Zadatak 241 (Đurđica, srednja škola)

Ako je $\sin^4 x - 2\cdot\sin^2 x = -1$ i $\cos^4 x - 2\cdot\cos^2 x = a$, koliko je a?

Rješenje 241

Ponovimo!

$$(a-b)^2 = a^2 - 2\cdot a\cdot b + b^2 \quad , \quad \left(a^n\right)^m = a^{n\cdot m} \quad , \quad \cos^2 x + \sin^2 x = 1.$$

Iz prve jednadžbe dobije se:

$$\sin^4 x - 2\cdot\sin^2 x = -1 \;\Rightarrow\; \sin^4 x - 2\cdot\sin^2 x + 1 = 0 \;\Rightarrow\; \left(\sin^2 x - 1\right)^2 = 0 \;\Rightarrow$$

$$\Rightarrow \left(\sin^2 x - 1\right)^2 = 0 \;/\sqrt{\ } \;\Rightarrow\; \sin^2 x - 1 = 0 \;\Rightarrow\; \sin^2 x = 1 \;\Rightarrow\; \sin^2 x = \cos^2 x + \sin^2 x \;\Rightarrow$$

$$\Rightarrow \sin^2 x = \cos^2 x + \sin^2 x \;\Rightarrow\; \cos^2 x = 0.$$

Sada se iz druge jednadžbe izračuna a.

$$\left.\begin{array}{l} \cos^2 x = 0 \\ \cos^4 x - 2\cdot\cos^2 x = a \end{array}\right\} \Rightarrow \left.\begin{array}{l} \cos^2 x = 0 \\ \left(\cos^2 x\right)^2 - 2\cdot\cos^2 x = a \end{array}\right\} \Rightarrow 0^2 - 2\cdot 0 = a \;\Rightarrow\; a = 0.$$

Vježba 241

Ako je $\sin^4 x - 2\cdot\sin^2 x = -1$ i $\cos^4 x - 2\cdot\cos^2 x = a - 2$, koliko je a?

Rezultat: 2.

Zadatak 242 (Sara, srednja škola)

Dokaži jednakost $tg^2\,\alpha - \sin^2\alpha = tg^2\,\alpha\cdot\sin^2\alpha$.

Rješenje 242

Ponovimo!

$$tg\,x = \frac{\sin x}{\cos x} \quad , \quad \left(\frac{a}{b}\right)^2 = \frac{a^2}{b^2} \quad , \quad \cos^2 x + \sin^2 x = 1 \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a\cdot d - b\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a\cdot b}{c} = a\cdot\frac{b}{c} = \frac{a}{c}\cdot b.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

1.inačica

$$tg^2\,\alpha - \sin^2\alpha = \frac{\sin^2\alpha}{\cos^2\alpha}\cdot\sin^2\alpha = \sin^2\alpha\cdot\left(\frac{1}{\cos^2\alpha} - 1\right) = \sin^2\alpha\cdot\left(\frac{1}{\cos^2\alpha} - \frac{1}{1}\right) =$$

$$= \sin^2\alpha\cdot\frac{1-\cos^2\alpha}{\cos^2\alpha} = \sin^2\alpha\cdot\frac{\sin^2\alpha}{\cos^2\alpha} = \sin^2\alpha\cdot tg^2\,\alpha = tg^2\,\alpha\cdot\sin^2\alpha.$$

2.inačica

$$tg^2\,\alpha - \sin^2\alpha = tg^2\,\alpha\cdot\left(1 - \frac{\sin^2\alpha}{tg^2\,\alpha}\right) = tg^2\,\alpha\cdot\left(1 - \frac{\sin^2\alpha}{\frac{\sin^2\alpha}{\cos^2\alpha}}\right) = tg^2\,\alpha\cdot\left(1 - \frac{\frac{\sin^2\alpha}{1}}{\frac{\sin^2\alpha}{\cos^2\alpha}}\right) =$$

$$=tg^2\alpha\cdot\left(1-\frac{\frac{\sin^2\alpha}{1}}{\frac{\sin^2\alpha}{\cos^2\alpha}}\right)=tg^2\alpha\cdot\left(1-\frac{\frac{1}{1}}{\frac{1}{\cos^2\alpha}}\right)=tg^2\alpha\cdot\cos^2\alpha.$$

3.inačica

$$tg^2\alpha-\sin^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}-\sin^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}-\frac{\sin^2\alpha}{1}=\frac{\sin^2\alpha-\sin^2\cdot\cos^2\alpha}{\cos^2\alpha}=$$

$$=\frac{\sin^2\alpha\cdot\left(1-\cos^2\alpha\right)}{\cos^2\alpha}=\frac{\sin^2\alpha\cdot\sin^2\alpha}{\cos^2\alpha}=\frac{\sin^2\alpha}{\cos^2\alpha}\cdot\sin^2\alpha=tg^2\alpha\cdot\sin^2\alpha.$$

Vježba 242

Dokaži jednakost $ctg^2\alpha-\cos^2\alpha=ctg^2\alpha\cdot\cos^2\alpha.$

Rezultat: Dokaz analogan.

Zadatak 243 (Marijana, srednja škola)

Zadano je $tg\,\alpha+ctg\,\alpha=m.$ Izračunajte zbroj $tg^2\alpha+ctg^2\alpha.$

$A.\ 1\qquad B.\ m^2\qquad C.\ m^2-2\qquad D.\ m^2+2$

Rješenje 243

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad tg\,\alpha\cdot ctg\,\alpha=1.$$

$$tg\,\alpha+ctg\,\alpha=m\ \Rightarrow\ \begin{bmatrix}\text{kvadriramo}\\\text{jednakost}\end{bmatrix}\Rightarrow\ tg\,\alpha+ctg\,\alpha=m\ /^2\ \Rightarrow\ (tg\,\alpha+ctg\,\alpha)^2=m^2\ \Rightarrow$$

$$\Rightarrow\ tg^2\alpha+2\cdot tg\,\alpha\cdot ctg\,\alpha+ctg^2\alpha=m^2\ \Rightarrow\ tg^2\alpha+2\cdot 1+ctg^2\alpha=m^2\ \Rightarrow$$

$$\Rightarrow\ tg^2\alpha+2+ctg^2\alpha=m^2\ \Rightarrow\ tg^2\alpha+ctg^2\alpha=m^2-2.$$

Odgovor je pod C.

Vježba 243

Zadano je $tg\,\alpha-ctg\,\alpha=m.$ Izračunajte zbroj $tg^2\alpha+ctg^2\alpha.$

$A.\ 1\qquad B.\ m^2\qquad C.\ m^2-2\qquad D.\ m^2+2$

Rezultat: D.

Zadatak 244 (Mario, srednja škola)

Pojednostavnite: $\dfrac{\cos^4x-2\cdot\sin x\cdot\cos x-\sin^4x}{1-tg\,2x}.$

Rješenje 244

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^2-b^2=(a+b)\cdot(a-b)\quad,\quad\sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha\quad,\quad\cos^2\alpha+\sin^2\alpha=1.$$

$$tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}\quad,\quad\cos^2\alpha-\sin^2\alpha=\cos 2\alpha\quad,\quad n=\frac{n}{1}\quad,\quad\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{\cos^4 x-2\cdot\sin x\cdot\cos x-\sin^4 x}{1-tg\,2x}=\frac{\cos^4 x-\sin^4 x-2\cdot\sin x\cdot\cos x}{1-tg\,2x}=\frac{\left(\cos^2 x\right)^2-\left(\sin^2 x\right)^2-\sin 2x}{1-\frac{\sin 2x}{\cos 2x}}=$$

$$=\frac{\left(\cos^2 x+\sin^2 x\right)\cdot\left(\cos^2 x-\sin^2 x\right)-\sin 2x}{\frac{1}{1}-\frac{\sin 2x}{\cos 2x}}=\frac{1\cdot\left(\cos^2 x-\sin^2 x\right)-\sin 2x}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=\frac{\cos^2 x-\sin^2 x-\sin 2x}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=$$

$$=\frac{\cos 2x-\sin 2x}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=\frac{\frac{\cos 2x-\sin 2x}{1}}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=\frac{\frac{\cos 2x-\sin 2x}{1}}{\frac{\cos 2x-\sin 2x}{\cos 2x}}=\frac{\frac{1}{1}}{\frac{1}{\cos 2x}}=\cos 2x.$$

Vježba 244

Pojednostavnite: $\frac{1-tg\,2x}{\cos^4 x-2\cdot\sin x\cdot\cos x-\sin^4 x}$.

Rezultat: $\frac{1}{\cos 2x}$.

Zadatak 245 (Petar, srednja škola)

Ako je $\cos 2x+2\cdot\cos x=0$, koliko je $\cos^2 x+\cos x$?

$A.\ \frac{1}{2}\qquad B.\ \frac{1}{3}\qquad C.\ \frac{1}{4}\qquad D.\ \frac{1}{5}$

Rješenje 245

Ponovimo!

$$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha\quad,\quad\cos^2\alpha+\sin^2\alpha=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Transformiramo zadanu jednadžbu.

$$\cos 2x+2\cdot\cos x=0\;\Rightarrow\;\cos^2 x-\sin^2 x+2\cdot\cos x=0\;\Rightarrow\;\cos^2 x-\left(1-\cos^2 x\right)+2\cdot\cos x=0\;\Rightarrow$$

$$\Rightarrow\;\cos^2 x-1+\cos^2 x+2\cdot\cos x=0\;\Rightarrow\;\cos^2 x+\cos^2 x+2\cdot\cos x=1\;\Rightarrow\;2\cdot\cos^2 x+2\cdot\cos x=1\;\Rightarrow$$

$$\Rightarrow\;2\cdot\cos^2 x+2\cdot\cos x=1\;/:2\;\Rightarrow\;\cos^2 x+\cos x=\frac{1}{2}.$$

Odgovor je pod A.

Vježba 245

Ako je $\cos 2x+2\cdot\cos x=1$, koliko je $\cos^2 x+\cos x$?

$A.\ 0\qquad B.\ 1\qquad C.\ 2\qquad D.\ 3$

Rezultat: B

Zadatak 246 (Petar, srednja škola)

Pojednostavnite: $\frac{\sin x - \sin 2x - \sin 4x + \sin 5x}{\cos x - \cos 2x - \cos 4x + \cos 5x}$.

A. $tg\, 2x$ *B.* $ctg\, 2x$ *C.* $tg\, 3x$ *D.* $ctg\, 3x$

Rješenje 246

Ponovimo!

$$tg\, \alpha = \frac{\sin \alpha}{\cos \alpha}.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Formule pretvorbe

$$\sin \alpha + \sin \beta = 2 \cdot \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2} \quad , \quad \cos \alpha + \cos \beta = 2 \cdot \cos \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}.$$

Parnost kosinusa

$$\cos(-\alpha) = \cos \alpha.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \neq 0 \; , \; n \neq 1.$$

$$\frac{\sin x - \sin 2x - \sin 4x + \sin 5x}{\cos x - \cos 2x - \cos 4x + \cos 5x} = \frac{(\sin x + \sin 5x) - (\sin 2x + \sin 4x)}{(\cos x + \cos 5x) - (\cos 2x + \cos 4x)} =$$

$$= \frac{2 \cdot \sin \frac{x + 5 \cdot x}{2} \cdot \cos \frac{x - 5 \cdot x}{2} - 2 \cdot \sin \frac{2 \cdot x + 4 \cdot x}{2} \cdot \cos \frac{2 \cdot x - 4 \cdot x}{2}}{2 \cdot \cos \frac{x + 5 \cdot x}{2} \cdot \cos \frac{x - 5 \cdot x}{2} - 2 \cdot \cos \frac{2 \cdot x + 4 \cdot x}{2} \cdot \cos \frac{2 \cdot x - 4 \cdot x}{2}} =$$

$$= \frac{2 \cdot \sin \frac{6 \cdot x}{2} \cdot \cos\left(-\frac{4 \cdot x}{2}\right) - 2 \cdot \sin \frac{6 \cdot x}{2} \cdot \cos\left(-\frac{2 \cdot x}{2}\right)}{2 \cdot \cos \frac{6 \cdot x}{2} \cdot \cos\left(-\frac{4 \cdot x}{2}\right) - 2 \cdot \cos \frac{6 \cdot x}{2} \cdot \cos\left(-\frac{2 \cdot x}{2}\right)} = \frac{2 \cdot \sin \frac{6 \cdot x}{2} \cdot \cos \frac{4 \cdot x}{2} - 2 \cdot \sin \frac{6 \cdot x}{2} \cdot \cos \frac{2 \cdot x}{2}}{2 \cdot \cos \frac{6 \cdot x}{2} \cdot \cos \frac{4 \cdot x}{2} - 2 \cdot \cos \frac{6 \cdot x}{2} \cdot \cos \frac{2 \cdot x}{2}} =$$

$$= \frac{2 \cdot \sin \frac{6 \cdot x}{2} \cdot \cos \frac{4 \cdot x}{2} - 2 \cdot \sin \frac{6 \cdot x}{2} \cdot \cos \frac{2 \cdot x}{2}}{2 \cdot \cos \frac{6 \cdot x}{2} \cdot \cos \frac{4 \cdot x}{2} - 2 \cdot \cos \frac{6 \cdot x}{2} \cdot \cos \frac{2 \cdot x}{2}} = \frac{2 \cdot \sin 3x \cdot \cos 2x - 2 \cdot \sin 3x \cdot \cos x}{2 \cdot \cos 3x \cdot \cos 2x - 2 \cdot \cos 3x \cdot \cos x} =$$

$$= \frac{2 \cdot \sin 3x \cdot (\cos 2x - \cos x)}{2 \cdot \cos 3x \cdot (\cos 2x - \cos x)} = \frac{2 \cdot \sin 3x \cdot (\cos 2x - \cos x)}{2 \cdot \cos 3x \cdot (\cos 2x - \cos x)} = \frac{\sin 3x}{\cos 3x} = tg\, 3x.$$

Odgovor je pod C.

Vježba 246

Pojednostavnite: $\frac{\cos x - \cos 2x - \cos 4x + \cos 5x}{\sin x - \sin 2x - \sin 4x + \sin 5x}$.

A. $tg\, 2x$ *B.* $ctg\, 2x$ *C.* $tg\, 3x$ *D.* $ctg\, 3x$

Rezultat: D.

Zadatak 247 (Petar, srednja škola)

Za svaki realni broj x vrijednost izraza $\left(1-\frac{1}{\cos^2 x}\right)\cdot\left(1-\frac{1}{\sin^2 x}\right)$ jednaka je:

$A.\ 4 \qquad B.\ 3 \qquad C.\ 2 \qquad D.\ 1$

Rješenje 247

Ponovimo!

$$n=\frac{n}{1} \quad , \quad \cos^2\alpha+\sin^2\alpha=1 \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d} \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

1.inačica

Najprije riješimo u zagradama.

$$\left(1-\frac{1}{\cos^2 x}\right)\cdot\left(1-\frac{1}{\sin^2 x}\right)=\left(\frac{1}{1}-\frac{1}{\cos^2 x}\right)\cdot\left(\frac{1}{1}-\frac{1}{\sin^2 x}\right)=\frac{\cos^2 x-1}{\cos^2 x}\cdot\frac{\sin^2 x-1}{\sin^2 x}=$$

$$=\frac{1-\sin^2 x-1}{\cos^2 x}\cdot\frac{1-\cos^2 x-1}{\sin^2 x}=\frac{1-\sin^2 x-1}{\cos^2 x}\cdot\frac{1-\cos^2 x-1}{\sin^2 x}=\frac{-\sin^2 x}{\cos^2 x}\cdot\frac{-\cos^2 x}{\sin^2 x}=\frac{\sin^2 x\cdot\cos^2 x}{\cos^2 x\cdot\sin^2 x}=$$

$$=\frac{\sin^2 x\cdot\cos^2 x}{\cos^2 x\cdot\sin^2 x}=1.$$

Odgovor je pod D.

2.inačica

Najprije pomnožimo zagrade.

$$\left(1-\frac{1}{\cos^2 x}\right)\cdot\left(1-\frac{1}{\sin^2 x}\right)=1-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{1}{\cos^2 x\cdot\sin^2 x}=\frac{1}{1}-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{1}{\cos^2 x\cdot\sin^2 x}=$$

$$=\frac{\cos^2 x\cdot\sin^2 x-\cos^2 x-\sin^2 x+1}{\cos^2 x\cdot\sin^2 x}=\frac{\cos^2 x\cdot\sin^2 x-\left(\cos^2 x+\sin^2 x\right)+1}{\cos^2 x\cdot\sin^2 x}=\frac{\cos^2 x\cdot\sin^2 x-1+1}{\cos^2 x\cdot\sin^2 x}=$$

$$=\frac{\cos^2 x\cdot\sin^2 x-1+1}{\cos^2 x\cdot\sin^2 x}=\frac{\cos^2 x\cdot\sin^2 x}{\cos^2 x\cdot\sin^2 x}=\frac{\cos^2 x\cdot\sin^2 x}{\cos^2 x\cdot\sin^2 x}=1.$$

Odgovor je pod D.

3.inačica

Najprije pomnožimo zagrade.

$$\left(1-\frac{1}{\cos^2 x}\right)\cdot\left(1-\frac{1}{\sin^2 x}\right)=1-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{1}{\cos^2 x\cdot\sin^2 x}=1-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{\cos^2 x+\sin^2 x}{\cos^2 x\cdot\sin^2 x}=$$

$$=1-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{\cos^2 x}{\cos^2 x\cdot\sin^2 x}+\frac{\sin^2 x}{\cos^2 x\cdot\sin^2 x}=1-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{\cos^2 x}{\cos^2 x\cdot\sin^2 x}+\frac{\sin^2 x}{\cos^2 x\cdot\sin^2 x}=$$

$$=1-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=1-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=1.$$

Odgovor je pod D.

Vježba 247

Za svaki realni broj x vrijednost izraza $\left(\frac{1}{\cos^2 x}-1\right)\cdot\left(\frac{1}{\sin^2 x}-1\right)$ jednaka je:

A. 4 *B.* 3 *C.* 2 *D.* 1

Rezultat: D.

Zadatak 248 (Marina, gimnazija)

Riješi jednadžbu: $\sin x\cdot\cos 2x=0.$

Rješenje 248

Ponovimo!

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a\cdot b=0 \Leftrightarrow a=0 \text{ ili } b=0 \text{ ili } a=b=0.$$

Trigonometrijska jednadžba

$$\sin x=a \;,\; |a|\le 1.$$

Skup rješenja jednadžbe

$$\sin x=a \;,\; |a|\le 1 \text{ je } \{x_0+k\cdot 2\cdot\pi : k\in Z\}\cup\{\pi-x_0+k\cdot 2\cdot\pi : k\in Z\}$$

gdje je $x_0\in R$ jedno rješenje te jednadžbe.

Specijalno:

$$\sin x=0 \Rightarrow x=\sin^{-1}0 \Rightarrow x=k\cdot\pi \;,\; k\in Z.$$

Trigonometrijska jednadžba

$$\cos x=a \;,\; |a|\le 1.$$

Postupak rješavanja:

$$\cos x=a$$

$$x_0=\cos^{-1}a \;,\; x_0\in R \text{ je jedno rješenje jednadžbe}$$

$$\cos x=\cos x_0 \Rightarrow x=\pm x_0+k\cdot 2\cdot\pi \;,\; k\in Z.$$

Specijalno:

$$\cos x=0 \Rightarrow x=\cos^{-1}0 \Rightarrow \left.\begin{matrix} x=\frac{\pi}{2}+k\cdot\pi \;,\; k\in Z \\ \text{ili} \\ x=(2\cdot k+1)\cdot\frac{\pi}{2} \;,\; k\in Z \end{matrix}\right\}.$$

Budući da je umnožak jednaka nuli, slijedi:

$$\sin x \cdot \cos 2x = 0 \Rightarrow \left.\begin{matrix} \sin x = 0 \\ \cos 2x = 0 \end{matrix}\right\}.$$

$\sin x = 0$	$\cos 2x = 0$
$x = \sin^{-1} 0$ $x = k \cdot \pi \ , \ k \in Z$	Supstitucija : $t = 2x$ $\cos t = 0$ $t = \cos^{-1} 0$ $t = (2 \cdot k + 1) \cdot \frac{\pi}{2}$ Vraćamo se na supstituciju $2 \cdot x = (2 \cdot k + 1) \cdot \frac{\pi}{2}$ $2 \cdot x = (2 \cdot k + 1) \cdot \frac{\pi}{2} \ /:2$ $x = (2 \cdot k + 1) \cdot \frac{\pi}{4}$

Vježba 248

Riješi jednadžbu $2 \cdot \sin x \cdot \cos 2x = 0.$

Rezultat: $x = k \cdot \pi \ , \ x = (2 \cdot k + 1) \cdot \frac{\pi}{4} \ , \ k \in Z.$

Zadatak 249 (Marina, gimnazija)

Riješi jednadžbu: $\sin^2 x = 2 \cdot \sin x.$

Rješenje 249

Ponovimo!

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a \cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Trigonometrijska jednadžba

$$\sin x = a \ , \ |a| \le 1.$$

Skup rješenja jednadžbe

$$\sin x = a \ , \ |a| \le 1 \text{ je } \{x_0 + k \cdot 2 \cdot \pi : k \in Z\} \cup \{\pi - x_0 + k \cdot 2 \cdot \pi : k \in Z\}$$

gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

Specijalno:

$$\sin x = 0 \Rightarrow x = \sin^{-1} 0 \Rightarrow x = k \cdot \pi \ , \ k \in Z.$$

$$\sin^2 x = 2 \cdot \sin x \Rightarrow \sin^2 x - 2 \cdot \sin x = 0 \Rightarrow \sin x \cdot (\sin x - 2) = 0 \Rightarrow \left.\begin{matrix} \sin x = 0 \\ \sin x - 2 = 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \sin x = 0 \\ \sin x = 2 \end{matrix}\right\}.$$

$\sin x = 0$	$\sin x = 2$
$x = \sin^{-1} 0$ $x = k \cdot \pi \ , \ k \in Z$	Nema rješenja jer vrijedi $\sin x = a \ , \ \|a\| \le 1.$

Vježba 249

Riješi jednadžbu: $\sin^2 x = 4 \cdot \sin x$.

Rezultat: $x = k \cdot \pi \; , \; k \in Z$

Zadatak 250 (Marina, gimnazija)

Riješi jednadžbu: $tg^2 x - \left(\sqrt{3} - 1\right) \cdot tg\, x - \sqrt{3} = 0$.

Rješenje 250

Ponovimo!

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2 \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad \sqrt{a^2} = a \; , \; a \ge 0.$$

$$(-a)^2 = a^2.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \ne 0 \; , \; n \ne 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Trigonometrijska jednadžba $tg\, x = a$

Skup rješenja jednadžbe $tg\, x = a, \; a \in R,$ je $\left\{ x_0 + k \cdot \pi : k \in Z \right\}$, gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$tg\, x = a \;\Rightarrow\; tg\, x = tg\, x_0 \;\Rightarrow\; x = x_0 + k \cdot \pi \; , \; k \in Z.$$

$$tg^2 x - \left(\sqrt{3} - 1\right) \cdot tg\, x - \sqrt{3} = 0 \;\Rightarrow\; \left[\begin{matrix} \text{supstitucija} \\ t = tg\, x \end{matrix}\right] \;\Rightarrow\; t^2 - \left(\sqrt{3} - 1\right) \cdot t - \sqrt{3} = 0 \;\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} t^2 - \left(\sqrt{3} - 1\right) \cdot t - \sqrt{3} = 0 \\ a = 1 \; , \; b = -\left(\sqrt{3} - 1\right) \; , \; c = -\sqrt{3} \end{matrix}\right\} \;\Rightarrow\; \left.\begin{matrix} a = 1 \; , \; b = -\left(\sqrt{3} - 1\right) \; , \; c = -\sqrt{3} \\ t_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{matrix}\right\} \;\Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{-\left(-\left(\sqrt{3} - 1\right)\right) \pm \sqrt{\left(-\left(\sqrt{3} - 1\right)\right)^2 - 4 \cdot 1 \cdot \left(-\sqrt{3}\right)}}{2 \cdot 1} \;\Rightarrow\; t_{1,2} = \frac{\sqrt{3} - 1 \pm \sqrt{\left(\sqrt{3} - 1\right)^2 + 4 \cdot \sqrt{3}}}{2} \;\Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{\sqrt{3} - 1 \pm \sqrt{\left(\sqrt{3}\right)^2 - 2 \cdot \sqrt{3} + 1 + 4 \cdot \sqrt{3}}}{2} \;\Rightarrow\; t_{1,2} = \frac{\sqrt{3} - 1 \pm \sqrt{\left(\sqrt{3}\right)^2 + 2 \cdot \sqrt{3} + 1}}{2} \;\Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{\sqrt{3} - 1 \pm \sqrt{\left(\sqrt{3} + 1\right)^2}}{2} \;\Rightarrow\; t_{1,2} = \frac{\sqrt{3} - 1 \pm \left(\sqrt{3} + 1\right)}{2} \;\Rightarrow\; \left.\begin{matrix} t_1 = \dfrac{\sqrt{3} - 1 + \left(\sqrt{3} + 1\right)}{2} \\ t_2 = \dfrac{\sqrt{3} - 1 - \left(\sqrt{3} + 1\right)}{2} \end{matrix}\right\} \;\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t_1 = \frac{\sqrt{3}-1+\sqrt{3}+1}{2} \\ t_2 = \frac{\sqrt{3}-1-\sqrt{3}-1}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = \frac{\sqrt{3}-1+\sqrt{3}+1}{2} \\ t_2 = \frac{\sqrt{3}-1-\sqrt{3}-1}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = \frac{2\cdot\sqrt{3}}{2} \\ t_2 = \frac{-2}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = \frac{2\cdot\sqrt{3}}{2} \\ t_2 = \frac{-2}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = \sqrt{3} \\ t_2 = -1 \end{array}\right\}.$$

Sada se vraćamo na supstituciju:

$t = tg\,x$, $t_1 = \sqrt{3}$	$t = tg\,x$, $t_2 = -1$
$tg\,x = \sqrt{3}$ $x = tg^{-1}\sqrt{3}$ $x = \frac{\pi}{3} + k\cdot\pi$, $k \in Z$	$tg\,x = -1$ $x = tg^{-1}(-1)$ $x = -\frac{\pi}{4} + k\cdot\pi$, $k \in Z$

Vježba 250

Riješi jednadžbu: $tg^2\,x + \left(1-\sqrt{3}\right)\cdot tg\,x - \sqrt{3} = 0.$

Rezultat: $x = \frac{\pi}{3} + k\cdot\pi$, $x = -\frac{\pi}{4} + k\cdot\pi$, $k \in Z.$

Zadatak 251 (Ana, gimnazija)

Rastavljanjem na faktore riješi jednadžbu: $ctg\,x + \cos x = 1 + ctg\,x\cdot\cos x.$

Rješenje 251

Ponovimo!

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a\cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Trigonometrijska jednadžba $tg\,x = a$

Skup rješenja jednadžbe $ctg\,x = a,\ a \in R,$ je $\left\{x_0 + k\cdot\pi : k \in Z\right\},$ gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$ctg\,x = a \Rightarrow ctg\,x = ctg\,x_0 \Rightarrow x = x_0 + k\cdot\pi \ , \ k \in Z.$$

Skup rješenja trigonometrijske jednadžbe

$$\cos x = a \ , \ |a| \le 1$$

je

$$\left\{\pm x_0 + k\cdot 2\cdot\pi \ , \ k \in Z\right\},$$

gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$ctg\,x + \cos x = 1 + ctg\,x\cdot\cos x \Rightarrow ctg\,x + \cos x - 1 - ctg\,x\cdot\cos x = 0 \Rightarrow \begin{bmatrix}\text{metoda} \\ \text{grupiranja}\end{bmatrix} \Rightarrow$$

$$\Rightarrow (ctg\,x - ctg\,x\cdot\cos x) + (\cos x - 1) = 0 \Rightarrow ctg\,x\cdot(1-\cos x) - (1-\cos x) = 0 \Rightarrow$$

$$\Rightarrow (1-\cos x)\cdot(ctg\,x - 1) = 0 \Rightarrow \left.\begin{array}{l} 1-\cos x = 0 \\ ctg\,x - 1 = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -\cos x = -1 \\ ctg\,x = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -\cos x = -1 \ /\cdot(-1) \\ ctg\,x = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \cos x = 1 \\ ctg\, x = 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = \cos^{-1} 1 \\ x = ctg^{-1} 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1 = k \cdot 2 \cdot \pi \;\;,\;\; k \in Z \\ x_2 = \frac{\pi}{4} + k \cdot \pi \;\;,\;\; k \in Z \end{matrix}\right\}.$$

Vježba 251

Rastavljanjem na faktore riješi jednadžbu: $ctg\, x + \cos x - \cos^2 x = \sin^2 x + ctg\, x \cdot \cos x.$

Rezultat: $x_1 = k \cdot 2 \cdot \pi \;,\; x_2 = \frac{\pi}{4} + k \cdot \pi \;,\; k \in Z.$

Zadatak 252 (Ana, gimnazija)

Riješi jednadžbu: $\sin x + \sin 2x = \cos x + 2 \cdot \cos^2 x.$

Rješenje 252

Ponovimo!

$$\sin 2\alpha = 2 \cdot \sin\alpha \cdot \cos\alpha \quad , \quad tg\,\alpha = \frac{\sin\alpha}{\cos\alpha}.$$

Da bi umnožak bio jednak nuli, dovoljno je da jedan faktor bude jednak nuli.

$$a \cdot b = 0 \;\Leftrightarrow\; a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Trigonometrijska jednadžba $tg\, x = a$

Skup rješenja jednadžbe $tg\, x = a,\; a \in R,$ je $\{ x_0 + k \cdot \pi : k \in Z \},$ gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$tg\, x = a \;\Rightarrow\; tg\, x = tg\, x_0 \;\Rightarrow\; x = x_0 + k \cdot \pi \;,\; k \in Z.$$

Skup rješenja trigonometrijske jednadžbe

$$\cos x = a \;,\; |a| \le 1$$

je

$$\{ \pm x_0 + k \cdot 2 \cdot \pi \;,\; k \in Z \},$$

gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$\sin x + \sin 2x = \cos x + 2 \cdot \cos^2 x \;\Rightarrow\; \sin x + \sin 2x - \cos x - 2 \cdot \cos^2 x = 0 \;\Rightarrow\; \begin{bmatrix} \text{metoda} \\ \text{grupiranja} \end{bmatrix} \Rightarrow$$

$$\Rightarrow \sin x + 2 \cdot \sin x \cdot \cos x - \cos x - 2 \cdot \cos^2 x = 0 \;\Rightarrow\; \sin x \cdot (1 + 2 \cdot \cos x) - \cos x \cdot (1 + 2 \cdot \cos x) = 0 \;\Rightarrow$$

$$\Rightarrow (1 + 2 \cdot \cos x) \cdot (\sin x - \cos x) = 0 \;\Rightarrow\; \left.\begin{matrix} 1 + 2 \cdot \cos x = 0 \\ \sin x - \cos x = 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2 \cdot \cos x = -1 \\ \sin x = \cos x \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} 2 \cdot \cos x = -1 \;/\cdot \frac{1}{2} \\ \sin x = \cos x \;/\cdot \frac{1}{\cos x} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos x = -\frac{1}{2} \\ \frac{\sin x}{\cos x} = 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos x = -\frac{1}{2} \\ tg\, x = 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = \cos^{-1}\left(-\frac{1}{2}\right) \\ x = tg^{-1} 1 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x_1 = \pm\frac{2\cdot\pi}{3} + k\cdot 2\cdot\pi \ , \ k \in Z \\ x_2 = \frac{\pi}{4} + k\cdot\pi \ , \ k \in Z \end{array}\right\}.$$

Vježba 252

Riješi jednadžbu: $\sin x + \sin 2x + 2\cdot\sin^2 x = \cos x + 2.$

Rezultat: $x_1 = \pm\frac{2\cdot\pi}{3} + k\cdot 2\cdot\pi \ , \ x_2 = \frac{\pi}{4} + k\cdot\pi \ , \ k \in Z.$

Zadatak 253 (Ines, srednja škola)

Razlomak $\frac{1+\sin^2\alpha - \cos^2\alpha}{1-\sin^2\alpha+\cos^2\alpha}$ jednak je:

$A.\ tg^2\alpha \qquad B.\ -1 \qquad C.\ ctg^2\alpha \qquad D.\ 1$

Rješenje 253

Ponovimo!

$$\cos^2 x + \sin^2 x = 1 \quad , \quad tg\, x = \frac{\sin x}{\cos x}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

$$\frac{1+\sin^2\alpha-\cos^2\alpha}{1-\sin^2\alpha+\cos^2\alpha} = \frac{\cos^2\alpha+\sin^2\alpha+\sin^2\alpha-\cos^2\alpha}{\cos^2\alpha+\sin^2\alpha-\sin^2\alpha+\cos^2\alpha} = \frac{\cos^2\alpha+\sin^2\alpha+\sin^2\alpha-\cos^2\alpha}{\cos^2\alpha+\sin^2\alpha-\sin^2\alpha+\cos^2\alpha} =$$

$$= \frac{2\cdot\sin^2\alpha}{2\cdot\cos^2\alpha} = \frac{2\cdot\sin^2\alpha}{2\cdot\cos^2\alpha} = \frac{\sin^2\alpha}{\cos^2\alpha} = tg^2\alpha.$$

Odgovor je pod A.

2.inačica

$$\frac{1+\sin^2\alpha-\cos^2\alpha}{1-\sin^2\alpha+\cos^2\alpha} = \frac{\left(1-\cos^2\alpha\right)+\sin^2\alpha}{\left(1-\sin^2\alpha\right)+\cos^2\alpha} = \frac{\sin^2\alpha+\sin^2\alpha}{\cos^2\alpha+\cos^2\alpha} = \frac{2\cdot\sin^2\alpha}{2\cdot\cos^2\alpha} = \frac{2\cdot\sin^2\alpha}{2\cdot\cos^2\alpha} =$$

$$= \frac{\sin^2\alpha}{\cos^2\alpha} = tg^2\alpha.$$

Odgovor je pod A.

Vježba 253

Razlomak $\frac{1-\sin^2\alpha+\cos^2\alpha}{1+\sin^2\alpha-\cos^2\alpha}$ jednak je:

$A.\ tg^2\alpha \qquad B.\ -1 \qquad C.\ ctg^2\alpha \qquad D.\ 1$

Rezultat: C.

Zadatak 254 (Ana, gimnazija)

Riješi jednadžbu: $tg\,x-\frac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}=0.$

Rješenje 254

Ponovimo!

$$\sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha \quad , \quad \cos^2\alpha+\sin^2\alpha=1 \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad \frac{a}{b}=\frac{\frac{a}{c}}{\frac{b}{c}} \quad , \quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n} \quad , \quad tg\,\alpha=\frac{\sin\alpha}{\cos\alpha}.$$

$$(a-b)^3=a^3-3\cdot a^2\cdot b+3\cdot a\cdot b^2-b^3 \quad , \quad tg\frac{\pi}{4}=1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Trigonometrijska jednadžba $tg\,x=a$

Skup rješenja jednadžbe $tg\,x=a,\ a\in R,$ je $\{x_0+k\cdot\pi : k\in Z\}$, gdje je $x_0\in R$ jedno rješenje te jednadžbe.

$$tg\,x=a \;\Rightarrow\; tg\,x=tg\,x_0 \;\Rightarrow\; x=x_0+k\cdot\pi \;,\; k\in Z.$$

1.inačica

$$tg\,x-\frac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}=0 \;\Rightarrow\; tg\,x-\frac{8\cdot\sin^2 x+3\cdot 2\cdot\sin x\cdot\cos x+\cos^2 x+\sin^2 x}{8\cdot\cos^2 x+3\cdot 2\cdot\sin x\cdot\cos x+\cos^2 x+\sin^2 x}=0 \;\Rightarrow$$

$$\Rightarrow\; tg\,x-\frac{9\cdot\sin^2 x+6\cdot\sin x\cdot\cos x+\cos^2 x}{9\cdot\cos^2 x+6\cdot\sin x\cdot\cos x+\sin^2 x}=0 \;\Rightarrow\; tg\,x-\frac{(3\cdot\sin x+\cos x)^2}{(3\cdot\cos x+\sin x)^2}=0 \;\Rightarrow$$

$$\Rightarrow\; tg\,x-\left(\frac{3\cdot\sin x+\cos x}{3\cdot\cos x+\sin x}\right)^2=0 \;\Rightarrow\; tg\,x-\left(\frac{\frac{3\cdot\sin x+\cos x}{\cos x}}{\frac{3\cdot\cos x+\sin x}{\cos x}}\right)^2=0 \;\Rightarrow\; tg\,x-\left(\frac{\frac{3\cdot\sin x}{\cos x}+\frac{\cos x}{\cos x}}{\frac{3\cdot\cos x}{\cos x}+\frac{\sin x}{\cos x}}\right)^2=0 \;\Rightarrow$$

$$\Rightarrow\; tg\,x-\left(\frac{3\cdot tg\,x+1}{3+tg\,x}\right)^2=0 \;\Rightarrow\; tg\,x-\frac{(3\cdot tg\,x+1)^2}{(3+tg\,x)^2}=0 \;\Rightarrow\; tg\,x-\frac{9\cdot tg^2 x+6\cdot tg\,x+1}{9+6\cdot tg\,x+tg^2 x}=0 \;\Rightarrow$$

$$\Rightarrow\; tg\,x-\frac{9\cdot tg^2 x+6\cdot tg\,x+1}{tg^2 x+6\cdot tg\,x+9}=0 \;\Rightarrow\; tg\,x-\frac{9\cdot tg^2 x+6\cdot tg\,x+1}{tg^2 x+6\cdot tg\,x+9}=0 \;/\cdot\left(tg^2 x+6\cdot tg\,x+9\right) \;\Rightarrow$$

$$\Rightarrow\; tg\,x\cdot\left(tg^2 x+6\cdot tg\,x+9\right)-\left(9\cdot tg^2 x+6\cdot tg\,x+1\right)=0 \;\Rightarrow$$

$$\Rightarrow\; tg^3 x+6\cdot tg^2 x+9\cdot tg\,x-9\cdot tg^2 x-6\cdot tg\,x-1=0 \;\Rightarrow\; tg^3 x-3\cdot tg^2 x+3\cdot tg\,x-1=0 \;\Rightarrow\; (tg\,x-1)^3=0 \;\Rightarrow$$

$$\Rightarrow\; (tg\,x-1)^3=0 \;/\sqrt[3]{\;} \;\Rightarrow\; tg\,x-1=0 \;\Rightarrow\; tg\,x=1 \;\Rightarrow\; x=tg^{-1}1 \;\Rightarrow\; x=\frac{\pi}{4}+k\cdot\pi \;,\; k\in Z.$$

2.inačica

$$tg\,x-\frac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}=0\ \Rightarrow\ \frac{\sin x}{\cos x}-\frac{8\cdot\sin^2 x+3\cdot 2\cdot\sin x\cdot\cos x+\cos^2 x+\sin^2 x}{8\cdot\cos^2 x+3\cdot 2\cdot\sin x\cdot\cos x+\cos^2 x+\sin^2 x}=0\ \Rightarrow$$

$$\Rightarrow\ \frac{\sin x}{\cos x}-\frac{9\cdot\sin^2 x+6\cdot\sin x\cdot\cos x+\cos^2 x}{9\cdot\cos^2 x+6\cdot\sin x\cdot\cos x+\sin^2 x}=0\ \Rightarrow$$

$$\Rightarrow\ \frac{\sin x}{\cos x}-\frac{9\cdot\sin^2 x+6\cdot\sin x\cdot\cos x+\cos^2 x}{9\cdot\cos^2 x+6\cdot\sin x\cdot\cos x+\sin^2 x}=0\ /\cdot\ \cos x\cdot\left(9\cdot\cos^2 x+6\cdot\sin x\cdot\cos x+\sin^2 x\right)\ \Rightarrow$$

$$\Rightarrow\ \sin x\cdot\left(9\cdot\cos^2 x+6\cdot\sin x\cdot\cos x+\sin^2 x\right)-\cos x\cdot\left(9\cdot\sin^2 x+6\cdot\sin x\cdot\cos x+\cos^2 x\right)=0\ \Rightarrow$$

$$\Rightarrow\ 9\cdot\sin x\cdot\cos^2 x+6\cdot\sin^2 x\cdot\cos x+\sin^3 x-9\cdot\cos x\cdot\sin^2 x-6\cdot\sin x\cdot\cos^2 x-\cos^3 x=0\ \Rightarrow$$

$$\Rightarrow\ 9\cdot\sin x\cdot\cos^2 x+6\cdot\sin^2 x\cdot\cos x+\sin^3 x-9\cdot\sin^2 x\cdot\cos x-6\cdot\sin x\cdot\cos^2 x-\cos^3 x=0\ \Rightarrow$$

$$\Rightarrow\ 3\cdot\sin x\cdot\cos^2 x-3\cdot\sin^2 x\cdot\cos x+\sin^3 x-\cos^3 x=0\ \Rightarrow$$

$$\Rightarrow\ 3\cdot\sin x\cdot\cos^2 x-3\cdot\sin^2 x\cdot\cos x+\sin^3 x-\cos^3 x=0\ /:\ \cos^3 x\ \Rightarrow$$

$$\Rightarrow\ \frac{3\cdot\sin x\cdot\cos^2 x}{\cos^3 x}-\frac{3\cdot\sin^2 x\cdot\cos x}{\cos^3 x}+\frac{\sin^3 x}{\cos^3 x}-\frac{\cos^3 x}{\cos^3 x}=0\ \Rightarrow$$

$$\Rightarrow\ \frac{3\cdot\sin x\cdot\cos^2 x}{\cos^3 x}-\frac{3\cdot\sin^2 x\cdot\cos x}{\cos^3 x}+\frac{\sin^3 x}{\cos^3 x}-\frac{\cos^3 x}{\cos^3 x}=0\ \Rightarrow\ \frac{3\cdot\sin x}{\cos x}-\frac{3\cdot\sin^2 x}{\cos^2 x}+\frac{\sin^3 x}{\cos^3 x}-1=0\ \Rightarrow$$

$$\Rightarrow\ 3\cdot\frac{\sin x}{\cos x}-3\cdot\left(\frac{\sin x}{\cos x}\right)^2+\left(\frac{\sin x}{\cos x}\right)^3-1=0\ \Rightarrow\ 3\cdot tg\,x-3\cdot tg^2 x+tg^3 x-1=0\ \Rightarrow$$

$$\Rightarrow\ tg^3 x-3\cdot tg^2 x+3\cdot tg\,x-1=0\ \Rightarrow\ (tg\,x-1)^3=0\ \Rightarrow$$

$$\Rightarrow\ (tg\,x-1)^3=0\ /\sqrt[3]{\ }\ \Rightarrow\ tg\,x-1=0\ \Rightarrow\ tg\,x=1\ \Rightarrow\ x=tg^{-1}1\ \Rightarrow\ x=\frac{\pi}{4}+k\cdot\pi\ ,\ k\in Z.$$

Vježba 254

Riješi jednadžbu: $\frac{1}{ctg\,x}=\frac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}.$

Rezultat: $x=\frac{\pi}{4}+k\cdot\pi\ ,\ k\in Z.$

Zadatak 255 (2A, TUPŠ)

Dokažimo identitet $\frac{1-2\cdot\sin^2\alpha}{2\cdot\cos^2\alpha-1}=1.$

Rješenje 255

Ponovimo!

$$\cos^2 x+\sin^2 x=1\ \Rightarrow\ \cos^2 x=1-\sin^2 x\ \Rightarrow\ \sin^2 x=1-\cos^2 x.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

$$\frac{1-2 \cdot \sin^2 \alpha}{2 \cdot \cos^2 \alpha - 1} = \left[\cos^2 \alpha = 1 - \sin^2 \alpha \right] = \frac{1-2 \cdot \sin^2 \alpha}{2 \cdot \left(1 - \sin^2 \alpha\right) - 1} = \frac{1-2 \cdot \sin^2 \alpha}{2 - 2 \cdot \sin^2 \alpha - 1} =$$

$$= \frac{1-2 \cdot \sin^2 \alpha}{1-2 \cdot \sin^2 \alpha} = \frac{1-2 \cdot \sin^2 \alpha}{1-2 \cdot \sin^2 \alpha} = 1.$$

2.inačica

$$\frac{1-2 \cdot \sin^2 \alpha}{2 \cdot \cos^2 \alpha - 1} = \left[\sin^2 \alpha = 1 - \cos^2 \alpha \right] = \frac{1 - 2 \cdot \left(1 - \cos^2 \alpha\right)}{2 \cdot \cos^2 \alpha - 1} = \frac{1 - 2 + 2 \cdot \cos^2 \alpha}{2 \cdot \cos^2 \alpha - 1} =$$

$$= \frac{2 \cdot \cos^2 \alpha - 1}{2 \cdot \cos^2 \alpha - 1} = \frac{2 \cdot \cos^2 \alpha - 1}{2 \cdot \cos^2 \alpha - 1} = 1.$$

3.inačica

$$\frac{1-2 \cdot \sin^2 \alpha}{2 \cdot \cos^2 \alpha - 1} = \left[1 = \cos^2 \alpha + \sin^2 \alpha \right] = \frac{\left(\cos^2 \alpha + \sin^2 \alpha\right) - 2 \cdot \sin^2 \alpha}{2 \cdot \cos^2 \alpha - \left(\cos^2 \alpha + \sin^2 \alpha\right)} = \frac{\cos^2 \alpha + \sin^2 \alpha - 2 \cdot \sin^2 \alpha}{2 \cdot \cos^2 \alpha - \cos^2 \alpha - \sin^2 \alpha} =$$

$$= \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha - \sin^2 \alpha} = \frac{\cos^2 \alpha - \sin^2 \alpha}{\cos^2 \alpha - \sin^2 \alpha} = 1.$$

Vježba 255

Dokažimo identitet $\frac{2 \cdot \cos^2 \alpha - 1}{1 - 2 \cdot \sin^2 \alpha} = 1.$

Rezultat: Dokaz analogan.

Zadatak 256 (Zvone, gimnazija)

Ako su a i b pozitivni brojevi te $a + b < \frac{\pi}{2}$, onda je $tg\, a \cdot tg\, b < 1.$ Dokažite!

Rješenje 256

Ponovimo!

$$\alpha < \beta \Rightarrow tg\, \alpha < tg\, \beta \quad , \quad ctg\, \alpha = tg \left(\frac{\pi}{2} - \alpha \right) \quad , \quad tg\, \alpha \cdot ctg\, \alpha = 1.$$

$$a < b \ , \ c > 0 \Rightarrow a \cdot c < b \cdot c \quad , \quad \alpha \in \left\langle 0, \frac{\pi}{2} \right\rangle \Rightarrow tg\, \alpha > 0.$$

$$a + b < \frac{\pi}{2} \Rightarrow a < \frac{\pi}{2} - b \Rightarrow tg\, a < tg \left(\frac{\pi}{2} - b \right) \Rightarrow tg\, a < ctg\, b \Rightarrow tg\, a < \frac{1}{tg\, b} \Rightarrow$$

$$\Rightarrow tg\, a < \frac{1}{tg\, b} \ / \cdot tg\, b \Rightarrow tg\, a \cdot tg\, b < 1.$$

Dokaz gotov.

Vježba 256

Ako su a i b pozitivni brojevi te $a+b<\frac{\pi}{2}$, onda je $tg\,a\cdot tg\,b-1<0.$ Dokažite!

Rezultat: Dokaz analogan.

Zadatak 257 (Zvone, gimnazija)

Dokažite da za sve realne brojeve x vrijedi nejednakost $\sin^3 x-\sin^6 x\le\frac{1}{4}.$

Rješenje 257

Ponovimo!

$$a\le b\ ,\ c<0\ \Rightarrow\ a\cdot c\ge b\cdot c\quad,\quad a^2\ge 0\ ,\ a\in R\quad,\quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$\left(a^n\right)^m=a^{n\cdot m}.$$

$$\sin^3 x-\sin^6 x\le\frac{1}{4}\ \Rightarrow\ \sin^3 x-\sin^6 x-\frac{1}{4}\le 0\ \Rightarrow\ -\sin^6 x+\sin^3 x-\frac{1}{4}\le 0\ \Rightarrow$$

$$\Rightarrow\ -\sin^6 x+\sin^3 x-\frac{1}{4}\le 0\ /\cdot(-4)\ \Rightarrow 4\cdot\sin^6 x-4\cdot\sin^3 x+1\ge 0\ \Rightarrow$$

$$\Rightarrow\ \left(2\cdot\sin^3 x\right)^2-2\cdot 2\cdot\sin^3 x+1\ge 0\ \Rightarrow\ \left(2\cdot\sin^3 x-1\right)^2\ge 0.$$

Vježba 257

Dokažite da za sve realne brojeve x vrijedi nejednakost $\sin^6 x-\sin^3 x+\frac{1}{4}\ge 0.$

Rezultat: Dokaz analogan.

Zadatak 258 (Zvone, gimnazija)

Dokažite da za sve realne brojeve x vrijedi nejednakost $\sin x\cdot\cos x\le\frac{1}{2}.$

Rješenje 258

Ponovimo!

$$a\le b\ ,\ c>0\ \Rightarrow\ a\cdot c\le b\cdot c\quad,\quad \cos^2\alpha+\sin^2\alpha=1\quad,\quad (a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$\sin x\cdot\cos x\le\frac{1}{2}\ \Rightarrow\ \sin x\cdot\cos x\le\frac{1}{2}\ /\cdot 2\ \Rightarrow\ 2\cdot\sin x\cdot\cos x\le 1\ \Rightarrow\ 2\cdot\sin x\cdot\cos x\le\cos^2 x+\sin^2 x\ \Rightarrow$$

$$\Rightarrow\ 2\cdot\sin x\cdot\cos x-\cos^2 x-\sin^2 x\le 0\ \Rightarrow\ -\cos^2 x+2\cdot\sin x\cdot\cos x-\sin^2 x\le 0\ \Rightarrow$$

$$\Rightarrow\ -\cos^2 x+2\cdot\sin x\cdot\cos x-\sin^2 x\le 0\ /\cdot(-1)\ \Rightarrow\ \cos^2 x-2\cdot\sin x\cdot\cos x+\sin^2 x\ge 0\ \Rightarrow$$

$$\Rightarrow\ (\cos x-\sin x)^2\ge 0.$$

Vježba 258

Dokažite da za sve realne brojeve x vrijedi nejednakost $2\cdot\sin x\cdot\cos x-1\le 0.$

Rezultat: Dokaz analogan.

Zadatak 259 (Dario, srednja škola)

Odredi mjeru u stupnjevima kuta mjere 1 rad.

Rješenje 259

Ponovimo!

$$1^0 = 60' \quad , \quad 1' = 60''.$$

Pretvorba radijana u stupnjeve

Ako je zadana mjera kuta u radijanima, tada se mjera u stupnjevima računa pomoću formule

$$\alpha^0 = \frac{180^0}{\pi} \cdot \alpha \ rad.$$

		$\alpha = 1 \ rad$	
		$\alpha = \frac{180^0}{\pi} \cdot 1 = 57.29577951^0$	
	Iznos	**Cijeli broj**	**Decimalni broj**
Stupnjevi	**57.29577951°**	**57 °**	**0.29577951°**
	Decimalni dio stupnjeva pretvaramo u minute tako da ga množimo sa 60. $0.29577951^0 = (0.29577951 \cdot 60)' = 17.7467706'$		
Minute	**17.7467706'**	**17 '**	**0.7467706'**
	Decimalni dio minuta pretvaramo u sekunde tako da ga množimo sa 60. $0.7467706' = (0.7467706 \cdot 60)'' = 44.806236''$		
Sekunde	**44.806236''**	**44 ''**	**0.806236''**
		zbog zaokruživanja **45 ''**	

$$1 \ rad = 57^0 17' 45''.$$

Vježba 259

Odredi mjeru u stupnjevima kuta mjere 2 rad.

Rezultat: 114° 35' 30''.

Zadatak 260 (Ivan, srednja škola)

Razlomak $\frac{\sin(\alpha+\beta)}{\cos\alpha \cdot \cos\beta}$ za sve α i β za koje je definiran identički je jednak:

$$A. \ tg\,\alpha + ctg\,\beta \qquad B. \ tg\,\alpha \cdot tg\,\beta \qquad C. \ tg\,\alpha + tg\,\beta \qquad D. \ 1$$

Rješenje 260

Ponovimo!

$$\sin(x+y) = \sin x \cdot \cos y + \cos x \cdot \sin y \quad , \quad \frac{a+b}{n} = \frac{a}{n} + \frac{b}{n} \quad , \quad tg\,x = \frac{\sin x}{\cos x}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\frac{\sin(\alpha+\beta)}{\cos\alpha \cdot \cos\beta} = \frac{\sin\alpha \cdot \cos\beta + \cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta} = \frac{\sin\alpha \cdot \cos\beta}{\cos\alpha \cdot \cos\beta} + \frac{\cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta} =$$

$$= \frac{\sin\alpha \cdot \cos\beta}{\cos\alpha \cdot \cos\beta} + \frac{\cos\alpha \cdot \sin\beta}{\cos\alpha \cdot \cos\beta} = \frac{\sin\alpha}{\cos\alpha} + \frac{\sin\beta}{\cos\beta} = tg\,\alpha + tg\,\beta.$$

Odgovor je pod C.

Vježba 260

Razlomak $\frac{\sin(\alpha-\beta)}{\cos\alpha\cdot\cos\beta}$ za sve α i β za koje je definiran identički je jednak:

$A.\ tg\ \alpha - ctg\ \beta \qquad B.\ tg\ \alpha \cdot tg\ \beta \qquad C.\ tg\ \alpha - tg\ \beta \qquad D.\ -1$

Rezultat: C.