Zadatak 361 (Tonka, gimnazija)

Odredite $x \in \left\langle 0, \frac{\pi}{2} \right\rangle$ za koji je $\cos^2 x - \sin(2 \cdot x) = 0.$ Rješenje zapišite zaokruženo na četiri decimale.

Rješenje 361

Ponovimo!

$$\sin(2 \cdot x) = 2 \cdot \sin x \cdot \cos x \quad , \quad tg\, x = \frac{\sin x}{\cos x}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a \cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Trigonometrijska jednadžba cos x = a

Jednadžba ima rješenje ako i samo ako je

$$-1 \le a \le 1.$$

Tada postoji jedinstven kut α u intervalu

$$-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$$

čiji je kosinus jednak a pa postoji jednadžba

$$\cos x = \cos \alpha$$

koja ima dva skupa rješenja:

$$\left. \begin{array}{l} x_1 = \alpha + k \cdot 2 \cdot \pi \ , \ k \in Z \\ x_2 = -\alpha + k \cdot 2 \cdot \pi \ , \ k \in Z \end{array} \right\} \text{ ili } x_{1,2} = \pm \alpha + k \cdot 2 \cdot \pi \ , \ k \in Z.$$

Trigonometrijska jednadžba tg x = a

Skup rješenja jednadžbe $tg\, x = a,\ a \in R,$ je $\left\{ x_0 + k \cdot \pi : k \in Z \right\},$ gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$tg\, x = a \Rightarrow tg\, x = tg\, x_0 \Rightarrow x = x_0 + k \cdot \pi \ , \ k \in Z.$$

Skup cijelih brojeva označavamo slovom Z, a zapisujemo

$$Z = \left\{ ..., -3, -2, -1, 0, 1, 2, 3, ... \right\}.$$

Zaokruživanje decimalnih brojeva

Ako je prva zanemarena znamenka jednaka 0, 1, 2, 3, 4 posljednja znamenka približne vrijednosti se ne mijenja.

$$0.374261 \Rightarrow \left[\begin{array}{l} \text{zaokruženo na} \\ \text{tri decimale} \end{array} \right] \Rightarrow 0.374$$

Ako je prva zanemarena znamenka jednaka 5, 6, 7, 8, 9 posljednja znamenka približne vrijednosti se povećava za 1.

$$0.374261 \Rightarrow \left[\begin{array}{l} \text{zaokruženo na} \\ \text{četiri decimale} \end{array} \right] \Rightarrow 0.3743$$

$$\cos^2 x - \sin(2 \cdot x) = 0 \Rightarrow \cos^2 x - 2 \cdot \sin x \cdot \cos x = 0 \Rightarrow \cos x \cdot (\cos x - 2 \cdot \sin x) = 0 \Rightarrow$$

$$\Rightarrow \left. \begin{array}{l} \cos x = 0 \\ \cos x - 2 \cdot \sin x = 0 \end{array} \right\}.$$

Riješimo jednadžbe!

- $\cos x = 0 \Rightarrow x = \cos^{-1} 0 \Rightarrow x = (2 \cdot k + 1) \cdot \frac{\pi}{2}, k \in Z.$

Ni jedno rješenje ne zadovoljava uvjet

$$x \in \left\langle 0, \frac{\pi}{2} \right\rangle \text{ ili } x \in \langle 0, 1.5708 \rangle.$$

- $\cos x - 2 \cdot \sin x = 0 \Rightarrow -2 \cdot \sin x = -\cos x \Rightarrow -2 \cdot \sin x = -\cos x \; / \cdot \left(-\frac{1}{2 \cdot \cos x} \right) \Rightarrow$

$$\Rightarrow \frac{\sin x}{\cos x} = \frac{1}{2} \Rightarrow tg\, x = \frac{1}{2} \Rightarrow x = tg^{-1}\left(\frac{1}{2}\right) + k \cdot \pi, k \in Z \Rightarrow$$

$$\Rightarrow x = 0.463647609 + k \cdot \pi, k \in Z.$$

Prema uvjetu zadatka rješenje je

$$x = 0.4636.$$

Vježba 361

Odredite $x \in \left\langle 0, \frac{\pi}{2} \right\rangle$ za koji je $\cos^2 x - \sin(2 \cdot x) = 0.$ Rješenje zapišite zaokruženo na pet decimala.

Rezultat: 0.46365.

Zadatak 362 (Sandra, maturantica)

Koliko korijena u intervalu $\left\langle -\frac{3 \cdot \pi}{2}, 5 \cdot \pi \right\rangle$ ima jednadžba $\cos \frac{x}{2} \cdot \cos(2 \cdot x) - \sin \frac{x}{2} \cdot \sin(2 \cdot x) = -1$?

Rješenje 362

Ponovimo!

$$\cos(x + y) = \cos x \cdot \cos y - \sin x \cdot \sin y \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

Trigonometrijska jednadžba cos x = a

Jednadžba ima rješenje ako i samo ako je

$$-1 \le a \le 1.$$

Tada postoji jedinstven kut α u intervalu

$$-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2}$$

čiji je kosinus jednak a pa postoji jednadžba

$$\cos x = \cos \alpha$$

koja ima dva skupa rješenja:

$$\left.\begin{array}{l} x_1 = \alpha + k \cdot 2 \cdot \pi \;,\; k \in Z \\ x_2 = -\alpha + k \cdot 2 \cdot \pi \;,\; k \in Z \end{array}\right\} \text{ ili } x_{1,2} = \pm \alpha + k \cdot 2 \cdot \pi \;,\; k \in Z.$$

Skup cijelih brojeva označavamo slovom Z, a zapisujemo

$$Z = \{ ..., -3, -2, -1, 0, 1, 2, 3, ... \}.$$

$$\frac{a}{b} \cdot \frac{b}{a} = 1 \quad , \quad a \cdot \frac{b}{c} = \frac{a \cdot b}{c} \quad , \quad a < b \Rightarrow a - c < b - c \;,\; c \in R.$$

$$a < b \;,\; c > 0 \;\Rightarrow\; \frac{a}{c} < \frac{b}{c} \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \;,\; n \neq 0 \;,\; n \neq 1.$$

Pretvaranje razlomka u mješoviti broj:

$$\frac{8}{5} = \left[\, 8 > 5 \,\right] = 1\frac{3}{5} \text{ jer je } 8:5 = 1 \text{ s ostatkom } 3.$$

$$\cos\frac{x}{2} \cdot \cos(2 \cdot x) - \sin\frac{x}{2} \cdot \sin(2 \cdot x) = -1 \;\Rightarrow\; \cos\left(\frac{x}{2} + 2 \cdot x\right) = -1 \;\Rightarrow\; \cos\left(\frac{x}{2} + \frac{2 \cdot x}{1}\right) = -1 \;\Rightarrow$$

$$\Rightarrow\; \cos\frac{x + 4 \cdot x}{2} = -1 \;\Rightarrow\; \cos\frac{5 \cdot x}{2} = -1 \;\Rightarrow\; \cos\frac{5 \cdot x}{2} = \cos\pi \;\Rightarrow\; \frac{5 \cdot x}{2} = \pi + k \cdot 2 \cdot \pi \;\Rightarrow$$

$$\Rightarrow\; \frac{5 \cdot x}{2} = \pi + k \cdot 2 \cdot \pi \;/ \cdot \frac{2}{5} \;\Rightarrow\; x = \frac{2 \cdot \pi}{5} + k \cdot \frac{4 \cdot \pi}{5},\; k \in Z.$$

Budući da korijeni jednadžbe moraju biti u intervalu $\left\langle -\frac{3 \cdot \pi}{2}, 5 \cdot \pi \right\rangle$, slijedi:

$$\left.\begin{matrix} x = \frac{2 \cdot \pi}{5} + k \cdot \frac{4 \cdot \pi}{5} \\ -\frac{3 \cdot \pi}{2} < x < 5 \cdot \pi \end{matrix}\right\} \;\Rightarrow\; -\frac{3 \cdot \pi}{2} < \frac{2 \cdot \pi}{5} + k \cdot \frac{4 \cdot \pi}{5} < 5 \cdot \pi \;\Rightarrow$$

$$\Rightarrow\; -\frac{3 \cdot \pi}{2} < \frac{2 \cdot \pi}{5} + k \cdot \frac{4 \cdot \pi}{5} < 5 \cdot \pi \;/ \cdot \frac{1}{\pi} \;\Rightarrow\; -\frac{3}{2} < \frac{2}{5} + k \cdot \frac{4}{5} < 5 \;\Rightarrow$$

$$\Rightarrow\; -\frac{3}{2} < \frac{2}{5} + k \cdot \frac{4}{5} < 5 \;/ - \frac{2}{5} \;\Rightarrow\; -\frac{3}{2} - \frac{2}{5} < \frac{2}{5} + k \cdot \frac{4}{5} - \frac{2}{5} < 5 - \frac{2}{5} \;\Rightarrow$$

$$\Rightarrow\; \frac{-15 - 4}{10} < \frac{2}{5} + k \cdot \frac{4}{5} - \frac{2}{5} < \frac{5}{1} - \frac{2}{5} \;\Rightarrow\; \frac{-19}{10} < k \cdot \frac{4}{5} < \frac{25 - 2}{5} \;\Rightarrow\; \frac{-19}{10} < k \cdot \frac{4}{5} < \frac{23}{5} \;\Rightarrow$$

$$\Rightarrow\; \frac{-19}{10} < k \cdot \frac{4}{5} < \frac{23}{5} \;/ \cdot \frac{5}{4} \;\Rightarrow\; \frac{-19}{10} \cdot \frac{5}{4} < k \cdot \frac{4}{5} \cdot \frac{5}{4} < \frac{23}{5} \cdot \frac{5}{4} \;\Rightarrow\; \frac{-19}{10} \cdot \frac{5}{4} < k \cdot \frac{4}{5} \cdot \frac{5}{4} < \frac{23}{5} \cdot \frac{5}{4} \;\Rightarrow$$

$$\Rightarrow\; \frac{-19}{8} < k < \frac{23}{4} \;\Rightarrow\; -2\frac{3}{8} < k < 5\frac{3}{4} \;\Rightarrow\; k \in \{-2, -1, 0, 1, 2, 3, 4, 5\}.$$

Ima 8 rješenja.

Vježba 362

Koliko korijena u intervalu $\left\langle -\frac{3 \cdot \pi}{2}, 5 \cdot \pi \right\rangle$ ima jednadžba

$$\sin\frac{x}{2} \cdot \sin(2 \cdot x) - \cos\frac{x}{2} \cdot \cos(2 \cdot x) = 1?$$

Rezultat: Ima 8 rješenja.

Zadatak 363 (Palčica, maturantica)

Točna vrijednost brojevnog izraza $\sin\frac{\pi}{12}+\sin\frac{5\cdot\pi}{12}$ jednaka je:

$$A.\ \frac{\sqrt{6}}{2}\qquad B.\ \frac{\sqrt{3}}{4}\qquad C.\ \frac{3}{2}\qquad D.\ \sqrt{3}$$

Rješenje 363

Ponovimo!

$$\sin\alpha+\sin\beta=2\cdot\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}\quad,\quad\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}\quad,\quad\frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$n=\frac{n}{1}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}\quad,\quad\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}.$$

$$\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}.$$

Parnost funkcije kosinus

$$\cos(-x)=\cos x.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\sin\frac{\pi}{12}+\sin\frac{5\cdot\pi}{12}=2\cdot\sin\frac{\frac{\pi}{12}+\frac{5\cdot\pi}{12}}{2}\cdot\cos\frac{\frac{\pi}{12}-\frac{5\cdot\pi}{12}}{2}=2\cdot\sin\frac{\frac{\pi+5\cdot\pi}{12}}{2}\cdot\cos\frac{\frac{\pi-5\cdot\pi}{12}}{2}=$$

$$=2\cdot\sin\frac{\frac{6\cdot\pi}{12}}{2}\cdot\cos\frac{\frac{-4\cdot\pi}{12}}{2}=2\cdot\sin\frac{\frac{6\cdot\pi}{12}}{2}\cdot\cos\frac{\frac{-4\cdot\pi}{12}}{2}=2\cdot\sin\frac{\frac{\pi}{2}}{2}\cdot\cos\frac{\frac{-\pi}{3}}{2}=2\cdot\sin\frac{\frac{\pi}{2}}{\frac{2}{1}}\cdot\cos\frac{\frac{-\pi}{3}}{\frac{2}{1}}=$$

$$=2\cdot\sin\frac{\pi}{4}\cdot\cos\left(-\frac{\pi}{6}\right)=2\cdot\sin\frac{\pi}{4}\cdot\cos\frac{\pi}{6}=2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}=2\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{2}}{1}\cdot\frac{\sqrt{3}}{2}=\frac{\sqrt{6}}{2}.$$

Odgovor je pod A.

Vježba 363

Točna vrijednost brojevnog izraza $\sin\frac{5\cdot\pi}{12}+\sin\frac{\pi}{12}$ jednaka je:

$$A.\ \frac{\sqrt{6}}{2}\qquad B.\ \frac{\sqrt{3}}{4}\qquad C.\ \frac{3}{2}\qquad D.\ \sqrt{3}$$

Rezultat: A.

Zadatak 364 (Vesna, gimnazija)

Ako je $tg(x)+ctg(x)=3$, koliko je $\frac{1}{\sin^2(x)}+\frac{1}{\cos^2(x)}$?

Rješenje 364

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad tg(x)\cdot ctg(x)=1\quad,\quad\cos^2(x)+\sin^2(x)=1.$$

$$tg(x)=\frac{\sin(x)}{\cos(x)} \quad , \quad ctg(x)=\frac{\cos(x)}{\sin(x)} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n} \quad , \quad a^n\cdot b^n=(a\cdot b)^n \quad , \quad n=\frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

1.inačica

Preoblikujemo zadanu jednadžbu.

$$tg(x)+ctg(x)=3 \Rightarrow tg(x)+ctg(x)=3 \ /^2 \Rightarrow (tg(x)+ctg(x))^2=3^2 \Rightarrow$$
$$\Rightarrow tg^2(x)+2\cdot tg(x)\cdot ctg(x)+ctg^2(x)=9 \Rightarrow tg^2(x)+2\cdot 1+ctg^2(x)=9 \Rightarrow$$
$$\Rightarrow tg^2(x)+2+ctg^2(x)=9 \Rightarrow tg^2(x)+ctg^2(x)=9-2 \Rightarrow tg^2(x)+ctg^2(x)=7.$$

Sada je:

$$\frac{1}{\sin^2(x)}+\frac{1}{\cos^2(x)}=\frac{\cos^2(x)+\sin^2(x)}{\sin^2(x)}+\frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)}=$$
$$=\frac{\cos^2(x)}{\sin^2(x)}+\frac{\sin^2(x)}{\sin^2(x)}+\frac{\cos^2(x)}{\cos^2(x)}+\frac{\sin^2(x)}{\cos^2(x)}=\frac{\cos^2(x)}{\sin^2(x)}+\frac{\sin^2(x)}{\sin^2(x)}+\frac{\cos^2(x)}{\cos^2(x)}+\frac{\sin^2(x)}{\cos^2(x)}=$$
$$=ctg^2(x)+1+1+tg^2(x)=tg^2(x)+ctg^2(x)+2=\left[\ tg^2(x)+ctg^2(x)=7\ \right]=7+2=9.$$

2.inačica

Preoblikujemo zadanu jednadžbu.

$$tg(x)+ctg(x)=3 \Rightarrow \frac{\sin(x)}{\cos(x)}+\frac{\cos(x)}{\sin(x)}=3 \Rightarrow \frac{\sin(x)}{\cos(x)}+\frac{\cos(x)}{\sin(x)}=3 \ /\cdot \cos(x)\cdot\sin(x) \Rightarrow$$
$$\Rightarrow \sin^2(x)+\cos^2(x)=3\cdot\cos(x)\cdot\sin(x) \Rightarrow 1=3\cdot\cos(x)\cdot\sin(x) \Rightarrow$$
$$\Rightarrow 3\cdot\cos(x)\cdot\sin(x)=1 \Rightarrow 3\cdot\cos(x)\cdot\sin(x)=1 \ /\cdot\frac{1}{3} \Rightarrow \cos(x)\cdot\sin(x)=\frac{1}{3}.$$

Sada je:

$$\frac{1}{\sin^2(x)}+\frac{1}{\cos^2(x)}=\frac{\cos^2(x)+\sin^2(x)}{\sin^2(x)\cdot\cos^2(x)}=\frac{1}{\sin^2(x)\cdot\cos^2(x)}=\frac{1}{(\sin(x)\cdot\cos(x))^2}=$$
$$=\left[\ \cos(x)+\sin(x)=\frac{1}{3}\ \right]=\frac{1}{\left(\frac{1}{3}\right)^2}=\frac{1}{\frac{1}{9}}=\frac{\frac{1}{1}}{\frac{1}{9}}=\frac{9}{1}=9.$$

3.inačica

Preoblikujemo zadanu jednadžbu.

$$tg(x)+ctg(x)=3 \Rightarrow \frac{\sin(x)}{\cos(x)}+\frac{\cos(x)}{\sin(x)}=3 \Rightarrow \frac{\sin^2(x)+\cos^2(x)}{\cos(x)\cdot\sin(x)}=3 \Rightarrow \frac{1}{\cos(x)\cdot\sin(x)}=3.$$

Sada je:

$$\frac{1}{\sin^2(x)}+\frac{1}{\cos^2(x)}=\frac{\cos^2(x)+\sin^2(x)}{\sin^2(x)\cdot\cos^2(x)}=\frac{1}{\sin^2(x)\cdot\cos^2(x)}=\frac{1}{(\sin(x)\cdot\cos(x))^2}=$$

$$=\left(\frac{1}{\cos(x)+\sin(x)}\right)^2=\left[\frac{1}{\cos(x)+\sin(x)}=3\right]=3^2=9.$$

Vježba 364

Ako je $tg(x)+ctg(x)-3=0$, koliko je $\frac{1}{\sin^2(x)}+\frac{1}{\cos^2(x)}$?

Rezultat: 9.

Zadatak 365 (Franjo, gimnazija)

Ako je $(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)=\cos\alpha\cdot\cos\beta\cdot\cos\gamma$, pojednostavnite izraz $A=(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)$.

Rješenje 365

Ponovimo!

$$n=\frac{n}{1} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \cos^2 x+\sin^2 x=1 \Rightarrow \cos^2 x=1-\sin^2 x.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}, \quad n\neq 0 \ , \ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$A=(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma) \Rightarrow A=\frac{(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)}{1} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l}\text{proširivanje}\\ \text{razlomka}\end{array}\right] \Rightarrow$$

$$\Rightarrow A=\frac{(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)}{1}\cdot\frac{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)} \Rightarrow$$

$$\Rightarrow A=\frac{(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)\cdot(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)} \Rightarrow$$

$$\Rightarrow A=\frac{(1-\sin\alpha)\cdot(1+\sin\alpha)\cdot(1-\sin\beta)\cdot(1+\sin\beta)\cdot(1-\sin\gamma)\cdot(1+\sin\gamma)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)} \Rightarrow$$

$$\Rightarrow A=\frac{\left(1-\sin^2\alpha\right)\cdot\left(1-\sin^2\beta\right)\cdot\left(1-\sin^2\gamma\right)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)} \Rightarrow A=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{c}\text{uvjet}\\ (1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)=\cos\alpha\cdot\cos\beta\cdot\cos\gamma\end{array}\right] \Rightarrow$$

$$\Rightarrow \; A=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{\cos\alpha\cdot\cos\beta\cdot\cos\gamma} \;\Rightarrow\; A=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{\cos\alpha\cdot\cos\beta\cdot\cos\gamma} \;\Rightarrow\; A=\cos\alpha\cdot\cos\beta\cdot\cos\gamma.$$

Vježba 365

Ako je $(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)-\cos\alpha\cdot\cos\beta\cdot\cos\gamma=0,$ pojednostavnite izraz $A=(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma).$

Rezultat: $A=\cos\alpha\cdot\cos\beta\cdot\cos\gamma.$

Zadatak 366 (Franjo, gimnazija)

Pojednostavnite izraz: $\dfrac{\sin^3 x+\cos^3 x}{\sin x+\cos x}+\dfrac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\dfrac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}.$

Rješenje 366

Ponovimo!

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right) \quad , \quad a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right).$$

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \; , \quad \cos^2\alpha+\sin^2\alpha=1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=\frac{(\sin x+\cos x)\cdot\left(\sin^2 x-\sin x\cdot\cos x+\cos^2 x\right)}{\sin x+\cos x}+$$

$$+\frac{(\sin x-\cos x)\cdot\left(\sin^2 x+\sin x\cdot\cos x+\cos^2 x\right)}{\sin x-\cos x}+\frac{\left(\sin^2 x\right)^2-\left(\cos^2 x\right)^2}{\sin^2 x-\cos^2 x}=$$

$$=\frac{(\sin x+\cos x)\cdot\left(\sin^2 x-\sin x\cdot\cos x+\cos^2 x\right)}{\sin x+\cos x}+$$

$$+\frac{(\sin x-\cos x)\cdot\left(\sin^2 x+\sin x\cdot\cos x+\cos^2 x\right)}{\sin x-\cos x}+\frac{\left(\sin^2 x-\cos^2 x\right)\cdot\left(\sin^2 x+\cos^2 x\right)}{\sin^2 x-\cos^2 x}=$$

$$=\sin^2 x-\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\sin x\cdot\cos x+\cos^2 x+$$

$$+\frac{\left(\sin^2 x-\cos^2 x\right)\cdot\left(\sin^2 x+\cos^2 x\right)}{\sin^2 x-\cos^2 x}=$$

$$=\sin^2 x-\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\cos^2 x=$$

$$=\sin^2 x-\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\sin x\cdot\cos x+\cos^2 x+\sin^2 x+\cos^2 x=$$

$$=\sin^2 x+\cos^2 x+\sin^2 x+\cos^2 x+\sin^2 x+\cos^2 x=3\cdot\sin^2 x+3\cdot\cos^2 x=$$

$$=3\cdot\left(\sin^2 x+\cos^2 x\right)=3\cdot 1=3.$$

Vježba 366

Pojednostavnite izraz: $\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\cos^4 x-\sin^4 x}{\cos^2 x-\sin^2 x}.$

Rezultat: 3.

Zadatak 367 (Doti, gimnazija)

Dokazati: $2\cdot\left(\sin^6 x+\cos^6 x\right)-3\cdot\left(\sin^4 x+\cos^4 x\right)+1=0.$

Rješenje 367

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right)\quad,\quad \cos^2\alpha+\sin^2\alpha=1.$$

$$a^2+2\cdot a\cdot b+b^2=(a+b)^2\quad,\quad a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$2\cdot\left(\sin^6 x+\cos^6 x\right)-3\cdot\left(\sin^4 x+\cos^4 x\right)+1=$$

$$=2\cdot\left(\left(\sin^2 x\right)^3+\left(\cos^2 x\right)^3\right)-3\cdot\left(\sin^4 x+\cos^4 x\right)+1=$$

$$=2\cdot\left(\sin^2 x+\cos^2 x\right)\cdot\left(\left(\sin^2 x\right)^2-\sin^2 x\cdot\cos^2 x+\left(\cos^2 x\right)^2\right)-3\cdot\left(\sin^4 x+\cos^4 x\right)+1=$$

$$=2\cdot 1\cdot\left(\sin^4 x-\sin^2 x\cdot\cos^2 x+\cos^4 x\right)-3\cdot\left(\sin^4 x+\cos^4 x\right)+1=$$

$$=2\cdot\left(\sin^4 x-\sin^2 x\cdot\cos^2 x+\cos^4 x\right)-3\cdot\left(\sin^4 x+\cos^4 x\right)+1=$$

$$=2\cdot\sin^4 x-2\cdot\sin^2 x\cdot\cos^2 x+2\cdot\cos^4 x-3\cdot\sin^4 x-3\cdot\cos^4 x+1=$$

$$=-\sin^4 x-2\cdot\sin^2 x\cdot\cos^2 x-\cos^4 x+1=-\left(\sin^4 x+2\cdot\sin^2 x\cdot\cos^2 x+\cos^4 x\right)+1=$$

$$=-\left(\left(\sin^2 x\right)^2+2\cdot\sin^2 x\cdot\cos^2 x+\left(\cos^2 x\right)^2\right)+1=-\left(\sin^2 x+\cos^2 x\right)^2+1=-1^2+1=-1+1=0.$$

2.inačica

Lijevu stranu postupno preoblikujemo na sljedeći način:

$$2\cdot\left(\sin^6 x+\cos^6 x\right)-3\cdot\left(\sin^4 x+\cos^4 x\right)+1=$$
$$=2\cdot\sin^6 x+2\cdot\cos^6 x-3\cdot\sin^4 x-3\cdot\cos^4 x+1=$$
$$=2\cdot\sin^6 x-2\cdot\sin^4 x+2\cdot\cos^6 x-2\cdot\cos^4 x-\sin^4 x-\cos^4 x+1=$$
$$=-2\cdot\sin^4 x\cdot\left(1-\sin^2 x\right)-2\cdot\cos^4 x\cdot\left(1-\cos^2 x\right)-\sin^4 x-\cos^4 x+1=$$
$$=-2\cdot\sin^4 x\cdot\cos^2 x-2\cdot\cos^4 x\cdot\sin^2 x-\sin^4 x-\cos^4 x+1=$$
$$=-2\cdot\sin^2 x\cdot\cos^2 x\cdot\left(\sin^2 x+\cos^2 x\right)-\sin^4 x-\cos^4 x+1=$$
$$=-2\cdot\sin^2 x\cdot\cos^2 x\cdot 1-\sin^4 x-\cos^4 x+1=-2\cdot\sin^2 x\cdot\cos^2 x-\sin^4 x-\cos^4 x+1=$$
$$=-\left(2\cdot\sin^2 x\cdot\cos^2 x+\sin^4 x+\cos^4 x\right)+1=-\left(\sin^4 x+2\cdot\sin^2 x\cdot\cos^2 x+\cos^4 x\right)+1=$$
$$=-\left(\left(\sin^2 x\right)^2+2\cdot\sin^2 x\cdot\cos^2 x+\left(\cos^2 x\right)^2\right)+1=-\left(\sin^2 x+\cos^2 x\right)^2+1=-1^2+1=-1+1=0.$$

Vježba 367

Dokazati: $2\cdot\left(\sin^6 x+\cos^6 x\right)+1=3\cdot\left(\sin^4 x+\cos^4 x\right).$

Rezultat: Dokaz analogan.

Zadatak 368 (Doti, gimnazija)

Izračunajte vrijednost izraza: $\cos^2 18^\circ+\cos^2 36^\circ+\cos^2 54^\circ+\cos^2 72^\circ.$

Rješenje 368

Ponovimo!

$$\sin\left(90^\circ-\alpha\right)=\cos\alpha \quad , \quad \cos\left(90^\circ-\alpha\right)=\sin\alpha \quad , \quad \sin^2\alpha+\cos^2\alpha=1.$$

$$\cos^2 18^\circ+\cos^2 36^\circ+\cos^2 54^\circ+\cos^2 72^\circ.$$
$$=\sin^2\left(90^\circ-18^\circ\right)+\sin^2\left(90^\circ-36^\circ\right)+\cos^2 54^\circ+\cos^2 72^\circ=$$
$$=\sin^2 72^\circ+\sin^2 54^\circ+\cos^2 54^\circ+\cos^2 72^\circ=$$
$$=\left(\sin^2 72^\circ+\cos^2 72^\circ\right)+\left(\sin^2 54^\circ+\cos^2 54^\circ\right)=1+1=2.$$

Vježba 368

Izračunajte vrijednost izraza: $\cos^2 19^\circ+\cos^2 33^\circ+\cos^2 57^\circ+\cos^2 71^\circ.$

Rezultat: 2.

Zadatak 369 (Doti, gimnazija)

Ako je $\alpha+\beta=\frac{\pi}{4}$ pokazati da je $(1+tg\,\alpha)\cdot(1+tg\,\beta)=2.$

Rješenje 369

Ponovimo!

$$tg\left(\alpha-\beta\right)=\frac{tg\,\alpha-tg\,\beta}{1+tg\,\alpha\cdot tg\,\beta} \quad , \quad tg\frac{\pi}{4}=1 \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Budući da je

$$\alpha+\beta=\frac{\pi}{4}\ \Rightarrow\ \beta=\frac{\pi}{4}-\alpha$$

pa vrijedi:

$$(1+tg\,\alpha)\cdot(1+tg\,\beta)=(1+tg\,\alpha)\cdot\left(1+tg\left(\frac{\pi}{4}-\alpha\right)\right)=(1+tg\,\alpha)\cdot\left(1+\frac{tg\,\frac{\pi}{4}-tg\,\alpha}{1+tg\,\frac{\pi}{4}\cdot tg\,\alpha}\right)=$$

$$=(1+tg\,\alpha)\cdot\left(1+\frac{1-tg\,\alpha}{1+1\cdot tg\,\alpha}\right)=(1+tg\,\alpha)\cdot\left(1+\frac{1-tg\,\alpha}{1+tg\,\alpha}\right)=(1+tg\,\alpha)\cdot\left(\frac{1}{1}+\frac{1-tg\,\alpha}{1+tg\,\alpha}\right)=$$

$$=(1+tg\,\alpha)\cdot\frac{1+tg\,\alpha+1-tg\,\alpha}{1+tg\,\alpha}=(1+tg\,\alpha)\cdot\frac{1+tg\,\alpha+1-tg\,\alpha}{1+tg\,\alpha}=(1+tg\,\alpha)\cdot\frac{2}{1+tg\,\alpha}=$$

$$=(1+tg\,\alpha)\cdot\frac{2}{1+tg\,\alpha}=2.$$

Vježba 369

Ako je $\alpha+\beta=\frac{\pi}{4}$ pokazati da je $(1+tg\,\alpha)\cdot(1+tg\,\beta)-2=0.$

Rezultat: Dokaz analogan.

Zadatak 370 (Ekipa ☺, gimnazija)

Ako je $\sin\left(\frac{\pi}{4}-x\right)=-\frac{2}{3},\ \frac{\pi}{4}<x<\frac{\pi}{2}$, izračunaj $\sin x$.

Rješenje 370

Ponovimo!

$$a<b\ ,\ c<0\ \Rightarrow\ a\cdot c>b\cdot c\quad,\quad a<b<c\ \Rightarrow\ c>b>a\quad,\quad a<b\ ,\ c\in R\ \Rightarrow\ a+c<b+c.$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad n=\frac{n}{1}\quad,\quad\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

$$\frac{a}{b}\cdot\frac{b}{a}=1\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad\left(\sqrt{a}\right)^2=a.$$

$$\sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b}\quad,\quad\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}\quad,\quad\cos^2 x+\sin^2 x=1\quad,\quad\sin\frac{\pi}{4}=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}.$$

U četvrtom kvadrantu $\left\langle\frac{3\cdot\pi}{2},\ 2\cdot\pi\right\rangle$ vrijedi $sin\,x<0$ i $\cos x>0$.

$$\sin(\alpha-\beta)=\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta\quad,\quad\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Uočimo da vrijedi:

$$\frac{\pi}{4}<x<\frac{\pi}{2} \Rightarrow \frac{\pi}{4}<x<\frac{\pi}{2} \;/\cdot(-1) \Rightarrow -\frac{\pi}{4}>-x>-\frac{\pi}{2} \Rightarrow -\frac{\pi}{4}>-x>-\frac{\pi}{2} \;/+\frac{\pi}{4} \Rightarrow$$

$$\Rightarrow -\frac{\pi}{4}+\frac{\pi}{4}>-x+\frac{\pi}{4}>-\frac{\pi}{2}+\frac{\pi}{4} \Rightarrow -\frac{\pi}{4}+\frac{\pi}{4}>\frac{\pi}{4}-x>\frac{-2\cdot\pi+\pi}{4} \Rightarrow$$

$$\Rightarrow 0>\frac{\pi}{4}-x>-\frac{\pi}{4} \Rightarrow -\frac{\pi}{4}<\frac{\pi}{4}-x<0 \Rightarrow \left[\begin{array}{l}\text{četvrti}\\ \text{kvadrant}\end{array}\right] \Rightarrow \left.\begin{array}{l}\sin\left(\frac{\pi}{4}-x\right)<0\\ \cos\left(\frac{\pi}{4}-x\right)>0\end{array}\right\}.$$

Preoblikujemo zadanu jednadžbu uporabom formule za sinus razlike.

$$\sin\left(\frac{\pi}{4}-x\right)=-\frac{2}{3} \Rightarrow \sin\frac{\pi}{4}\cdot\cos x-\cos\frac{\pi}{4}\cdot\sin x=-\frac{2}{3} \Rightarrow \frac{\sqrt{2}}{2}\cdot\cos x-\frac{\sqrt{2}}{2}\cdot\sin x=-\frac{2}{3} \Rightarrow$$

$$\Rightarrow \frac{\sqrt{2}}{2}\cdot\cos x-\frac{\sqrt{2}}{2}\cdot\sin x=-\frac{2}{3} \;/\cdot\frac{2}{\sqrt{2}} \Rightarrow \cos x-\sin x=-\frac{4}{3\cdot\sqrt{2}} \Rightarrow \left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right] \Rightarrow$$

$$\Rightarrow \cos x-\sin x=-\frac{4}{3\cdot\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \cos x-\sin x=-\frac{4\cdot\sqrt{2}}{3\cdot\left(\sqrt{2}\right)^2} \Rightarrow \cos x-\sin x=-\frac{4\cdot\sqrt{2}}{3\cdot 2} \Rightarrow$$

$$\Rightarrow \cos x-\sin x=-\frac{4\cdot\sqrt{2}}{3\cdot 2} \Rightarrow \cos x-\sin x=-\frac{2\cdot\sqrt{2}}{3}.$$

Budući da su trigonometrijske funkcije sin x i cos x povezane relacijom

$$\cos^2 x+\sin^2 x=1,$$

vrijedi:

$$\cos^2\left(\frac{\pi}{4}-x\right)+\sin^2\left(\frac{\pi}{4}-x\right)=1 \Rightarrow \cos^2\left(\frac{\pi}{4}-x\right)=1-\sin^2\left(\frac{\pi}{4}-x\right) \Rightarrow$$

$$\Rightarrow \left[\sin\left(\frac{\pi}{4}-x\right)=-\frac{2}{3}\right] \Rightarrow \cos^2\left(\frac{\pi}{4}-x\right)=1-\left(-\frac{2}{3}\right)^2 \Rightarrow \cos^2\left(\frac{\pi}{4}-x\right)=1-\frac{4}{9} \Rightarrow$$

$$\Rightarrow \cos^2\left(\frac{\pi}{4}-x\right)=\frac{1}{1}-\frac{4}{9} \Rightarrow \cos^2\left(\frac{\pi}{4}-x\right)=\frac{9-4}{9} \Rightarrow \cos^2\left(\frac{\pi}{4}-x\right)=\frac{5}{9} \Rightarrow$$

$$\Rightarrow \cos^2\left(\frac{\pi}{4}-x\right)=\frac{5}{9} \;/\sqrt{\ } \Rightarrow \cos\left(\frac{\pi}{4}-x\right)=\pm\sqrt{\frac{5}{9}} \Rightarrow \cos\left(\frac{\pi}{4}-x\right)=\pm\frac{\sqrt{5}}{\sqrt{9}} \Rightarrow$$

$$\Rightarrow \cos\left(\frac{\pi}{4}-x\right)=\pm\frac{\sqrt{5}}{3} \Rightarrow \left[\begin{array}{c}\text{četvrti kvadrant}\\ -\frac{\pi}{4}<\frac{\pi}{4}-x<0\end{array}\right] \Rightarrow \cos\left(\frac{\pi}{4}-x\right)=\frac{\sqrt{5}}{3}.$$

Preoblikujemo tu jednadžbu uporabom formule za kosinus razlike.

$$\cos\left(\frac{\pi}{4}-x\right)=\frac{\sqrt{5}}{3} \Rightarrow \cos\frac{\pi}{4}\cdot\cos x+\sin\frac{\pi}{4}\cdot\sin x=\frac{\sqrt{5}}{3} \Rightarrow \frac{\sqrt{2}}{2}\cdot\cos x+\frac{\sqrt{2}}{2}\cdot\sin x=\frac{\sqrt{5}}{3} \Rightarrow$$

$$\Rightarrow \frac{\sqrt{2}}{2}\cdot\cos x+\frac{\sqrt{2}}{2}\cdot\sin x=\frac{\sqrt{5}}{3} \;/\cdot\frac{2}{\sqrt{2}} \Rightarrow \cos x+\sin x=\frac{2\cdot\sqrt{5}}{3\cdot\sqrt{2}} \Rightarrow \left[\begin{array}{l}\text{racionalizacija}\\ \text{nazivnika}\end{array}\right] \Rightarrow$$

$$\Rightarrow \cos x+\sin x=\frac{2\cdot\sqrt{5}}{3\cdot\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \Rightarrow \cos x+\sin x=\frac{2\cdot\sqrt{10}}{3\cdot\left(\sqrt{2}\right)^2} \Rightarrow \cos x+\sin x=\frac{2\cdot\sqrt{10}}{3\cdot 2} \Rightarrow$$

$$\Rightarrow \cos x+\sin x=\frac{2\cdot\sqrt{10}}{3\cdot 2} \Rightarrow \cos x+\sin x=\frac{\sqrt{10}}{3}.$$

Iz sustava trigonometrijskih jednadžba dobijemo sin x.

$$\left.\begin{aligned}\cos x-\sin x&=-\frac{2\cdot\sqrt{2}}{3}\\ \cos x+\sin x&=\frac{\sqrt{10}}{3}\end{aligned}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda suprotnih}\\ \text{koeficijenata}\end{array}\right] \Rightarrow \left.\begin{aligned}\cos x-\sin x&=-\frac{2\cdot\sqrt{2}}{3}\ /\cdot(-1)\\ \cos x+\sin x&=\frac{\sqrt{10}}{3}\end{aligned}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{aligned}-\cos x+\sin x&=\frac{2\cdot\sqrt{2}}{3}\\ \cos x+\sin x&=\frac{\sqrt{10}}{3}\end{aligned}\right\} \Rightarrow 2\cdot\sin x=\frac{2\cdot\sqrt{2}}{3}+\frac{\sqrt{10}}{3} \Rightarrow 2\cdot\sin x=\frac{2\cdot\sqrt{2}+\sqrt{10}}{3} \Rightarrow$$

$$\Rightarrow 2\cdot\sin x=\frac{2\cdot\sqrt{2}+\sqrt{10}}{3}\ /\cdot\frac{1}{2} \Rightarrow \sin x=\frac{2\cdot\sqrt{2}+\sqrt{10}}{6}.$$

Vježba 370

Ako je $\sin\left(\frac{\pi}{4}-x\right)+\frac{2}{3}=0,\ \frac{\pi}{4}<x<\frac{\pi}{2}$, izračunaj $\sin x$.

Rezultat: $\sin x=\frac{2\cdot\sqrt{2}+\sqrt{10}}{6}.$

Zadatak 371 (Marija, gimnazija)

Ako je $tg\ x+tg\ y=25,\ ctg\ x+ctg\ y=30$, koliko je $tg\left(x+y\right)$?

Rješenje 371

Ponovimo!

$$ctg\ \alpha=\frac{1}{tg\ \alpha}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad n=\frac{n}{1}.$$

$$\frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\left.\begin{aligned}tg\ x+tg\ y&=25\\ ctg\ x+ctg\ y&=30\end{aligned}\right\} \Rightarrow \left.\begin{aligned}tg\ x+tg\ y&=25\\ \frac{1}{tg\ x}+\frac{1}{tg\ y}&=30\end{aligned}\right\} \Rightarrow \left.\begin{aligned}tg\ x+tg\ y&=25\\ \frac{tg\ y+tg\ x}{tg\ x\cdot tg\ y}&=30\end{aligned}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda}\\ \text{zamjene}\end{array}\right] \Rightarrow$$

$$\Rightarrow \frac{25}{tg\ x\cdot tg\ y}=30 \Rightarrow \frac{25}{tg\ x\cdot tg\ y}=\frac{30}{1} \Rightarrow \frac{tg\ x\cdot tg\ y}{25}=\frac{1}{30} \Rightarrow \frac{tg\ x\cdot tg\ y}{25}=\frac{1}{30}\ /\cdot 25 \Rightarrow$$

$$\Rightarrow \; tg\,x \cdot tg\,y = \frac{25}{30} \;\Rightarrow\; tg\,x \cdot tg\,y = \frac{25}{30} \;\Rightarrow\; tg\,x \cdot tg\,y = \frac{5}{6}.$$

Dalje izravno računamo:

$$tg\left(x+y\right) = \frac{tg\,x + tg\,y}{1 - tg\,x \cdot tg\,y} = \begin{bmatrix} tg\,x + tg\,y = 25 \\ tg\,x \cdot tg\,y = \frac{5}{6} \end{bmatrix} = \frac{25}{1-\frac{5}{6}} = \frac{25}{\frac{1}{1}-\frac{5}{6}} = \frac{25}{\frac{6-5}{6}} = \frac{25}{\frac{1}{6}} = \frac{\frac{25}{1}}{\frac{1}{6}} = \frac{150}{1} = 150.$$

Vježba 371

Ako je $tg\,x + tg\,y - 25 = 0,\; ctg\,x + ctg\,y - 30 = 0,$ koliko je $tg\left(x+y\right)$?

Rezultat: 150.

Zadatak 372 (Marija, gimnazija)

Ako je $\alpha + \beta = \frac{\pi}{4},$ koliko je $\left(1 + tg\,\alpha\right) \cdot \left(1 + tg\,\beta\right)$?

Rješenje 372

Ponovimo!

$$tg\left(x+y\right) = \frac{tg\,x + tg\,y}{1 - tg\,x \cdot tg\,y} \quad , \quad tg\,\frac{\pi}{4} = 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot \left(b+c\right) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot \left(b+c\right).$$

Množenje zagrada

$$\left(a+b\right) \cdot \left(c+d\right) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

$$\alpha + \beta = \frac{\pi}{4} \;\Rightarrow\; tg\left(\alpha+\beta\right) = tg\,\frac{\pi}{4} \;\Rightarrow\; \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha \cdot tg\,\beta} = 1 \;\Rightarrow\; \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha \cdot tg\,\beta} = 1 \;/ \cdot \left(1 - tg\,\alpha \cdot tg\,\beta\right) \;\Rightarrow$$

$$\Rightarrow tg\,\alpha + tg\,\beta = 1 - tg\,\alpha \cdot tg\,\beta.$$

Dalje slijedi:

$$\left(1 + tg\,\alpha\right) \cdot \left(1 + tg\,\beta\right) = 1 + tg\,\beta + tg\,\alpha + tg\,\alpha \cdot tg\,\beta = 1 + \left(tg\,\beta + tg\,\alpha\right) + tg\,\alpha \cdot tg\,\beta =$$

$$= 1 + \left(tg\,\alpha + tg\,\beta\right) + tg\,\alpha \cdot tg\,\beta = \begin{bmatrix} \text{uvjet} \\ tg\,\alpha + tg\,\beta = 1 - tg\,\alpha \cdot tg\,\beta \end{bmatrix} =$$

$$= 1 + 1 - tg\,\alpha \cdot tg\,\beta + tg\,\alpha \cdot tg\,\beta = 1 + 1 - tg\,\alpha \cdot tg\,\beta + tg\,\alpha \cdot tg\,\beta = 2.$$

Vježba 372

Ako je $\alpha + \beta - \frac{\pi}{4} = 0,$ koliko je $\left(1 + tg\,\alpha\right) \cdot \left(1 + tg\,\beta\right)$?

Rezultat: 2.

Zadatak 373 (Marija, gimnazija)

Ako je $x + y = \frac{\pi}{2},\; x \neq 0,$ koliko je $\frac{tg\left(x-y\right)}{tg\,x - tg\,y}$?

Rješenje 373

Ponovimo!

$$\frac{a}{b} = \frac{1}{b} \cdot a \quad , \quad tg\left(\alpha-\beta\right) = \frac{tg\,\alpha - tg\,\beta}{1 + tg\,\alpha \cdot tg\,\beta} \quad , \quad tg\left(\frac{\pi}{2} - \alpha\right) = ctg\,\alpha \quad , \quad tg\,\alpha \cdot ctg\,\alpha = 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$\frac{tg\left(x-y\right)}{tg\ x - tg\ y} = \frac{1}{tg\ x - tg\ y} \cdot tg\left(x-y\right) = \frac{1}{tg\ x - tg\ y} \cdot \frac{tg\ x - tg\ y}{1 + tg\ x \cdot tg\ y} = \frac{1}{tg\ x - tg\ y} \cdot \frac{tg\ x - tg\ y}{1 + tg\ x \cdot tg\ y} =$$

$$= \frac{1}{1 + tg\ x \cdot tg\ y} = \left[\begin{array}{c} \text{uvjet} \\ x + y = \frac{\pi}{2} \quad \Rightarrow \quad y = \frac{\pi}{2} - x \end{array} \right] = \frac{1}{1 + tg\ x \cdot tg\left(\frac{\pi}{2} - x\right)} = \frac{1}{1 + tg\ x \cdot ctg\ x} = \frac{1}{1+1} = \frac{1}{2}.$$

Vježba 373

Ako je $x + y - \frac{\pi}{2} = 0,\ x \neq 0,$ koliko je $\frac{tg\left(x-y\right)}{tg\ x - tg\ y}$?

Rezultat: $\frac{1}{2}.$

Zadatak 374 (Luka, gimnazija)

Pojednostavnite: $\frac{1 - \sin\alpha \cdot \cos\alpha}{\cos\alpha \cdot \left(\frac{1}{\cos\alpha} - \frac{1}{\sin\alpha}\right)} \cdot \frac{\sin^2\alpha - \cos^2\alpha}{\sin^3\alpha + \cos^3\alpha}.$

Rješenje 374

Ponovimo!

$$\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad a^2 - b^2 = \left(a-b\right) \cdot \left(a+b\right) \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad n = \frac{n}{1}.$$

$$a^3 + b^3 = \left(a+b\right) \cdot \left(a^2 - a \cdot b + b^2\right) \quad , \quad \sin^2 x + \cos^2 x = 1 \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$\frac{1 - \sin\alpha \cdot \cos\alpha}{\cos\alpha \cdot \left(\frac{1}{\cos\alpha} - \frac{1}{\sin\alpha}\right)} \cdot \frac{\sin^2\alpha - \cos^2\alpha}{\sin^3\alpha + \cos^3\alpha} =$$

$$= \frac{1 - \sin\alpha \cdot \cos\alpha}{\cos\alpha \cdot \frac{\sin\alpha - \cos\alpha}{\cos\alpha \cdot \sin\alpha}} \cdot \frac{\left(\sin\alpha - \cos\alpha\right) \cdot \left(\sin\alpha + \cos\alpha\right)}{\left(\sin\alpha + \cos\alpha\right) \cdot \left(\sin^2\alpha - \sin\alpha \cdot \cos\alpha + \cos^2\alpha\right)} =$$

$$= \frac{1 - \sin\alpha \cdot \cos\alpha}{\cos\alpha \cdot \frac{\sin\alpha - \cos\alpha}{\cos\alpha \cdot \sin\alpha}} \cdot \frac{\left(\sin\alpha - \cos\alpha\right) \cdot \left(\sin\alpha + \cos\alpha\right)}{\left(\sin\alpha + \cos\alpha\right) \cdot \left(\sin^2\alpha - \sin\alpha \cdot \cos\alpha + \cos^2\alpha\right)} =$$

$$=\frac{1-\sin\alpha\cdot\cos\alpha}{\frac{\sin\alpha-\cos\alpha}{\sin\alpha}}\cdot\frac{\sin\alpha-\cos\alpha}{\sin^2\alpha-\sin\alpha\cdot\cos\alpha+\cos^2\alpha}=\frac{\frac{1-\sin\alpha\cdot\cos\alpha}{1}}{\frac{\sin\alpha-\cos\alpha}{\sin\alpha}}\cdot\frac{\sin\alpha-\cos\alpha}{\sin^2\alpha+\cos^2\alpha-\sin\alpha\cdot\cos\alpha}=$$

$$=\frac{\sin\alpha\cdot(1-\sin\alpha\cdot\cos\alpha)}{\sin\alpha-\cos\alpha}\cdot\frac{\sin\alpha-\cos\alpha}{1-\sin\alpha\cdot\cos\alpha}=\frac{\sin\alpha\cdot(1-\sin\alpha\cdot\cos\alpha)}{\sin\alpha-\cos\alpha}\cdot\frac{\sin\alpha-\cos\alpha}{1-\sin\alpha\cdot\cos\alpha}=\sin\alpha.$$

Vježba 374

Pojednostavnite : $\dfrac{\sin\alpha\cdot\cos\alpha-1}{\cos\alpha\cdot\left(\frac{1}{\cos\alpha}-\frac{1}{\sin\alpha}\right)}\cdot\dfrac{\cos^2\alpha-\sin^2\alpha}{\sin^3\alpha+\cos^3\alpha}.$

Rezultat: $\sin\alpha.$

Zadatak 375 (Luka, gimnazija)

Dokazati da je $\alpha+\beta=\frac{\pi}{4}$, ako je $tg\,\alpha=\frac{n}{n+1}$, $tg\,\beta=\frac{1}{2\cdot n+1}$.

Rješenje 375

Ponovimo!

$$tg\,(x+y)=\frac{tg\,x+tg\,y}{1-tg\,x\cdot tg\,y}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad n=\frac{n}{1}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad tg\frac{\pi}{4}=1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$tg\,(\alpha+\beta)=\frac{tg\,\alpha+tg\,\beta}{1-tg\,\alpha\cdot tg\,\beta}\Rightarrow\left[\begin{array}{l}tg\,\alpha=\frac{n}{n+1}\\ tg\,\beta=\frac{1}{2\cdot n+1}\end{array}\right]\Rightarrow tg\,(\alpha+\beta)=\frac{\frac{n}{n+1}+\frac{1}{2\cdot n+1}}{1-\frac{n}{n+1}\cdot\frac{1}{2\cdot n+1}}\Rightarrow$$

$$\Rightarrow tg\,(\alpha+\beta)=\frac{\frac{n\cdot(2\cdot n+1)+n+1}{(n+1)\cdot(2\cdot n+1)}}{1-\frac{n}{(n+1)\cdot(2\cdot n+1)}}\Rightarrow tg\,(\alpha+\beta)=\frac{\frac{2\cdot n^2+n+n+1}{(n+1)\cdot(2\cdot n+1)}}{\frac{1}{1}-\frac{n}{(n+1)\cdot(2\cdot n+1)}}\Rightarrow$$

$$\Rightarrow tg\,(\alpha+\beta)=\frac{\frac{2\cdot n^2+2\cdot n+1}{(n+1)\cdot(2\cdot n+1)}}{\frac{(n+1)\cdot(2\cdot n+1)-n}{(n+1)\cdot(2\cdot n+1)}}\Rightarrow tg\,(\alpha+\beta)=\frac{\frac{2\cdot n^2+2\cdot n+1}{(n+1)\cdot(2\cdot n+1)}}{\frac{(n+1)\cdot(2\cdot n+1)-n}{(n+1)\cdot(2\cdot n+1)}}\Rightarrow$$

$$\Rightarrow tg(\alpha+\beta)=\frac{\frac{2\cdot n^2+2\cdot n+1}{1}}{\frac{(n+1)\cdot(2\cdot n+1)-n}{1}} \Rightarrow tg(\alpha+\beta)=\frac{2\cdot n^2+2\cdot n+1}{(n+1)\cdot(2\cdot n+1)-n} \Rightarrow$$

$$\Rightarrow tg(\alpha+\beta)=\frac{2\cdot n^2+2\cdot n+1}{2\cdot n^2+n+2\cdot n+1-n} \Rightarrow tg(\alpha+\beta)=\frac{2\cdot n^2+2\cdot n+1}{2\cdot n^2+n+2\cdot n+1-n} \Rightarrow$$

$$\Rightarrow tg(\alpha+\beta)=\frac{2\cdot n^2+2\cdot n+1}{2\cdot n^2+2\cdot n+1} \Rightarrow tg(\alpha+\beta)=\frac{2\cdot n^2+2\cdot n+1}{2\cdot n^2+2\cdot n+1} \Rightarrow tg(\alpha+\beta)=1 \Rightarrow$$

$$\Rightarrow \alpha+\beta=tg^{-1}1 \Rightarrow \alpha+\beta=\frac{\pi}{4}.$$

Vježba 375

Dokazati da je $\alpha+\beta=\frac{\pi}{4}$, ako je $tg\,\alpha-\frac{n}{n+1}=0,\ tg\,\beta-\frac{1}{2\cdot n+1}=0.$

Rezultat: Dokaz analogan.

Zadatak 376 (Ivana, gimnazija)

Ako je $x=\frac{1}{\cos\alpha\cdot\cos\beta},\ y=\frac{tg\,\alpha}{\cos\beta},\ z=tg\,\beta$ izračunati vrijednost izraza $A=x^2-y^2-z^2.$

Rješenje 376

Ponovimo!

$$\frac{1}{\cos^2 x}=1+tg^2 x \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \frac{a}{b}=\frac{1}{b}\cdot a.$$

$$\frac{a\cdot b}{c\cdot d}=\frac{a}{c}\cdot\frac{b}{d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$\left.\begin{array}{l} x=\frac{1}{\cos\alpha\cdot\cos\beta} \\ y=\frac{tg\,\alpha}{\cos\beta} \\ z=tg\,\beta \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=\frac{1}{\cos\alpha\cdot\cos\beta}\ /^2 \\ y=\frac{tg\,\alpha}{\cos\beta}\ /^2 \\ z=tg\,\beta\ /^2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2=\frac{1}{\cos^2\alpha\cdot\cos^2\beta} \\ y^2=\frac{tg^2\alpha}{\cos^2\beta} \\ z^2=tg^2\beta \end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l} x^2=\dfrac{1}{\cos^2\alpha}\cdot\dfrac{1}{\cos^2\beta} \\ \Rightarrow\ y^2=\dfrac{1}{\cos^2\beta}\cdot tg^2\alpha \\ z^2=tg^2\beta \end{array}\right\} \Rightarrow \left[\dfrac{1}{\cos^2 x}=1+tg^2 x\right] \Rightarrow \left.\begin{array}{l} x^2=\left(1+tg^2\alpha\right)\cdot\left(1+tg^2\beta\right) \\ y^2=\left(1+tg^2\beta\right)\cdot tg^2\alpha \\ z^2=tg^2\beta \end{array}\right\}.$$

Sada je:

$$A=x^2-y^2-z^2 \Rightarrow A=\left(1+tg^2\alpha\right)\cdot\left(1+tg^2\beta\right)-\left(1+tg^2\beta\right)\cdot tg^2\alpha-tg^2\beta \Rightarrow$$

$$\Rightarrow A=1+tg^2\beta+tg^2\alpha+tg^2\alpha\cdot tg^2\beta-tg^2\alpha-tg^2\alpha\cdot tg^2\beta-tg^2\beta \Rightarrow$$

$$\Rightarrow A=1+tg^2\beta+tg^2\alpha+tg^2\alpha\cdot tg^2\beta-tg^2\alpha-tg^2\alpha\cdot tg^2\beta-tg^2\beta \Rightarrow A=1.$$

Vježba 376

Ako je $x=\dfrac{1}{\cos\alpha\cdot\cos\beta}$, $y=\dfrac{tg\,\alpha}{\cos\beta}$, $z=tg\,\beta$ izračunati vrijednost izraza $A=x^2-\left(y^2+z^2\right)$.

Rezultat: $A=1.$

Zadatak 377 (Josip, gimnazija)

Broj $\dfrac{\pi}{8}$ rješenje je jednadžbe $\dfrac{1}{\sin^2 x}+\dfrac{1}{\cos^2 x}=a$ ako je:

$A.\ a=8$ $B.\ a=6$ $C.\ a=4$ $D.\ a=2$

Rješenje 377

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \cos^2 x+\sin^2 x=1\quad,\quad a^n\cdot b^n=(a\cdot b)^n\quad,\quad 2\cdot\sin x\cdot\cos x=\sin(2\cdot x).$$

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad,\quad \left(\sqrt{a}\right)^2=a\quad,\quad \sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=a \Rightarrow \frac{\cos^2 x+\sin^2 x}{\sin^2 x\cdot\cos^2 x}=a \Rightarrow \frac{1}{\sin^2 x\cdot\cos^2 x}=a \Rightarrow \left[\begin{array}{l}\text{proširimo}\\ \text{razlomak}\\ \text{brojem } 4\end{array}\right] \Rightarrow$$

$$\Rightarrow \frac{4}{4\cdot\sin^2 x\cdot\cos^2 x}=a \Rightarrow \frac{4}{(2\cdot\sin x\cdot\cos x)^2}=a \Rightarrow \frac{4}{\sin^2(2\cdot x)}=a \Rightarrow \left[x=\frac{\pi}{8}\right] \Rightarrow$$

$$\Rightarrow \frac{4}{\sin^2\left(2\cdot\frac{\pi}{8}\right)}=a \Rightarrow \frac{4}{\sin^2\left(2\cdot\frac{\pi}{8}\right)}=a \Rightarrow \frac{4}{\sin^2\left(\frac{\pi}{4}\right)}=a \Rightarrow \frac{4}{\left(\frac{\sqrt{2}}{2}\right)^2}=a \Rightarrow$$

$$\Rightarrow \frac{\frac{4}{1}}{\frac{2}{4}}=a \Rightarrow \frac{16}{2}=a \Rightarrow \frac{16}{2}=a \Rightarrow 8=a \Rightarrow a=8.$$

Odgovor je pod A.

Vježba 377

Broj $\frac{\pi}{8}$ rješenje je jednadžbe $\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=2\cdot a$ ako je:

A. $a=8$ *B.* $a=6$ *C.* $a=4$ *D.* $a=2$

Rezultat: C.

Zadatak 378 (Josip, gimnazija)

Broj $\frac{3\cdot\pi}{4}$ rješenje je jednadžbe $\sin^2 x+a\cdot\sin x\cdot\cos x+\cos^2 x=0$ ako je:

A. $a=-1$ *B.* $a=2$ *C.* $a=\frac{1}{2}$ *D.* $a=1$

Rješenje 378

Ponovimo!

$$\cos^2 x+\sin^2 x=1 \quad , \quad 2\cdot\sin x\cdot\cos x=\sin(2\cdot x) \quad , \quad \sin\frac{3\cdot\pi}{4}=-1.$$

$$\frac{a}{b}=1 \Rightarrow a=b \quad , \quad n=\frac{n}{1}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \ , \ n\neq 0 \ , \ n\neq 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$\sin^2 x+a\cdot\sin x\cdot\cos x+\cos^2 x=0 \Rightarrow \sin^2 x+\cos^2 x+a\cdot\sin x\cdot\cos x=0 \Rightarrow$$

$$\Rightarrow 1+a\cdot\sin x\cdot\cos x=0 \Rightarrow 1+\frac{a\cdot\sin x\cdot\cos x}{1}=0 \Rightarrow \left[\begin{array}{l}\text{proširimo}\\ \text{razlomak}\\ \text{brojem } 2\end{array}\right] \Rightarrow$$

$$\Rightarrow 1+\frac{2\cdot a\cdot \sin x\cdot \cos x}{2}=0 \Rightarrow 1+\frac{a\cdot 2\cdot \sin x\cdot \cos x}{2}=0 \Rightarrow 1+\frac{a\cdot \sin(2\cdot x)}{2}=0 \Rightarrow$$

$$\Rightarrow \left[x=\frac{3\cdot\pi}{4}\right] \Rightarrow 1+\frac{a\cdot\sin\left(2\cdot\frac{3\cdot\pi}{4}\right)}{2}=0 \Rightarrow 1+\frac{a\cdot\sin\left(2\cdot\frac{3\cdot\pi}{4}\right)}{2}=0 \Rightarrow 1+\frac{a\cdot\sin\left(\frac{3\cdot\pi}{2}\right)}{2}=0 \Rightarrow$$

$$\Rightarrow 1+\frac{a\cdot(-1)}{2}=0 \Rightarrow 1-\frac{a}{2}=0 \Rightarrow 1=\frac{a}{2} \Rightarrow \frac{a}{2}=1 \Rightarrow a=2.$$

Odgovor je pod B.

Vježba 378

Broj $\frac{3\cdot\pi}{4}$ rješenje je jednadžbe $\sin^2 x+2\cdot a\cdot\sin x\cdot\cos x+\cos^2 x=0$ ako je:

$A.\ a=-1 \qquad B.\ a=2 \qquad C.\ a=\frac{1}{2} \qquad D.\ a=1$

Rezultat: D.

Zadatak 379 (Anita, gimnazija)

Dokaži identitet: $\frac{1-\cos^4 x}{\cos^2 x}=\sin^2 x+tg^2 x.$

Rješenje 379

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \cos^2 x+\sin^2 x=1.$$

$$\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n} \quad , \quad tg\, x=\frac{\sin x}{\cos x}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{1-\cos^4 x}{\cos^2 x}=\frac{1^2-\left(\cos^2 x\right)^2}{\cos^2 x}=\frac{\left(1-\cos^2 x\right)\cdot\left(1+\cos^2 x\right)}{\cos^2 x}=\frac{\sin^2 x\cdot\left(1+\cos^2 x\right)}{\cos^2 x}=$$

$$=\frac{\sin^2 x+\sin^2 x\cdot\cos^2 x}{\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}+\frac{\sin^2 x\cdot\cos^2 x}{\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}+\frac{\sin^2 x\cdot\cos^2 x}{\cos^2 x}=$$

$$=tg^2 x+\sin^2 x=\sin^2 x+tg^2 x.$$

2.inačica

$$\frac{1-\cos^4 x}{\cos^2 x}=\frac{\sin^2 x+\cos^2 x-\cos^4 x}{\cos^2 x}=\frac{\sin^2 x+\cos^2 x\cdot\left(1-\cos^2 x\right)}{\cos^2 x}=\frac{\sin^2 x+\cos^2 x\cdot\sin^2 x}{\cos^2 x}=$$

$$=\frac{\sin^2 x}{\cos^2 x}+\frac{\sin^2 x\cdot\cos^2 x}{\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}+\frac{\sin^2 x\cdot\cos^2 x}{\cos^2 x}=$$

$$=tg^2 x+\sin^2 x=\sin^2 x+tg^2 x.$$

Vježba 379

Dokaži identitet: $\frac{1-\cos^4 x}{\cos^2 x}-\sin^2 x-tg^2 x=0.$

Rezultat: Dokaz analogan.

Zadatak 380 (Anita, gimnazija)

Dokaži identitet: $(1-ctg\,x)^2+(1+ctg\,x)^2=\frac{2}{\sin^2 x}.$

Rješenje 380

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad ctg\,x=\frac{\cos x}{\sin x}.$$

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \cos^2 x+\sin^2 x=1 \quad , \quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$(1-ctg\,x)^2+(1+ctg\,x)^2=1-2\cdot ctg\,x+ctg^2 x+1+2\cdot ctg\,x+ctg^2 x=$$

$$=1-2\cdot ctg\,x+ctg^2 x+1+2\cdot ctg\,x+ctg^2 x=1+ctg^2 x+1+ctg^2 x=2+2\cdot ctg^2 x=$$

$$=2\cdot\left(1+ctg^2 x\right)=2\cdot\left(\frac{1}{1}+\frac{\cos^2 x}{\sin^2 x}\right)=2\cdot\frac{\sin^2 x+\cos^2 x}{\sin^2 x}=2\cdot\frac{1}{\sin^2 x}=\frac{2}{\sin^2 x}.$$

Vježba 380

Dokaži identitet: $(1-ctg\,x)^2+(1+ctg\,x)^2-\frac{2}{\sin^2 x}=0.$

Rezultat: Dokaz analogan.