Zadatak 401 (Marija, gimnazija)

Dokažite identitet: $tg(2\cdot\alpha)=\frac{2\cdot tg\,\alpha}{1-tg^2\,\alpha}$.

Rješenje 401

Ponovimo!

$$tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}\quad,\quad \sin(2\cdot\alpha)=2\cdot\sin\alpha\cdot\cos\alpha\quad,\quad \cos(2\cdot\alpha)=\cos^2\alpha-\sin^2\alpha.$$

$$a^1=a\quad,\quad \frac{a^n}{a^m}=a^{n-m}\quad,\quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n\quad,\quad \frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$tg(2\cdot\alpha)=\frac{\sin(2\cdot\alpha)}{\cos(2\cdot\alpha)}=\frac{2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha-\sin^2\alpha}=\frac{\frac{2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha}}{\frac{\cos^2\alpha-\sin^2\alpha}{\cos^2\alpha}}=\frac{\frac{2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha}}{\frac{\cos^2\alpha}{\cos^2\alpha}-\frac{\sin^2\alpha}{\cos^2\alpha}}=$$

$$=\frac{\frac{2\cdot\sin\alpha}{\cos\alpha}}{\frac{\cos^2\alpha}{\cos^2\alpha}-\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{2\cdot tg\,\alpha}{1-\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{2\cdot tg\,\alpha}{1-\left(\frac{\sin\alpha}{\cos\alpha}\right)^2}=\frac{2\cdot tg\,\alpha}{1-tg^2\,\alpha}.$$

Vježba 401

Dokažite identitet: $\frac{1}{2}\cdot tg(2\cdot\alpha)=\frac{tg\,\alpha}{1-tg^2\,\alpha}$.

Rezultat: Dokaz analogan.

Zadatak 402 (Ana, gimnazija)

Pokaži da je $\frac{1-2\cdot\cos^2\alpha}{\sin\alpha\cdot\cos\alpha}=tg\,\alpha-ctg\,\alpha$.

Rješenje 402

Ponovimo!

$$\cos^2 x+\sin^2 x=1\quad,\quad \frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}\quad,\quad tg\,x=\frac{\sin x}{\cos x}\quad,\quad ctg\,x=\frac{\cos x}{\sin x}.$$

$$a^1=a\quad,\quad \frac{a^n}{a^m}=a^{n-m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

1.inačica

$$\frac{1 - 2 \cdot \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{\cos^2 \alpha + \sin^2 \alpha - 2 \cdot \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{\sin^2 \alpha - \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} =$$

$$= \frac{\sin^2 \alpha}{\sin \alpha \cdot \cos \alpha} - \frac{\cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{\sin^2 \alpha}{\sin \alpha \cdot \cos \alpha} - \frac{\cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\sin \alpha} = tg\,\alpha - ctg\,\alpha.$$

2.inačica

$$\frac{1 - 2 \cdot \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{1 - \cos^2 \alpha - \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{\left(1 - \cos^2 \alpha\right) - \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{\sin^2 \alpha - \cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} =$$

$$= \frac{\sin^2 \alpha}{\sin \alpha \cdot \cos \alpha} - \frac{\cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{\sin^2 \alpha}{\sin \alpha \cdot \cos \alpha} - \frac{\cos^2 \alpha}{\sin \alpha \cdot \cos \alpha} = \frac{\sin \alpha}{\cos \alpha} - \frac{\cos \alpha}{\sin \alpha} = tg\,\alpha - ctg\,\alpha.$$

Vježba 402

Pokaži da je $\frac{2 \cdot \cos^2 \alpha - 1}{\sin \alpha \cdot \cos \alpha} = ctg\,\alpha - tg\,\alpha.$

Rezultat: Dokaz analogan.

Zadatak 403 (Matija, gimnazija)

Pokaži da je $\frac{\sin \alpha + \sin \beta}{\cos \alpha + \cos \beta} = tg\,\frac{\alpha + \beta}{2}.$

Rješenje 403

Ponovimo!

$$\sin x + \sin y = 2 \cdot \sin \frac{x + y}{2} \cdot \cos \frac{x - y}{2} \quad , \quad \cos x + \cos y = 2 \cdot \cos \frac{x + y}{2} \cdot \cos \frac{x - y}{2}.$$

$$tg\,x = \frac{\sin x}{\cos x} \quad , \quad ctg\,x = \frac{\cos x}{\sin x}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$\frac{\sin \alpha + \sin \beta}{\cos \alpha + \cos \beta} = \frac{2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}}{2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}} = \frac{2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}}{2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}} =$$

$$= \frac{\sin \frac{\alpha + \beta}{2}}{\cos \frac{\alpha + \beta}{2}} = tg\,\frac{\alpha + \beta}{2}.$$

Vježba 403

Pokaži da je $\frac{\cos\alpha+\cos\beta}{\sin\alpha+\sin\beta}=ctg\,\frac{\alpha+\beta}{2}.$

Rezultat: Dokaz analogan.

Zadatak 404 (Matija, gimnazija)

Koja sveza postoji između kutova α, β i γ ako vrijedi $tg\,\alpha+tg\,\beta+tg\,\gamma=tg\,\alpha\cdot tg\,\beta\cdot tg\,\gamma.$

Rješenje 404

Ponovimo!

$$tg\,(x+y)=\frac{tg\,x+tg\,y}{1-tg\,x\cdot tg\,y}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Trigonometrijska jednadžba tg x = a

Skup rješenja jednadžbe $tg\,x=a,\ a\in R,$ je $\{x_\circ+k\cdot\pi : k\in Z\},$ gdje je $x_\circ\in R$ jedno rješenje te jednadžbe.

$$tg\,x=a \Rightarrow tg\,x=tg\,x_\circ \Rightarrow x=x_\circ+k\cdot\pi \ ,\ k\in Z.$$

Skup cijelih brojeva označavamo slovom Z, a zapisujemo

$$Z=\{\ldots,-3,-2,-1,0,1,2,3,\ldots\}.$$

Funkciju y = f(x) definiranu u simetričnom području – a ≤ x ≤ a nazivamo:

- parnom, ako je f(– x) = f(x)
- neparnom, ako je f(– x) = – f(x).
- Funkcija f(x) = tg x je neparna:

$$tg\,(-x)=-tg\,x.$$

Preoblikujemo zadanu relaciju.

$$tg\,\alpha+tg\,\beta+tg\,\gamma=tg\,\alpha\cdot tg\,\beta\cdot tg\,\gamma \Rightarrow tg\,\alpha-tg\,\alpha\cdot tg\,\beta\cdot tg\,\gamma=-tg\,\beta-tg\,\gamma \Rightarrow$$

$$\Rightarrow tg\,\alpha\cdot(1-tg\,\beta\cdot tg\,\gamma)=-(tg\,\beta+tg\,\gamma) \Rightarrow$$

$$\Rightarrow tg\,\alpha\cdot(1-tg\,\beta\cdot tg\,\gamma)=-(tg\,\beta+tg\,\gamma)\ /\cdot\frac{1}{1-tg\,\beta\cdot tg\,\gamma} \Rightarrow tg\,\alpha=-\frac{tg\,\beta+tg\,\gamma}{1-tg\,\beta\cdot tg\,\gamma} \Rightarrow$$

$$\Rightarrow tg\,\alpha=-tg\,(\beta+\gamma) \Rightarrow tg\,\alpha=tg\,(-(\beta+\gamma)) \Rightarrow \alpha=-(\beta+\gamma)+k\cdot\pi \Rightarrow$$

$$\Rightarrow \alpha=-\beta-\gamma+k\cdot\pi \Rightarrow \alpha+\beta+\gamma=k\cdot\pi,\ k\in Z.$$

Vježba 404

Koja sveza postoji između kutova α, β i γ ako vrijedi $tg\,\alpha+tg\,\beta+tg\,\gamma-tg\,\alpha\cdot tg\,\beta\cdot tg\,\gamma=0.$

Rezultat: Dokaz analogan.

Zadatak 405 (Matija, gimnazija)

Dokaži. $tg\,\alpha+ctg\,\alpha=\frac{1}{\sin\alpha\cdot\cos\alpha},\ \sin\alpha\neq 0,\ \cos\alpha\neq 0.$

Rješenje 405

Ponovimo!

$$tg\,x=\frac{\sin x}{\cos x} \quad , \quad ctg\,x=\frac{\cos x}{\sin x} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \cos^2\alpha+\sin^2\alpha=1.$$

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$tg\,\alpha + ctg\,\alpha = \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha\cdot\sin\alpha} = \frac{1}{\cos\alpha\cdot\sin\alpha} = \frac{1}{\sin\alpha\cdot\cos\alpha}.$$

Vježba 405

Dokaži. $ctg\,\alpha + tg\,\alpha = \frac{1}{\sin\alpha\cdot\cos\alpha}$, $\sin\alpha \ne 0$, $\cos\alpha \ne 0$.

Rezultat: Dokaz analogan.

Zadatak 406 (Iva, gimnazija)

Dokaži. $\frac{\sin\alpha}{1-\cos\alpha} = \frac{1+\cos\alpha}{\sin\alpha}$, $\sin\alpha \ne 0$.

Rješenje 406

Ponovimo!

$$\frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d} \quad , \quad (a-b)\cdot(a+b) = a^2 - b^2 \quad , \quad \cos^2\alpha + \sin^2\alpha = 1.$$

$$a^1 = a \quad , \quad \frac{a^n}{a^m} = a^{n-m}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a\cdot n}{b\cdot n} \; , \; n \ne 0 \; , \; n \ne 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \; , \; n \ne 0 \; , \; n \ne 1.$$

$$\frac{\sin\alpha}{1-\cos\alpha} = \left[\begin{array}{l}\text{proširimo razlomak} \\ \text{s } \frac{1+\cos\alpha}{1+\cos\alpha}\end{array}\right] = \frac{\sin\alpha}{1-\cos}\cdot\frac{1+\cos\alpha}{1+\cos\alpha} = \frac{\sin\alpha\cdot(1+\cos\alpha)}{(1-\cos\alpha)\cdot(1+\cos\alpha)} =$$

$$= \frac{\sin\alpha\cdot(1+\cos\alpha)}{1-\cos^2\alpha} = \frac{\sin\alpha\cdot(1+\cos\alpha)}{\sin^2\alpha} = \frac{\sin\alpha\cdot(1+\cos\alpha)}{\sin^2\alpha} = \frac{1+\cos\alpha}{\sin\alpha}.$$

Vježba 406

Dokaži. $\frac{1-\cos\alpha}{\sin\alpha} = \frac{\sin\alpha}{1+\cos\alpha}$, $\sin\alpha \ne 0$.

Rezultat: Dokaz analogan.

Zadatak 407 (Iva, gimnazija)

Dokaži. $tg\,\alpha + ctg\,\alpha = \frac{ctg\,\alpha}{\cos^2\alpha}$, $\cos\alpha \ne 0$.

Rješenje 407

Ponovimo!

$$tg\,x = \frac{\sin x}{\cos x} \quad , \quad ctg\,x = \frac{\cos x}{\sin x} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a\cdot d + b\cdot c}{b\cdot d} \quad , \quad \cos^2\alpha + \sin^2\alpha = 1.$$

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad a \cdot \frac{1}{b} = \frac{a}{b}.$$

$$n = \frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b} = \frac{a \cdot n}{b \cdot n} \ , \ n \neq 0 \ , \ n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

$$tg\,\alpha + ctg\,\alpha = \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha} = \frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha \cdot \sin\alpha} = \frac{1}{\cos\alpha \cdot \sin\alpha} = \left[\begin{array}{l}\text{proširimo razlomak} \\ \text{s } \cos\alpha\end{array}\right] =$$

$$= \frac{1}{\cos\alpha \cdot \sin\alpha} \cdot \frac{\cos\alpha}{\cos\alpha} = \frac{\cos\alpha}{\cos^2\alpha \cdot \sin\alpha} = \frac{\cos\alpha}{\sin\alpha} \cdot \frac{1}{\cos^2\alpha} = ctg\,\alpha \cdot \frac{1}{\cos^2\alpha} = \frac{ctg\,\alpha}{\cos^2\alpha}.$$

2.inačica

$$tg\,\alpha + ctg\,\alpha = ctg\,\alpha \cdot \left(\frac{tg\,\alpha}{ctg\,\alpha} + 1\right) = ctg\,\alpha \cdot \left(\frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\cos\alpha}{\sin\alpha}} + 1\right) = ctg\,\alpha \cdot \left(\frac{\sin^2\alpha}{\cos^2\alpha} + 1\right) =$$

$$= ctg\,\alpha \cdot \left(\frac{\sin^2\alpha}{\cos^2\alpha} + \frac{1}{1}\right) = ctg\,\alpha \cdot \frac{\sin^2\alpha + \cos^2\alpha}{\cos^2\alpha} = ctg\,\alpha \cdot \frac{1}{\cos^2\alpha} = \frac{ctg\,\alpha}{\cos^2\alpha}.$$

Vježba 407

Dokaži. $ctg\,\alpha + tg\,\alpha = \dfrac{ctg\,\alpha}{\cos^2\alpha}, \ \cos\alpha \neq 0.$

Rezultat: Dokaz analogan.

Zadatak 408 (Miroslav, srednja škola)

Pojednostavni: $\dfrac{1+\sin\alpha}{\cos^2\alpha} - \dfrac{4-2\cdot\sin\alpha}{\cos\alpha} - \dfrac{2\cdot\sin\alpha - \cos\alpha - 1}{1-\sin\alpha} + \dfrac{3\cdot\cos\alpha}{1+\sin\alpha}.$

Rješenje 408

Ponovimo!

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

$$\cos^2 x + \sin^2 x = 1 \quad , \quad \frac{a}{n} - \frac{b}{n} = \frac{a-b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\frac{1+\sin\alpha}{\cos^2\alpha} - \frac{4-2\cdot\sin\alpha}{\cos\alpha} - \frac{2\cdot\sin\alpha-\cos\alpha-1}{1-\sin\alpha} + \frac{3\cdot\cos\alpha}{1+\sin\alpha} =$$

$$= \left(\frac{1+\sin\alpha}{\cos^2\alpha} - \frac{4-2\cdot\sin\alpha}{\cos\alpha}\right) - \left(\frac{2\cdot\sin\alpha-\cos\alpha-1}{1-\sin\alpha} - \frac{3\cdot\cos\alpha}{1+\sin\alpha}\right) =$$

$$= \frac{1+\sin\alpha-\cos\alpha\cdot(4-2\cdot\sin\alpha)}{\cos^2\alpha} - \frac{(1+\sin\alpha)\cdot(2\cdot\sin\alpha-\cos\alpha-1)-3\cdot\cos\alpha\cdot(1-\sin\alpha)}{(1-\sin\alpha)\cdot(1+\sin\alpha)} =$$

$$= \frac{1+\sin\alpha-4\cdot\cos\alpha+2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha} -$$

$$- \frac{2\cdot\sin\alpha-\cos\alpha-1+2\cdot\sin^2\alpha-\sin\alpha\cdot\cos\alpha-\sin\alpha-3\cdot\cos\alpha+3\cdot\sin\alpha\cdot\cos\alpha}{(1-\sin\alpha)\cdot(1+\sin\alpha)} =$$

$$= \frac{1+\sin\alpha-4\cdot\cos\alpha+2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha} - \frac{\sin\alpha-4\cdot\cos\alpha+2\cdot\sin^2\alpha+2\cdot\sin\alpha\cdot\cos\alpha-1}{1-\sin^2\alpha} =$$

$$= \frac{1+\sin\alpha-4\cdot\cos\alpha+2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha} - \frac{\sin\alpha-4\cdot\cos\alpha+2\cdot\sin^2\alpha+2\cdot\sin\alpha\cdot\cos\alpha-1}{\cos^2\alpha} =$$

$$= \frac{1+\sin\alpha-4\cdot\cos\alpha+2\cdot\sin\alpha\cdot\cos\alpha-\left(\sin\alpha-4\cdot\cos\alpha+2\cdot\sin^2\alpha+2\cdot\sin\alpha\cdot\cos\alpha-1\right)}{\cos^2\alpha} =$$

$$= \frac{1+\sin\alpha-4\cdot\cos\alpha+2\cdot\sin\alpha\cdot\cos\alpha-\sin\alpha+4\cdot\cos\alpha-2\cdot\sin^2\alpha-2\cdot\sin\alpha\cdot\cos\alpha+1}{\cos^2\alpha} =$$

$$= \frac{1+\sin\alpha-4\cdot\cos\alpha+2\cdot\sin\alpha\cdot\cos\alpha-\sin\alpha+4\cdot\cos\alpha-2\cdot\sin^2\alpha-2\cdot\sin\alpha\cdot\cos\alpha+1}{\cos^2\alpha} =$$

$$= \frac{1-2\cdot\sin^2\alpha+1}{\cos^2\alpha} = \frac{2-2\cdot\sin^2\alpha}{\cos^2\alpha} = \frac{2\cdot\left(1-\sin^2\alpha\right)}{\cos^2\alpha} = \frac{2\cdot\cos^2\alpha}{\cos^2\alpha} = \frac{2\cdot\cos^2\alpha}{\cos^2\alpha} = 2.$$

Vježba 408

Pojednostavni: $\frac{1+\cos^2\alpha}{\sin^2\alpha} + \frac{1+3\cdot\cos\alpha}{\sin\alpha} - \frac{2\cdot\sin\alpha+2\cdot\cos\alpha-1}{1-\cos\alpha} - \frac{1-\sin\alpha}{1+\cos\alpha}.$

Rezultat: 1.

Zadatak 409 (Rex, gimnazija)

Pokaži da se iz sustava $\sin x+\sin y=2\cdot a$, $\cos x+\cos y=2\cdot b$, $\cos(x-y)=-4\cdot a\cdot b$ eliminacijom x i y dobije $(a+b)^2=\frac{1}{2}$.

Rješenje 409

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \left.\begin{array}{l} a=b \\ c=d \end{array}\right\} \Rightarrow a+c=b+d.$$

$$\cos^2 x+\sin^2 x=1 \quad , \quad \cos(x-y)=\cos x\cdot\cos y+\sin x\cdot\sin y.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\left.\begin{array}{l} \sin x+\sin y=2\cdot a \\ \cos x+\cos y=2\cdot b \\ \cos(x-y)=-4\cdot a\cdot b \end{array}\right\} \Rightarrow \left.\begin{array}{l} \sin x+\sin y=2\cdot a \ /^2 \\ \cos x+\cos y=2\cdot b \ /^2 \\ \cos(x-y)=-4\cdot a\cdot b \ /\cdot(-2) \end{array}\right\} \Rightarrow \left.\begin{array}{l} (\sin x+\sin y)^2=4\cdot a^2 \\ (\cos x+\cos y)^2=4\cdot b^2 \\ -2\cdot\cos(x-y)=8\cdot a\cdot b \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{c} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow (\sin x+\sin y)^2+(\cos x+\cos y)^2-2\cdot\cos(x-y)=4\cdot a^2+4\cdot b^2+8\cdot a\cdot b \Rightarrow$$

$$\Rightarrow \sin^2 x+2\cdot\sin x\cdot\sin y+\sin^2 y+\cos^2 x+2\cdot\cos x\cdot\cos y+\cos^2 y-2\cdot\cos(x-y)=$$
$$=4\cdot a^2+4\cdot b^2+8\cdot a\cdot b \Rightarrow$$

$$\Rightarrow \sin^2 x+2\cdot\sin x\cdot\sin y+\sin^2 y+\cos^2 x+2\cdot\cos x\cdot\cos y+\cos^2 y-2\cdot\cos x\cdot\cos y-2\cdot\sin x\cdot\sin y=$$
$$=4\cdot a^2+4\cdot b^2+8\cdot a\cdot b \Rightarrow$$

$$\Rightarrow \sin^2 x+2\cdot\sin x\cdot\sin y+\sin^2 y+\cos^2 x+2\cdot\cos x\cdot\cos y+\cos^2 y-2\cdot\cos x\cdot\cos y-2\cdot\sin x\cdot\sin y=$$
$$=4\cdot a^2+4\cdot b^2+8\cdot a\cdot b \Rightarrow$$

$$\Rightarrow \sin^2 x+\sin^2 y+\cos^2 x+\cos^2 y=4\cdot a^2+4\cdot b^2+8\cdot a\cdot b \Rightarrow$$

$$\Rightarrow \left(\sin^2 x+\cos^2 x\right)+\left(\sin^2 y+\cos^2 y\right)=4\cdot\left(a^2+b^2+2\cdot a\cdot b\right) \Rightarrow$$

$$\Rightarrow 1+1=4\cdot\left(a^2+2\cdot a\cdot b+b^2\right) \Rightarrow 2=4\cdot(a+b)^2 \Rightarrow 4\cdot(a+b)^2=2 \Rightarrow$$

$$\Rightarrow 4\cdot(a+b)^2=2 \ /:4 \Rightarrow (a+b)^2=\frac{2}{4} \Rightarrow (a+b)^2=\frac{2}{4} \Rightarrow (a+b)^2=\frac{1}{2}.$$

Vježba 409

Pokaži da se iz sustava $\sin x + \sin y = 2 \cdot a$, $\cos x + \cos y = 2 \cdot b$, $\cos(x - y) = -4 \cdot a \cdot b$ eliminacijom x i y dobije $2 \cdot (a + b)^2 - 1 = 0.$

Rezultat: Dokaz analogan.

Zadatak 410 (Tihomir, gimnazija)

Pojednostavite $\sin^3 \alpha \cdot ctg\,\alpha + \cos^3 \alpha.$

Rješenje 410

Ponovimo!

$$ctg\,\alpha = \frac{\cos\alpha}{\sin\alpha} \quad , \quad tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} \quad , \quad \frac{a^n}{a^m} = a^{n-m} \quad , \quad \cos^2\alpha + \sin^2\alpha = 1.$$

$$a^1 = a \quad , \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$\sin^3\alpha \cdot ctg\,\alpha + \cos^3\alpha = \sin^3\alpha \cdot \frac{\cos\alpha}{\sin\alpha} + \cos^3\alpha = \sin^3\alpha \cdot \frac{\cos\alpha}{\sin\alpha} + \cos^3\alpha =$$

$$= \sin^2\alpha \cdot \cos\alpha + \cos^3\alpha = \cos\alpha \cdot \left(\sin^2\alpha + \cos^2\alpha\right) = \cos\alpha \cdot 1 = \cos\alpha.$$

Vježba 410

Pojednostavite $\sin^3\alpha + \cos^3\alpha \cdot tg\,\alpha.$

Rezultat: $\sin\alpha.$

Zadatak 411 (Željka, gimnazija)

Dokaži da ne ovisi od x izraz $\sin^4\alpha + \cos^4\alpha + 2 \cdot \sin^2\alpha \cdot \cos^4\alpha + 2 \cdot \sin^4\alpha \cdot \cos^2\alpha.$

Rješenje 411

Ponovimo!

$$\cos^2 x + \sin^2 x = 1 \quad , \quad \left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^2 + 2 \cdot a \cdot b + b^2 = (a + b)^2.$$

$$a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$\sin^4\alpha + \cos^4\alpha + 2 \cdot \sin^2\alpha \cdot \cos^4\alpha + 2 \cdot \sin^4\alpha \cdot \cos^2\alpha =$$

$$= \sin^4\alpha + \cos^4\alpha + 2 \cdot \sin^2\alpha \cdot \cos^2\alpha \cdot \left(\cos^2\alpha + \sin^2\alpha\right) =$$

$$= \sin^4\alpha + \cos^4\alpha + 2 \cdot \sin^2\alpha \cdot \cos^2\alpha \cdot 1 = \sin^4\alpha + \cos^4\alpha + 2 \cdot \sin^2\alpha \cdot \cos^2\alpha =$$

$$=\sin^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha=\left(\sin^2\alpha\right)^2+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\left(\cos^2\alpha\right)^2=$$

$$=\left(\sin^2\alpha+\cos^2\alpha\right)^2=1^2=1.$$

2.inačica

$$\sin^4\alpha+\cos^4\alpha+2\cdot\sin^2\alpha\cdot\cos^4\alpha+2\cdot\sin^4\alpha\cdot\cos^2\alpha=$$

$$=\sin^4\alpha+\cos^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha-2\cdot\sin^2\alpha\cdot\cos^2\alpha+2\cdot\sin^2\alpha\cdot\cos^4\alpha+2\cdot\sin^4\alpha\cdot\cos^2\alpha=$$

$$=\sin^4\alpha+\cos^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha-2\cdot\sin^2\alpha\cdot\cos^2\alpha+2\cdot\sin^2\alpha\cdot\cos^4\alpha+2\cdot\sin^4\alpha\cdot\cos^2\alpha=$$

$$=\left(\sin^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha\right)-2\cdot\sin^2\alpha\cdot\cos^2\alpha+2\cdot\sin^2\alpha\cdot\cos^4\alpha+2\cdot\sin^4\alpha\cdot\cos^2\alpha=$$

$$=\left(\left(\sin^2\alpha\right)^2+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\left(\cos^2\alpha\right)^2\right)-2\cdot\sin^2\alpha\cdot\cos^2\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha\cdot\left(\cos^2\alpha+\sin^2\alpha\right)=$$

$$=\left(\sin^2\alpha+\cos^2\alpha\right)^2-2\cdot\sin^2\alpha\cdot\cos^2\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha\cdot 1=$$

$$=1^2-2\cdot\sin^2\alpha\cdot\cos^2\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha=1-2\cdot\sin^2\alpha\cdot\cos^2\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha=1.$$

Vježba 411

Dokaži da ne ovisi od x izraz $\sin^4\alpha+\cos^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha.$

Rezultat: Dokaz analogan.

Zadatak 412 (Vlado, gimnazija)

Ako je $\alpha+\beta=60^\circ$ i $tg\,\alpha=\frac{\sqrt{3}}{2}$, onda je $tg\,\beta$:

$A.\ \frac{\sqrt{3}}{5}$ $B.\ \frac{\sqrt{3}}{3}$ $C.\ \frac{\sqrt{3}}{2}$ $D.\ \sqrt{3}$

Rješenje 412

Ponovimo!

$$tg\left(x+y\right)=\frac{tg\,x+tg\,y}{1-tg\,x\cdot tg\,y}\quad,\quad tg\,60^\circ=\sqrt{3}\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}.$$

$$\left(\sqrt{a}\right)^2=a\quad,\quad \frac{a}{b}\cdot b=a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$tg\left(\alpha+\beta\right)=\frac{tg\,\alpha+tg\,\beta}{1-tg\,\alpha\cdot tg\,\beta}\Rightarrow\left[\begin{array}{l}\alpha+\beta=60^\circ\\ tg\,\alpha=\frac{\sqrt{3}}{2}\end{array}\right]\Rightarrow tg\,60^\circ=\frac{\frac{\sqrt{3}}{2}+tg\,\beta}{1-\frac{\sqrt{3}}{2}\cdot tg\,\beta}\Rightarrow\sqrt{3}=\frac{\frac{\sqrt{3}}{2}+tg\,\beta}{1-\frac{\sqrt{3}}{2}\cdot tg\,\beta}\Rightarrow$$

$$\Rightarrow \sqrt{3}=\frac{\frac{\sqrt{3}}{2}+tg\,\beta}{1-\frac{\sqrt{3}}{2}\cdot tg\,\beta} \;/\cdot\left(1-\frac{\sqrt{3}}{2}\cdot tg\,\beta\right) \Rightarrow \sqrt{3}\cdot\left(1-\frac{\sqrt{3}}{2}\cdot tg\,\beta\right)=\frac{\sqrt{3}}{2}+tg\,\beta \Rightarrow$$

$$\Rightarrow \sqrt{3}-\frac{\left(\sqrt{3}\right)^2}{2}\cdot tg\,\beta=\frac{\sqrt{3}}{2}+tg\,\beta \Rightarrow \sqrt{3}-\frac{3}{2}\cdot tg\,\beta=\frac{\sqrt{3}}{2}+tg\,\beta \Rightarrow$$

$$\Rightarrow \sqrt{3}-\frac{3}{2}\cdot tg\,\beta=\frac{\sqrt{3}}{2}+tg\,\beta \;/\cdot 2 \Rightarrow 2\cdot\sqrt{3}-3\cdot tg\,\beta=\sqrt{3}+2\cdot tg\,\beta \Rightarrow$$

$$\Rightarrow -3\cdot tg\,\beta-2\cdot tg\,\beta=\sqrt{3}-2\cdot\sqrt{3} \Rightarrow -5\cdot tg\,\beta=-\sqrt{3} \Rightarrow$$

$$\Rightarrow -5\cdot tg\,\beta=-\sqrt{3} \;/:(-5) \Rightarrow tg\,\beta=\frac{\sqrt{3}}{5}.$$

Odgovor je pod A.

Vježba 412

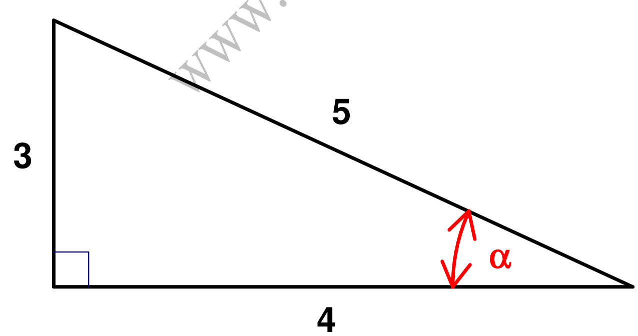
Ako je $\alpha+\beta=60^\circ$ i $tg\,\beta=\frac{\sqrt{3}}{2}$, onda je $tg\,\alpha$:

$A.\ \frac{\sqrt{3}}{5}$ $\quad B.\ \frac{\sqrt{3}}{3}$ $\quad C.\ \frac{\sqrt{3}}{2}$ $\quad D.\ \sqrt{3}$

Rezultat: A.

Zadatak 413 (MR18, gimnazija)

Koliko iznosi sin (2 · α) sa slike?



$A.\ \frac{24}{25}$ $\quad B.\ \frac{3}{4}$ $\quad C.\ \frac{4}{5}$ $\quad D.\ \frac{3}{25}$

Rješenje 413

Ponovimo!

$$\sin(2\cdot\alpha)=2\cdot\sin\alpha\cdot\cos\alpha \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Trokut je dio ravnine omeđen s tri dužine. Te dužine zovemo stranice trokuta.
Pravokutni trokuti imaju jedan pravi kut (kut od 90º). Stranice koje zatvaraju pravi kut zovu se katete, a najdulja stranica je hipotenuza pravokutnog trokuta.
Sinus šiljastog kuta pravokutnog trokuta jednak je omjeru duljine katete nasuprot tog kuta i duljine hipotenuze.
Kosinus šiljastog kuta pravokutnog trokuta jednak je omjeru duljine katete uz taj kut i duljine hipotenuze.

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \neq 0 \; , \; n \neq 1.$$

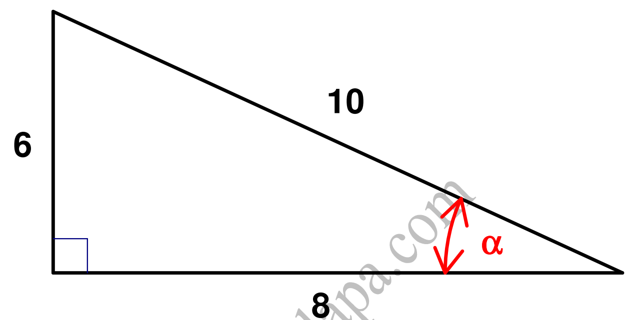
$$\sin(2 \cdot \alpha) = 2 \cdot \sin\alpha \cdot \cos\alpha \; \Rightarrow \; \begin{bmatrix} \sin\alpha = \frac{3}{5} \\ \cos\alpha = \frac{4}{5} \end{bmatrix} \; \Rightarrow \; \sin(2 \cdot \alpha) = 2 \cdot \frac{3}{5} \cdot \frac{4}{5} \; \Rightarrow \; \sin(2 \cdot \alpha) = \frac{2}{1} \cdot \frac{3}{5} \cdot \frac{4}{5} \; \Rightarrow$$

$$\Rightarrow \; \sin(2 \cdot \alpha) = \frac{24}{25}.$$

Odgovor je pod A.

Vježba 413

Koliko iznosi sin (2 · α) sa slike?



$A. \; \frac{24}{25} \qquad B. \; \frac{3}{4} \qquad C. \; \frac{4}{5} \qquad D. \; \frac{3}{25}$

Rezultat: A.

Zadatak 414 (MR18, gimnazija)

Nađite vrijednosti sin α, cos α, tg α i ctg α za α = 15° bez uporabe džepnog računala.

Rješenje 414

Ponovimo!

$$\sin\alpha = \sqrt{\frac{1-\cos(2 \cdot \alpha)}{2}} \quad , \quad \cos\alpha = \sqrt{\frac{1+\cos(2 \cdot \alpha)}{2}} \; \text{ za } 0^\circ < \alpha < 90^\circ.$$

$$tg\,\alpha = \sqrt{\frac{1-\cos(2 \cdot \alpha)}{1+\cos(2 \cdot \alpha)}} \quad , \quad ctg\,\alpha = \sqrt{\frac{1+\cos(2 \cdot \alpha)}{1-\cos(2 \cdot \alpha)}} \; \text{ za } 0^\circ < \alpha < 90^\circ.$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \quad , \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad , \quad (a-b) \cdot (a+b) = a^2 - b^2 \quad , \quad \sqrt{a^2} = a \; , \; a \geq 0.$$

$$tg\,x = \frac{\sin x}{\cos x} \quad , \quad ctg\,x = \frac{\cos x}{\sin x} \quad , \quad ctg\,x = \frac{1}{tg\,x} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Računamo sin 15º.

$$\sin 15^\circ=\sqrt{\frac{1-\cos\left(2\cdot 15^\circ\right)}{2}}\Rightarrow \sin 15^\circ=\sqrt{\frac{1-\cos 30^\circ}{2}}\Rightarrow \sin 15^\circ=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}\Rightarrow$$

$$\Rightarrow \sin 15^\circ=\sqrt{\frac{\frac{1}{1}-\frac{\sqrt{3}}{2}}{2}}\Rightarrow \sin 15^\circ=\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2}{1}}}\Rightarrow \sin 15^\circ=\sqrt{\frac{2-\sqrt{3}}{4}}\Rightarrow$$

$$\Rightarrow \sin 15^\circ=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}}\Rightarrow \sin 15^\circ=\frac{\sqrt{2-\sqrt{3}}}{2}.$$

Računamo cos 15º.

$$\cos 15^\circ=\sqrt{\frac{1+\cos\left(2\cdot 15^\circ\right)}{2}}\Rightarrow \cos 15^\circ=\sqrt{\frac{1+\cos 30^\circ}{2}}\Rightarrow \cos 15^\circ=\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}\Rightarrow$$

$$\Rightarrow \cos 15^\circ=\sqrt{\frac{\frac{1}{1}+\frac{\sqrt{3}}{2}}{2}}\Rightarrow \cos 15^\circ=\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}}\Rightarrow \cos 15^\circ=\sqrt{\frac{2+\sqrt{3}}{4}}\Rightarrow$$

$$\Rightarrow \cos 15^\circ=\frac{\sqrt{2+\sqrt{3}}}{\sqrt{4}}\Rightarrow \cos 15^\circ=\frac{\sqrt{2+\sqrt{3}}}{2}.$$

Računamo tg 15º.
1.inačica

$$tg\,15^\circ=\sqrt{\frac{1-\cos\left(2\cdot 15^\circ\right)}{1+\cos\left(2\cdot 15^\circ\right)}}\Rightarrow tg\,15^\circ=\sqrt{\frac{1-\cos 30^\circ}{1+\cos 30^\circ}}\Rightarrow tg\,15^\circ=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}\Rightarrow$$

$$\Rightarrow tg\,15^\circ=\sqrt{\frac{\frac{1}{1}-\frac{\sqrt{3}}{2}}{\frac{1}{1}+\frac{\sqrt{3}}{2}}}\Rightarrow tg\,15^\circ=\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}}\Rightarrow tg\,15^\circ=\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2+\sqrt{3}}{2}}}\Rightarrow tg\,15^\circ=\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\Rightarrow$$

$$\Rightarrow \left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]\Rightarrow tg\,15^\circ=\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}}\Rightarrow tg\,15^\circ=\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{2^2-\left(\sqrt{3}\right)^2}}\Rightarrow$$

$$\Rightarrow tg\,15^\circ=\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{4-3}}\Rightarrow tg\,15^\circ=\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{1}}\Rightarrow tg\,15^\circ=\sqrt{\left(2-\sqrt{3}\right)^2}\Rightarrow tg\,15^\circ=2-\sqrt{3}.$$

2.inačica

$$tg\,15^\circ=\frac{\sin 15^\circ}{\cos 15^\circ}\Rightarrow\left[\begin{array}{l}\sin 15^\circ=\frac{\sqrt{2-\sqrt{3}}}{2}\\ \cos 15^\circ=\frac{\sqrt{2+\sqrt{3}}}{2}\end{array}\right]\Rightarrow tg\,15^\circ=\frac{\frac{\sqrt{2-\sqrt{3}}}{2}}{\frac{\sqrt{2+\sqrt{3}}}{2}}\Rightarrow$$

$$\Rightarrow tg\,15^\circ=\frac{\frac{\sqrt{2-\sqrt{3}}}{2}}{\frac{\sqrt{2+\sqrt{3}}}{2}}\Rightarrow tg\,15^\circ=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{2+\sqrt{3}}}\Rightarrow tg\,15^\circ=\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}\Rightarrow\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]\Rightarrow$$

$$\Rightarrow tg\,15^\circ=\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}\cdot\frac{2-\sqrt{3}}{2-\sqrt{3}}}\Rightarrow tg\,15^\circ=\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{2^2-\left(\sqrt{3}\right)^2}}\Rightarrow$$

$$\Rightarrow tg\,15^\circ=\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{4-3}}\Rightarrow tg\,15^\circ=\sqrt{\frac{\left(2-\sqrt{3}\right)^2}{1}}\Rightarrow tg\,15^\circ=\sqrt{\left(2-\sqrt{3}\right)^2}\Rightarrow tg\,15^\circ=2-\sqrt{3}.$$

Računamo ctg 15º.

1.inačica

$$ctg\,15^\circ=\sqrt{\frac{1+\cos\left(2\cdot 15^\circ\right)}{1-\cos\left(2\cdot 15^\circ\right)}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{1+\cos 30^\circ}{1-\cos 30^\circ}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}}\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\sqrt{\frac{\frac{1}{1}+\frac{\sqrt{3}}{2}}{\frac{1}{1}-\frac{\sqrt{3}}{2}}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2-\sqrt{3}}{2}}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2-\sqrt{3}}{2}}}\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\Rightarrow\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]\Rightarrow ctg\,15^\circ=\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}}\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\sqrt{\frac{\left(2+\sqrt{3}\right)^2}{2^2-\left(\sqrt{3}\right)^2}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{\left(2+\sqrt{3}\right)^2}{4-3}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{\left(2+\sqrt{3}\right)^2}{1}}\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\sqrt{\left(2+\sqrt{3}\right)^2}\Rightarrow ctg\,15^\circ=2+\sqrt{3}.$$

2.inačica

$$ctg\,15^\circ=\frac{\cos 15^\circ}{\sin 15^\circ}\Rightarrow\left[\begin{array}{l}\cos 15^\circ=\frac{\sqrt{2+\sqrt{3}}}{2}\\ \sin 15^\circ=\frac{\sqrt{2-\sqrt{3}}}{2}\end{array}\right]\Rightarrow ctg\,15^\circ=\frac{\frac{\sqrt{2+\sqrt{3}}}{2}}{\frac{\sqrt{2-\sqrt{3}}}{2}}\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\frac{\frac{\sqrt{2+\sqrt{3}}}{2}}{\frac{\sqrt{2-\sqrt{3}}}{2}}\Rightarrow ctg\,15^\circ=\frac{\sqrt{2+\sqrt{3}}}{\sqrt{2-\sqrt{3}}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}\Rightarrow\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{\left(2+\sqrt{3}\right)^2}{2^2-\left(\sqrt{3}\right)^2}}\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\sqrt{\frac{\left(2+\sqrt{3}\right)^2}{4-3}}\Rightarrow ctg\,15^\circ=\sqrt{\frac{\left(2+\sqrt{3}\right)^2}{1}}\Rightarrow ctg\,15^\circ=\sqrt{\left(2+\sqrt{3}\right)^2}\Rightarrow ctg\,15^\circ=2+\sqrt{3}.$$

3.inačica

$$ctg\,15^\circ=\frac{1}{tg\,15^\circ}\Rightarrow\left[\,tg\,15^\circ=2-\sqrt{3}\,\right]\Rightarrow ctg\,15^\circ=\frac{1}{2-\sqrt{3}}\Rightarrow\left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right]\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\frac{1}{2-\sqrt{3}}\cdot\frac{2+\sqrt{3}}{2+\sqrt{3}}\Rightarrow ctg\,15^\circ=\frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}\Rightarrow ctg\,15^\circ=\frac{2+\sqrt{3}}{4-3}\Rightarrow$$

$$\Rightarrow ctg\,15^\circ=\frac{2+\sqrt{3}}{1}\Rightarrow ctg\,15^\circ=2+\sqrt{3}.$$

Vježba 414

Nađite vrijednosti sin α, cos α, tg α i ctg α za α = 22º 30' bez uporabe džepnog računala.

Rezultat: $\sin 22^\circ 30'=\frac{\sqrt{2-\sqrt{2}}}{2}$, $\cos 22^\circ 30'=\frac{\sqrt{2+\sqrt{2}}}{2}$

$tg\,22^\circ 30'=\sqrt{2}-1$, $ctg\,22^\circ 30'=\sqrt{2}+1.$

Zadatak 415 (Vinko, srednja škola)

Izraz $\frac{2\cdot\cos^2 x+\sin\left(2\cdot x\right)}{2\cdot\sin^2 x+\sin\left(2\cdot x\right)}$ jednak je:

A. $\sin x$ *B.* $\cos x$ *C.* $tg\,x$ *D.* $ctg\,x$

Rješenje 415

Ponovimo!

$$\sin\left(2\cdot\alpha\right)=2\cdot\sin\alpha\cdot\cos\alpha\quad,\quad a^1=a\quad,\quad a^n:a^m=a^{n-m}.$$

$$tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}\quad,\quad ctg\,\alpha=\frac{\cos\alpha}{\sin\alpha}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{2\cdot\cos^2 x+\sin(2\cdot x)}{2\cdot\sin^2 x+\sin(2\cdot x)}=\frac{2\cdot\cos^2 x+2\cdot\sin x\cdot\cos x}{2\cdot\sin^2 x+2\cdot\sin x\cdot\cos x}=\frac{2\cdot\cos x\cdot(\cos x+\sin x)}{2\cdot\sin x\cdot(\sin x+\cos x)}=$$

$$=\frac{2\cdot\cos x\cdot(\cos x+\sin x)}{2\cdot\sin x\cdot(\sin x+\cos x)}=\frac{\cos x}{\sin x}=ctg\ x.$$

Odgovor je pod D.

Vježba 415

Izraz $\frac{2\cdot\sin^2 x+\sin(2\cdot x)}{2\cdot\cos^2 x+\sin(2\cdot x)}$ jednak je:

A. $\sin x$ *B.* $\cos x$ *C.* $tg\ x$ *D.* $ctg\ x$

Rezultat: C.

Zadatak 416 (Ivona, gimnazija)

Dokažite: $\frac{\sin(3\cdot x)}{\cos x}+\frac{\cos(3\cdot x)}{\sin x}=2\cdot ctg(2\cdot x).$

Rješenje 416

Ponovimo!

$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}$, $\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta$, $2\cdot\sin\alpha\cdot\cos\alpha=\sin(2\cdot\alpha).$

$$ctg\ \alpha=\frac{\cos\alpha}{\sin\alpha}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{\sin(3\cdot x)}{\cos x}+\frac{\cos(3\cdot x)}{\sin x}=\frac{\sin(3\cdot x)\cdot\sin x+\cos(3\cdot x)\cdot\cos x}{\cos x\cdot\sin x}=$$

$$=\frac{\cos(3\cdot x)\cdot\cos x+\sin(3\cdot x)\cdot\sin x}{\sin x\cdot\cos x}=\frac{\cos(3\cdot x-x)}{\sin x\cdot\cos x}=\frac{\cos(2\cdot x)}{\sin x\cdot\cos x}=\left[\begin{array}{l}\text{proširiti razlomak}\\ \text{brojem } 2\end{array}\right]=$$

$$=\frac{2\cdot\cos(2\cdot x)}{2\cdot\sin x\cdot\cos x}=\frac{2\cdot\cos(2\cdot x)}{\sin(2\cdot x)}=2\cdot\frac{\cos(2\cdot x)}{\sin(2\cdot x)}=2\cdot ctg(2\cdot x).$$

Vježba 416

Dokažite: $\frac{\cos(3\cdot x)}{\sin x}+\frac{\sin(3\cdot x)}{\cos x}=2\cdot ctg(2\cdot x).$

Rezultat: Dokaz analogan.

Zadatak 417 (Ivona, gimnazija)

Ako je $\sin\alpha=0.8,\ \frac{\pi}{2}<\alpha<\pi,\ \cos\beta=\frac{7}{25},\ \frac{3\cdot\pi}{2}<\beta<2\cdot\pi,$ koliko je $tg(\alpha+\beta)$?

Rješenje 417

Ponovimo!

$$\cos^2 x+\sin^2 x=1 \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad tg\, x=\frac{\sin x}{\cos x}.$$

$$\sin(x+y)=\sin x\cdot\cos y+\cos x\cdot\sin y \quad , \quad \cos(x+y)=\cos x\cdot\cos y-\sin x\cdot\sin y.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Decimalni broj piše se u obliku decimalnog razlomka tako da se u brojnik napiše zadani decimalni broj bez decimalne točke, a u nazivnik se napiše dekadska jedinica (10, 100, 1000, 10000, 100000, …) koja ima toliko nula koliko decimalni broj ima decimala (znamenaka na decimalnom mjestu, tj. iza decimalne točke ili decimalnog zareza).

	I. kvadrant $\left\langle 0, \frac{\pi}{2}\right\rangle$	**II. kvadrant** $\left\langle \frac{\pi}{2}, \pi\right\rangle$	**III. kvadrant** $\left\langle \pi, \frac{3\cdot\pi}{2}\right\rangle$	**IV. kvadrant** $\left\langle \frac{3\cdot\pi}{2}, 2\cdot\pi\right\rangle$
sin	+	+	—	—
cos	+	—	—	+
tg	+	—	+	—
ctg	+	—	+	—

Najprije izračunamo sin β i cos α.

$$\left.\begin{array}{l}\cos^2\alpha+\sin^2\alpha=1\\ \cos^2\beta+\sin^2\beta=1\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos^2\alpha+0.8^2=1\\ \left(\frac{7}{25}\right)^2+\sin^2\beta=1\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos^2\alpha+0.64=1\\ \frac{49}{625}+\sin^2\beta=1\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\cos^2\alpha=1-0.64\\ \sin^2\beta=1-\frac{49}{625}\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos^2\alpha=0.36\\ \sin^2\beta=\frac{1}{1}-\frac{49}{625}\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos^2\alpha=\frac{36}{100}\\ \sin^2\beta=\frac{625-49}{625}\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos^2\alpha=\frac{36}{100}\\ \sin^2\beta=\frac{576}{625}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\cos^2\alpha=\frac{9}{25}\\ \sin^2\beta=\frac{576}{625}\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos^2\alpha=\frac{9}{25}\ /\sqrt{}\\ \sin^2\beta=\frac{576}{625}\ /\sqrt{}\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos\alpha=\pm\sqrt{\frac{9}{25}}\\ \sin\beta=\pm\sqrt{\frac{576}{625}}\end{array}\right\} \Rightarrow \left.\begin{array}{l}\cos\alpha=\pm\frac{3}{5}\\ \sin\beta=\pm\frac{24}{25}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l}\frac{\pi}{2}<\alpha<\pi\\ \frac{3\cdot\pi}{2}<\beta<2\cdot\pi\end{array}\right] \Rightarrow \left.\begin{array}{l}\cos\alpha=-\frac{3}{5}\\ \sin\beta=-\frac{24}{25}\end{array}\right\}.$$

Sada računamo tg (α + β).

$$tg(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} \Rightarrow tg(\alpha+\beta)=\frac{\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta}{\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \sin\alpha = 0.8 \ \ , \ \cos\beta = \frac{7}{25} \\ \cos\alpha = -\frac{3}{5} \ \ , \ \sin\beta = -\frac{24}{25} \end{array}\right] \Rightarrow tg(\alpha+\beta) = \frac{0.8\cdot\frac{7}{25}+\left(-\frac{3}{5}\right)\cdot\left(-\frac{24}{25}\right)}{-\frac{3}{5}\cdot\frac{7}{25}-0.8\cdot\left(-\frac{24}{25}\right)} \Rightarrow$$

$$\Rightarrow tg(\alpha+\beta) = \frac{\frac{8}{10}\cdot\frac{7}{25}+\frac{3}{5}\cdot\frac{24}{25}}{-\frac{3}{5}\cdot\frac{7}{25}+\frac{8}{10}\cdot\frac{24}{25}} \Rightarrow tg(\alpha+\beta) = \frac{\frac{8}{10}\cdot\frac{7}{25}+\frac{3}{5}\cdot\frac{24}{25}}{-\frac{3}{5}\cdot\frac{7}{25}+\frac{8}{10}\cdot\frac{24}{25}} \Rightarrow$$

$$\Rightarrow tg(\alpha+\beta) = \frac{\frac{4}{5}\cdot\frac{7}{25}+\frac{3}{5}\cdot\frac{24}{25}}{-\frac{3}{5}\cdot\frac{7}{25}+\frac{4}{5}\cdot\frac{24}{25}} \Rightarrow tg(\alpha+\beta) = \frac{\frac{28}{125}+\frac{72}{125}}{-\frac{21}{125}+\frac{96}{125}} \Rightarrow tg(\alpha+\beta) = \frac{\frac{28+72}{125}}{\frac{-21+96}{125}} \Rightarrow$$

$$\Rightarrow tg(\alpha+\beta) = \frac{\frac{100}{125}}{\frac{75}{125}} \Rightarrow tg(\alpha+\beta) = \frac{\frac{100}{125}}{\frac{75}{125}} \Rightarrow tg(\alpha+\beta) = \frac{\frac{100}{1}}{\frac{75}{1}} \Rightarrow tg(\alpha+\beta) = \frac{100}{75} \Rightarrow$$

$$\Rightarrow tg(\alpha+\beta) = \frac{100}{75} \Rightarrow tg(\alpha+\beta) = \frac{4}{3}.$$

Vježba 417

Ako je $\cos\alpha = -0.6$, $\frac{\pi}{2} < \alpha < \pi$, $\cos\beta = \frac{7}{25}$, $\frac{3\cdot\pi}{2} < \beta < 2\cdot\pi$, koliko je $tg(\alpha+\beta)$?

Rezultat: $tg(\alpha+\beta) = \frac{4}{3}.$

Zadatak 418 (Matej, gimnazija)

Ako je $x + y = \frac{3\cdot\pi}{4}$, koliko je $(1+ctg\ x)\cdot(1+ctg\ y)$?

Rješenje 418

Ponovimo!

$$ctg(\alpha+\beta) = \frac{ctg\ \alpha\cdot ctg\ \beta - 1}{ctg\ \alpha + ctg\ \beta} \quad , \quad ctg\left(\frac{3\cdot\pi}{4}\right) = -1 \quad , \quad ctg\left(\frac{7\cdot\pi}{4}\right) = -1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d) = a\cdot c + a\cdot d + b\cdot c + b\cdot d.$$

$$ctg(x+y) = \frac{ctg\ x\cdot ctg\ y - 1}{ctg\ x + ctg\ y} \Rightarrow \frac{ctg\ x\cdot ctg\ y - 1}{ctg\ x + ctg\ y} = ctg(x+y) \Rightarrow \left[\ x + y = \frac{3\cdot\pi}{4}\ \right] \Rightarrow$$

$$\Rightarrow \frac{ctg\ x\cdot ctg\ y - 1}{ctg\ x + ctg\ y} = ctg\left(\frac{3\cdot\pi}{4}\right) \Rightarrow \frac{ctg\ x\cdot ctg\ y - 1}{ctg\ x + ctg\ y} = -1 \Rightarrow$$

$$\Rightarrow \frac{ctg\ x\cdot ctg\ y - 1}{ctg\ x + ctg\ y} = -1 \ /\cdot\ (ctg\ x + ctg\ y) \Rightarrow ctg\ x\cdot ctg\ y - 1 = -1\cdot(ctg\ x + ctg\ y) \Rightarrow$$

$$\Rightarrow ctg\ x\cdot ctg\ y - 1 = -ctg\ x - ctg\ y \Rightarrow ctg\ x\cdot ctg\ y + ctg\ x + ctg\ y = 1.$$

Sada možemo zadani izraz izračunati na dva načina.

1.inačica

$$(1+ctg\,x)\cdot(1+ctg\,y)=1+ctg\,y+ctg\,x+ctg\,x\cdot ctg\,y=$$

$$=\left[\,ctg\,x\cdot ctg\,y+ctg\,x+ctg\,y=1\,\right]=1+1=2.$$

2.inačica

$$ctg\,x\cdot ctg\,y+ctg\,x+ctg\,y=1\;\Rightarrow\;ctg\,x\cdot ctg\,y+ctg\,x+ctg\,y=1\;/+1\;\Rightarrow$$

$$\Rightarrow\;ctg\,x\cdot ctg\,y+ctg\,x+ctg\,y+1=1+1\;\Rightarrow\;ctg\,x\cdot ctg\,y+ctg\,x+ctg\,y+1=2\;\Rightarrow$$

$$\Rightarrow\;(ctg\,x\cdot ctg\,y+ctg\,x)+(ctg\,y+1)=2\;\Rightarrow\;ctg\,x\cdot(ctg\,y+1)+(ctg\,y+1)=2\;\Rightarrow$$

$$\Rightarrow\;(ctg\,y+1)\cdot(ctg\,x+1)=2\;\Rightarrow\;(1+ctg\,x)\cdot(1+ctg\,y)=2.$$

Vježba 418

Ako je $x+y=\frac{7\cdot\pi}{4}$, koliko je $(1+ctg\,x)\cdot(1+ctg\,y)$?

Rezultat: 2.

Zadatak 419 (Ivan, gimnazija)

Dokaži identičnost $\dfrac{tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cdot(1+\sin\alpha)}{\sin\alpha}=ctg\,\alpha.$

Rješenje 419

Ponovimo!

$$tg\,\alpha=\frac{\sin(2\cdot\alpha)}{1+\cos(2\cdot\alpha)}\quad,\quad\sin\left(\frac{\pi}{2}-\alpha\right)=\cos\alpha\quad,\quad\cos\left(\frac{\pi}{2}-\alpha\right)=\sin\alpha\quad,\quad ctg\,\alpha=\frac{\cos\alpha}{\sin\alpha}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\;,\;n\neq 0\;,\;n\neq 1.$$

$$\frac{tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cdot(1+\sin\alpha)}{\sin\alpha}=\frac{\dfrac{\sin\left(2\cdot\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right)}{1+\cos\left(2\cdot\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right)}\cdot(1+\sin\alpha)}{\sin\alpha}=$$

$$=\frac{\dfrac{\sin\left(\frac{\pi}{2}-\alpha\right)}{1+\cos\left(\frac{\pi}{2}-\alpha\right)}\cdot(1+\sin\alpha)}{\sin\alpha}=\frac{\dfrac{\cos\alpha}{1+\sin\alpha}\cdot(1+\sin\alpha)}{\sin\alpha}=\frac{\dfrac{\cos\alpha}{1+\sin\alpha}\cdot(1+\sin\alpha)}{\sin\alpha}=\frac{\cos\alpha}{\sin\alpha}=ctg\,\alpha.$$

Vježba 419

Dokaži identičnost $\dfrac{tg\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\cdot(1+\sin\alpha)}{\cos\alpha}=1.$

Rezultat: Dokaz analogan.

Zadatak 420 (Božidar, srednja škola)

Odredite $x \in \langle 0, \pi \rangle$ za koji su $\frac{1}{tg\,x}, \frac{1}{\sin x}, tg\,x$ uzastopni članovi aritmetičkog niza.

Rješenje 420

Ponovimo!

$$tg\,x = \frac{\sin x}{\cos x} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \quad , \quad a \cdot \frac{b}{c} = \frac{a \cdot b}{c} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$\cos^2 x + \sin^2 x = 1 \quad , \quad \frac{a}{b} = \frac{c}{d} \Rightarrow a \cdot d = b \cdot c.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a \cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Trigonometrijska jednadžba cos x = a

Jednadžba ima rješenje ako i samo ako je

$$-1 \le a \le 1.$$

Tada postoji jedinstven kut α u intervalu

$$-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$$

čiji je kosinus jednak a pa postoji jednadžba

$$\cos x = \cos \alpha$$

koja ima dva skupa rješenja:

$$\left. \begin{array}{l} x_1 = \alpha + k \cdot 2 \cdot \pi \ , \ k \in Z \\ x_2 = -\alpha + k \cdot 2 \cdot \pi \ , \ k \in Z \end{array} \right\} \text{ ili } x_{1,2} = \pm \alpha + k \cdot 2 \cdot \pi \ , \ k \in Z.$$

Trigonometrijska jednadžba $\sin x = a \ , \ |a| \le 1$

Skup rješenja jednadžbe $\sin x = a, |a| \le 1,$ je $\{ x_0 + k \cdot 2 \cdot \pi : k \in Z \} \cup \{ \pi - x_0 + k \cdot 2 \cdot \pi : k \in Z \}$

gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$\sin x = \sin x_0 \Rightarrow x = x_0.$$

Niz (slijed) je aritmetički ako je razlika svakog člana niza (osim prvog) i člana ispred njega stalna i iznosi d.

$$a_2 - a_1 = d \ , \ a_3 - a_2 = d \ , \ a_4 - a_3 = d \ , \ a_5 - a_4 = d \ , \ a_6 - a_5 = d \ , \ \ldots \ , \ a_n - a_{n-1} = d \ \ldots$$
$$a_n - a_{n-1} = d \ , \ n \ge 2.$$

Broj d naziva se razlika (diferencija) aritmetičkog niza. Aritmetički niz je jednoznačno određen ako znamo prvi član a_1 i razliku d.

Svaki član aritmetičkog niza (osim prvog) jednak je aritmetičkoj sredini dvaju susjednih članova niza (prethodnika i sljedbenika)

$$a_n = \frac{a_{n-1} + a_{n+1}}{2} \Rightarrow 2 \cdot a_n = a_{n-1} + a_{n+1}.$$

Budući da su $\frac{1}{tg\ x}, \frac{1}{\sin x}, tg\ x$ uzastopni članovi aritmetičkog niza, slijedi:

$$2 \cdot \frac{1}{\sin x} = \frac{1}{tg\ x} + tg\ x \Rightarrow \frac{2}{\sin x} = \frac{1}{\frac{\sin x}{\cos x}} + \frac{\sin x}{\cos x} \Rightarrow \frac{2}{\sin x} = \frac{\frac{1}{1}}{\frac{\sin x}{\cos x}} + \frac{\sin x}{\cos x} \Rightarrow$$

$$\Rightarrow \frac{2}{\sin x} = \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \Rightarrow \frac{2}{\sin x} = \frac{\cos^2 x + \sin^2 x}{\sin x \cdot \cos x} \Rightarrow \frac{2}{\sin x} = \frac{1}{\sin x \cdot \cos x} \Rightarrow$$

$$\Rightarrow 2 \cdot \sin x \cdot \cos x = \sin x \Rightarrow 2 \cdot \sin x \cdot \cos x - \sin x = 0 \Rightarrow \sin x \cdot (2 \cdot \cos x - 1) = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} \sin x = 0 \\ 2 \cdot \cos x - 1 = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x = \sin^{-1} 0 \\ 2 \cdot \cos x = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x = k \cdot \pi, \ \ k \in Z \ \text{nema smisla} \\ 2 \cdot \cos x = 1 \ /:2 \end{array}\right\} \Rightarrow \cos x = \frac{1}{2} \Rightarrow$$

$$\Rightarrow x = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow x = \frac{\pi}{3}.$$

Vježba 420

Odredite $x \in \left\langle 0^\circ, 180^\circ \right\rangle$ za koji su $\frac{1}{tg\ x}, \frac{1}{\sin x}, tg\ x$ uzastopni članovi aritmetičkog niza.

Rezultat: 60º.