Zadatak 321 (Petra, gimnazija)

Ako je $\cos\alpha = \frac{x}{y+z}$, $\cos\beta = \frac{y}{z+x}$, $\cos\gamma = \frac{z}{x+y}$, tada je vrijednost izraza $S = tg^2\frac{\alpha}{2} + tg^2\frac{\beta}{2} + tg^2\frac{\gamma}{2}$ jednaka:

$$A.\ \frac{2}{3} \qquad B.\ \frac{3}{2} \qquad C.\ 2 \qquad D.\ 1$$

Rješenje 321

Ponovimo!

$$tg^2\frac{x}{2} = \frac{1-\cos x}{1+\cos x} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a\cdot d + b\cdot c}{b\cdot d} \quad , \quad \frac{a}{n} + \frac{b}{n} = \frac{a+b}{n}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$S = tg^2\frac{\alpha}{2} + tg^2\frac{\beta}{2} + tg^2\frac{\gamma}{2} = \frac{1-\cos\alpha}{1+\cos\alpha} + \frac{1-\cos\beta}{1+\cos\beta} + \frac{1-\cos\gamma}{1+\cos\gamma} =$$

$$= \frac{1-\frac{x}{y+z}}{1+\frac{x}{y+z}} + \frac{1-\frac{y}{z+x}}{1+\frac{y}{z+x}} + \frac{1-\frac{z}{x+y}}{1+\frac{z}{x+y}} = \frac{\frac{1}{1}-\frac{x}{y+z}}{\frac{1}{1}+\frac{x}{y+z}} + \frac{\frac{1}{1}-\frac{y}{z+x}}{\frac{1}{1}+\frac{y}{z+x}} + \frac{\frac{1}{1}-\frac{z}{x+y}}{\frac{1}{1}+\frac{z}{x+y}} =$$

$$= \frac{\frac{y+z-x}{y+z}}{\frac{y+z+x}{y+z}} + \frac{\frac{z+x-y}{z+x}}{\frac{z+x+y}{z+x}} + \frac{\frac{x+y-z}{x+y}}{\frac{x+y+z}{x+y}} = \frac{\frac{y+z-x}{y+z}}{\frac{y+z+x}{y+z}} + \frac{\frac{z+x-y}{z+x}}{\frac{z+x+y}{z+x}} + \frac{\frac{x+y-z}{x+y}}{\frac{x+y+z}{x+y}} =$$

$$= \frac{\frac{y+z-x}{1}}{\frac{y+z+x}{1}} + \frac{\frac{z+x-y}{1}}{\frac{z+x+y}{1}} + \frac{\frac{x+y-z}{1}}{\frac{x+y+z}{1}} = \frac{y+z-x}{y+z+x} + \frac{z+x-y}{z+x+y} + \frac{x+y-z}{x+y+z} =$$

$$= \frac{y+z-x}{x+y+z} + \frac{z+x-y}{x+y+z} + \frac{x+y-z}{x+y+z} = \frac{y+z-x+z+x-y+x+y-z}{x+y+z} = \frac{y+z-x+z+x-y+x+y-z}{x+y+z} =$$

$$= \frac{z+x+y}{x+y+z} = \frac{x+y+z}{x+y+z} = 1.$$

Odgovor je pod D.

Vježba 321

Ako je $\cos\alpha = \frac{1}{5}$, $\cos\beta = \frac{1}{2}$, $\cos\gamma = 1$, tada je vrijednost izraza $S = tg^2\frac{\alpha}{2} + tg^2\frac{\beta}{2} + tg^2\frac{\gamma}{2}$ jednaka:

$$A.\ \frac{2}{3} \qquad B.\ \frac{3}{2} \qquad C.\ 2 \qquad D.\ 1$$

Rezultat: D.

Zadatak 322 (Marko, gimnazija)

Vrijednost izraza $\cos^2 3 + \cos^2 1 - \cos 4 \cdot \cos 2$ je jednaka:

$$A.\ -1 \qquad B.\ 1 \qquad C.\ -2 \qquad D.\ 2$$

Rješenje 322

Ponovimo!

$$\cos^2 x = \frac{1+\cos 2x}{2} \quad , \quad \cos x \cdot \cos y = \frac{1}{2} \cdot \left[\cos\left(x+y\right) + \cos\left(x-y\right)\right] \quad , \quad \frac{a+b}{n} = \frac{a}{n} + \frac{b}{n}.$$

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} \cdot c = \frac{a \cdot c}{b}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot \left(b+c\right) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot \left(b+c\right).$$

$$\cos^2 3 + \cos^2 1 - \cos 4 \cdot \cos 2 = \frac{1+\cos\left(2 \cdot 3\right)}{2} + \frac{1+\cos\left(2 \cdot 1\right)}{2} - \frac{1}{2} \cdot \left[\cos\left(4+2\right) + \cos\left(4-2\right)\right] =$$

$$= \frac{1+\cos 6}{2} + \frac{1+\cos 2}{2} - \frac{1}{2} \cdot \left[\cos 6 + \cos 2\right] = \frac{1}{2} + \frac{\cos 6}{2} + \frac{1}{2} + \frac{\cos 2}{2} - \frac{\cos 6}{2} - \frac{\cos 2}{2} =$$

$$= \frac{1}{2} + \frac{\cos 6}{2} + \frac{1}{2} + \frac{\cos 2}{2} - \frac{\cos 6}{2} - \frac{\cos 2}{2} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = \frac{2}{2} = 1.$$

Odgovor je pod B.

Vježba 322

Vrijednost izraza $\cos^2 4 + \cos^2 3 - \cos 7 \cdot \cos 1$ je jednaka:

$$A.\ -1 \qquad B.\ 1 \qquad C.\ -2 \qquad D.\ 2$$

Rezultat: B.

Zadatak 323 (Nely, gimnazija)

Ako je $tg\ x + ctg\ x = 4$, koliko je $\sin x + \cos x$?

Rješenje 323

Ponovimo!

$$tg\ x = \frac{\sin x}{\cos x} \quad , \quad ctg\ x = \frac{\cos x}{\sin x} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$\cos^2 \alpha + \sin^2 \alpha = 1 \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c} \quad , \quad \left(a+b\right)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Najprije preoblikujemo zadanu jednadžbu.

$$tg\ x + ctg\ x = 4 \Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = 4 \Rightarrow \frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x} = 4 \Rightarrow \frac{1}{\cos x \cdot \sin x} = 4 \Rightarrow$$

$$\Rightarrow \frac{1}{\sin x\cdot\cos x}=4 \Rightarrow \frac{1}{\sin x\cdot\cos x}=\frac{4}{1} \Rightarrow \sin x\cdot\cos x=\frac{1}{4}.$$

Označimo da je

$$y=\sin x+\cos x.$$

Kvadriranjem tog izraza dobije se:

$$y=\sin x+\cos x \Rightarrow y=\sin x+\cos x \;/^2 \Rightarrow y^2=(\sin x+\cos x)^2 \Rightarrow$$

$$\Rightarrow y^2=\sin^2 x+2\cdot\sin x\cdot\cos x+\cos^2 x \Rightarrow y^2=\sin^2 x+\cos^2 x+2\cdot\sin x\cdot\cos x \Rightarrow$$

$$\Rightarrow y^2=1+2\cdot\sin x\cdot\cos x \Rightarrow \left[\sin x\cdot\cos x=\frac{1}{4}\right] \Rightarrow y^2=1+2\cdot\frac{1}{4} \Rightarrow y^2=1+2\cdot\frac{1}{4} \Rightarrow$$

$$\Rightarrow y^2=1+\frac{1}{2} \Rightarrow y^2=\frac{1}{1}+\frac{1}{2} \Rightarrow y^2=\frac{2+1}{2} \Rightarrow y^2=\frac{3}{2} \Rightarrow y^2=\frac{3}{2} \;/\sqrt{\;} \Rightarrow y=\pm\sqrt{\frac{3}{2}}.$$

Dakle, vrijedi:

$$\sin x+\cos x=\pm\sqrt{\frac{3}{2}}.$$

Vježba 323

Ako je $tg\,x+\frac{1}{tg\,x}=4,$ koliko je $\sin x+\cos x$?

Rezultat: $\sin x+\cos x=\pm\sqrt{\frac{3}{2}}.$

Zadatak 324 (Nely, gimnazija)

Dokaži identitet: $\left[\left(\frac{1+\cos x}{\sin x}\right)^2+1\right]:\frac{1+\cos x}{\sin^2 x}=2.$

Rješenje 324

Ponovimo!

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad \frac{a}{b}:\frac{c}{d}=\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad n=\frac{n}{1}.$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \cos^2\alpha+\sin^2\alpha=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

$$\left[\left(\frac{1+\cos x}{\sin x}\right)^2+1\right]:\frac{1+\cos x}{\sin^2 x}=\left[\frac{(1+\cos x)^2}{\sin^2 x}+1\right]\cdot\frac{\sin^2 x}{1+\cos x}=\left[\frac{(1+\cos x)^2}{\sin^2 x}+\frac{1}{1}\right]\cdot\frac{\sin^2 x}{1+\cos x}=$$

$$=\frac{(1+\cos x)^2+\sin^2 x}{\sin^2 x}\cdot\frac{\sin^2 x}{1+\cos x}=\frac{(1+\cos x)^2+\sin^2 x}{\sin^2 x}\cdot\frac{\sin^2 x}{1+\cos x}=\frac{(1+\cos x)^2+\sin^2 x}{1}\cdot\frac{1}{1+\cos x}=$$

$$=\frac{(1+\cos x)^2+\sin^2 x}{1+\cos x}=\frac{1+2\cdot\cos x+\cos^2 x+\sin^2 x}{1+\cos x}=\frac{1+2\cdot\cos x+\left(\cos^2 x+\sin^2 x\right)}{1+\cos x}=$$

$$=\frac{1+2\cdot\cos x+1}{1+\cos x}=\frac{2+2\cdot\cos x}{1+\cos x}=\frac{2\cdot(1+\cos x)}{1+\cos x}=\frac{2\cdot(1+\cos x)}{1+\cos x}=2.$$

Vježba 324

Dokaži identitet: $\left[1+\left(\frac{1+\cos x}{\sin x}\right)^2\right]:\frac{1+\cos x}{1-\cos^2 x}=2.$

Rezultat: Dokaz analogan.

Zadatak 325 (Nely, gimnazija)

Ako je $tg\ x+ctg\ x=3$, koliko je $\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}$?

Rješenje 325

Ponovimo!

$$tg\ x=\frac{\sin x}{\cos x}\quad,\quad ctg\ x=\frac{\cos x}{\sin x}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\cos^2\alpha+\sin^2\alpha=1\quad,\quad a^n\cdot b^n=(a\cdot b)^n\quad,\quad \frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n.$$

Najprije preoblikujemo zadanu jednadžbu.

$$tg\ x+ctg\ x=3\ \Rightarrow\ \frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=3\ \Rightarrow\ \frac{\sin^2 x+\cos^2 x}{\cos x\cdot\sin x}=3\ \Rightarrow\ \frac{1}{\cos x\cdot\sin x}=3.$$

Sada je:

$$\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=\frac{\cos^2 x+\sin^2 x}{\sin^2 x\cdot\cos^2 x}=\frac{1}{\sin^2 x\cdot\cos^2 x}=\frac{1}{(\sin x\cdot\cos x)^2}=\left(\frac{1}{\sin x\cdot\cos x}\right)^2=$$

$$=\left[\frac{1}{\sin x\cdot\cos x}=3\right]=3^2=9.$$

Vježba 325

Ako je $tg\ x+\frac{1}{tg\ x}=3$, koliko je $\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}$?

Rezultat: 9.

Zadatak 326 (Anita, gimnazija)

Ako je $\sin x+\cos x=a$, tada je $\sin^4 x+\cos^4 x$ jednako:

$A.\ \frac{a^4-1}{2}\qquad B.\ \frac{a^4+1}{2}\qquad C.\ \frac{1-2\cdot a^2-a^4}{2}\qquad D.\ \frac{1+2\cdot a^2-a^4}{2}$

Rješenje 326

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad \sin^2 x+\cos^2 x=1\quad,\quad (a\cdot b)^n=a^n\cdot b^n\quad,\quad n=\frac{n}{1}.$$

$$\left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Najprije preoblikujemo zadanu jednadžbu.

$$\sin x+\cos x=a \ \Rightarrow \ \sin x+\cos x=a \ /^{2} \ \Rightarrow \ (\sin x+\cos x)^{2}=a^{2} \ \Rightarrow$$

$$\Rightarrow \ \sin^{2}x+2\cdot\sin x\cdot\cos x+\cos^{2}x=a^{2} \ \Rightarrow \ \sin^{2}x+\cos^{2}x+2\cdot\sin x\cdot\cos x=a^{2} \ \Rightarrow$$

$$\Rightarrow \ 1+2\cdot\sin x\cdot\cos x=a^{2} \ \Rightarrow \ 2\cdot\sin x\cdot\cos x=a^{2}-1 \ \Rightarrow \ 2\cdot\sin x\cdot\cos x=a^{2}-1 \ /^{2} \ \Rightarrow$$

$$\Rightarrow \ (2\cdot\sin x\cdot\cos x)^{2}=\left(a^{2}-1\right)^{2} \ \Rightarrow \ 4\cdot\sin^{2}x\cdot\cos^{2}x=\left(a^{2}-1\right)^{2}.$$

Sada polazimo od trigonometrijske relacije koju kvadriramo:

$$\sin^{2}x+\cos^{2}x=1 \ \Rightarrow \ \sin^{2}x+\cos^{2}x=1 \ /^{2} \ \Rightarrow \ \left(\sin^{2}x+\cos^{2}x\right)^{2}=1 \ \Rightarrow$$

$$\Rightarrow \ \left(\sin^{2}x\right)^{2}+2\cdot\sin^{2}x\cdot\cos^{2}x+\left(\cos^{2}x\right)^{2}=1 \ \Rightarrow \ \sin^{4}x+2\cdot\sin^{2}x\cdot\cos^{2}x+\cos^{4}x=1 \ \Rightarrow$$

$$\Rightarrow \ \sin^{4}x+\cos^{4}x=1-2\cdot\sin^{2}x\cdot\cos^{2}x \ \Rightarrow \ \sin^{4}x+\cos^{4}x=1-\frac{2\cdot\sin^{2}x\cdot\cos^{2}x}{1} \ \Rightarrow$$

$$\Rightarrow \ \sin^{4}x+\cos^{4}x=1-\frac{2\cdot 2\cdot\sin^{2}x\cdot\cos^{2}x}{2\cdot 1} \ \Rightarrow \ \sin^{4}x+\cos^{4}x=1-\frac{4\cdot\sin^{2}x\cdot\cos^{2}x}{2} \ \Rightarrow$$

$$\Rightarrow \ \left[4\cdot\sin^{2}x\cdot\cos^{2}x=\left(a^{2}-1\right)^{2}\right] \ \Rightarrow \ \sin^{4}x+\cos^{4}x=1-\frac{\left(a^{2}-1\right)^{2}}{2} \ \Rightarrow$$

$$\Rightarrow \ \sin^{4}x+\cos^{4}x=1-\frac{\left(a^{2}\right)^{2}-2\cdot a^{2}+1}{2} \ \Rightarrow \ \sin^{4}x+\cos^{4}x=1-\frac{a^{4}-2\cdot a^{2}+1}{2} \ \Rightarrow$$

$$\Rightarrow \ \sin^{4}x+\cos^{4}x=\frac{1}{1}-\frac{a^{4}-2\cdot a^{2}+1}{2} \ \Rightarrow \ \sin^{4}x+\cos^{4}x=\frac{2-\left(a^{4}-2\cdot a^{2}+1\right)}{2} \ \Rightarrow$$

$$\Rightarrow \ \sin^{4}x+\cos^{4}x=\frac{2-a^{4}+2\cdot a^{2}-1}{2} \ \Rightarrow \ \sin^{4}x+\cos^{4}x=\frac{1+2\cdot a^{2}-a^{4}}{2}.$$

Odgovor je pod D.

Vježba 326

Ako je $\sin x+\cos x=1$, tada je $\sin^{4}x+\cos^{4}x$ jednako:

A. 0 *B.* 1 *C.* 2 *D.* 4

Rezultat: B.

Zadatak 327 (Toni, elektrotehnička škola)

Ako je $\sin t = \frac{17}{35}$, $t \in \left\langle 0, \frac{\pi}{2} \right\rangle$, koliko je $ctg\, t$?

Rješenje 327

Ponovimo!

$$\sin^2 x + \cos^2 x = 1 \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}} \quad , \quad \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b} \quad , \quad ctg\, x = \frac{\cos x}{\sin x} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

$$ctg^2 x + 1 = \frac{1}{\sin^2 x}.$$

	I. kvadrant	II. kvadrant	III. kvadrant	IV. kvadrant
sin	+	+	—	—
cos	+	—	—	+
tg	+	—	+	—
ctg	+	—	+	—

1.inačica

Najprije izračunamo cos t pa onda ctg t.

$$\cos^2 t + \sin^2 t = 1 \Rightarrow \cos^2 t + \left(\frac{17}{35}\right)^2 = 1 \Rightarrow \cos^2 t + \frac{17^2}{35^2} = 1 \Rightarrow \cos^2 t + \frac{289}{1225} = 1 \Rightarrow$$

$$\Rightarrow \cos^2 t = 1 - \frac{289}{1225} \Rightarrow \cos^2 t = \frac{1}{1} - \frac{289}{1225} \Rightarrow \cos^2 t = \frac{1225 - 289}{1225} \Rightarrow \cos^2 t = \frac{936}{1225} \Rightarrow$$

$$\Rightarrow \cos^2 t = \frac{936}{1225} \;/\sqrt{\;} \Rightarrow \cos t = \pm\sqrt{\frac{936}{1225}} \Rightarrow \cos t = \pm\frac{\sqrt{936}}{\sqrt{1225}} \Rightarrow \cos t = \pm\frac{\sqrt{936}}{35} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{djelomično} \\ \text{korjenovanje} \\ \text{u brojniku} \end{bmatrix} \Rightarrow \cos t = \pm\frac{\sqrt{36 \cdot 26}}{35} \Rightarrow \cos t = \pm\frac{\sqrt{36} \cdot \sqrt{26}}{35} \Rightarrow \cos t = \pm\frac{6 \cdot \sqrt{26}}{35} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} t \in \left\langle 0, \frac{\pi}{2} \right\rangle \\ \cos t > 0 \end{bmatrix} \Rightarrow \cos t = \frac{6 \cdot \sqrt{26}}{35}.$$

Sada je:

$$ctg\, t = \frac{\cos t}{\sin t} \Rightarrow ctg\, t = \frac{\frac{6 \cdot \sqrt{26}}{35}}{\frac{17}{35}} \Rightarrow ctg\, t = \frac{\frac{6 \cdot \sqrt{26}}{35}}{\frac{17}{35}} \Rightarrow ctg\, t = \frac{\frac{6 \cdot \sqrt{26}}{1}}{\frac{17}{1}} \Rightarrow ctg\, t = \frac{6 \cdot \sqrt{26}}{17}.$$

2.inačica

$$ctg^2 t+1=\frac{1}{\sin^2 t} \Rightarrow ctg^2 t=\frac{1}{\sin^2 t}-1 \Rightarrow ctg^2 t=\frac{1}{\left(\frac{17}{36}\right)^2}-1 \Rightarrow ctg^2 t=\frac{1}{\frac{17^2}{36^2}}-1 \Rightarrow$$

$$\Rightarrow ctg^2 t=\frac{1}{\frac{289}{1225}}-1 \Rightarrow ctg^2 t=\frac{\frac{1}{1}}{\frac{289}{1225}}-1 \Rightarrow ctg^2 t=\frac{1225}{289}-1 \Rightarrow ctg^2 t=\frac{1225}{289}-\frac{1}{1} \Rightarrow$$

$$\Rightarrow ctg^2 t=\frac{1225-289}{289} \Rightarrow ctg^2 t=\frac{936}{289} \Rightarrow ctg^2 t=\frac{936}{289} \;/\sqrt{\;} \Rightarrow ctg\, t=\pm\sqrt{\frac{936}{289}} \Rightarrow$$

$$\Rightarrow ctg\, t=\pm\frac{\sqrt{936}}{\sqrt{289}} \Rightarrow ctg\, t=\pm\frac{\sqrt{936}}{17} \Rightarrow \left[\begin{matrix}\text{djelomično}\\ \text{korjenovanje}\\ \text{u brojniku}\end{matrix}\right] \Rightarrow ctg\, t=\pm\frac{\sqrt{36\cdot 26}}{17} \Rightarrow$$

$$\Rightarrow ctg\, t=\pm\frac{\sqrt{36}\cdot\sqrt{26}}{17} \Rightarrow ctg\, t=\pm\frac{6\cdot\sqrt{26}}{17} \Rightarrow \left[\begin{matrix} t\in\left\langle 0,\ \frac{\pi}{2}\right\rangle \\ ctg\, t>0\end{matrix}\right] \Rightarrow ctg\, t=\frac{6\cdot\sqrt{26}}{17}.$$

Vježba 327

Ako je $\sin t=\frac{\sqrt{2}}{2},\ t\in\left\langle 0,\ \frac{\pi}{2}\right\rangle$, koliko je $ctg\, t$?

Rezultat: 1.

Zadatak 328 (Toni, elektrotehnička škola)

Izračunajte $\frac{\sin x}{1+\cos x}+\frac{\sin x}{1-\cos x}$.

Rješenje 328

Ponovimo!

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad (a-b)\cdot(a+b)=a^2-b^2 \quad , \quad \sin^2 x+\cos^2 x=1.$$

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{\sin x}{1+\cos x}+\frac{\sin x}{1-\cos x}=\frac{\sin x\cdot(1-\cos x)+\sin x\cdot(1+\cos x)}{(1+\cos x)\cdot(1-\cos x)}=$$

$$=\frac{\sin x-\sin x\cdot\cos x+\sin x+\sin x\cdot\cos x}{1-\cos^2 x}=\frac{\sin x-\sin x\cdot\cos x+\sin x+\sin x\cdot\cos x}{\sin^2 x}=$$

$$=\frac{\sin x+\sin x}{\sin^2 x}=\frac{2\cdot\sin x}{\sin^2 x}=\frac{2\cdot\sin x}{\sin^2 x}=\frac{2}{\sin x}.$$

2.inačica

$$\frac{\sin x}{1+\cos x}+\frac{\sin x}{1-\cos x}=\sin x\cdot\left(\frac{1}{1+\cos x}+\frac{1}{1-\cos x}\right)=\sin x\cdot\frac{1-\cos x+1+\cos x}{(1+\cos x)\cdot(1-\cos x)}=$$

$$=\sin x\cdot\frac{1-\cos x+1+\cos x}{1-\cos^2 x}=\sin x\cdot\frac{1+1}{1-\cos^2 x}=\sin x\cdot\frac{2}{\sin^2 x}=\sin x\cdot\frac{2}{\sin^2 x}=\frac{2}{\sin x}.$$

Vježba 328

Izračunajte $\frac{\sin x}{1-\cos x}+\frac{\sin x}{1+\cos x}.$

Rezultat: $\frac{2}{\sin x}.$

Zadatak 329 (Toni, elektrotehnička škola)

Koliko je $\frac{5\cdot\sin x+6\cdot\cos x}{\cos x-\sin x}$, ako je $ctg\,t=\frac{1}{2}$?

Rješenje 329

Ponovimo!

$$\frac{a}{b}=\frac{\frac{a}{c}}{\frac{b}{c}}\ ,\quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}\ ,\quad \frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}\ ,\quad ctg\,x=\frac{\cos x}{\sin x}.$$

$$n=\frac{n}{1}\ ,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\ ,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{5\cdot\sin x+6\cdot\cos x}{\cos x-\sin x}=\left[\begin{array}{l}\text{brojnik i nazivnik}\\ \text{podijelimo sa }\sin x\end{array}\right]=\frac{\frac{5\cdot\sin x+6\cdot\cos x}{\sin x}}{\frac{\cos x-\sin x}{\sin x}}=\frac{\frac{5\cdot\sin x}{\sin x}+\frac{6\cdot\cos x}{\sin x}}{\frac{\cos x}{\sin x}-\frac{\sin x}{\sin x}}=$$

$$=\frac{\frac{5\cdot\sin x}{\sin x}+\frac{6\cdot\cos x}{\sin x}}{\frac{\cos x}{\sin x}-\frac{\sin x}{\sin x}}=\frac{5+\frac{6\cdot\cos x}{\sin x}}{\frac{\cos x}{\sin x}-1}=\frac{5+6\cdot\frac{\cos x}{\sin x}}{\frac{\cos x}{\sin x}-1}=\frac{5+6\cdot ctg\,x}{ctg\,x-1}=\left[ctg\,x=\frac{1}{2}\right]=\frac{5+6\cdot\frac{1}{2}}{\frac{1}{2}-1}=$$

$$=\frac{5+6\cdot\frac{1}{2}}{\frac{1}{2}-1}=\frac{5+3}{\frac{1}{2}-1}=\frac{8}{\frac{1}{2}-\frac{1}{1}}=\frac{8}{\frac{1-2}{2}}=\frac{8}{-\frac{1}{2}}=\frac{\frac{8}{1}}{-\frac{1}{2}}=-\frac{16}{1}=-16.$$

Vježba 329

Koliko je $\frac{6 \cdot \cos x + 5 \cdot \sin x + 6 \cdot \cos x}{\cos x - \sin x}$, ako je $ctg\ t = \frac{1}{2}$?

Rezultat: – 16.

Zadatak 330 (Paula, glazbena škola)

Provjerite točnost jednakosti $(\sin x + tg\ x) \cdot (\cos x + ctg\ x) = (1 + \sin x) \cdot (1 + \cos x)$.

Rješenje 330

Ponovimo!

Množenje zagrada

$$(a+b) \cdot (c+d) = a \cdot c + a \cdot d + b \cdot c + b \cdot d.$$

$$tg\ x = \frac{\sin x}{\cos x} \quad , \quad ctg\ x = \frac{\cos x}{\sin x} \quad , \quad b \cdot \frac{a}{b} = a \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$tg\ x \cdot ctg\ x = 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

$$(\sin x + tg\ x) \cdot (\cos x + ctg\ x) = \sin x \cdot \cos x + \sin x \cdot ctg\ x + tg\ x \cdot \cos x + tg\ x \cdot ctg\ x =$$

$$= \sin x \cdot \cos x + \sin x \cdot \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \cdot \cos x + 1 =$$

$$= \sin x \cdot \cos x + \sin x \cdot \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \cdot \cos x + 1 = \sin x \cdot \cos x + \cos x + \sin x + 1 =$$

$$= (\sin x \cdot \cos x + \cos x) + (\sin x + 1) = (\sin x \cdot \cos x + \cos x) + (\sin x + 1) = \cos x \cdot (\sin x + 1) + (\sin x + 1) =$$

$$= \cos x \cdot (\sin x + 1) + (\sin x + 1) = (\sin x + 1) \cdot (\cos x + 1) = (1 + \sin x) \cdot (1 + \cos x).$$

2.inačica

$$(\sin x + tg\ x) \cdot (\cos x + ctg\ x) = \left(\sin x + \frac{\sin x}{\cos x}\right) \cdot \left(\cos x + \frac{\cos x}{\sin x}\right) = \left(\frac{\sin x}{1} + \frac{\sin x}{\cos x}\right) \cdot \left(\frac{\cos x}{1} + \frac{\cos x}{\sin x}\right) =$$

$$= \frac{\sin x \cdot \cos x + \sin x}{\cos x} \cdot \frac{\sin x \cdot \cos x + \cos x}{\sin x} = \frac{\sin x \cdot \cos x + \sin x}{\cos x} \cdot \frac{\sin x \cdot \cos x + \cos x}{\sin x} =$$

$$= \frac{\sin x \cdot (\cos x + 1)}{\cos x} \cdot \frac{\cos x \cdot (\sin x + 1)}{\sin x} = \frac{\sin x \cdot (\cos x + 1)}{\cos x} \cdot \frac{\cos x \cdot (\sin x + 1)}{\sin x} =$$

$$= (\cos x + 1) \cdot (\sin x + 1) = (1 + \sin x) \cdot (1 + \cos x).$$

Vježba 330

Provjerite točnost jednakosti $(\cos x + ctg\ x) \cdot (\sin x + tg\ x) = (1 + \sin x) \cdot (1 + \cos x)$.

Rezultat: Dokaz analogan.

Zadatak 331 (Paula, glazbena škola)

Provjerite točnost jednakosti $\frac{tg\,x+ctg\,x-2}{tg\,x+ctg\,x+2}=\left(\frac{tg\,x-1}{tg\,x+1}\right)^2.$

Rješenje 331

Ponovimo!

$$tg\,x\cdot ctg\,x=1\ \Rightarrow\ ctg\,x=\frac{1}{tg\,x}\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad a^2-2\cdot a\cdot b+b^2=(a-b)^2\quad,\quad a^2+2\cdot a\cdot b+b^2=(a+b)^2.$$

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{tg\,x+ctg\,x-2}{tg\,x+ctg\,x+2}=\frac{tg\,x+\frac{1}{tg\,x}-2}{tg\,x+\frac{1}{tg\,x}+2}=\frac{\frac{tg\,x}{1}+\frac{1}{tg\,x}-\frac{2}{1}}{\frac{tg\,x}{1}+\frac{1}{tg\,x}+\frac{2}{1}}=\frac{\frac{tg^2\,x+1-2\cdot tg\,x}{tg\,x}}{\frac{tg^2\,x+1+2\cdot tg\,x}{tg\,x}}=\frac{\frac{tg^2\,x-2\cdot tg\,x+1}{tg\,x}}{\frac{tg^2\,x+2\cdot tg\,x+1}{tg\,x}}=$$

$$=\frac{\frac{tg^2\,x-2\cdot tg\,x+1}{tg\,x}}{\frac{tg^2\,x+2\cdot tg\,x+1}{tg\,x}}=\frac{\frac{tg^2\,x-2\cdot tg\,x+1}{1}}{\frac{tg^2\,x+2\cdot tg\,x+1}{1}}=\frac{tg^2\,x-2\cdot tg\,x+1}{tg^2\,x+2\cdot tg\,x+1}=\frac{(tg\,x-1)^2}{(tg\,x+1)^2}=\left(\frac{tg\,x-1}{tg\,x+1}\right)^2.$$

Vježba 331

Provjerite točnost jednakosti $\frac{tg\,x+ctg\,x+2}{tg\,x+ctg\,x-2}=\left(\frac{tg\,x+1}{tg\,x-1}\right)^2.$

Rezultat: Dokaz analogan.

Zadatak 332 (Paula, glazbena škola)

Provjerite točnost jednakosti $3\cdot\sin^4 x-2\cdot\sin^6 x=1-3\cdot\cos^4 x+2\cdot\cos^6 x.$

Rješenje 332

Ponovimo!

$$a^n:a^m=a^{n-m}\quad,\quad \cos^2 x+\sin^2 x=1\quad,\quad \left(a^n\right)^m=a^{n\cdot m}.$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad a^1=a\quad,\quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$3\cdot\sin^4 x-2\cdot\sin^6 x=2\cdot\sin^4 x+\sin^4 x-2\cdot\sin^6 x=2\cdot\sin^4 x-2\cdot\sin^6 x+\sin^4 x=$$

$$=\left(2\cdot\sin^4 x-2\cdot\sin^6 x\right)+\sin^4 x=2\cdot\sin^4 x\cdot\left(1-\sin^2 x\right)+\sin^4 x=2\cdot\sin^4 x\cdot\cos^2 x+\sin^4 x=$$

$$=2\cdot\sin^4 x\cdot\cos^2 x+\sin^4 x=\sin^4 x\cdot\left(2\cdot\cos^2 x+1\right)=\left(\sin^2 x\right)^2\cdot\left(2\cdot\cos^2 x+1\right)=$$

$$=\left(1-\cos^2 x\right)^2\cdot\left(2\cdot\cos^2 x+1\right)=\left(1-2\cdot\cos^2 x+\left(\cos^2 x\right)^2\right)\cdot\left(2\cdot\cos^2 x+1\right)=$$

$$=\left(1-2\cdot\cos^2 x+\cos^4 x\right)\cdot\left(2\cdot\cos^2 x+1\right)=$$

$$=2\cdot\cos^2 x+1-4\cdot\cos^4 x-2\cdot\cos^2 x+2\cdot\cos^6 x+\cos^4 x=$$

$$=2\cdot\cos^2 x+1-4\cdot\cos^4 x-2\cdot\cos^2 x+2\cdot\cos^6 x+\cos^4 x=1-4\cdot\cos^4 x+2\cdot\cos^6 x+\cos^4 x=$$

$$=1-3\cdot\cos^4 x+2\cdot\cos^6 x.$$

Vježba 332

Provjerite točnost jednakosti $3\cdot\sin^4 x-2\cdot\sin^6 x-\cos^2 x=\sin^2 x-3\cdot\cos^4 x+2\cdot\cos^6 x.$

Rezultat: Dokaz analogan.

Zadatak 333 (Paula, glazbena škola)

Provjerite točnost jednakosti $\dfrac{\sin x\cdot\cos x}{\cos^2 x-\sin^2 x}=\dfrac{tg\, x}{1-tg^2 x}.$

Rješenje 333

Ponovimo!

$$\frac{a}{b}=\frac{\frac{a}{c}}{\frac{b}{c}}\quad,\quad\frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}\quad,\quad tg\, x=\frac{\sin x}{\cos x}\quad,\quad tg^2 x=\left(\frac{\sin x}{\cos x}\right)^2=\frac{\sin^2 x}{\cos^2 x}.$$

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\quad,\quad n\neq 0\quad,\quad n\neq 1.$$

$$\frac{\sin x\cdot\cos x}{\cos^2 x-\sin^2 x}=\frac{\frac{\sin x\cdot\cos x}{\cos^2 x}}{\frac{\cos^2 x-\sin^2 x}{\cos^2 x}}=\frac{\frac{\sin x\cdot\cos x}{\cos^2 x}}{\frac{\cos^2 x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}}=\frac{\frac{\sin x\cdot\cos x}{\cos^2 x}}{\frac{\cos^2 x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}}=$$

$$=\frac{\frac{\sin x}{\cos x}}{1-\frac{\sin^2 x}{\cos^2 x}}=\frac{tg\, x}{1-tg^2 x}.$$

Vježba 333

Provjerite točnost jednakosti $\frac{\cos^2 x-\sin^2 x}{\sin x\cdot\cos x}=\frac{1-tg^2 x}{tg\, x}.$

Rezultat: Dokaz analogan.

Zadatak 334 (Paula, glazbena škola)

Provjerite točnost jednakosti $\frac{\sin^2 x}{\sin x-\cos x}-\frac{\sin x+\cos x}{tg^2 x-1}=\sin x+\cos x.$

Rješenje 334

Ponovimo!

$$tg\, x=\frac{\sin x}{\cos x}\quad,\quad tg^2 x=\left(\frac{\sin x}{\cos x}\right)^2=\frac{\sin^2 x}{\cos^2 x}\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\frac{\sin^2 x}{\sin x-\cos x}-\frac{\sin x+\cos x}{tg^2 x-1}=\frac{\sin^2 x}{\sin x-\cos x}-\frac{\sin x+\cos x}{\frac{\sin^2 x}{\cos^2 x}-1}=\frac{\sin^2 x}{\sin x-\cos x}-\frac{\sin x+\cos x}{\frac{\sin^2 x}{\cos^2 x}-\frac{1}{1}}=$$

$$=\frac{\sin^2 x}{\sin x-\cos x}-\frac{\sin x+\cos x}{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}=\frac{\sin^2 x}{\sin x-\cos x}-\frac{\frac{\sin x+\cos x}{1}}{\frac{\sin^2 x-\cos^2 x}{\cos^2 x}}=$$

$$=\frac{\sin^2 x}{\sin x-\cos x}-\frac{\cos^2 x\cdot(\sin x+\cos x)}{\sin^2 x-\cos^2 x}=\frac{\sin^2 x}{\sin x-\cos x}-\frac{\cos^2 x\cdot(\sin x+\cos x)}{(\sin x-\cos x)\cdot(\sin x+\cos x)}=$$

$$=\frac{\sin^2 x}{\sin x-\cos x}-\frac{\cos^2 x\cdot(\sin x+\cos x)}{(\sin x-\cos x)\cdot(\sin x+\cos x)}=\frac{\sin^2 x}{\sin x-\cos x}-\frac{\cos^2 x}{\sin x-\cos x}=$$

$$=\frac{\sin^2 x-\cos^2 x}{\sin x-\cos x}=\frac{(\sin x-\cos x)\cdot(\sin x+\cos x)}{\sin x-\cos x}=\frac{(\sin x-\cos x)\cdot(\sin x+\cos x)}{\sin x-\cos x}=$$

$$=\sin x+\cos x.$$

Vježba 334

Provjerite točnost jednakosti $\frac{\sin^2 x}{\sin x-\cos x}+\frac{\sin x+\cos x}{1-tg^2 x}=\sin x+\cos x.$

Rezultat: Dokaz analogan.

Zadatak 335 (Dominik, srednja škola)

Odredite sve realne brojeve x za koje vrijede nejednakosti $-1 \le \frac{\sqrt{3} \cdot \sin x}{2 + \cos x} \le 1.$

Rješenje 335

Ponovimo!

Za realni broj x njegova je apsolutna vrijednost (modul) broj $|x|$ koji određujemo na ovaj način:

$$|x| = \begin{cases} x, & x \ge 0 \\ -x, & x < 0. \end{cases}$$

Ako je broj x pozitivan ili nula, tada je on jednak svojoj apsolutnoj vrijednosti. Za svaki x, $x \ge 0$, vrijedi $|x| = x$.

Ako je x negativan broj, njegova apsolutna vrijednost je suprotan broj – x koji je pozitivan. Za svaki x, $x < 0$, je $|x| = -x$.

$$|x| \le a \ , \ a > 0 \Rightarrow -a \le x \le a \quad , \quad \left|\frac{a}{b}\right| = \frac{|a|}{|b|} \ , \ b \ne 0 \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2.$$

$$\cos^2 x + \sin^2 x = 1 \quad , \quad a^2 \ge 0 \ , \ a \in R \quad , \quad (a \cdot b)^n = a^n \cdot b^n \quad , \quad \left(\sqrt{a}\right)^2 = a.$$

$$a \le b \ , \ c > 0 \Rightarrow a \cdot c \le b \cdot c \quad , \quad a \le b \ , \ c < 0 \Rightarrow a \cdot c \ge b \cdot c.$$

Polazne nejednadžbe preoblikujemo na ovaj način:

$$-1 \le \frac{\sqrt{3} \cdot \sin x}{2 + \cos x} \le 1 \Rightarrow \left|\frac{\sqrt{3} \cdot \sin x}{2 + \cos x}\right| \le 1 \Rightarrow \frac{\left|\sqrt{3} \cdot \sin x\right|}{|2 + \cos x|} \le 1 \Rightarrow$$

$$\Rightarrow \frac{\left|\sqrt{3} \cdot \sin x\right|}{|2 + \cos x|} \le 1 \ / \cdot |2 + \cos x| \Rightarrow \left|\sqrt{3} \cdot \sin x\right| \le |2 + \cos x| \Rightarrow$$

$$\Rightarrow \left|\sqrt{3} \cdot \sin x\right| \le |2 + \cos x| \ /^2 \Rightarrow \left(\sqrt{3} \cdot \sin x\right)^2 \le (2 + \cos x)^2 \Rightarrow$$

$$\Rightarrow \left(\sqrt{3}\right)^2 \cdot \sin^2 x \le 4 + 4 \cdot \cos x + \cos^2 x \Rightarrow 3 \cdot \sin^2 x \le 4 + 4 \cdot \cos x + \cos^2 x \Rightarrow$$

$$\Rightarrow 3 \cdot \left(1 - \cos^2 x\right) \le 4 + 4 \cdot \cos x + \cos^2 x \Rightarrow 3 - 3 \cdot \cos^2 x \le 4 + 4 \cdot \cos x + \cos^2 x \Rightarrow$$

$$\Rightarrow 3 - 3 \cdot \cos^2 x - 4 - 4 \cdot \cos x - \cos^2 x \le 0 \Rightarrow -4 \cdot \cos^2 x - 4 \cdot \cos x - 1 \le 0 \Rightarrow$$

$$\Rightarrow -4 \cdot \cos^2 x - 4 \cdot \cos x - 1 \le 0 \ / \cdot (-1) \Rightarrow 4 \cdot \cos^2 x + 4 \cdot \cos x + 1 \ge 0 \Rightarrow$$

$$\Rightarrow (2 \cdot \cos x + 1)^2 \ge 0.$$

Budući da je kvadrat realnog broja nenegativan broj, posljednja nejednadžba vrijedi za svaki realan broj x.

Vježba 335

Odredite sve realne brojeve x za koje vrijede nejednakosti $-1 \le \frac{\sqrt{3} \cdot \cos x}{2 + \sin x} \le 1.$

Rezultat: Dokaz analogan.

Zadatak 336 (Paula, maturantica)

Provjerite jednakost $\frac{\sin x\cdot tg\,x}{\sin x+tg\,x}=\frac{tg\,x-\sin x}{\sin x\cdot tg\,x}$.

Rješenje 336

Ponovimo!

$$tg\,x=\frac{\sin x}{\cos x}\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad \cos^2 x+\sin^2 x=1.$$

$$a^2-b^2=(a-b)\cdot(a+b)\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Preoblikujemo lijevu stranu jednakosti.

$$\frac{\sin x\cdot tg\,x}{\sin x+tg\,x}=\frac{\sin x\cdot\frac{\sin x}{\cos x}}{\sin x+\frac{\sin x}{\cos x}}=\frac{\frac{\sin^2 x}{\cos x}}{\sin x\cdot\left(1+\frac{1}{\cos x}\right)}=\frac{\frac{1-\cos^2 x}{\cos x}}{\sin x\cdot\left(\frac{1}{1}+\frac{1}{\cos x}\right)}=\frac{\frac{(1-\cos x)\cdot(1+\cos x)}{\cos x}}{\sin x\cdot\frac{\cos x+1}{\cos x}}=$$

$$=\frac{\frac{(1-\cos x)\cdot(1+\cos x)}{\cos x}}{\sin x\cdot\frac{\cos x+1}{\cos x}}=\frac{\frac{1-\cos x}{\cos x}}{\sin x\cdot\frac{1}{\cos x}}=\frac{\frac{1}{\cos x}-\frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}}=\frac{\frac{1}{\cos x}-\frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}}=\frac{\frac{1}{\cos x}-1}{tg\,x}=$$

$$=\left[\begin{array}{l}\text{proširimo}\\ \text{razlomak sa }\sin x\end{array}\right]=\frac{\frac{1}{\cos x}-1}{tg\,x}\cdot\frac{\sin x}{\sin x}=\frac{\frac{\sin x}{\cos x}-\sin x}{tg\,x\cdot\sin x}=\frac{tg\,x-\sin x}{\sin x\cdot tg\,x}.$$

Vježba 336

Provjerite jednakost $\frac{tg\,x-\sin x}{\sin x\cdot tg\,x}=\frac{\sin x\cdot tg\,x}{\sin x+tg\,x}$.

Rezultat: Dokaz analogan.

Zadatak 337 (Nikolina, gimnazija)

Ako je $\cos(a+b)=\frac{1}{3}$, $\cos(a-b)=\frac{1}{5}$, koliko je $tg\,a\cdot tg\,b$?

Rješenje 337

Ponovimo!

$$tg\,x=\frac{\sin x}{\cos x}\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Formule pretvorbe umnoška trigonometrijskih funkcija u zbroj

$$\sin x\cdot\sin y=\frac{1}{2}\cdot\left(\cos\left(x-y\right)-\cos\left(x+y\right)\right)\quad,\quad\cos x\cdot\cos y=\frac{1}{2}\cdot\left(\cos\left(x+y\right)+\cos\left(x-y\right)\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$tg\,a\cdot tg\,b=\frac{\sin a}{\cos a}\cdot\frac{\sin b}{\cos b}=\frac{\sin a\cdot\sin b}{\cos a\cdot\cos b}=\frac{\frac{1}{2}\cdot\left(\cos\left(a-b\right)-\cos\left(a+b\right)\right)}{\frac{1}{2}\cdot\left(\cos\left(a+b\right)+\cos\left(a-b\right)\right)}=$$

$$=\frac{\frac{1}{2}\cdot\left(\cos\left(a-b\right)-\cos\left(a+b\right)\right)}{\frac{1}{2}\cdot\left(\cos\left(a+b\right)+\cos\left(a-b\right)\right)}=\frac{\cos\left(a-b\right)-\cos\left(a+b\right)}{\cos\left(a+b\right)+\cos\left(a-b\right)}=\begin{bmatrix}\cos\left(a-b\right)=\frac{1}{5}\\ \cos\left(a+b\right)=\frac{1}{3}\end{bmatrix}=\frac{\frac{1}{5}-\frac{1}{3}}{\frac{1}{3}+\frac{1}{5}}=$$

$$=\frac{\frac{3-5}{15}}{\frac{5+3}{15}}=\frac{-\frac{2}{15}}{\frac{8}{15}}=\frac{-\frac{2}{15}}{\frac{8}{15}}=\frac{-\frac{2}{1}}{\frac{8}{1}}=\frac{-2}{8}=-\frac{2}{8}=-\frac{1}{4}.$$

Vježba 337

Ako je $\cos\left(a+b\right)=\frac{1}{3}$, $\cos\left(a-b\right)=\frac{1}{5}$, koliko je $\frac{1}{tg\,a\cdot tg\,b}$?

Rezultat: -4.

Zadatak 338 (Jele, gimnazija)

Vrijednost brojevnog izraza $\sin^2 1^0+\sin^2 2^0+\sin^2 3^0+\ldots+\sin^2 89^0$ jednaka je:

A. 1.5 *B.* 89 *C.* 0 *D.* 44.5

Rješenje 338

Ponovimo!

$$\sin\alpha=\cos\left(90^0-\alpha\right)\quad,\quad\sin^2\alpha+\cos^2\alpha=1\quad,\quad\sin 45^0=\frac{\sqrt{2}}{2}.$$

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad,\quad\left(\sqrt{a}\right)^2=a.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot\left(b+c\right)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot\left(b+c\right).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Uočimo da je za prirodne brojeve od 1 do 89 srednji broj 45.

$$1,\ 2,\ 3,\ \ldots,\ 42,\ 43,\ 44,\ 45,\ 46,\ 47,\ 48,\ \ldots,\ 87,\ 88,\ 89.$$

Preoblikujemo zadani brojevni izraz.

$$\sin^2 1^0+\sin^2 2^0+\sin^2 3^0+\ldots+\sin^2 89^0=$$

$$=\sin^2 1^0+\ldots+\sin^2 44^0+\sin^2 45^0+\sin^2 46^0+\ldots+\sin^2 89^0=$$

$$=\left(\sin^2 1^0+\ldots+\sin^2 44^0\right)+\sin^2 45^0+\left(\sin^2 46^0+\ldots+\sin^2 89^0\right)=$$

$$=\left(\sin^2 1^0+\sin^2 89^0\right)+\left(\sin^2 2^0+\sin^2 88^0\right)+\ldots+\left(\sin^2 44^0+\sin^2 46^0\right)+\sin^2 45^0=$$

$$=\left(\sin^2 1^0+\cos^2 1^0\right)+\left(\sin^2 2^0+\cos^2 2^0\right)+\ldots+\left(\sin^2 44^0+\cos^2 44^0\right)+\sin^2 45^0=$$

$$=\underbrace{1+1+\ldots+1}_{44\ \text{puta}}+\sin^2 45^0=44\cdot 1+\left(\frac{\sqrt{2}}{2}\right)^2=44+\frac{2}{4}=44+\frac{2}{4}=44+\frac{1}{2}=44.5.$$

Odgovor je pod D.

Vježba 338

Vrijednost brojevnog izraza $\cos^2 1^0+\cos^2 2^0+\cos^2 3^0+\ldots+\cos^2 89^0$ jednaka je:

A. 1.5 *B.* 89 *C.* 0 *D.* 44.5

Rezultat: D.

Zadatak 339 (Tina, gimnazija)

Izračunaj vrijednost izraza $\cos^2 18^0+\cos^2 36^0+\cos^2 54^0+\cos^2 72^0.$

Rješenje 339

Ponovimo!

$$\cos\alpha=\sin\left(90^0-\alpha\right)\quad,\quad \sin^2\alpha+\cos^2\alpha=1.$$

$$\cos^2 18^0+\cos^2 36^0+\cos^2 54^0+\cos^2 72^0=$$

$$=\cos^2 18^0+\cos^2 36^0+\sin^2\left(90^0-54^0\right)+\sin^2\left(90^0-72^0\right)=$$

$$=\cos^2 18^0+\cos^2 36^0+\sin^2 36^0+\sin^2 18^0=\cos^2 18^0+\sin^2 18^0+\cos^2 36^0+\sin^2 36^0=$$

$$=\left(\cos^2 18^0+\sin^2 18^0\right)+\left(\cos^2 36^0+\sin^2 36^0\right)=1+1=2.$$

Vježba 339

Izračunaj vrijednost izraza $\cos^2 10^0+\cos^2 30^0+\cos^2 60^0+\cos^2 80^0.$

Rezultat: 2.

Zadatak 340 (Tina, gimnazija)

Ako je $x=\dfrac{1}{\cos\alpha\cdot\cos\beta},\ y=\dfrac{tg\,\alpha}{\cos\beta},\ z=tg\,\beta,$ izračunati vrijednost izraza $x^2-y^2-z^2.$

Rješenje 340

Ponovimo!

$$\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \frac{a\cdot c}{b\cdot d}=\frac{a}{b}\cdot\frac{c}{d} \quad , \quad \frac{1}{\cos^2\alpha}=1+tg^2\alpha.$$

$$\frac{1}{ctg^2\alpha}=tg^2\alpha.$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\left.\begin{array}{l} x=\dfrac{1}{\cos\alpha\cdot\cos\beta} \\ y=\dfrac{tg\,\alpha}{\cos\beta} \\ z=tg\,\beta \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=\dfrac{1}{\cos\alpha\cdot\cos\beta}\ /^2 \\ y=\dfrac{tg\,\alpha}{\cos\beta}\ /^2 \\ z=tg\,\beta\ /^2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2=\dfrac{1}{\cos^2\alpha\cdot\cos^2\beta} \\ y^2=\dfrac{tg^2\alpha}{\cos^2\beta} \\ z^2=tg^2\beta \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x^2=\dfrac{1}{\cos^2\alpha}\cdot\dfrac{1}{\cos^2\beta} \\ y^2=tg^2\alpha\cdot\dfrac{1}{\cos^2\beta} \\ z^2=tg^2\beta \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2=\left(1+tg^2\alpha\right)\cdot\left(1+tg^2\beta\right) \\ y^2=tg^2\alpha\cdot\left(1+tg^2\beta\right) \\ z^2=tg^2\beta \end{array}\right\}.$$

Sada je:

$$x^2-y^2-z^2=\left(1+tg^2\alpha\right)\cdot\left(1+tg^2\beta\right)-tg^2\alpha\cdot\left(1+tg^2\beta\right)-tg^2\beta=$$

$$=1+tg^2\beta+tg^2\alpha+tg^2\alpha\cdot tg^2\beta-tg^2\alpha-tg^2\alpha\cdot tg^2\beta-tg^2\beta=$$

$$=1+tg^2\beta+tg^2\alpha+tg^2\alpha\cdot tg^2\beta-tg^2\alpha-tg^2\alpha\cdot tg^2\beta-tg^2\beta=1.$$

Vježba 340

Ako je $x=\dfrac{1}{\cos\alpha\cdot\cos\beta}$, $y=\dfrac{1}{ctg\,\alpha\cdot\cos\beta}$, $z=\dfrac{1}{ctg\,\beta}$, izračunati vrijednost izraza $x^2-y^2-z^2$.

Rezultat: 1.