Zadatak 301 (Ivek, tehnička škola)

Dokazati identitet: $tg\,6x - tg\,4x - tg\,2x = tg\,6x \cdot tg\,4x \cdot tg\,2x.$

Rješenje 301

Ponovimo!

$$tg\,(x+y)=\frac{tg\,x+tg\,y}{1-tg\,x\cdot tg\,y} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}\cdot c=a\cdot\frac{c}{b}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Preoblikujemo lijevu stranu identiteta.

$$tg\,6x-tg\,4x-tg\,2x=tg\,(4x+2x)-tg\,4x-tg\,2x=tg\,(4x+2x)-(tg\,4x+tg\,2x)=$$

$$=\frac{tg\,4x+tg\,2x}{1-tg\,4x\cdot tg\,2x}-(tg\,4x+tg\,2x)=\frac{tg\,4x+tg\,2x}{1-tg\,4x\cdot tg\,2x}-\frac{tg\,4x+tg\,2x}{1}=$$

$$=\frac{tg\,4x+tg\,2x}{1-tg\,4x\cdot tg\,2x}-\frac{tg\,4x+tg\,2x}{1}=$$

$$=(tg\,4x+tg\,2x)\cdot\left(\frac{1}{1-tg\,4x\cdot tg\,2x}-\frac{1}{1}\right)=(tg\,4x+tg\,2x)\cdot\frac{1-(1-tg\,4x\cdot tg\,2x)}{1-tg\,4x\cdot tg\,2x}=$$

$$=(tg\,4x+tg\,2x)\cdot\frac{1-1+tg\,4x\cdot tg\,2x}{1-tg\,4x\cdot tg\,2x}=(tg\,4x+tg\,2x)\cdot\frac{1-1+tg\,4x\cdot tg\,2x}{1-tg\,4x\cdot tg\,2x}=$$

$$=(tg\,4x+tg\,2x)\cdot\frac{tg\,4x\cdot tg\,2x}{1-tg\,4x\cdot tg\,2x}=\frac{tg\,4x+tg\,2x}{1}\cdot\frac{tg\,4x\cdot tg\,2x}{1-tg\,4x\cdot tg\,2x}=\frac{tg\,4x+tg\,2x}{1-tg\,4x\cdot tg\,2x}\cdot\frac{tg\,4x\cdot tg\,2x}{1}=$$

$$=tg\,6x\cdot\frac{tg\,4x\cdot tg\,2x}{1}=tg\,6x\cdot tg\,4x\cdot tg\,2x.$$

2.inačica

Polazimo od identiteta

$$tg\,(x+y)=\frac{tg\,x+tg\,y}{1-tg\,x\cdot tg\,y}.$$

Uočimo da je

$$tg\,6x=tg\,(4x+2x).$$

Tada vrijedi:

$$tg\,6x=\frac{tg\,4x+tg\,2x}{1-tg\,4x\cdot tg\,2x} \Rightarrow tg\,6x=\frac{tg\,4x+tg\,2x}{1-tg\,4x\cdot tg\,2x} \;/\cdot(1-tg\,4x\cdot tg\,2x) \Rightarrow$$

$$\Rightarrow tg\,6x\cdot(1-tg\,4x\cdot tg\,2x)=tg\,4x+tg\,2x \Rightarrow tg\,6x-tg\,6x\cdot tg\,4x\cdot tg\,2x=tg\,4x+tg\,2x \Rightarrow$$

$$\Rightarrow tg\,6x-tg\,4x-tg\,2x=tg\,6x\cdot tg\,4x\cdot tg\,2x.$$

Vježba 301

Dokazati identitet: $tg\,3x-tg\,2x-tg\,x=tg\,3x\cdot tg\,2x\cdot tg\,x.$

Rezultat: Dokaz analogan.

Zadatak 302 (Girl in the black, gimnazija)

Dokazati identitet: $4\cdot\sin x\cdot\cos x-8\cdot\sin^3 x\cdot\cos x=\sin 4x.$

Rješenje 302

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1 \quad , \quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha \quad , \quad \cos 2\alpha=\cos^2\alpha-\sin^2\alpha.$$

$$\cos 2\alpha=1-2\cdot\sin^2\alpha \quad , \quad a^1=a \quad , \quad a^n:a^m=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Preoblikujemo lijevu stranu identiteta.

$$4\cdot\sin x\cdot\cos x-8\cdot\sin^3 x\cdot\cos x=4\cdot\sin x\cdot\cos x\cdot\left(1-2\cdot\sin^2 x\right)=\left[1-2\cdot\sin^2 x=\cos 2x\right]=$$

$$=4\cdot\sin x\cdot\cos x\cdot\cos 2x=2\cdot2\cdot\sin x\cdot\cos x\cdot\cos 2x=2\cdot(2\cdot\sin x\cdot\cos x)\cdot\cos 2x=2\cdot\sin 2x\cdot\cos 2x=$$

$$=\sin(2\cdot2x)=\sin 4x.$$

2.inačica

Preoblikujemo lijevu stranu identiteta.

$$4\cdot\sin x\cdot\cos x-8\cdot\sin^3 x\cdot\cos x=4\cdot\sin x\cdot\cos x\cdot\left(1-2\cdot\sin^2 x\right)=$$

$$=4\cdot\sin x\cdot\cos x\cdot\left(\cos^2 x+\sin^2 x-2\cdot\sin^2 x\right)=4\cdot\sin x\cdot\cos x\cdot\left(\cos^2 x-\sin^2 x\right)=$$

$$=\left[\cos^2 x-\sin^2 x=\cos 2x\right]=4\cdot\sin x\cdot\cos x\cdot\cos 2x=2\cdot2\cdot\sin x\cdot\cos x\cdot\cos 2x=$$

$$=2\cdot(2\cdot\sin x\cdot\cos x)\cdot\cos 2x=2\cdot\sin 2x\cdot\cos 2x=\sin(2\cdot2x)=\sin 4x.$$

Vježba 302

Dokazati identitet: $4\cdot\sin x\cdot\cos x-4\cdot\left(1-\cos^2 x\right)\cdot\sin 2x=\sin 4x.$

Rezultat: Dokaz analogan.

Zadatak 303 (Girl in the black, gimnazija)

Dokazati identitet: $\dfrac{\sin 3x}{\sin x}-\dfrac{\cos 3x}{\cos x}=2.$

Rješenje 303

Ponovimo!

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \sin(\alpha-\beta)=\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta \quad , \quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq0 \ , \ n\neq1.$$

Preoblikujemo lijevu stranu identiteta.

$$\frac{\sin 3x}{\sin x}-\frac{\cos 3x}{\cos x}=\frac{\sin 3x\cdot\cos x-\cos 3x\cdot\sin x}{\sin x\cdot\cos x}=\frac{\sin(3x-x)}{\sin x\cdot\cos x}=\frac{\sin 2x}{\sin x\cdot\cos x}=$$

$$=\frac{2\cdot\sin x\cdot\cos x}{\sin x\cdot\cos x}=\frac{2\cdot\sin x\cdot\cos x}{\sin x\cdot\cos x}=2.$$

Vježba 303

Dokazati identitet: $\frac{\cos 3x}{\cos x}-\frac{\sin 3x}{\sin x}=-2.$

Rezultat: Dokaz analogan.

Zadatak 304 (DVD, gimnazija)

Provjeri jednakost $\frac{1}{\sin 10^{0}}-4\cdot\sin 70^{0}=2.$

Rješenje 304

Ponovimo!

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\sin x\cdot\sin y=\frac{1}{2}\cdot\left[\cos(x-y)-\cos(x+y)\right].$$

$$\cos\left(90^{0}-x\right)=\sin x\quad,\quad\cos 60^{0}=\frac{1}{2}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Preoblikujemo lijevu stranu jednakosti uporabom formule za transformaciju umnoška sinusa.

$$\frac{1}{\sin 10^{0}}-4\cdot\sin 70^{0}=\frac{1}{\sin 10^{0}}-\frac{4\cdot\sin 70^{0}}{1}=\frac{1-4\cdot\sin 70^{0}\cdot\sin 10^{0}}{\sin 10^{0}}=$$

$$=\frac{1-4\cdot\frac{1}{2}\cdot\left[\cos\left(70^{0}-10^{0}\right)-\cos\left(70^{0}+10^{0}\right)\right]}{\sin 10^{0}}=\frac{1-4\cdot\frac{1}{2}\cdot\left[\cos\left(70^{0}-10^{0}\right)-\cos\left(70^{0}+10^{0}\right)\right]}{\sin 10^{0}}=$$

$$=\frac{1-2\cdot\left[\cos 60^{0}-\cos 80^{0}\right]}{\sin 10^{0}}=\frac{1-2\cdot\cos 60^{0}+2\cdot\cos 80^{0}}{\sin 10^{0}}=\frac{1-2\cdot\frac{1}{2}+2\cdot\cos 80^{0}}{\sin 10^{0}}=$$

$$=\frac{1-2\cdot\frac{1}{2}+2\cdot\cos 80^{0}}{\sin 10^{0}}=\frac{1-1+2\cdot\cos 80^{0}}{\sin 10^{0}}=\frac{1-1+2\cdot\cos 80^{0}}{\sin 10^{0}}=\frac{2\cdot\cos 80^{0}}{\sin 10^{0}}=$$

$$=\frac{2\cdot\cos\left(90^{0}-10^{0}\right)}{\sin 10^{0}}=\frac{2\cdot\sin 10^{0}}{\sin 10^{0}}=\frac{2\cdot\sin 10^{0}}{\sin 10^{0}}=2.$$

Vježba 304

Provjeri jednakost $4\cdot\sin 70^{0}-\frac{1}{\sin 10^{0}}-2=0.$

Rezultat: Jednakost valjana.

Zadatak 305 (DVD, gimnazija)

Pojednostavimo li izraz $S=\frac{2\cdot\left(\sin 2\alpha+2\cdot\cos^2\alpha-1\right)}{\cos\alpha-\sin\alpha-\cos 3\alpha+\sin 3\alpha}$, dobit ćemo:

$A.\ S=\frac{1}{\sin\alpha}$ $\quad B.\ S=\frac{1}{\cos\alpha}$ $\quad C.\ S=\sin\alpha$ $\quad D.\ S=\cos\alpha$

Rješenje 305

Ponovimo!

$$\cos^2 x+\sin^2 x=1 \quad , \quad \cos 2x=\cos^2 x-\sin^2 x \quad , \quad \sin(-x)=-\sin x.$$

$$\cos x-\cos y=-2\cdot\sin\frac{x-y}{2}\cdot\sin\frac{x+y}{2} \quad , \quad \sin x-\sin y=2\cdot\sin\frac{x-y}{2}\cdot\cos\frac{x+y}{2}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Preoblikujemo brojnik i nazivnik zadanog razlomka.

$$S=\frac{2\cdot\left(\sin 2\alpha+2\cdot\cos^2\alpha-1\right)}{\cos\alpha-\sin\alpha-\cos 3\alpha+\sin 3\alpha} \Rightarrow S=\frac{2\cdot\left(\sin 2\alpha+2\cdot\cos^2\alpha-\left(\cos^2\alpha+\sin^2\alpha\right)\right)}{(\cos\alpha-\cos 3\alpha)+(\sin 3\alpha-\sin\alpha)} \Rightarrow$$

$$\Rightarrow S=\frac{2\cdot\left(\sin 2\alpha+2\cdot\cos^2\alpha-\cos^2\alpha-\sin^2\alpha\right)}{-2\cdot\sin\frac{\alpha-3\alpha}{2}\cdot\sin\frac{\alpha+3\alpha}{2}+2\cdot\sin\frac{3\alpha-\alpha}{2}\cdot\cos\frac{3\alpha+\alpha}{2}} \Rightarrow$$

$$\Rightarrow S=\frac{2\cdot\left(\sin 2\alpha+\cos^2\alpha-\sin^2\alpha\right)}{-2\cdot\sin\frac{-2\alpha}{2}\cdot\sin\frac{4\alpha}{2}+2\cdot\sin\frac{2\alpha}{2}\cdot\cos\frac{4\alpha}{2}} \Rightarrow$$

$$\Rightarrow S=\frac{2\cdot(\sin 2\alpha+\cos 2\alpha)}{-2\cdot\sin\frac{-2\alpha}{2}\cdot\sin\frac{4\alpha}{2}+2\cdot\sin\frac{2\alpha}{2}\cdot\cos\frac{4\alpha}{2}} \Rightarrow$$

$$\Rightarrow S=\frac{2\cdot(\sin 2\alpha+\cos 2\alpha)}{-2\cdot\sin(-\alpha)\cdot\sin 2\alpha+2\cdot\sin\alpha\cdot\cos 2\alpha} \Rightarrow S=\frac{2\cdot(\sin 2\alpha+\cos 2\alpha)}{2\cdot\sin\alpha\cdot\sin 2\alpha+2\cdot\sin\alpha\cdot\cos 2\alpha} \Rightarrow$$

$$\Rightarrow S=\frac{2\cdot(\sin 2\alpha+\cos 2\alpha)}{2\cdot\sin\alpha\cdot(\sin 2\alpha+\cos 2\alpha)} \Rightarrow S=\frac{2\cdot(\sin 2\alpha+\cos 2\alpha)}{2\cdot\sin\alpha\cdot(\sin 2\alpha+\cos 2\alpha)} \Rightarrow S=\frac{1}{\sin\alpha}.$$

Odgovor je pod A.

Vježba 305

Pojednostavimo li izraz $S=\frac{2\cdot\left(1-\sin 2\alpha-2\cdot\cos^2\alpha\right)}{\sin\alpha+\cos 3\alpha-\cos\alpha-\sin 3\alpha}$, dobit ćemo:

$A.\ S=\frac{1}{\sin\alpha}$ $\quad B.\ S=\frac{1}{\cos\alpha}$ $\quad C.\ S=\sin\alpha$ $\quad D.\ S=\cos\alpha$

Rezultat: A.

Zadatak 306 (DVD, gimnazija)

Ako je $(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)=\cos\alpha\cdot\cos\beta\cdot\cos\gamma,$ pojednostavnite izraz $A=(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma).$

Rješenje 306

Ponovimo!

$$n=\frac{n}{1}\quad,\quad(a-b)\cdot(a+b)=a^2-b^2\quad,\quad\cos^2x+\sin^2x=1.$$

$$a^1=a\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad a^n:a^m=a^{n-m}.$$

$$\cos x-\cos y=-2\cdot\sin\frac{x-y}{2}\cdot\sin\frac{x+y}{2}\quad,\quad\sin x-\sin y=2\cdot\sin\frac{x-y}{2}\cdot\cos\frac{x+y}{2}.$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq0\ ,\ n\neq1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq0\ ,\ n\neq1.$$

Uporabom pretpostavke dobije se:

$$A=(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)\ \Rightarrow\ A=\frac{(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)}{1}\ \Rightarrow$$

$$\Rightarrow\ A=\frac{(1-\sin\alpha)\cdot(1-\sin\beta)\cdot(1-\sin\gamma)}{1}\cdot\frac{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}\ \Rightarrow$$

$$\Rightarrow\ A=\frac{(1-\sin\alpha)\cdot(1+\sin\alpha)\cdot(1-\sin\beta)\cdot(1+\sin\beta)\cdot(1-\sin\gamma)\cdot(1+\sin\gamma)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}\ \Rightarrow$$

$$\Rightarrow\ A=\frac{\left(1-\sin^2\alpha\right)\cdot\left(1-\sin^2\beta\right)\cdot\left(1-\sin^2\gamma\right)}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}\ \Rightarrow\ A=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)}\ \Rightarrow$$

$$\Rightarrow\ \left[\begin{matrix}\text{pretpostavka}\\(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)=\cos\alpha\cdot\cos\beta\cdot\cos\gamma\end{matrix}\right]\ \Rightarrow$$

$$\Rightarrow\ A=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{\cos\alpha\cdot\cos\beta\cdot\cos\gamma}\ \Rightarrow\ A=\frac{\cos^2\alpha\cdot\cos^2\beta\cdot\cos^2\gamma}{\cos\alpha\cdot\cos\beta\cdot\cos\gamma}\ \Rightarrow\ A=\cos\alpha\cdot\cos\beta\cdot\cos\gamma.$$

Vježba 306

Ako je $(1+\sin\alpha)\cdot(1+\sin\beta)\cdot(1+\sin\gamma)-\cos\alpha\cdot\cos\beta\cdot\cos\gamma=0,$ pojednostavnite izraz $A=-(\sin\alpha-1)\cdot(\sin\beta-1)\cdot(\sin\gamma-1).$

Rezultat: $A=\cos\alpha\cdot\cos\beta\cdot\cos\gamma.$

Zadatak 307 (Hrvoje, gimnazija)

Ako je $tg^2 x + ctg^2 x = a,\ tg^4 x + ctg^4 x = b,\ a\ x \neq k \cdot \frac{\pi}{2},\ k \in Z,$ dokazati da je $a^2 - b = 2.$

Rješenje 307

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad a^n \cdot b^n = (a \cdot b)^n \quad , \quad tg\, x \cdot ctg\, x = 1.$$

Transformiramo prvu jednakost kvadriranjem.

$$tg^2 x + ctg^2 x = a \Rightarrow tg^2 x + ctg^2 x = a \ /^2 \Rightarrow \left(tg^2 x + ctg^2 x\right)^2 = a^2 \Rightarrow$$

$$\Rightarrow \left(tg^2 x\right)^2 + 2 \cdot tg^2 x \cdot ctg^2 x + \left(ctg^2 x\right)^2 = a^2 \Rightarrow tg^4 x + 2 \cdot (tg\, x \cdot ctg\, x) + ctg^4 x = a^2 \Rightarrow$$

$$\Rightarrow tg^4 x + 2 \cdot 1 + ctg^4 x = a^2 \Rightarrow tg^4 x + 2 + ctg^4 x = a^2 \Rightarrow tg^4 x + ctg^4 x = a^2 - 2.$$

Sada je:

$$\left.\begin{array}{l} tg^4 x + ctg^4 x = a^2 - 2 \\ tg^4 x + ctg^4 x = b \ \text{uvjet} \end{array}\right\} \Rightarrow a^2 - 2 = b \Rightarrow a^2 - b = 2.$$

Vježba 307

Ako je $tg^2 x + ctg^2 x = a,\ tg^4 x + ctg^4 x = b^2,\ a\ x \neq k \cdot \frac{\pi}{2},\ k \in Z,$ dokazati da je

$a^2 - b^2 = 2.$

Rezultat: Dokaz analogan.

Zadatak 308 (Hrvoje, gimnazija)

Dokaži jednakost $\frac{\sin\alpha + \cos\alpha}{\sin\alpha - \cos\alpha} - \frac{1 + 2 \cdot \cos^2\alpha}{\cos^2\alpha \cdot \left(tg^2\alpha - 1\right)} = \frac{2}{1 + tg\,\alpha}.$

Rješenje 308

Ponovimo!

$$tg\, x = \frac{\sin x}{\cos x} \quad , \quad \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2} \quad , \quad b \cdot \frac{a}{b} = a \quad , \quad a^2 - b^2 = (a-b) \cdot (a+b).$$

$$\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad \cos^2 x + \sin^2 x = 1 \quad , \quad (a+b) = a^2 + 2 \cdot a \cdot b + b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Preoblikujemo lijevu stranu jednakosti.

$$\frac{\sin\alpha + \cos\alpha}{\sin\alpha - \cos\alpha} - \frac{1 + 2 \cdot \cos^2\alpha}{\cos^2\alpha \cdot \left(tg^2\alpha - 1\right)} = \frac{\sin\alpha + \cos\alpha}{\sin\alpha - \cos\alpha} - \frac{1 + 2 \cdot \cos^2\alpha}{\cos^2\alpha \cdot \left(\frac{\sin^2 x}{\cos^2 x} - 1\right)} =$$

$$=\frac{\sin\alpha+\cos\alpha}{\sin\alpha-\cos\alpha}-\frac{1+2\cdot\cos^2\alpha}{\sin^2\alpha-\cos^2 x}=\frac{\sin\alpha+\cos\alpha}{\sin\alpha-\cos\alpha}-\frac{1+2\cdot\cos^2\alpha}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=$$

$$=\frac{(\sin\alpha+\cos\alpha)^2-\left(1+2\cdot\cos^2\alpha\right)}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=\frac{\sin^2\alpha+2\cdot\sin\alpha\cdot\cos\alpha+\cos^2\alpha-1-2\cdot\cos^2\alpha}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=$$

$$=\frac{\left(\sin^2\alpha+\cos^2\alpha\right)+2\cdot\sin\alpha\cdot\cos\alpha-1-2\cdot\cos^2\alpha}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=\frac{1+2\cdot\sin\alpha\cdot\cos\alpha-1-2\cdot\cos^2\alpha}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=$$

$$=\frac{1+2\cdot\sin\alpha\cdot\cos\alpha-1-2\cdot\cos^2\alpha}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=\frac{2\cdot\sin\alpha\cdot\cos\alpha-2\cdot\cos^2\alpha}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=$$

$$=\frac{2\cdot\cos\alpha\cdot(\sin\alpha-\cos\alpha)}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=\frac{2\cdot\cos\alpha\cdot(\sin\alpha-\cos\alpha)}{(\sin\alpha-\cos\alpha)\cdot(\sin\alpha+\cos\alpha)}=\frac{2\cdot\cos\alpha}{\sin\alpha+\cos\alpha}=$$

$$=\frac{2\cdot\cos\alpha}{\cos\alpha\cdot\left(\frac{\sin\alpha}{\cos\alpha}+1\right)}=\frac{2\cdot\cos\alpha}{\cos\alpha\cdot\left(\frac{\sin\alpha}{\cos\alpha}+1\right)}=\frac{2}{\frac{\sin\alpha}{\cos\alpha}+1}=\frac{2}{tg\,\alpha+1}=\frac{2}{1+tg\,\alpha}.$$

Vježba 308

Dokaži jednakost $\frac{\sin\alpha+\cos\alpha}{\sin\alpha-\cos\alpha}+\frac{1+2\cdot\cos^2\alpha}{\cos^2\alpha\cdot\left(1-tg^2\alpha\right)}=\frac{2}{1+tg\,\alpha}.$

Rezultat: Dokaz analogan.

Zadatak 309 (Hrvoje, gimnazija)

Ako je $\alpha+\beta=\frac{\pi}{4}$, pokazati da je $(1+tg\,\alpha)\cdot(1+tg\,\beta)=2.$

Rješenje 309

Ponovimo!

$$tg\,(x-y)=\frac{tg\,x-tg\,y}{1+tg\,x\cdot tg\,y}\quad,\quad tg\frac{\pi}{4}=1\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad b\cdot\frac{a}{b}=a.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Budući da je

$$\alpha+\beta=\frac{\pi}{4}\ \Rightarrow\ \beta=\frac{\pi}{4}-\alpha,$$

slijedi

$$(1+tg\,\alpha)\cdot(1+tg\,\beta)=(1+tg\,\alpha)\cdot\left(1+tg\left(\frac{\pi}{4}-\alpha\right)\right)=(1+tg\,\alpha)\cdot\left(1+\frac{tg\frac{\pi}{4}-tg\,\alpha}{1+tg\frac{\pi}{4}\cdot tg\,\alpha}\right)=$$

$$=(1+tg\,\alpha)\cdot\left(1+\frac{1-tg\,\alpha}{1+1\cdot tg\,\alpha}\right)=(1+tg\,\alpha)\cdot\left(1+\frac{1-tg\,\alpha}{1+tg\,\alpha}\right)=(1+tg\,\alpha)\cdot\left(\frac{1}{1}+\frac{1-tg\,\alpha}{1+tg\,\alpha}\right)=$$

$$=(1+tg\,\alpha)\cdot\frac{1+tg\,\alpha+1-tg\,\alpha}{1+tg\,\alpha}=(1+tg\,\alpha)\cdot\frac{1+tg\,\alpha+1-tg\,\alpha}{1+tg\,\alpha}=(1+tg\,\alpha)\cdot\frac{2}{1+tg\,\alpha}=$$

$$=(1+tg\,\alpha)\cdot\frac{2}{1+tg\,\alpha}=2.$$

Vježba 309

Ako je $\alpha+\beta=\frac{\pi}{4}$, pokazati da je $tg\,\alpha+tg\,\beta+tg\,\alpha\cdot tg\,\beta=1$.

Rezultat: Dokaz analogan.

Zadatak 310 (Tina, gimnazija)

Dokaži: $tg\,x+tg\,y=\frac{\sin(x+y)}{\cos x\cdot\cos y}$.

Rješenje 310

Ponovimo!

$$tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta.$$

$$\sin(\alpha-\beta)=\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta.$$

$$tg\,x+tg\,y=\frac{\sin x}{\cos x}+\frac{\sin y}{\cos y}=\frac{\sin x\cdot\cos y+\cos x\cdot\sin y}{\cos x\cdot\cos y}=\frac{\sin(x+y)}{\cos x\cdot\cos y}.$$

Vježba 310

Dokaži: $tg\,x-tg\,y=\frac{\sin(x-y)}{\cos x\cdot\cos y}$.

Rezultat: Dokaz analogan.

Zadatak 311 (Tina, gimnazija)

Dokaži: $1-tg\,x\cdot tg\,y=\frac{\cos(x+y)}{\cos x\cdot\cos y}$.

Rješenje 311

Ponovimo!

$$tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta\quad,\quad\cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta.$$

$$1-tg\,x\cdot tg\,y=1-\frac{\sin x}{\cos x}\cdot\frac{\sin y}{\cos y}=1-\frac{\sin x\cdot\sin y}{\cos x\cdot\cos y}=\frac{1}{1}-\frac{\sin x\cdot\sin y}{\cos x\cdot\cos y}=\frac{\cos x\cdot\cos y-\sin x\cdot\sin y}{\cos x\cdot\cos y}=$$

$$=\frac{\cos(x+y)}{\cos x\cdot\cos y}.$$

Vježba 311

Dokaži: $1+tg\,x\cdot tg\,y=\frac{\cos(x-y)}{\cos x\cdot\cos y}$.

Rezultat: Dokaz analogan.

Zadatak 312 (Marko, tehnička škola)

Ako je $\sin x=\frac{4}{5}$ i $\sin y=\frac{12}{13}$, $x,\ y\in\left\langle 0,\ \frac{\pi}{2}\right\rangle$, koliko je $\sin(x+y)$?

Rješenje 312

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1\quad,\quad\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad,\quad n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

	Predznak vrijednosti funkcije			
	sin x	**cos x**	**tg x**	**ctg x**
I. kvadrant	+	+	+	+
II. kvadrant	+	–	–	–
III. kvadrant	–	–	+	+
IV. kvadrant	–	+	–	–

$$\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

$$\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta\quad,\quad\sin(\alpha-\beta)=\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta.$$

$$\left.\begin{array}{l}\cos^2 x+\sin^2 x=1\\ \cos^2 y+\sin^2 y=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x+\left(\frac{4}{5}\right)^2=1\\ \cos^2 y+\left(\frac{12}{13}\right)^2=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x+\frac{4^2}{5^2}=1\\ \cos^2 y+\frac{12^2}{13^2}=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x+\frac{16}{25}=1\\ \cos^2 y+\frac{144}{169}=1\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}\cos^2 x=1-\frac{16}{25}\\ \cos^2 y=1-\frac{144}{169}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x=\frac{1}{1}-\frac{16}{25}\\ \cos^2 y=\frac{1}{1}-\frac{144}{169}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x=\frac{25-16}{25}\\ \cos^2 y=\frac{169-144}{169}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x=\frac{9}{25}\\ \cos^2 y=\frac{25}{169}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}\cos^2 x=\frac{9}{25}\ /\sqrt{\ }\\ \cos^2 y=\frac{25}{169}\ /\sqrt{\ }\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos x=\pm\sqrt{\frac{9}{25}}\\ \cos y=\pm\sqrt{\frac{25}{169}}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos x=\pm\frac{\sqrt{9}}{\sqrt{25}}\\ \cos y=\pm\frac{\sqrt{25}}{\sqrt{169}}\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos x=\pm\frac{3}{5}\\ \cos y=\pm\frac{5}{13}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left[\begin{array}{c}x,\ y\in\left\langle 0,\ \frac{\pi}{2}\right\rangle\ ,\ \text{prvi kvadrant}\\ \cos x>0\ ,\ \cos y>0\end{array}\right]\Rightarrow\left.\begin{array}{l}\cos x=\frac{3}{5}\\ \cos y=\frac{5}{13}\end{array}\right\}.$$

Sada je:

$$\sin(x+y)=\sin x\cdot\cos y+\cos x\cdot\sin y=\left[\begin{array}{l}\sin x=\frac{4}{5}\ ,\ \sin y=\frac{12}{13}\\ \cos x=\frac{3}{5}\ ,\ \cos y=\frac{5}{13}\end{array}\right]=$$

$$=\frac{4}{5}\cdot\frac{5}{13}+\frac{3}{5}\cdot\frac{12}{13}=\frac{4}{5}\cdot\frac{5}{13}+\frac{3}{5}\cdot\frac{12}{13}=\frac{4}{13}+\frac{3}{5}\cdot\frac{12}{13}=\frac{4}{13}+\frac{36}{65}=\frac{20+36}{65}=\frac{56}{65}.$$

Vježba 312

Ako je $\sin x=\frac{4}{5}$ i $\sin y=\frac{12}{13}$, $x,\ y\in\left\langle 0,\ \frac{\pi}{2}\right\rangle$, koliko je $\sin(x-y)$?

Rezultat: $-\frac{16}{65}.$

Zadatak 313 (Marko, tehnička škola)

Ako je $\sin x=\frac{5\cdot\sqrt{3}}{14}$ i $\cos y=-\frac{3\cdot\sqrt{3}}{14}$, $x\in\left\langle 0,\ \frac{\pi}{2}\right\rangle$, $y\in\left\langle \frac{\pi}{2},\ \pi\right\rangle$, koliko je $\sin(x+y)$?

Rješenje 313

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1\quad,\quad\left(\frac{a}{b}\right)^2=\frac{a^2}{b^2}\quad,\quad(a\cdot b)^2=a^2\cdot b^2\quad,\quad\left(\sqrt{a}\right)^2=a.$$

$$n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}\quad,\quad\sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}\quad,\quad\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

$$\frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad\frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

	Predznak vrijednosti funkcije			
	sin x	cos x	tg x	ctg x
I. kvadrant	+	+	+	+
II. kvadrant	+	–	–	–
III. kvadrant	–	–	+	+
IV. kvadrant	–	+	–	–

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\quad,\quad n\neq 0\quad,\quad n\neq 1.$$

$$\sin(\alpha+\beta)=\sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta\quad,\quad\sin(\alpha-\beta)=\sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta.$$

$$\left.\begin{array}{l}\cos^2 x+\sin^2 x=1\\ \cos^2 y+\sin^2 y=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x+\left(\frac{5\cdot\sqrt{3}}{14}\right)^2=1\\ \left(-\frac{3\cdot\sqrt{3}}{14}\right)^2+\sin^2 y=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x+\frac{\left(5\cdot\sqrt{3}\right)^2}{14^2}=1\\ \frac{\left(3\cdot\sqrt{3}\right)^2}{14^2}+\sin^2 y=1\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}\cos^2 x+\frac{5^2\cdot\left(\sqrt{3}\right)^2}{14^2}=1\\ \frac{3^2\cdot\left(\sqrt{3}\right)^2}{14^2}+\sin^2 y=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x+\frac{25\cdot 3}{196}=1\\ \frac{9\cdot 3}{196}+\sin^2 y=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos^2 x+\frac{75}{196}=1\\ \frac{27}{196}+\sin^2 y=1\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \cos^2 x = 1 - \frac{75}{196} \\ \sin^2 y = 1 - \frac{27}{196} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos^2 x = \frac{1}{1} - \frac{75}{196} \\ \sin^2 y = \frac{1}{1} - \frac{27}{196} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos^2 x = \frac{196-75}{196} \\ \sin^2 y = \frac{195-27}{196} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos^2 x = \frac{121}{196} \\ \sin^2 y = \frac{169}{196} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \cos^2 x = 1 - \frac{75}{196} \\ \sin^2 y = 1 - \frac{27}{196} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos^2 x = \frac{1}{1} - \frac{75}{196} \\ \sin^2 y = \frac{1}{1} - \frac{27}{196} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos^2 x = \frac{196-75}{196} \\ \sin^2 y = \frac{195-27}{196} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos^2 x = \frac{121}{196} \\ \sin^2 y = \frac{169}{196} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \cos^2 x = \frac{121}{196} \ /\sqrt{\ } \\ \sin^2 y = \frac{169}{196} \ /\sqrt{\ } \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos x = \pm\sqrt{\frac{121}{196}} \\ \sin y = \pm\sqrt{\frac{169}{196}} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos x = \pm\frac{\sqrt{121}}{\sqrt{196}} \\ \sin y = \pm\frac{\sqrt{169}}{\sqrt{196}} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos x = \pm\frac{11}{14} \\ \sin y = \pm\frac{13}{14} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix} x \in \left\langle 0, \frac{\pi}{2} \right\rangle \ , \text{ prvi kvadrant } , \ \cos x > 0 \\ y \in \left\langle \frac{\pi}{2}, 0 \right\rangle \ , \text{ drugi kvadrant } , \ \sin y > 0 \end{bmatrix} \Rightarrow \left.\begin{matrix} \cos x = \frac{11}{14} \\ \sin y = \frac{13}{14} \end{matrix}\right\}.$$

Sada je:

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y = \begin{bmatrix} \sin x = \frac{5\cdot\sqrt{3}}{14} \ , \ \sin y = \frac{13}{14} \\ \cos x = \frac{11}{14} \ , \ \cos y = -\frac{3\cdot\sqrt{3}}{14} \end{bmatrix} =$$

$$= \frac{5\cdot\sqrt{3}}{14} \cdot \left(-\frac{3\cdot\sqrt{3}}{14}\right) + \frac{11}{14} \cdot \frac{13}{14} = \frac{-15\cdot\left(\sqrt{3}\right)^2}{196} + \frac{143}{196} = \frac{-15\cdot 3}{196} + \frac{143}{196} = \frac{-45}{196} + \frac{143}{196} =$$

$$= \frac{-45+143}{196} = \frac{98}{196} = \frac{98}{196} = \frac{1}{2}.$$

Vježba 313

Ako je $\sin x = \frac{5\cdot\sqrt{3}}{14}$ i $\cos y = -\frac{3\cdot\sqrt{3}}{14}$, $x \in \left\langle 0, \frac{\pi}{2} \right\rangle$, $y \in \left\langle \frac{\pi}{2}, \pi \right\rangle$, koliko je $\sin(x-y)$?

Rezultat: $-\frac{47}{49}.$

Zadatak 314 (Edijeva Pepeljuga ☺, gimnazija)

Pojednostavni: $2 - \sin^2 x - \cos^2 x.$

Rješenje 314

Ponovimo!

$$\cos^2 \alpha + \sin^2 \alpha = 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

1.inačica

$$2-\sin^2 x-\cos^2 x=2\cdot 1-\sin^2 x-\cos^2 x=2\cdot\left(\cos^2 x+\sin^2 x\right)-\sin^2 x-\cos^2 x=$$
$$=2\cdot\cos^2 x+2\cdot\sin^2 x-\sin^2 x-\cos^2 x=\cos^2 x+\sin^2 x=1.$$

2.inačica

$$2-\sin^2 x-\cos^2 x=2-\left(\sin^2 x+\cos^2 x\right)=2-1=1.$$

Vježba 314

Pojednostavni: $3-\sin^2 x-\cos^2 x.$

Rezultat: 2.

Zadatak 315 (Mario, gimnazija)

Riješi jednadžbu: $tg\, x-\dfrac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}=0.$

Rješenje 315

Ponovimo!

$$\sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha\quad,\quad\cos^2\alpha+\sin^2\alpha=1\quad,\quad\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n\quad,\quad\frac{a}{b}=\frac{\frac{a}{c}}{\frac{b}{c}}.$$

$$tg\, x=\frac{\sin x}{\cos x}\quad,\quad\left(\frac{a}{b}\right)^n=\frac{a^n}{b^n}\quad,\quad(a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Proširiti razlomak znači brojnik i nazivnik tog razlomka pomnožiti istim brojem različitim od nule i jedinice

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\quad,\quad n\neq 0\quad,\quad n\neq 1.$$

$$\frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}\quad,\quad(a-b)^3=a^3-3\cdot a^2\cdot b+3\cdot a\cdot b^2-b^3.$$

Trigonometrijska jednadžba $tg\, x=a$

Skup rješenja jednadžbe $tg\, x=a,\ a\in R,$ je $\left\{x_0+k\cdot\pi : k\in Z\right\},$ gdje je $x_0\in R$ jedno rješenje te jednadžbe.

$$tg\, x=a\ \Rightarrow\ tg\, x=tg\, x_0\ \Rightarrow\ x=x_0+k\cdot\pi\quad,\quad k\in Z.$$

$$tg\, x-\frac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}=0\ \Rightarrow\ tg\, x-\frac{8\cdot\sin^2 x+3\cdot 2\cdot\sin x\cdot\cos x+\cos^2 x+\sin^2 x}{8\cdot\cos^2 x+3\cdot 2\cdot\sin x\cdot\cos x+\cos^2 x+\sin^2 x}=0\ \Rightarrow$$

$$\Rightarrow\ tg\, x-\frac{9\cdot\sin^2 x+6\cdot\sin x\cdot\cos x+\cos^2 x}{9\cdot\cos^2 x+6\cdot\sin x\cdot\cos x+\sin^2 x}=0\ \Rightarrow\ tg\, x-\frac{(3\cdot\sin x+\cos x)^2}{(3\cdot\cos x+\sin x)^2}=0\ \Rightarrow$$

$$\Rightarrow\ tg\, x-\left(\frac{3\cdot\sin x+\cos x}{3\cdot\cos x+\sin x}\right)^2=0\ \Rightarrow\ tg\, x-\left(\frac{\frac{3\cdot\sin x+\cos x}{\cos x}}{\frac{3\cdot\cos x+\sin x}{\cos x}}\right)^2=0\ \Rightarrow$$

$$\Rightarrow tg\,x-\left(\frac{\frac{3\cdot\sin x}{\cos x}+\frac{\cos x}{\cos x}}{\frac{3\cdot\cos x}{\cos x}+\frac{\sin x}{\cos x}}\right)^2=0 \Rightarrow tg\,x-\left(\frac{\frac{3\cdot\sin x}{\cos x}+\frac{\cos x}{\cos x}}{\frac{3\cdot\cos x}{\cos x}+\frac{\sin x}{\cos x}}\right)^2=0 \Rightarrow tg\,x-\left(\frac{3\cdot tg\,x+1}{3+tg\,x}\right)^2=0 \Rightarrow$$

$$\Rightarrow tg\,x-\frac{(3\cdot tg\,x+1)^2}{(3+tg\,x)^2}=0 \Rightarrow tg\,x-\frac{9\cdot tg^2\,x+6\cdot tg\,x+1}{9+6\cdot tg\,x+tg^2\,x}=0 \Rightarrow$$

$$\Rightarrow tg\,x-\frac{9\cdot tg^2\,x+6\cdot tg\,x+1}{9+6\cdot tg\,x+tg^2\,x}=0 \;/\cdot\left(9+6\cdot tg\,x+tg^2\,x\right) \Rightarrow$$

$$\Rightarrow tg\,x\cdot\left(9+6\cdot tg\,x+tg^2\,x\right)-\left(9\cdot tg^2\,x+6\cdot tg\,x+1\right)=0 \Rightarrow$$

$$\Rightarrow 9\cdot tg\,x+6\cdot tg^2\,x+tg^3\,x-9\cdot tg^2\,x-6\cdot tg\,x-1=0 \Rightarrow tg^3\,x-3\cdot tg^2\,x+3\cdot tg\,x-1=0 \Rightarrow$$

$$\Rightarrow (tg\,x-1)^3=0 \Rightarrow (tg\,x-1)^3=0 \;/\sqrt[3]{\;} \Rightarrow tg\,x-1=0 \Rightarrow tg\,x=1 \Rightarrow x=tg^{-1}1 \Rightarrow$$

$$\Rightarrow x=\frac{\pi}{4}+k\cdot\pi \;,\; k\in Z.$$

Vježba 315

Riješi jednadžbu: $\frac{\sin x}{\cos x}=\frac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}.$

Rezultat: $x=\frac{\pi}{4}+k\cdot\pi \;,\; k\in Z.$

Zadatak 316 (Božidar, gimnazija)

Riješi jednadžbu: $tg\left(x^2-x\right)\cdot ctg\,2=1.$

Rješenje 316

Ponovimo!

$$ctg\,\alpha=\frac{1}{tg\,\alpha} \quad,\quad a\ge b \;,\; c>0 \Rightarrow a\cdot c\ge b\cdot c \quad,\quad tg\frac{\pi}{4}=1.$$

Rješenja kvadratne jednadžbe

$$a\cdot x^2+b\cdot x+c=0$$

su brojevi

$$x_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \Rightarrow \left.\begin{matrix} x_1=\frac{-b+\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \\ x_2=\frac{-b-\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Trigonometrijska jednadžba $tg\,x=a$

Skup rješenja jednadžbe $tg\,x=a,\; a\in R,$ je $\left\{x_0+k\cdot\pi : k\in Z\right\},$ gdje je $x_0\in R$ jedno rješenje te jednadžbe.

$$tg\, x = a \Rightarrow tg\, x = tg\, x_0 \Rightarrow x = x_0 + k\cdot\pi \ ,\ k \in Z.$$

$$tg\left(x^2 - x\right)\cdot ctg\, 2 = 1 \Rightarrow tg\left(x^2 - x\right)\cdot\frac{1}{tg\, 2} = 1 \Rightarrow tg\left(x^2 - x\right)\cdot\frac{1}{tg\, 2} = 1 \ /\cdot tg\, 2 \Rightarrow$$

$$\Rightarrow tg\left(x^2 - x\right) = tg\, 2 \Rightarrow x^2 - x = 2 + k\cdot\pi \ ,\ k = 0, \pm1, \pm2, \pm3, \ldots.$$

Riješimo kvadratnu jednadžbu:

$$x^2 - x = 2 + k\cdot\pi \Rightarrow$$

$$\Rightarrow x^2 - x - 2 - k\cdot\pi = 0 \Rightarrow x^2 - x - (2 + k\cdot\pi) = 0 \Rightarrow \left.\begin{matrix} x^2 - x - (2 + k\cdot\pi) = 0 \\ a = 1\ ,\ b = -1\ ,\ c = -(2 + k\cdot\pi) \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a = 1\ ,\ b = -1\ ,\ c = -(2 + k\cdot\pi) \\ x_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a} \end{matrix}\right\} \Rightarrow x_{1,2} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\cdot 1\cdot(-(2 + k\cdot\pi))}}{2\cdot 1} \Rightarrow$$

$$\Rightarrow x_{1,2} = \frac{1 \pm \sqrt{1 + 4\cdot(2 + k\cdot\pi)}}{2} \Rightarrow x_{1,2} = \frac{1 \pm \sqrt{1 + 8 + 4\cdot k\cdot\pi}}{2} \Rightarrow x_{1,2} = \frac{1 \pm \sqrt{9 + 4\cdot k\cdot\pi}}{2}.$$

Da bi jednadžba imala realna rješenja izraz pod korijenom mora biti jednak ili veći od nule.

$$9 + 4\cdot k\cdot\pi \ge 0 \Rightarrow 4\cdot k\cdot\pi \ge -9 \Rightarrow 4\cdot k\cdot\pi \ge -9 \ /\cdot\frac{1}{4\cdot\pi} \Rightarrow k \ge -\frac{9}{4\cdot\pi} \Rightarrow k \ge -0.716.$$

Rješenja će biti realna, ako je $k \ge 0$.

$$x_{1,2} = \frac{1 \pm \sqrt{9 + 4\cdot k\cdot\pi}}{2}\ ,\ k = 0, 1, 2, 3, \ldots.$$

Vježba 316

Riješi jednadžbu: $tg\left(x^2 - x\right)\cdot ctg\, 2 - tg\,\frac{\pi}{4} = 0.$

Rezultat: $x_{1,2} = \frac{1 \pm \sqrt{9 + 4\cdot k\cdot\pi}}{2}\ ,\ k = 0, 1, 2, 3, \ldots.$

Zadatak 317 (Palčica, gimnazija)

Koristeći adicijske formule dokaži sljedeću jednakost: $tg\left(\frac{\pi}{4} + x\right) = \frac{\cos x + \sin x}{\cos x - \sin x}.$

Rješenje 317

Ponovimo!

$$tg(\alpha + \beta) = \frac{tg\,\alpha + tg\,\beta}{1 - tg\,\alpha\cdot tg\,\beta}\quad,\quad tg\,\frac{\pi}{4} = 1\quad,\quad tg\,\alpha = \frac{\sin\alpha}{\cos\alpha}\quad,\quad n = \frac{n}{1}.$$

$$\frac{a}{b} + \frac{c}{d} = \frac{a\cdot d + b\cdot c}{b\cdot d}\quad,\quad \frac{a}{b} - \frac{c}{d} = \frac{a\cdot d - b\cdot c}{b\cdot d}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a\cdot d}{b\cdot c}\quad,\quad \sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}.$$

$$\sin(\alpha + \beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta\quad,\quad \cos(\alpha + \beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b + c) = a\cdot b + a\cdot c\quad,\quad a\cdot b + a\cdot c = a\cdot(b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

1.inačica

$$tg\left(\frac{\pi}{4}+x\right) = \frac{tg\frac{\pi}{4}+tg\,x}{1-tg\frac{\pi}{4}\cdot tg\,x} = \frac{1+tg\,x}{1-1\cdot tg\,x} = \frac{1+tg\,x}{1-tg\,x} = \frac{1+\frac{\sin x}{\cos x}}{1-\frac{\sin x}{\cos x}} = \frac{\frac{1}{1}+\frac{\sin x}{\cos x}}{\frac{1}{1}-\frac{\sin x}{\cos x}} = \frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}} =$$

$$= \frac{\frac{\cos x+\sin x}{\cos x}}{\frac{\cos x-\sin x}{\cos x}} = \frac{\frac{\cos x+\sin x}{1}}{\frac{\cos x-\sin x}{1}} = \frac{\cos x+\sin x}{\cos x-\sin x}.$$

2.inačica

$$tg\left(\frac{\pi}{4}+x\right) = \frac{\sin\left(\frac{\pi}{4}+x\right)}{\cos\left(\frac{\pi}{4}+x\right)} = \frac{\sin\frac{\pi}{4}\cdot\cos x+\cos\frac{\pi}{4}\cdot\sin x}{\cos\frac{\pi}{4}\cdot\cos x-\sin\frac{\pi}{4}\cdot\sin x} = \frac{\frac{\sqrt{2}}{2}\cdot\cos x+\frac{\sqrt{2}}{2}\cdot\sin x}{\frac{\sqrt{2}}{2}\cdot\cos x-\frac{\sqrt{2}}{2}\cdot\sin x} =$$

$$= \frac{\frac{\sqrt{2}}{2}\cdot(\cos x+\sin x)}{\frac{\sqrt{2}}{2}\cdot(\cos x-\sin x)} = \frac{\frac{\sqrt{2}}{2}\cdot(\cos x+\sin x)}{\frac{\sqrt{2}}{2}\cdot(\cos x-\sin x)} = \frac{\cos x+\sin x}{\cos x-\sin x}.$$

Vježba 317

Koristeći adicijske formule dokaži sljedeću jednakost: $\dfrac{1}{tg\left(\frac{\pi}{4}+x\right)} = \dfrac{\cos x-\sin x}{\cos x+\sin x}.$

Rezultat: Dokaz analogan.

Zadatak 318 (Palčica, gimnazija)

Pojednostavni izraz: $\dfrac{\sin(2x+y)-\sin(2x-y)+\sin y}{\cos(2x+y)+\cos(2x-y)+\cos y}$, pri čemu je $\cos y \neq 0,\ \cos 2x \neq -\frac{1}{2}.$

Rješenje 318

Ponovimo!

$$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta+\cos\alpha\cdot\sin\beta \quad , \quad \sin(\alpha-\beta) = \sin\alpha\cdot\cos\beta-\cos\alpha\cdot\sin\beta.$$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta \quad , \quad \cos(\alpha-\beta) = \cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta.$$

$$tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} \quad , \quad ctg\,\alpha = \frac{\cos\alpha}{\sin\alpha}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c = a\cdot(b+c).$$

$$\frac{\sin(2x+y)-\sin(2x-y)+\sin y}{\cos(2x+y)+\cos(2x-y)+\cos y} = \frac{\sin 2x\cdot\cos y+\cos 2x\cdot\sin y-(\sin 2x\cdot\cos y-\cos 2x\cdot\sin y)+\sin y}{\cos 2x\cdot\cos y-\sin 2x\cdot\sin y+\cos 2x\cdot\cos y+\sin 2x\cdot\sin y+\cos y} =$$

$$=\frac{\sin 2x\cdot\cos y+\cos 2x\cdot\sin y-\sin 2x\cdot\cos y+\cos 2x\cdot\sin y+\sin y}{\cos 2x\cdot\cos y-\sin 2x\cdot\sin y+\cos 2x\cdot\cos y+\sin 2x\cdot\sin y+\cos y}=$$

$$=\frac{\sin 2x\cdot\cos y+\cos 2x\cdot\sin y-\sin 2x\cdot\cos y+\cos 2x\cdot\sin y+\sin y}{\cos 2x\cdot\cos y-\sin 2x\cdot\sin y+\cos 2x\cdot\cos y+\sin 2x\cdot\sin y+\cos y}=\frac{\cos 2x\cdot\sin y+\cos 2x\cdot\sin y+\sin y}{\cos 2x\cdot\cos y+\cos 2x\cdot\cos y+\cos y}=$$

$$=\frac{2\cdot\cos 2x\cdot\sin y+\sin y}{2\cdot\cos 2x\cdot\cos y+\cos y}=\frac{\sin y\cdot(2\cdot\cos 2x+1)}{\cos y\cdot(2\cdot\cos 2x+1)}=\frac{\sin y\cdot(2\cdot\cos 2x+1)}{\cos y\cdot(2\cdot\cos 2x+1)}=\frac{\sin y}{\cos y}=tg\ y.$$

Vježba 318

Pojednostavni izraz: $\frac{\cos(2x+y)+\cos(2x-y)+\cos y}{\sin(2x+y)-\sin(2x-y)+\sin y}$, pri čemu je $\sin y\neq 0,\ \cos 2x\neq -\frac{1}{2}$.

Rezultat: $ctg\ y.$

Zadatak 319 (Palčica, gimnazija)

Ako je $m=tg\ x+\sin x,\ n=tg\ x-\sin x$, koliko je $\frac{m-n}{m+n}$?

Rješenje 319

Ponovimo!

$$tg\ \alpha=\frac{\sin\alpha}{\cos\alpha}\quad,\quad n=\frac{n}{1}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\quad,\quad n\neq 0\quad,\quad n\neq 1.$$

$$\frac{m-n}{m+n}=\frac{tg\ x+\sin x-(tg\ x-\sin x)}{tg\ x+\sin x+tg\ x-\sin x}=\frac{tg\ x+\sin x-tg\ x+\sin x}{tg\ x+\sin x+tg\ x-\sin x}=\frac{tg\ x+\sin x-tg\ x+\sin x}{tg\ x+\sin x+tg\ x-\sin x}=$$

$$=\frac{\sin x+\sin x}{tg\ x+tg\ x}=\frac{2\cdot\sin x}{2\cdot tg\ x}=\frac{2\cdot\sin x}{2\cdot tg\ x}=\frac{\sin x}{tg\ x}=\frac{\frac{\sin x}{1}}{\frac{\sin x}{\cos x}}=\frac{\frac{\sin x}{1}}{\frac{\sin x}{\cos x}}=\frac{\frac{1}{1}}{\frac{1}{\cos x}}=\cos x.$$

Vježba 319

Ako je $m=tg\ x+\sin x,\ n=tg\ x-\sin x$, koliko je $\frac{m+n}{m-n}$?

Rezultat: $\frac{1}{\cos x}.$

Zadatak 320 (Mateo, gimnazija)

$$\text{Ako je } x=\frac{\sqrt{2}\cdot\cos\alpha-2\cdot\cos\left(\frac{\pi}{4}+\alpha\right)}{2\cdot\sin\left(\frac{\pi}{4}+\alpha\right)-\sqrt{2}\cdot\sin\alpha}, \text{ tada je } x \text{ jednako:}$$

$A.\ x=\sin\alpha \qquad B.\ x=\cos\alpha \qquad C.\ x=tg\,\alpha \qquad D.\ x=ctg\,\alpha$

Rješenje 320

Ponovimo!

$$\cos\frac{\pi}{4}=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2} \quad , \quad tg\,x=\frac{\sin x}{\cos x} \quad , \quad ctg\,x=\frac{\cos x}{\sin x}.$$

$$\cos(x+y)=\cos x\cdot\cos y-\sin x\cdot\sin y \quad , \quad \sin(x+y)=\sin x\cdot\cos y+\cos x\cdot\sin y.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

$$x=\frac{\sqrt{2}\cdot\cos\alpha-2\cdot\cos\left(\frac{\pi}{4}+\alpha\right)}{2\cdot\sin\left(\frac{\pi}{4}+\alpha\right)-\sqrt{2}\cdot\sin\alpha}=\frac{\sqrt{2}\cdot\cos\alpha-2\cdot\left(\cos\frac{\pi}{4}\cdot\cos\alpha-\sin\frac{\pi}{4}\cdot\sin\alpha\right)}{2\cdot\left(\sin\frac{\pi}{4}\cdot\cos\alpha+\cos\frac{\pi}{4}\cdot\sin\alpha\right)-\sqrt{2}\cdot\sin\alpha}=$$

$$=\frac{\sqrt{2}\cdot\cos\alpha-2\cdot\cos\frac{\pi}{4}\cdot\cos\alpha+2\cdot\sin\frac{\pi}{4}\cdot\sin\alpha}{2\cdot\sin\frac{\pi}{4}\cdot\cos\alpha+2\cdot\cos\frac{\pi}{4}\cdot\sin\alpha-\sqrt{2}\cdot\sin\alpha}=\frac{\sqrt{2}\cdot\cos\alpha-2\cdot\frac{\sqrt{2}}{2}\cdot\cos\alpha+2\cdot\frac{\sqrt{2}}{2}\cdot\sin\alpha}{2\cdot\frac{\sqrt{2}}{2}\cdot\cos\alpha+2\cdot\frac{\sqrt{2}}{2}\cdot\sin\alpha-\sqrt{2}\cdot\sin\alpha}=$$

$$=\frac{\sqrt{2}\cdot\cos\alpha-2\cdot\frac{\sqrt{2}}{2}\cdot\cos\alpha+2\cdot\frac{\sqrt{2}}{2}\cdot\sin\alpha}{2\cdot\frac{\sqrt{2}}{2}\cdot\cos\alpha+2\cdot\frac{\sqrt{2}}{2}\cdot\sin\alpha-\sqrt{2}\cdot\sin\alpha}=\frac{\sqrt{2}\cdot\cos\alpha-\sqrt{2}\cdot\cos\alpha+\sqrt{2}\cdot\sin\alpha}{\sqrt{2}\cdot\cos\alpha+\sqrt{2}\cdot\sin\alpha-\sqrt{2}\cdot\sin\alpha}=$$

$$=\frac{\sqrt{2}\cdot\cos\alpha-\sqrt{2}\cdot\cos\alpha+\sqrt{2}\cdot\sin\alpha}{\sqrt{2}\cdot\cos\alpha+\sqrt{2}\cdot\sin\alpha-\sqrt{2}\cdot\sin\alpha}=\frac{\sqrt{2}\cdot\sin\alpha}{\sqrt{2}\cdot\cos\alpha}=\frac{\sqrt{2}\cdot\sin\alpha}{\sqrt{2}\cdot\cos\alpha}=\frac{\sin\alpha}{\cos\alpha}=tg\,\alpha.$$

Odgovor je pod C.

Vježba 320

$$\text{Ako je } x=\frac{2\cdot\sin\left(\frac{\pi}{4}+\alpha\right)-\sqrt{2}\cdot\sin\alpha}{\sqrt{2}\cdot\cos\alpha-2\cdot\cos\left(\frac{\pi}{4}+\alpha\right)}, \text{ tada je } x \text{ jednako:}$$

$A.\ x=\sin\alpha \qquad B.\ x=\cos\alpha \qquad C.\ x=tg\,\alpha \qquad D.\ x=ctg\,\alpha$

Rezultat: D.