Zadatak 381 (Anita, gimnazija)

Pojednostavni razlomak $\frac{(1-\sin x-\cos x)\cdot(1-\sin x+\cos x)}{\sin x\cdot(1-\sin x)}$.

Rješenje 381

Ponovimo!

$$(a-b)\cdot(a+b)=a^2-b^2 \quad , \quad (a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad \cos^2 x+\sin^2 x=1.$$

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

1.inačica

$$\frac{(1-\sin x-\cos x)\cdot(1-\sin x+\cos x)}{\sin x\cdot(1-\sin x)}=\frac{((1-\sin x)-\cos x)\cdot((1-\sin x)+\cos x)}{\sin x\cdot(1-\sin x)}=$$

$$=\frac{(1-\sin x)^2-\cos^2 x}{\sin x\cdot(1-\sin x)}=\frac{1-2\cdot\sin x+\sin^2 x-\cos^2 x}{\sin x\cdot(1-\sin x)}=\frac{1-\cos^2 x-2\cdot\sin x+\sin^2 x}{\sin x\cdot(1-\sin x)}=$$

$$=\frac{\sin^2 x-2\cdot\sin x+\sin^2 x}{\sin x\cdot(1-\sin x)}=\frac{-2\cdot\sin x+2\cdot\sin^2 x}{\sin x\cdot(1-\sin x)}=\frac{-2\cdot\sin x\cdot(1-\sin x)}{\sin x\cdot(1-\sin x)}=$$

$$=\frac{-2\cdot\sin x\cdot(1-\sin x)}{\sin x\cdot(1-\sin x)}=-2.$$

2.inačica

$$\frac{(1-\sin x-\cos x)\cdot(1-\sin x+\cos x)}{\sin x\cdot(1-\sin x)}=$$

$$=\frac{1-\sin x+\cos x-\sin x+\sin^2 x-\sin x\cdot\cos x-\cos x+\cos x\cdot\sin x-\cos^2 x}{\sin x\cdot(1-\sin x)}=$$

$$=\frac{1-\sin x+\cos x-\sin x+\sin^2 x-\sin x\cdot\cos x-\cos x+\cos x\cdot\sin x-\cos^2 x}{\sin x\cdot(1-\sin x)}=$$

$$=\frac{1-\sin x-\sin x+\sin^2 x-\cos^2 x}{\sin x\cdot(1-\sin x)}=\frac{1-2\cdot\sin x+\sin^2 x-\cos^2 x}{\sin x\cdot(1-\sin x)}=$$

$$=\frac{1-\cos^2 x-2\cdot\sin x+\sin^2 x}{\sin x\cdot(1-\sin x)}=\frac{\sin^2 x-2\cdot\sin x+\sin^2 x}{\sin x\cdot(1-\sin x)}=$$

$$=\frac{-2\cdot\sin x+2\cdot\sin^2 x}{\sin x\cdot(1-\sin x)}=\frac{-2\cdot\sin x\cdot(1-\sin x)}{\sin x\cdot(1-\sin x)}=\frac{-2\cdot\sin x\cdot(1-\sin x)}{\sin x\cdot(1-\sin x)}=-2.$$

Vježba 381

Pojednostavni razlomak $\dfrac{(1-\sin x-\cos x)\cdot(1-\sin x+\cos x)}{-\sin x\cdot(\sin x-1)}$.

Rezultat: $-2.$

Zadatak 382 (Maturant, gimnazija)

Pojednostavni razlomak $\dfrac{\cos^4 x-\sin(2\cdot x)-\sin^4 x}{1-tg(2\cdot x)}$.

Rješenje 382

Ponovimo!

$$n=\frac{n}{1}\quad,\quad tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad\cos^2\alpha+\sin^2\alpha=1\quad,\quad\cos(2\cdot\alpha)=\cos^2\alpha-\sin^2\alpha.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\;,\;n\neq 0\;,\;n\neq 1.$$

$$\frac{\cos^4 x-\sin(2\cdot x)-\sin^4 x}{1-tg(2\cdot x)}=\frac{\cos^4 x-\sin^4 x-\sin(2\cdot x)}{\frac{1}{1}-\frac{\sin(2\cdot x)}{\cos(2\cdot x)}}=\frac{\left(\cos^2 x\right)^2-\left(\sin^2 x\right)^2-\sin(2\cdot x)}{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{\cos(2\cdot x)}}=$$

$$=\frac{\left(\cos^2 x-\sin^2 x\right)\cdot\left(\cos^2 x+\sin^2 x\right)-\sin(2\cdot x)}{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{\cos(2\cdot x)}}=\frac{\left(\cos^2 x-\sin^2 x\right)\cdot 1-\sin(2\cdot x)}{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{\cos(2\cdot x)}}=$$

$$=\frac{\cos^2 x-\sin^2 x-\sin(2\cdot x)}{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{\cos(2\cdot x)}}=\frac{\cos(2\cdot x)-\sin(2\cdot x)}{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{\cos(2\cdot x)}}=\frac{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{1}}{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{\cos(2\cdot x)}}=$$

$$=\frac{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{1}}{\frac{\cos(2\cdot x)-\sin(2\cdot x)}{\cos(2\cdot x)}}=\frac{\frac{1}{1}}{\frac{1}{\cos(2\cdot x)}}=\cos(2\cdot x).$$

Vježba 382

Pojednostavni razlomak $\frac{\sin^4 x + \sin(2 \cdot x) - \cos^4 x}{tg(2 \cdot x) - 1}$.

Rezultat: $\cos(2 \cdot x)$.

Zadatak 383 (Ana, ekonomska škola)

Pojednostavni izraz: $\cos\left(\frac{\pi}{4} + \alpha\right) \cdot \cos\left(\frac{\pi}{12} - \alpha\right) - \sin\left(\frac{\pi}{4} + \alpha\right) \cdot \sin\left(\frac{\pi}{12} - \alpha\right)$.

Rješenje 383

Ponovimo!

$$\cos x \cdot \cos y - \sin x \cdot \sin y = \cos(x + y) \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \cos\frac{\pi}{3} = \frac{1}{2}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\cos\left(\frac{\pi}{4} + \alpha\right) \cdot \cos\left(\frac{\pi}{12} - \alpha\right) - \sin\left(\frac{\pi}{4} + \alpha\right) \cdot \sin\left(\frac{\pi}{12} - \alpha\right) =$$

$$\cos\left(\left(\frac{\pi}{4} + \alpha\right) + \left(\frac{\pi}{12} - \alpha\right)\right) = \cos\left(\frac{\pi}{4} + \alpha + \frac{\pi}{12} - \alpha\right) = \cos\left(\frac{\pi}{4} + \alpha + \frac{\pi}{12} - \alpha\right) = \cos\left(\frac{\pi}{4} + \frac{\pi}{12}\right) =$$

$$= \cos\frac{3 \cdot \pi + \pi}{12} = \cos\frac{4 \cdot \pi}{12} = \cos\frac{4 \cdot \pi}{12} = \cos\frac{\pi}{3} = \frac{1}{2}.$$

Vježba 383

Pojednostavni izraz: $\sin\left(\frac{\pi}{4} + \alpha\right) \cdot \sin\left(\frac{\pi}{12} - \alpha\right) - \cos\left(\frac{\pi}{4} + \alpha\right) \cdot \cos\left(\frac{\pi}{12} - \alpha\right)$.

Rezultat: $-\frac{1}{2}$.

Zadatak 384 (Bruno, gimnazija)

Dokaži: $\cos^2\left(\frac{\pi}{4} + \alpha\right) + \cos^2\left(\frac{\pi}{4} - \alpha\right) = 1$.

Rješenje 384

Ponovimo!

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x \quad , \quad \cos^2 x + \sin^2 x = 1 \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Uočimo da je

$$\cos\left(\frac{\pi}{4} + \alpha\right) = \sin\left(\frac{\pi}{2} - \left(\frac{\pi}{4} + \alpha\right)\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{4} - \alpha\right) = \sin\left(\frac{2 \cdot \pi - \pi}{4} - \alpha\right) = \sin\left(\frac{\pi}{4} - \alpha\right).$$

Sada vrijedi:

$$\cos^2\left(\frac{\pi}{4}+\alpha\right)+\cos^2\left(\frac{\pi}{4}-\alpha\right)=\left[\cos\left(\frac{\pi}{4}+\alpha\right)=\sin\left(\frac{\pi}{4}-\alpha\right)\right]=$$

$$=\sin^2\left(\frac{\pi}{4}-\alpha\right)+\cos^2\left(\frac{\pi}{4}-\alpha\right)=1.$$

Vježba 384

Dokaži: $\cos^2\left(\frac{\pi}{4}+\alpha\right)+\cos^2\left(\frac{\pi}{4}-\alpha\right)-1=0.$

Rezultat: Dokaz analogan.

Zadatak 385 (Bruno, gimnazija)

Izračunaj bez uporabe računala vrijednost brojevnog izraza $\sin\frac{\pi}{10}\cdot\cos\frac{\pi}{5}$.

Rješenje 385

Ponovimo!

$$n=\frac{n}{1}\quad,\quad 2\cdot\sin x\cdot\cos x=\sin(2\cdot x)\quad,\quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\sin x=\cos\left(\frac{\pi}{2}-x\right).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\sin\frac{\pi}{10}\cdot\cos\frac{\pi}{5}=\frac{\sin\frac{\pi}{10}\cdot\cos\frac{\pi}{5}}{1}=\left[\begin{array}{l}\text{proširimo razlomak}\\ \text{s } 2\cdot\cos\frac{\pi}{10}\end{array}\right]=\frac{2\cdot\cos\frac{\pi}{10}\cdot\sin\frac{\pi}{10}\cdot\cos\frac{\pi}{5}}{2\cdot\cos\frac{\pi}{10}}=$$

$$=\frac{\left(2\cdot\sin\frac{\pi}{10}\cdot\cos\frac{\pi}{10}\right)\cdot\cos\frac{\pi}{5}}{2\cdot\cos\frac{\pi}{10}}=\frac{\sin\left(2\cdot\frac{\pi}{10}\right)\cdot\cos\frac{\pi}{5}}{2\cdot\cos\frac{\pi}{10}}=\frac{\sin\left(2\cdot\frac{\pi}{10}\right)\cdot\cos\frac{\pi}{5}}{2\cdot\cos\frac{\pi}{10}}=$$

$$=\frac{\sin\frac{\pi}{5}\cdot\cos\frac{\pi}{5}}{2\cdot\cos\frac{\pi}{10}}=\left[\begin{array}{l}\text{proširimo}\\ \text{razlomak s } 2\end{array}\right]=\frac{2\cdot\sin\frac{\pi}{5}\cdot\cos\frac{\pi}{5}}{2\cdot 2\cdot\cos\frac{\pi}{10}}=\frac{\sin\left(2\cdot\frac{\pi}{5}\right)}{4\cdot\cos\frac{\pi}{10}}=\frac{\sin\frac{2\cdot\pi}{5}}{4\cdot\cos\frac{\pi}{10}}=$$

$$=\left[\sin x=\cos\left(\frac{\pi}{2}-x\right)\right]=\frac{\cos\left(\frac{\pi}{2}-\frac{2\cdot\pi}{5}\right)}{4\cdot\cos\frac{\pi}{10}}=\frac{\cos\frac{5\cdot\pi-4\cdot\pi}{10}}{4\cdot\cos\frac{\pi}{10}}=\frac{\cos\frac{\pi}{10}}{4\cdot\cos\frac{\pi}{10}}=$$

$$=\frac{\cos\frac{\pi}{10}}{4\cdot\cos\frac{\pi}{10}}=\frac{1}{4}.$$

Vježba 385

Izračunaj bez uporabe računala vrijednost brojevnog izraza $\cos\frac{\pi}{5}\cdot\sin\frac{\pi}{10}$.

Rezultat: $\frac{1}{4}$.

Zadatak 386 (Dona, gimnazija)

Dokažite identitet: $\frac{\sin^3 x-\cos^3 x}{1+\sin x\cdot\cos x}=\sin x-\cos x.$

Rješenje 386

Ponovimo!

$$a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right)\quad,\quad\cos^2 x+\sin^2 x=1.$$

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right).$$

Identitet je jednakost dvaju matematičkih izraza koja ostaje točna ako se umjesto općih brojeva (slova) u nju uvrste ma koje vrijednosti.

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\quad,\quad n\neq 0\quad,\quad n\neq 1.$$

$$\frac{\sin^3 x-\cos^3 x}{1+\sin x\cdot\cos x}=\frac{(\sin x-\cos x)\cdot\left(\sin^2 x+\sin x\cdot\cos x+\cos^2 x\right)}{1+\sin x\cdot\cos x}=$$

$$=\frac{(\sin x-\cos x)\cdot\left(\sin^2 x+\cos^2 x+\sin x\cdot\cos x\right)}{1+\sin x\cdot\cos x}=\frac{(\sin x-\cos x)\cdot(1+\sin x\cdot\cos x)}{1+\sin x\cdot\cos x}=$$

$$=\frac{(\sin x-\cos x)\cdot(1+\sin x\cdot\cos x)}{1+\sin x\cdot\cos x}=\sin x-\cos x.$$

Vježba 386

Dokažite identitet: $\frac{\sin^3 x+\cos^3 x}{1-\sin x\cdot\cos x}=\sin x+\cos x.$

Rezultat: Dokaz analogan.

Zadatak 387 (Dona, gimnazija)

Zapiši relaciju između x i y nezavisnu o parametru t ako je $\begin{cases} x = \sin t + \cos t \\ y = \sin t \cdot \cos t \end{cases}.$

Rješenje 387

Ponovimo!

$$(a+b)^2 = a^2 + 2\cdot a\cdot b + b^2 \quad , \quad \cos^2 x + \sin^2 x = 1.$$

Parametar

Vladimir Anić, Ivo Goldstein, Rječnik stranih riječi, Novi Liber, Zagreb, 2002.

Veličina, obično realna varijabla, čije vrijednosti služe za razlikovanje elemenata nekog skupa točaka funkcija, jednadžbi ili drugih matematičkih objekata.

Bratoljub Klaić, Rječnik stranih riječi, Nakladni zavod MH, Zagreb, 1983.

Veličina o kojoj ovisi funkcija ili oblik krivulje.

$$\left.\begin{matrix} x = \sin t + \cos t \\ y = \sin t \cdot \cos t \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = \sin t + \cos t \;/^2 \\ y = \sin t \cdot \cos t \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2 = (\sin t + \cos t)^2 \\ y = \sin t \cdot \cos t \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x^2 = \sin^2 t + 2\cdot \sin t \cdot \cos t + \cos^2 t \\ y = \sin t \cdot \cos t \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2 = \sin^2 t + \cos^2 t + 2\cdot \sin t \cdot \cos t \\ y = \sin t \cdot \cos t \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x^2 = 1 + 2\cdot \sin t \cdot \cos t \\ y = \sin t \cdot \cos t \end{matrix}\right\} \Rightarrow x^2 = 1 + 2\cdot y \Rightarrow 1 + 2\cdot y = x^2 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x^2 = 1 + 2\cdot \sin t \cdot \cos t \\ y = \sin t \cdot \cos t \end{matrix}\right\} \Rightarrow x^2 = 1 + 2\cdot y \Rightarrow 1 + 2\cdot y = x^2 \Rightarrow$$

$$\Rightarrow 2\cdot y = x^2 - 1 \Rightarrow 2\cdot y = x^2 - 1 \;/\cdot \frac{1}{2} \Rightarrow y = \frac{1}{2}\cdot\left(x^2 - 1\right).$$

Vježba 387

Zapiši relaciju između x i y nezavisnu o parametru t ako je $\begin{cases} x = 3\cdot \cos t \\ y = 3\cdot \sin t \end{cases}.$

Rezultat: $x^2 + y^2 = 9.$

Zadatak 388 (Asterix, gimnazija)

Duljina dana (u satima) u nekom mjestu izražena je formulom

$D(t) = 3.2\cdot \sin\left(\frac{2\cdot\pi}{365}\cdot(t-79)\right) + 12,$ pri čemu je t dan u godini (t = 0 je 1. siječnja). Kolika je duljina dana 25. siječnja?

Rješenje 388

Ponovimo!

$$1 \; dan = 24 \; h.$$

Varijabla t je dan u godini i određuje se na ovaj način:

t = 0 je 1. siječnja

t = 1 je 2. siječnja

t = 2 je 3. siječnja

t = 24 je 25. siječnja.

Duljina dana (u satima) iznosi:

$$\left.\begin{array}{l} t = 24 \\ D(t) = 3.2 \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (t - 79)\right) + 12 \end{array}\right\} \Rightarrow D(24) = 3.2 \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (24 - 79)\right) + 12 \Rightarrow$$

$$\Rightarrow \left[\begin{array}{c} \text{džepno} \\ \text{računalo} \\ RAD \end{array}\right] \Rightarrow D(24) = 9.4 \ h.$$

Vježba 388

Duljina dana (u satima) u nekom mjestu izražena je formulom $D(t) = 3.2 \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (t - 79)\right) + 12,$ pri čemu je t dan u godini (t = 0 je 1. siječnja). Kolika je duljina dana 1. siječnja?

Rezultat: $8.87 \ h.$

Zadatak 389 (Asterix, gimnazija)

Duljina dana (u satima) u nekom mjestu izražena je formulom $D(t) = 3.2 \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (t - 79)\right) + 12,$ pri čemu je t dan u godini (t = 0 je 1. siječnja). Koji je dan najdulji?

Rješenje 389

Ponovimo!

$$\sin\frac{\pi}{2} = 1 \quad , \quad \sin\alpha = \sin\beta \Rightarrow \alpha = \beta + k \cdot 2 \cdot \pi \ \text{ i } \ \alpha = \pi - \beta + k \cdot 2 \cdot \pi, \ k \in Z.$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \frac{a}{b} \cdot \frac{b}{a} = 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Svaki se razlomak (racionalan broj) može napisati kao decimalni broj (brojnik podijelimo nazivnikom).

Koeficijent 3.2 uz trigonometrijsku funkciju sinus pozitivan je broj pa funkcija D(t) ima najveću vrijednost kada je sinus jednak 1 (to je maksimalna vrijednost funkcije sinus).

$$\sin\left(\frac{2 \cdot \pi}{365} \cdot (t - 79)\right) = 1 \Rightarrow \sin\left(\frac{2 \cdot \pi}{365} \cdot (t - 79)\right) = \sin\frac{\pi}{2} \Rightarrow \frac{2 \cdot \pi}{365} \cdot (t - 79) = \frac{\pi}{2} \Rightarrow$$

$$\Rightarrow \frac{2 \cdot \pi}{365} \cdot (t - 79) = \frac{\pi}{2} \ / \cdot \frac{365}{2 \cdot \pi} \Rightarrow t - 79 = \frac{\pi}{2} \cdot \frac{365}{2 \cdot \pi} \Rightarrow t - 79 = \frac{\pi}{2} \cdot \frac{365}{2 \cdot \pi} \Rightarrow t - 79 = \frac{365}{4} \Rightarrow$$

$$\Rightarrow t = \frac{365}{4} + 79 \Rightarrow t = \frac{365}{4} + \frac{79}{1} \Rightarrow t = \frac{365 + 316}{4} \Rightarrow t = \frac{681}{4} \Rightarrow t \approx 171.$$

Brojeći dane po kalendaru (nije prijestupna godina) dobije se nadnevak 21. lipnja (to je po astronomskim mjerenjima ljetni solsticij, suncostaj).

	SIJEČANJ	VELJAČA	OŽUJAK	TRAVANJ	SVIBANJ	LIPANJ
Pon	6 13 20 27	3 10 17 24	3 10 17 24 31	7 14 21 28	5 12 19 26	2 9 16 23 30
Uto	7 14 21 28	4 11 18 25	4 11 18 25	1 8 15 22 29	6 13 20 27	3 10 17 24
Sri	1 8 15 22 29	5 12 19 26	5 12 19 26	2 9 16 23 30	7 14 21 28	4 11 18 25
Čet	2 9 16 23 30	6 13 20 27	6 13 20 27	3 10 17 24	1 8 15 22 29	5 12 19 26
Pet	3 10 17 24 31	7 14 21 28	7 14 21 28	4 11 18 25	2 9 16 23 30	6 13 20 27
Sub	4 11 18 25	1 8 15 22	1 8 15 22 29	5 12 19 26	3 10 17 24 31	7 14 21 28
Ned	5 12 19 26	2 9 16 23	2 9 16 23 30	6 13 20 27	4 11 18 25	1 8 15 22 29
Tje	1 2 3 4 5	5 6 7 8 9	9 10 11 12 13 14	14 15 16 17 18	18 19 20 21 22	22 23 24 25 26 27

	SRPANJ	KOLOVOZ	RUJAN	LISTOPAD	STUDENI	PROSINAC
Pon	7 14 21 28	4 11 18 25	1 8 15 22 29	6 13 20 27	3 10 17 24	1 8 15 22 29
Uto	1 8 15 22 29	5 12 19 26	2 9 16 23 30	7 14 21 28	4 11 18 25	2 9 16 23 30
Sri	2 9 16 23 30	6 13 20 27	3 10 17 24	1 8 15 22 29	5 12 19 26	3 10 17 24 31
Čet	3 10 17 24 31	7 14 21 28	4 11 18 25	2 9 16 23 30	6 13 20 27	4 11 18 25
Pet	4 11 18 25	1 8 15 22 29	5 12 19 26	3 10 17 24 31	7 14 21 28	5 12 19 26
Sub	5 12 19 26	2 9 16 23 30	6 13 20 27	4 11 18 25	1 8 15 22 29	6 13 20 27
Ned	6 13 20 27	3 10 17 24 31	7 14 21 28	5 12 19 26	2 9 16 23 30	7 14 21 28
Tje	27 28 29 30 31	31 32 33 34 35	36 37 38 39 40	40 41 42 43 44	44 45 46 47 48	49 50 51 52 1

Vježba 389

Odredi maksimalnu vrijednost funkcije $D(x) = 4 \cdot \sin x + 9.$

Rezultat: 13.

Zadatak 390 (Asterix, gimnazija)

Duljina dana (u satima) u nekom mjestu izražena je formulom

$D(t) = 3.2 \cdot \sin\left(\frac{2 \cdot \pi}{365} \cdot (t - 79)\right) + 12,$ pri čemu je t dan u godini (t = 0 je 1. siječnja). Koji je dan najkraći?

Rješenje 390

Ponovimo!

$$\sin\frac{3 \cdot \pi}{2} = -1 \quad , \quad \sin\alpha = \sin\beta \;\Rightarrow\; \alpha = \beta + k \cdot 2 \cdot \pi \;\text{ i }\; \alpha = \pi - \beta + k \cdot 2 \cdot \pi,\; k \in Z.$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \frac{a}{b} \cdot \frac{b}{a} = 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \;,\; n \neq 0 \;,\; n \neq 1.$$

Svaki se razlomak (racionalan broj) može napisati kao decimalni broj (brojnik podijelimo nazivnikom).

Koeficijent 3.2 uz trigonometrijsku funkciju sinus pozitivan je broj pa funkcija D(t) ima najmanju vrijednost kada je sinus jednak – 1 (to je minimalna vrijednost funkcije sinus).

$$\sin\left(\frac{2 \cdot \pi}{365} \cdot (t - 79)\right) = -1 \;\Rightarrow\; \sin\left(\frac{2 \cdot \pi}{365} \cdot (t - 79)\right) = \sin\frac{3 \cdot \pi}{2} \;\Rightarrow\; \frac{2 \cdot \pi}{365} \cdot (t - 79) = \frac{3 \cdot \pi}{2} \;\Rightarrow$$

$$\Rightarrow \frac{2 \cdot \pi}{365} \cdot (t - 79) = \frac{3 \cdot \pi}{2} \;/ \cdot \frac{365}{2 \cdot \pi} \;\Rightarrow\; t - 79 = \frac{3 \cdot \pi}{2} \cdot \frac{365}{2 \cdot \pi} \;\Rightarrow\; t - 79 = \frac{3 \cdot \cancel{\pi}}{2} \cdot \frac{365}{2 \cdot \cancel{\pi}} \;\Rightarrow\; t - 79 = \frac{1095}{4} \;\Rightarrow$$

$$\Rightarrow t = \frac{1095}{4} + 79 \;\Rightarrow\; t = \frac{1095}{4} + \frac{79}{1} \;\Rightarrow\; t = \frac{1095 + 316}{4} \;\Rightarrow\; t = \frac{1411}{4} \;\Rightarrow\; t \approx 353.$$

Brojeći dane po kalendaru (nije prijestupna godina) dobije se nadnevak 20. prosinca (to je po astronomskim mjerenjima zimski solsticij, suncostaj).

	SIJEČANJ	VELJAČA	OŽUJAK	TRAVANJ	SVIBANJ	LIPANJ
Pon	6 13 20 27	3 10 17 24	3 10 17 24 31	7 14 21 28	5 12 19 26	2 9 16 23 30
Uto	7 14 21 28	4 11 18 25	4 11 18 25	1 8 15 22 29	6 13 20 27	3 10 17 24
Sri	1 8 15 22 29	5 12 19 26	5 12 19 26	2 9 16 23 30	7 14 21 28	4 11 18 25
Čet	2 9 16 23 30	6 13 20 27	6 13 20 27	3 10 17 24	1 8 15 22 29	5 12 19 26
Pet	3 10 17 24 31	7 14 21 28	7 14 21 28	4 11 18 25	2 9 16 23 30	6 13 20 27
Sub	4 11 18 25	1 8 15 22	1 8 15 22 29	5 12 19 26	3 10 17 24 31	7 14 21 28
Ned	5 12 19 26	2 9 16 23	2 9 16 23 30	6 13 20 27	4 11 18 25	1 8 15 22 29
Tje	1 2 3 4 5	5 6 7 8 9	9 10 11 12 13 14	14 15 16 17 18	18 19 20 21 22	22 23 24 25 26 27

	SRPANJ	KOLOVOZ	RUJAN	LISTOPAD	STUDENI	PROSINAC
Pon	7 14 21 28	4 11 18 25	1 8 15 22 29	6 13 20 27	3 10 17 24	1 8 15 22 29
Uto	1 8 15 22 29	5 12 19 26	2 9 16 23 30	7 14 21 28	4 11 18 25	2 9 16 23 30
Sri	2 9 16 23 30	6 13 20 27	3 10 17 24	1 8 15 22 29	5 12 19 26	3 10 17 24 31
Čet	3 10 17 24 31	7 14 21 28	4 11 18 25	2 9 16 23 30	6 13 20 27	4 11 18 25
Pet	4 11 18 25	1 8 15 22 29	5 12 19 26	3 10 17 24 31	7 14 21 28	5 12 19 26
Sub	5 12 19 26	2 9 16 23 30	6 13 20 27	4 11 18 25	1 8 15 22 29	6 13 20 27
Ned	6 13 20 27	3 10 17 24 31	7 14 21 28	5 12 19 26	2 9 16 23 30	7 14 21 28
Tje	27 28 29 30 31	31 32 33 34 35	36 37 38 39 40	40 41 42 43 44	44 45 46 47 48	49 50 51 52 1

Vježba 390

Odredi minimalnu vrijednost funkcije $D(x)=4\cdot\sin x+9.$

Rezultat: 5.

Zadatak 391 (2A, TUPŠ)

Dokažite identitet: $\dfrac{\cos\alpha}{1-\sin\alpha}-\dfrac{\cos\alpha}{1+\sin\alpha}=2\cdot tg\,\alpha.$

Rješenje 391

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad(a-b)\cdot(a+b)=a^2-b^2.$$

$$\cos^2\alpha+\sin^2\alpha=1\quad,\quad tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\,,\ n\neq0\ ,\ n\neq1.$$

Identitet je jednakost dvaju matematičkih izraza koja ostaje točna ako se umjesto općih brojeva (slova) u nju uvrste ma koje vrijednosti.

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{\cos\alpha}{1-\sin\alpha}-\frac{\cos\alpha}{1+\sin\alpha}=\frac{\cos\alpha\cdot(1+\sin\alpha)-\cos\alpha\cdot(1-\sin\alpha)}{(1-\sin\alpha)\cdot(1+\sin\alpha)}=$$

$$=\frac{\cos\alpha+\cos\alpha\cdot\sin\alpha-\cos\alpha+\cos\alpha\cdot\sin\alpha}{1-\sin^2\alpha}=\frac{\cos\alpha+\cos\alpha\cdot\sin\alpha-\cos\alpha+\cos\alpha\cdot\sin\alpha}{\cos^2\alpha}=$$

$$=\frac{\cos\alpha\cdot\sin\alpha+\cos\alpha\cdot\sin\alpha}{\cos^2\alpha}=\frac{2\cdot\cos\alpha\cdot\sin\alpha}{\cos^2\alpha}=\frac{2\cdot\cos\alpha\cdot\sin\alpha}{\cos^2\alpha}=2\cdot\frac{\sin\alpha}{\cos\alpha}=2\cdot tg\,\alpha.$$

2.inačica

$$\frac{\cos\alpha}{1-\sin\alpha}-\frac{\cos\alpha}{1+\sin\alpha}=\cos\alpha\cdot\left(\frac{1}{1-\sin\alpha}-\frac{1}{1+\sin\alpha}\right)=\cos\alpha\cdot\frac{1+\sin\alpha-(1-\sin\alpha)}{(1-\sin\alpha)\cdot(1+\sin\alpha)}=$$

$$=\cos\alpha\cdot\frac{1+\sin\alpha-1+\sin\alpha}{(1-\sin\alpha)\cdot(1+\sin\alpha)}=\cos\alpha\cdot\frac{1+\sin\alpha-1+\sin\alpha}{1-\sin^2\alpha}=\cos\alpha\cdot\frac{\sin\alpha+\sin\alpha}{\cos^2\alpha}=$$

$$= \cos\alpha \cdot \frac{2 \cdot \sin\alpha}{\cos^2\alpha} = \cos\alpha \cdot \frac{2 \cdot \sin\alpha}{\cos^{2}\alpha} = 2 \cdot \frac{\sin\alpha}{\cos\alpha} = 2 \cdot tg\,\alpha.$$

Hvala kolegi Tiboru Pejiću na uočenoj pogrešci u zadatku.
(Sanja Varošanec, Matematika 2, str. 104, zadatak 7c, Element, Zagreb, 2012.)

Vježba 391

Dokažite identitet: $\frac{\cos\alpha}{\sin\alpha - 1} + \frac{\cos\alpha}{\sin\alpha + 1} = -2 \cdot tg\,\alpha.$

Rezultat: Dokaz analogan.

Zadatak 392 (Ante, tehnička škola)

Ako je $\sin\alpha + \cos\beta = m$ i $\cos\alpha - \cos\beta = n$ koliko je $\sin(2 \cdot \alpha)$?

$A.\ m^2 + n^2 \qquad B.\ (m-n)^2 + 1 \qquad C.\ (m+n)^2 + 1 \qquad D.\ (m+n)^2 - 1$

Rješenje 392

Ponovimo!

$$\left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow a + c = b + d \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad \sin^2\alpha + \cos^2\alpha = 1.$$

$$\sin(2 \cdot \alpha) = 2 \cdot \sin\alpha \cdot \cos\alpha.$$

$$\left.\begin{array}{l} \sin\alpha + \cos\beta = m \\ \cos\alpha - \cos\beta = n \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow \sin\alpha + \cos\beta + \cos\alpha - \cos\beta = m + n \Rightarrow$$

$$\Rightarrow \sin\alpha + \cos\beta + \cos\alpha - \cos\beta = m + n \Rightarrow \sin\alpha + \cos\alpha = m + n \Rightarrow \sin\alpha + \cos\alpha = m + n \ /^2 \Rightarrow$$

$$\Rightarrow (\sin\alpha + \cos\alpha)^2 = (m+n)^2 \Rightarrow \sin^2\alpha + 2 \cdot \sin\alpha \cdot \cos\alpha + \cos^2\alpha = (m+n)^2 \Rightarrow$$

$$\Rightarrow \sin^2\alpha + \cos^2\alpha + 2 \cdot \sin\alpha \cdot \cos\alpha = (m+n)^2 \Rightarrow 1 + 2 \cdot \sin\alpha \cdot \cos\alpha = (m+n)^2 \Rightarrow$$

$$\Rightarrow 1 + \sin(2 \cdot \alpha) = (m+n)^2 \Rightarrow \sin(2 \cdot \alpha) = (m+n)^2 - 1.$$

Odgovor je pod D.

Vježba 392

Ako je $\sin\alpha + \cos\beta = m$ i $\cos\beta - \cos\alpha + n = 0$ koliko je $\sin(2 \cdot \alpha)$?

$A.\ m^2 + n^2 \qquad B.\ (m-n)^2 + 1 \qquad C.\ (m+n)^2 + 1 \qquad D.\ (m+n)^2 - 1$

Rezultat: D.

Zadatak 393 (Ante, tehnička škola)

Zadana je funkcija $f(x) = \sin^4 x + \cos^4 x.$ Ako je $\sin(2 \cdot \alpha) = \frac{2}{3}$, nađite $f(\alpha)$.

Rješenje 393

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^2 + 2 \cdot a \cdot b + b^2 = (a+b)^2 \quad , \quad \sin^2 x + \cos^2 x = 1.$$

$$\sin(2\cdot x)=2\cdot\sin x\cdot\cos x \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad n=\frac{n}{1}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \ , \ n\neq 0 \ , \ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$f(x)=\sin^4 x+\cos^4 x \Rightarrow f(\alpha)=\sin^4\alpha+\cos^4\alpha \Rightarrow$$

$$\Rightarrow f(\alpha)=\sin^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha-2\cdot\sin^2\alpha\cdot\cos^2\alpha \Rightarrow$$

$$\Rightarrow f(\alpha)=\left(\sin^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha\right)-\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{1} \Rightarrow$$

$$\Rightarrow f(\alpha)=\left(\sin^2\alpha+\cos^2\alpha\right)^2-\frac{4\cdot\sin^2\alpha\cdot\cos^2\alpha}{2} \Rightarrow f(\alpha)=1^2-\frac{(2\cdot\sin\alpha\cdot\cos\alpha)^2}{2} \Rightarrow$$

$$\Rightarrow f(\alpha)=1-\frac{(\sin(2\cdot\alpha))^2}{2} \Rightarrow \left[\sin(2\cdot\alpha)=\frac{2}{3}\right] \Rightarrow f(\alpha)=1-\frac{\left(\frac{2}{3}\right)^2}{2} \Rightarrow f(\alpha)=1-\frac{\frac{4}{9}}{\frac{2}{1}} \Rightarrow$$

$$\Rightarrow f(\alpha)=1-\frac{\frac{4}{9}}{\frac{2}{1}} \Rightarrow f(\alpha)=1-\frac{\frac{2}{9}}{\frac{1}{1}} \Rightarrow f(\alpha)=1-\frac{2}{9} \Rightarrow f(\alpha)=\frac{1}{1}-\frac{2}{9} \Rightarrow$$

$$\Rightarrow f(\alpha)=\frac{9-2}{9} \Rightarrow f(\alpha)=\frac{7}{9}.$$

Vježba 393

Zadana je funkcija $f(x)=\cos^4 x+\sin^4 x$. Ako je $\sin(2\cdot\alpha)=\frac{2}{3}$, nađite $f(\alpha)$.

Rezultat: $\frac{7}{9}$.

Zadatak 394 (Veseli maturanti :), gimnazija)

Dokažite: $\frac{1}{\sin^2\alpha\cdot\cos^2\alpha}=tg^2\alpha+ctg^2\alpha+2.$

Rješenje 394

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1 \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n} \quad , \quad \frac{a^n}{a^m}=a^{n-m} \quad , \quad tg\,\alpha=\frac{\sin\alpha}{\cos\alpha} \quad , \quad ctg\,\alpha=\frac{\cos\alpha}{\sin\alpha} \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\frac{1}{\sin^2\alpha\cdot\cos^2\alpha}=\frac{1^2}{\sin^2\alpha\cdot\cos^2\alpha}=\frac{\left(\sin^2\alpha+\cos^2\alpha\right)^2}{\sin^2\alpha\cdot\cos^2\alpha}=$$

$$=\frac{\left(\sin^2\alpha\right)^2+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\left(\cos^2\alpha\right)^2}{\sin^2\alpha\cdot\cos^2\alpha}=\frac{\sin^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\cos^4\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=$$

$$=\frac{\sin^4\alpha}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{\cos^4\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=$$

$$=\frac{\sin^4\alpha}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{\cos^4\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=\frac{\sin^2\alpha}{\cos^2\alpha}+2+\frac{\cos^2\alpha}{\sin^2\alpha}=$$

$$=tg^2\alpha+2+ctg^2\alpha=tg^2\alpha+ctg^2\alpha+2.$$

Vježba 394

Dokažite: $\frac{\sin\alpha}{\sin^3\alpha\cdot\cos^2\alpha}=tg^2\alpha+ctg^2\alpha+2.$

Rezultat: Dokaz analogan.

Zadatak 395 (Veseli maturanti :), gimnazija)

Napišite u jednostavnijem obliku: $\frac{(1+\sin\alpha)^2+\cos^2\alpha}{(1+\sin\alpha)^2-\cos^2\alpha}.$

Rješenje 395

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \cos^2\alpha+\sin^2\alpha=1 \quad , \quad a^1=a \quad , \quad a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$\frac{(1+\sin\alpha)^2+\cos^2\alpha}{(1+\sin\alpha)^2-\cos^2\alpha} = \frac{1+2\cdot\sin\alpha+\sin^2\alpha+\cos^2\alpha}{1+2\cdot\sin\alpha+\sin^2\alpha-\cos^2\alpha} = \frac{1+2\cdot\sin\alpha+\left(\sin^2\alpha+\cos^2\alpha\right)}{2\cdot\sin\alpha+\sin^2\alpha+\left(1-\cos^2\alpha\right)} =$$

$$= \frac{1+2\cdot\sin\alpha+1}{2\cdot\sin\alpha+\sin^2\alpha+\sin^2\alpha} = \frac{2+2\cdot\sin\alpha}{2\cdot\sin\alpha+2\cdot\sin^2\alpha} = \frac{2+2\cdot\sin\alpha}{\sin\alpha\cdot(2+2\cdot\sin\alpha)} =$$

$$= \frac{2+2\cdot\sin\alpha}{\sin\alpha\cdot(2+2\cdot\sin\alpha)} = \frac{1}{\sin\alpha}.$$

Vježba 395

Napišite u jednostavnijem obliku: $\frac{(1+\cos\alpha)^2+\sin^2\alpha}{(1+\cos\alpha)^2-\sin^2\alpha}$.

Rezultat: $\frac{1}{\cos\alpha}$.

Zadatak 396 (Veseli maturanti :), gimnazija)

Napišite u jednostavnijem obliku: $\frac{(1-\sin\alpha)^2+\cos^2\alpha}{(1-\sin\alpha)^2-\cos^2\alpha}$.

Rješenje 396

Ponovimo!

$$(a-b)^2 = a^2 - 2\cdot a\cdot b + b^2 \quad , \quad \cos^2\alpha+\sin^2\alpha = 1 \quad , \quad a^1 = a \quad , \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$\frac{(1-\sin\alpha)^2+\cos^2\alpha}{(1-\sin\alpha)^2-\cos^2\alpha} = \frac{1-2\cdot\sin\alpha+\sin^2\alpha+\cos^2\alpha}{1-2\cdot\sin\alpha+\sin^2\alpha-\cos^2\alpha} = \frac{1-2\cdot\sin\alpha+\left(\sin^2\alpha+\cos^2\alpha\right)}{-2\cdot\sin\alpha+\sin^2\alpha+\left(1-\cos^2\alpha\right)} =$$

$$= \frac{1-2\cdot\sin\alpha+1}{-2\cdot\sin\alpha+\sin^2\alpha+\sin^2\alpha} = \frac{2-2\cdot\sin\alpha}{-2\cdot\sin\alpha+2\cdot\sin^2\alpha} = \frac{2-2\cdot\sin\alpha}{-\sin\alpha\cdot(2-2\cdot\sin\alpha)} =$$

$$= \frac{2-2\cdot\sin\alpha}{-\sin\alpha\cdot(2-2\cdot\sin\alpha)} = -\frac{1}{\sin\alpha} = \frac{-1}{\sin\alpha}.$$

Vježba 396

Napišite u jednostavnijem obliku: $\frac{(1-\cos\alpha)^2+\sin^2\alpha}{(1-\cos\alpha)^2-\sin^2\alpha}$.

Rezultat: $\frac{-1}{\cos\alpha}$.

Zadatak 397 (Veseli maturanti :), gimnazija)

Pojednostavni: $\frac{1-\sin\alpha-\cos^2\alpha}{1-\sin\alpha}$.

Rješenje 397

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1 \quad , \quad a^1=a \quad , \quad a^n : a^m = a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

1.inačica

$$\frac{1-\sin\alpha-\cos^2\alpha}{1-\sin\alpha}=\frac{\left(1-\cos^2\alpha\right)-\sin\alpha}{1-\sin\alpha}=\frac{\sin^2\alpha-\sin\alpha}{1-\sin\alpha}=$$

$$=\frac{-\sin\alpha\cdot(1-\sin\alpha)}{1-\sin\alpha}=\frac{-\sin\alpha\cdot(1-\sin\alpha)}{1-\sin\alpha}=-\sin\alpha.$$

2.inačica

$$\frac{1-\sin\alpha-\cos^2\alpha}{1-\sin\alpha}=\frac{\cos^2\alpha+\sin^2\alpha-\sin\alpha-\cos^2\alpha}{1-\sin\alpha}=\frac{\cos^2\alpha+\sin^2\alpha-\sin\alpha-\cos^2\alpha}{1-\sin\alpha}=$$

$$=\frac{\sin^2\alpha-\sin\alpha}{1-\sin\alpha}=\frac{-\sin\alpha\cdot(1-\sin\alpha)}{1-\sin\alpha}=\frac{-\sin\alpha\cdot(1-\sin\alpha)}{1-\sin\alpha}=-\sin\alpha.$$

Vježba 397

Pojednostavni: $\frac{1-\cos\alpha-\sin^2\alpha}{1-\cos\alpha}$.

Rezultat: $-\cos\alpha$.

Zadatak 398 (Tonka, gimnazija)

Ako je α + β + γ = 90º, a nijedan od kutova nije takav da je njegov tangens beskonačan, onda je $tg\,\alpha\cdot tg\,\beta+tg\,\beta\cdot tg\,\gamma+tg\,\gamma\cdot tg\,\alpha=1$.

Rješenje 398

Ponovimo!

$$tg\left(90^\circ-x\right)=ctg\,x \quad , \quad tg\,x=\frac{1}{ctg\,x} \quad , \quad tg\,(x+y)=\frac{tg\,x+tg\,y}{1-tg\,x\cdot tg\,y}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\alpha+\beta+\gamma=90^\circ \Rightarrow \alpha+\beta=90^\circ-\gamma \Rightarrow tg(\alpha+\beta)=tg\left(90^\circ-\gamma\right) \Rightarrow tg(\alpha+\beta)=ctg\,\gamma \Rightarrow$$

$$\Rightarrow tg(\alpha+\beta)=\frac{1}{tg\,\gamma} \Rightarrow \frac{tg\,\alpha+tg\,\beta}{1-tg\,\alpha\cdot tg\,\beta}=\frac{1}{tg\,\gamma} \Rightarrow$$

$$\Rightarrow \frac{tg\,\alpha+tg\,\beta}{1-tg\,\alpha\cdot tg\,\beta}=\frac{1}{tg\,\gamma}\;/\cdot\,tg\,\gamma\cdot(1-tg\,\alpha\cdot tg\,\beta) \Rightarrow (tg\,\alpha+tg\,\beta)\cdot tg\,\gamma=1-tg\,\alpha\cdot tg\,\beta \Rightarrow$$

$$\Rightarrow tg\,\alpha\cdot tg\,\gamma+tg\,\beta\cdot tg\,\gamma=1-tg\,\alpha\cdot tg\,\beta \Rightarrow tg\,\alpha\cdot tg\,\beta+tg\,\alpha\cdot tg\,\gamma+tg\,\beta\cdot tg\,\gamma=1 \Rightarrow$$

$$\Rightarrow tg\,\alpha\cdot tg\,\beta+tg\,\beta\cdot tg\,\gamma+tg\,\gamma\cdot tg\,\alpha=1.$$

Vježba 398

Ako je α + β + γ = 90º, a nijedan od kutova nije takav da je njegov tangens beskonačan, onda je $tg\,\alpha\cdot tg\,\beta+tg\,\beta\cdot tg\,\gamma+tg\,\gamma\cdot tg\,\alpha-1=0.$

Rezultat: Dokaz analogan.

Zadatak 399 (Tonka, gimnazija)

Nađi relaciju između veličina α i β, ako je $tg\,\beta=\frac{tg\,\alpha-1}{tg\,\alpha+1}.$

Rješenje 399

Ponovimo!

$$tg(x-y)=\frac{tg\,x-tg\,y}{1+tg\,x\cdot tg\,y} \quad , \quad tg\,45^\circ=1 \quad , \quad a\cdot\frac{1}{b}=\frac{a}{b}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$tg\,\beta=\frac{tg\,\alpha-1}{tg\,\alpha+1} \Rightarrow tg\,\beta=\frac{tg\,\alpha-1}{tg\,\alpha+1}\;/\cdot\,(tg\,\alpha+1) \Rightarrow tg\,\beta\cdot(tg\,\alpha+1)=tg\,\alpha-1 \Rightarrow$$

$$\Rightarrow tg\,\beta\cdot tg\,\alpha+tg\,\beta=tg\,\alpha-1 \Rightarrow 1+tg\,\beta\cdot tg\,\alpha=tg\,\alpha-tg\,\beta \Rightarrow$$

$$\Rightarrow tg\,\alpha-tg\,\beta=1+tg\,\beta\cdot tg\,\alpha \Rightarrow tg\,\alpha-tg\,\beta=1+tg\,\alpha\cdot tg\,\beta\;/\cdot\,\frac{1}{1+tg\,\alpha\cdot tg\,\beta} \Rightarrow$$

$$\Rightarrow \frac{tg\,\alpha-tg\,\beta}{1+tg\,\alpha\cdot tg\,\beta}=1 \Rightarrow tg(\alpha-\beta)=1 \Rightarrow \alpha-\beta=tg^{-1}(1) \Rightarrow \alpha-\beta=45^\circ.$$

Vježba 399

Nađi relaciju između veličina α i β, ako je $tg\,\beta=\frac{1-tg\,\alpha}{1+tg\,\alpha}.$

Rezultat: $\alpha+\beta=45^\circ.$

Zadatak 400 (Marija, gimnazija)

Dokažite identitet: $\sin(2\cdot\alpha)=\frac{2\cdot tg\,\alpha}{1+tg^2\,\alpha}.$

Rješenje 400

Ponovimo!

$$n=\frac{n}{1}\quad,\quad \sin(2\cdot\alpha)=2\cdot\sin\alpha\cdot\cos\alpha\quad,\quad \cos^2\alpha+\sin^2\alpha=1\quad,\quad tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}.$$

$$a^1=a\quad,\quad \frac{a^n}{a^m}=a^{n-m}\quad,\quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n\quad,\quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\sin(2\cdot\alpha)=\frac{\sin(2\cdot\alpha)}{1}=\frac{2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha+\sin^2\alpha}=\frac{\frac{2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha}}{\frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha}}=\frac{\frac{2\cdot\sin\alpha\cdot\cos\alpha}{\cos^2\alpha}}{\frac{\cos^2\alpha}{\cos^2\alpha}+\frac{\sin^2\alpha}{\cos^2\alpha}}=$$

$$=\frac{\frac{2\cdot\sin\alpha}{\cos\alpha}}{\frac{\cos^2\alpha}{\cos^2\alpha}+\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{2\cdot tg\,\alpha}{1+\frac{\sin^2\alpha}{\cos^2\alpha}}=\frac{2\cdot tg\,\alpha}{1+\left(\frac{\sin\alpha}{\cos\alpha}\right)^2}=\frac{2\cdot tg\,\alpha}{1+tg^2\,\alpha}.$$

Vježba 400

Dokažite identitet: $\sin 4=\frac{2\cdot tg\,2}{1+tg^2\,2}.$

Rezultat: Dokaz analogan.