frac{3}{4} - \frac{1}{1}}{\frac{3}{4} + \frac{1}{7}} \Rightarrow \operatorname{ctg}(\alpha + \beta) = \frac{\frac{3 - 4}{4}}{\frac{3 + 4}{28}} \Rightarrow \operatorname{ctg}(\alpha + \beta) = \frac{-\frac{1}{4}}{\frac{7}{28}} \Rightarrow$$



$$\Rightarrow \operatorname{ctg}(\alpha + \beta) = \frac{-\frac{25}{28}}{\frac{25}{28}} \Rightarrow \operatorname{ctg}(\alpha + \beta) = -1 \Rightarrow \alpha + \beta = \operatorname{ctg}^{-1}(-1) \Rightarrow \alpha + \beta = \frac{3 \cdot \pi}{4}.$$

### Vježba 437

Ako je  $\operatorname{ctg} \alpha = \frac{3}{4}$ ,  $\operatorname{ctg} \beta = \frac{1}{7}$ ,  $0 < \alpha < \frac{\pi}{2}$ ,  $0 < \beta < \frac{\pi}{2}$ , izračunati  $\frac{1}{3} \cdot (\alpha + \beta)$ .

**Rezultat:**  $\frac{\pi}{4}$ .

### Zadatak 438 (Tomislav, gimnazija)

Za koji realan broj  $t$  vrijedi  $\frac{\sin x - \sin^3 x}{1 + \cos(2 \cdot x)} = t \cdot \sin x$  za svaki  $x \neq \frac{k \cdot \pi}{2}$ ,  $k \in \mathbb{Z}$ ?

### Rješenje 438

Ponovimo!

$$a^1 = a, \quad a^n : a^m = a^{n-m}, \quad \cos^2 x + \sin^2 x = 1, \quad \cos(2 \cdot x) = \cos^2 x - \sin^2 x.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c, \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b}, \quad n \neq 0, \quad n \neq 1.$$

Skup cijelih brojeva označavamo slovom  $\mathbb{Z}$ , a zapisujemo

$$\mathbb{Z} = \{ \dots, -3, -2, -1, 0, 1, 2, 3, \dots \}.$$

$$\begin{aligned} \frac{\sin x - \sin^3 x}{1 + \cos(2 \cdot x)} = t \cdot \sin x &\Rightarrow \frac{\sin x \cdot (1 - \sin^2 x)}{\cos^2 x + \sin^2 x + \cos^2 x - \sin^2 x} = t \cdot \sin x \Rightarrow \\ &\Rightarrow \frac{\sin x \cdot \cos^2 x}{\cos^2 x + \sin^2 x + \cos^2 x - \sin^2 x} = t \cdot \sin x \Rightarrow \frac{\sin x \cdot \cos^2 x}{\cos^2 x + \cos^2 x} = t \cdot \sin x \Rightarrow \\ &\Rightarrow \frac{\sin x \cdot \cos^2 x}{2 \cdot \cos^2 x} = t \cdot \sin x \Rightarrow \frac{\sin x \cdot \cos^2 x}{2 \cdot \cos^2 x} = t \cdot \sin x \Rightarrow \frac{\sin x}{2} = t \cdot \sin x \Rightarrow \\ &\Rightarrow \frac{\sin x}{2} = t \cdot \sin x \Rightarrow t \cdot \sin x = \frac{\sin x}{2} \Rightarrow t \cdot \sin x = \frac{\sin x}{2} \quad /: \sin x \Rightarrow t = \frac{1}{2}. \end{aligned}$$

### Vježba 438

Za koji realan broj  $t$  vrijedi  $\frac{\sin x - \sin^3 x}{1 + \cos(2 \cdot x)} - t \cdot \sin x = 0$  za svaki  $x \neq \frac{k \cdot \pi}{2}$ ,  $k \in \mathbb{Z}$ ?

**Rezultat:**  $\frac{1}{2}$ .

### Zadatak 439 (Marija, gimnazija)

Ako su  $a, b, c$  tri pozitivna broja,  $0 < \alpha < \pi$  i  $a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha$ , pokažite da je  $|b - c| < a < b + c$ .

### Rješenje 439

Ponovimo!

$$\cos 0 = 1, \quad \cos \pi = -1, \quad a < b, c > 0 \Rightarrow a \cdot c < b \cdot c, \quad a^2 - 2 \cdot a \cdot b + b^2 = (a-b)^2.$$

$$\sqrt{a^2} = |a|, \quad \sqrt{a^2} = a, a \geq 0, \quad a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2.$$

$$a > b, c > 0 \Rightarrow a \cdot c > b \cdot c.$$

Za realni broj  $x$  njegova je apsolutna vrijednost (modul) broj  $|x|$  koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj  $x$  pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki  $x, x \geq 0$ , vrijedi  $|x| = x$ .

Ako je  $x$  negativan broj, njegova apsolutna vrijednost je suprotan broj  $-x$  koji je pozitivan. Za svaki  $x, x < 0$ , je  $|x| = -x$ .

Budući da je

$$0 < \alpha < \pi,$$

slijedi

$$-1 < \cos \alpha < 1.$$

Preoblikujemo zadanu jednakost.

$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos \alpha \Rightarrow 2 \cdot b \cdot c \cdot \cos \alpha = b^2 + c^2 - a^2 \Rightarrow$$

$$\Rightarrow 2 \cdot b \cdot c \cdot \cos \alpha = b^2 + c^2 - a^2 \quad / \cdot \frac{1}{2 \cdot b \cdot c} \Rightarrow \cos \alpha = \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c}.$$

Prvi slučaj

$$\cos \alpha < 1 \Rightarrow \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c} < 1 \Rightarrow [b \cdot c > 0] \Rightarrow \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c} < 1 \quad / \cdot 2 \cdot b \cdot c \Rightarrow$$

$$\Rightarrow b^2 + c^2 - a^2 < 2 \cdot b \cdot c \Rightarrow b^2 - 2 \cdot b \cdot c + c^2 < a^2 \Rightarrow (b-c)^2 < a^2 \Rightarrow$$

$$\Rightarrow (b-c)^2 < a^2 \quad / \sqrt{\quad} \Rightarrow \sqrt{(b-c)^2} < \sqrt{a^2} \Rightarrow |b-c| < a.$$

Drugi slučaj

$$\cos \alpha > -1 \Rightarrow \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c} > -1 \Rightarrow [b \cdot c > 0] \Rightarrow \frac{b^2 + c^2 - a^2}{2 \cdot b \cdot c} > -1 \quad / \cdot 2 \cdot b \cdot c \Rightarrow$$

$$\Rightarrow b^2 + c^2 - a^2 > -2 \cdot b \cdot c \Rightarrow b^2 + 2 \cdot b \cdot c + c^2 > a^2 \Rightarrow (b+c)^2 > a^2 \Rightarrow$$

$$\Rightarrow (b+c)^2 > a^2 \quad / \sqrt{\quad} \Rightarrow \sqrt{(b+c)^2} > \sqrt{a^2} \Rightarrow b+c > a \Rightarrow a < b+c.$$

Konačno je:

$$\left. \begin{array}{l} |b-c| < a \\ a < b+c \end{array} \right\} \Rightarrow |b-c| < a < b+c.$$

### Vježba 439

Ako su  $a, b, c$  tri pozitivna broja,  $0 < \beta < \pi$  i  $b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos \beta$ , pokažite da je  $|a-c| < b < a+c$ .

**Rezultat:** Dokaz analogan.