Zadatak 181 (Marina, ekonomska škola)

Za svaki realni broj x vrijednost izraza $\left(1-\frac{1}{\cos^2 x}\right)\cdot\left(1-\frac{1}{\sin^2 x}\right)$ jednaka je:

$A)\ 4 \qquad B)\ 3 \qquad C)\ 2 \qquad D)\ 1$

Rješenje 181

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1 \quad , \quad \cos^2\alpha=1-\sin^2\alpha \quad , \quad \sin^2\alpha=1-\cos^2\alpha.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada.

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

1.inačica

$$\left(1-\frac{1}{\cos^2 x}\right)\cdot\left(1-\frac{1}{\sin^2 x}\right)=\left(\frac{1}{1}-\frac{1}{\cos^2 x}\right)\cdot\left(\frac{1}{1}-\frac{1}{\sin^2 x}\right)=\frac{\cos^2 x-1}{\cos^2 x}\cdot\frac{\sin^2 x-1}{\sin^2 x}=$$

$$=\frac{-\left(1-\cos^2 x\right)}{\cos^2 x}\cdot\frac{-\left(1-\sin^2 x\right)}{\sin^2 x}=\frac{-\sin^2 x}{\cos^2 x}\cdot\frac{-\cos^2 x}{\sin^2 x}=\frac{\sin^2 x}{\cos^2 x}\cdot\frac{\cos^2 x}{\sin^2 x}=\frac{\sin^2 x}{\cos^2 x}\cdot\frac{\cos^2 x}{\sin^2 x}=1.$$

Odgovor je pod D.

2.inačica

$$\left(1-\frac{1}{\cos^2 x}\right)\cdot\left(1-\frac{1}{\sin^2 x}\right)=1-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{1}{\cos^2 x\cdot\sin^2 x}=$$

$$=\frac{1}{1}-\frac{1}{\sin^2 x}-\frac{1}{\cos^2 x}+\frac{1}{\cos^2 x\cdot\sin^2 x}=\frac{\cos^2 x\cdot\sin^2 x-\cos^2 x-\sin^2 x+1}{\cos^2 x\cdot\sin^2 x}=$$

$$=\frac{\cos^2 x\cdot\sin^2 x-\left(\cos^2 x+\sin^2 x\right)+1}{\cos^2 x\cdot\sin^2 x}=\frac{\cos^2 x\cdot\sin^2 x-1+1}{\cos^2 x\cdot\sin^2 x}=\frac{\cos^2 x\cdot\sin^2 x-1+1}{\cos^2 x\cdot\sin^2 x}=$$

$$=\frac{\cos^2 x\cdot\sin^2 x}{\cos^2 x\cdot\sin^2 x}=\frac{\cos^2 x\cdot\sin^2 x}{\cos^2 x\cdot\sin^2 x}=1.$$

Odgovor je pod D.

Vježba 181

Za svaki realni broj x vrijednost izraza $\left(\frac{1}{\cos^2 x}-1\right)\cdot\left(\frac{1}{\sin^2 x}-1\right)$ jednaka je:

$A)\ 4 \qquad B)\ 3 \qquad C)\ 2 \qquad D)\ 1$

Rezultat: D.

Zadatak 182 (Željka, gimnazija)

Udaljenost svake dvije susjedne nultočke funkcije $f(x)=a\cdot\sin bx\cdot\cos bx$ jednaka je $\frac{3\cdot\pi}{4}$. Tada je:

$$A)\ b=\frac{1}{3}\qquad B)\ b=\frac{3}{4}\qquad C)\ b=\frac{2}{3}\qquad D)\ b=\frac{4}{3}$$

Rješenje 182

Ponovimo!

$$\sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha\quad,\quad\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0.$$

Trigonometrijska jednadžba sin x = a, $|a|\leq 1$

Skup rješenja jednadžbe sin x = a, $|a|\leq 1$, je $\{x_0+k\cdot 2\cdot\pi : k\in Z\}\cup\{\pi-x_0+k\cdot 2\cdot\pi : k\in Z\}$ gdje je $x_0\in R$ jedno rješenje te jednadžbe.

Nultočka grafa je točka u kojoj graf siječe os apscisa (dakle y = 0). Vrijednost x za koju je f(x) = 0 zove se nulište funkcije. Najčešće se za oba pojma koristi izraz nultočka.

Računamo nultočke funkcije.

$$f(x)=0\ \Rightarrow\ a\cdot\sin bx\cdot\cos bx=0\ \Rightarrow\ a\cdot\sin bx\cdot\cos bx=0\ /:a\ \Rightarrow\ \sin bx\cdot\cos bx=0\ \Rightarrow$$

$$\Rightarrow\ \frac{2\cdot\sin bx\cdot\cos bx}{2}=0\ \Rightarrow\ \frac{2\cdot\sin bx\cdot\cos bx}{2}=0\ \Rightarrow\ \frac{\sin 2bx}{2}=0\ \Rightarrow\ \frac{\sin 2bx}{2}=0\ /\cdot 2\ \Rightarrow$$

$$\Rightarrow\ \sin 2bx=0\ \Rightarrow\ 2\cdot b\cdot x=\sin^{-1}0\ \Rightarrow\ 2\cdot b\cdot x=k\cdot\pi\ \Rightarrow\ 2\cdot b\cdot x=k\cdot\pi\ /\cdot\frac{1}{2\cdot b}\ \Rightarrow$$

$$\Rightarrow\ x=\frac{k\cdot\pi}{2\cdot b},\ k\in Z.$$

Zbog jednostavnosti promatramo susjedna rješenja za k = 0 i k = 1.

$$\left.\begin{array}{l} k=0\ ,\ x_1=\frac{k\cdot\pi}{2\cdot b} \\ k=1\ ,\ x_2=\frac{k\cdot\pi}{2\cdot b}\end{array}\right\}\Rightarrow\left.\begin{array}{l} x_1=\frac{0\cdot\pi}{2\cdot b} \\ x_2=\frac{1\cdot\pi}{2\cdot b}\end{array}\right\}\Rightarrow\left.\begin{array}{l} x_1=0 \\ x_2=\frac{\pi}{2\cdot b}\end{array}\right\}.$$

Budući da je udaljenost svake dvije susjedne nultočke jednaka $\frac{3\cdot\pi}{4}$, slijedi:

$$\left.\begin{array}{l} x_1=0\ ,\ x_2=\frac{\pi}{2\cdot b} \\ x_2-x_1=\frac{3\cdot\pi}{4}\end{array}\right\}\Rightarrow\ \frac{\pi}{2\cdot b}-0=\frac{3\cdot\pi}{4}\ \Rightarrow\ \frac{\pi}{2\cdot b}=\frac{3\cdot\pi}{4}\ \Rightarrow\ 6\cdot\pi\cdot b=4\cdot\pi\ \Rightarrow$$

$$\Rightarrow\ 6\cdot\pi\cdot b=4\cdot\pi\ /\cdot\frac{1}{6\cdot\pi}\ \Rightarrow\ b=\frac{4\cdot\pi}{6\cdot\pi}\ \Rightarrow\ b=\frac{4\cdot\pi}{6\cdot\pi}\ \Rightarrow\ b=\frac{4}{6}\ \Rightarrow\ b=\frac{4}{6}\ \Rightarrow\ b=\frac{2}{3}.$$

Odgovor je pod C.

Vježba 182

Udaljenost svake dvije susjedne nultočke funkcije $f(x)=a\cdot\sin bx\cdot\cos bx$ jednaka je $\frac{\pi}{2}$. Tada je:

$$A)\ b=\frac{1}{2}\qquad B)\ b=1\qquad C)\ b=2\qquad D)\ b=\frac{2}{3}$$

Rezultat: B.

Zadatak 183 (Mateja, medicinska škola)

Za svaki realni broj x vrijednost izraza $(tg\,x+ctg\,x)^2-(tg\,x-ctg\,x)^2$ jednaka je:

$A)\ 1 \qquad B)\ 2 \qquad C)\ 3 \qquad D)\ 4$

Rješenje 183

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad -a+a=0 \quad , \quad (a+b)^2=a^2+2\cdot a\cdot b+b^2.$$

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$(a-b)^2=(b-a)^2 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$tg\,\alpha\cdot ctg\,\alpha=1 \quad , \quad tg\,\alpha=\frac{\sin\alpha}{\cos\alpha} \quad , \quad ctg\,\alpha=\frac{\cos\alpha}{\sin\alpha} \quad , \quad \cos^2\alpha+\sin^2\alpha=1.$$

$$\cos^2\alpha-\sin^2\alpha=\cos 2\alpha \quad , \quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

Uporabit ćemo formulu za razliku kvadrata.

$$(tg\,x+ctg\,x)^2-(tg\,x-ctg\,x)^2=\big((tg\,x+ctg\,x)-(tg\,x-ctg\,x)\big)\cdot\big((tg\,x+ctg\,x)+(tg\,x-ctg\,x)\big)=$$

$$=(tg\,x+ctg\,x-tg\,x+ctg\,x)\cdot(tg\,x+ctg\,x+tg\,x-ctg\,x)=$$

$$=(tg\,x+ctg\,x-tg\,x+ctg\,x)\cdot(tg\,x+ctg\,x+tg\,x-ctg\,x)=2\cdot ctg\,x\cdot 2\cdot tg\,x=4\cdot ctg\,x\cdot tg\,x=4\cdot 1=4.$$

Odgovor je pod D.

2.inačica

Uporabit ćemo formule za kvadrat zbroja i kvadrat razlike.

$$(tg\,x+ctg\,x)^2-(tg\,x-ctg\,x)^2=tg^2x+2\cdot tg\,x\cdot ctg\,x+ctg^2x-\left(tg^2x-2\cdot tg\,x\cdot ctg\,x+ctg^2x\right)=$$

$$=tg^2x+2\cdot tg\,x\cdot ctg\,x+ctg^2x-tg^2x+2\cdot tg\,x\cdot ctg\,x-ctg^2x=$$

$$=tg^2x+2\cdot tg\,x\cdot ctg\,x+ctg^2x-tg^2x+2\cdot tg\,x\cdot ctg\,x-ctg^2x=4\cdot tg\,x\cdot ctg\,x=4\cdot 1=4.$$

Odgovor je pod D.

3.inačica

$$(tg\,x+ctg\,x)^2-(tg\,x-ctg\,x)^2=\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)^2-\left(\frac{\sin x}{\cos x}-\frac{\cos x}{\sin x}\right)^2=$$

$$=\left(\frac{\sin^2x+\cos^2x}{\cos x\cdot\sin x}\right)^2-\left(\frac{\sin^2x-\cos^2x}{\cos x\cdot\sin x}\right)^2=\left(\frac{1}{\cos x\cdot\sin x}\right)^2-\left(\frac{\cos^2x-\sin^2x}{\cos x\cdot\sin x}\right)^2=$$

$$=\left(\frac{2}{2\cdot\cos x\cdot\sin x}\right)^2-\left(\frac{\cos^2x-\sin^2x}{\cos x\cdot\sin x}\right)^2=\left(\frac{2}{\sin 2x}\right)^2-\left(\frac{\cos 2x}{\cos x\cdot\sin x}\right)^2=\left(\frac{2}{\sin 2x}\right)^2-\left(\frac{2\cdot\cos 2x}{2\cdot\cos x\cdot\sin x}\right)^2=$$

$$=\left(\frac{2}{\sin 2x}\right)^2-\left(\frac{2\cdot\cos 2x}{\sin 2x}\right)^2=\frac{4}{\sin^2 2x}-\frac{4\cdot\cos^2 2x}{\sin^2 2x}=\frac{4-4\cdot\cos^2 2x}{\sin^2 2x}=\frac{4\cdot\left(1-\cos^2 2x\right)}{\sin^2 2x}=$$

$$=\frac{4\cdot\sin^2 2x}{\sin^2 2x}=\frac{4\cdot\sin^2 2x}{\sin^2 2x}=4.$$

Odgovor je pod D.

Vježba 183

Za svaki realni broj x vrijednost izraza $(tg\,x+ctg\,x)^2-(ctg\,x-tg\,x)^2$ jednaka je:

$A)\ 1 \qquad B)\ 2 \qquad C)\ 3 \qquad D)\ 4$

Rezultat: D.

Zadatak 184 (Mateja, medicinska škola)

Vrijednost brojevnog izraza $\sin\frac{\pi}{12}\cdot\sin\frac{5\cdot\pi}{12}-\cos\frac{\pi}{12}\cdot\cos\frac{5\cdot\pi}{12}$ jednaka je:

$A)\ 0 \qquad B)\ -1 \qquad C)\ 1 \qquad D)\ 0.5$

Rješenje 184

Ponovimo!

$$\cos(\alpha+\beta)=\cos\alpha\cdot\cos\beta-\sin\alpha\cdot\sin\beta \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n} \quad , \quad \cos\frac{\pi}{2}=0.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\sin\frac{\pi}{12}\cdot\sin\frac{5\cdot\pi}{12}-\cos\frac{\pi}{12}\cdot\cos\frac{5\cdot\pi}{12}=-\left(\cos\frac{\pi}{12}\cdot\cos\frac{5\cdot\pi}{12}-\sin\frac{\pi}{12}\cdot\sin\frac{5\cdot\pi}{12}\right)=-\cos\left(\frac{\pi}{12}+\frac{5\cdot\pi}{12}\right)=-\cos\frac{6\cdot\pi}{12}=$$

$$=-\cos\frac{6\cdot\pi}{12}=-\cos\frac{\pi}{2}=0.$$

Odgovor je pod A.

Vježba 184

Vrijednost brojevnog izraza $\sin\frac{3\cdot\pi}{14}\cdot\sin\frac{4\cdot\pi}{14}-\cos\frac{3\cdot\pi}{14}\cdot\cos\frac{4\cdot\pi}{14}$ jednaka je:

$A)\ 0 \qquad B)\ -1 \qquad C)\ 1 \qquad D)\ 0.5$

Rezultat: A.

Zadatak 185 (Željko, srednja škola)

Broj $\frac{\pi}{8}$ rješenje je jednadžbe $\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=a$ ako je:

$A)\ a=8 \qquad B)\ a=6 \qquad C)\ a=4 \qquad D)\ a=2$

Rješenje 185

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1 \quad , \quad (a\cdot b)^n=a^n\cdot b^n \quad , \quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha \quad , \quad \sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}.$$

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\ ,\ n\neq 0 \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad \left(\sqrt{2}\right)^2=2 \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=a \Rightarrow \frac{\cos^2 x+\sin^2 x}{\sin^2 x\cdot\cos^2 x}=a \Rightarrow \frac{1}{\sin^2 x\cdot\cos^2 x}=a \Rightarrow \frac{4}{4\cdot\sin^2 x\cdot\cos^2 x}=a \Rightarrow$$

$$\Rightarrow \frac{4}{(2\cdot\sin x\cdot\cos x)^2}=a \Rightarrow \frac{4}{\sin^2 2x}=a \Rightarrow \frac{4}{\sin^2\left(2\cdot\frac{\pi}{8}\right)}=a \Rightarrow \frac{4}{\sin^2\left(2\cdot\frac{\pi}{8}\right)}=a \Rightarrow$$

$$\Rightarrow \frac{4}{\sin^2\frac{\pi}{4}}=a \Rightarrow \frac{4}{\left(\frac{\sqrt{2}}{2}\right)^2}=a \Rightarrow \frac{4}{\frac{\left(\sqrt{2}\right)^2}{2^2}}=a \Rightarrow \frac{4}{\frac{2}{4}}=a \Rightarrow \frac{\frac{4}{1}}{\frac{2}{4}}=a \Rightarrow \frac{16}{2}=a \Rightarrow a=8.$$

Odgovor je pod A.

Vježba 185

Broj $\frac{\pi}{12}$ rješenje je jednadžbe $\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=a$ ako je:

$A)\ a=8 \qquad B)\ a=16 \qquad C)\ a=12 \qquad D)\ a=6$

Rezultat: B.

Zadatak 186 (Željko, srednja škola)

Broj $\frac{3\cdot\pi}{4}$ rješenje je jednadžbe $\sin^2 x+a\cdot\sin x\cdot\cos x+\cos^2 x=0$ ako je:

$A)\ a=-1 \qquad B)\ a=2 \qquad C)\ a=\frac{1}{2} \qquad D)\ a=1$

Rješenje 186

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1 \quad , \quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha \quad , \quad a=\frac{a\cdot n}{n}\ ,\ n\neq 0 \quad , \quad \sin\frac{3\cdot\pi}{2}=-1.$$

$$\sin^2 x+a\cdot\sin x\cdot\cos x+\cos^2 x=0 \Rightarrow \sin^2 x+\cos^2 x+a\cdot\sin x\cdot\cos x=0 \Rightarrow$$

$$\Rightarrow 1+a\cdot\sin x\cdot\cos x=0 \Rightarrow a\cdot\sin x\cdot\cos x=-1 \Rightarrow \frac{a\cdot 2\cdot\sin x\cdot\cos x}{2}=-1 \Rightarrow \frac{a\cdot\sin 2x}{2}=-1 \Rightarrow$$

$$\Rightarrow \frac{a\cdot\sin 2x}{2}=-1\ /\cdot\frac{2}{\sin 2x} \Rightarrow a=\frac{-2}{\sin 2x} \Rightarrow a=\frac{-2}{\sin\left(2\cdot\frac{3\cdot\pi}{4}\right)} \Rightarrow a=\frac{-2}{\sin\left(2\cdot\frac{3\cdot\pi}{4}\right)} \Rightarrow$$

$$\Rightarrow a=\frac{-2}{\sin\frac{3\cdot\pi}{2}} \Rightarrow a=\frac{-2}{-1} \Rightarrow a=2.$$

Odgovor je pod B.

Vježba 186

Broj $\frac{3\cdot\pi}{4}$ rješenje je jednadžbe $\sin^2 x+a\cdot\sin x\cdot\cos x+\cos^2 x=0$ ako je:

$A)\ a=-1 \qquad B)\ a=-4 \qquad C)\ a=\frac{1}{2} \qquad D)\ a=1$

Rezultat: B.

Zadatak 187 (Alen, gimnazija)

Ako za kutove α i β trokuta ABC vrijedi

$$\frac{\sin^2 \alpha}{\sin^2 \beta} = \frac{tg\, \alpha}{tg\, \beta},$$

tada je trokut jednakokračan ili pravokutan. Dokaži!

Rješenje 187

Ponovimo!

Trokut je dio ravnine omeđen s tri dužine koje zovemo stranice trokuta.

Nasuprot jednakim stranicama nalaze se jednaki kutovi.

$$\begin{cases} a = b \Rightarrow \alpha = \beta \\ a = c \Rightarrow \alpha = \gamma \\ b = c \Rightarrow \beta = \gamma \\ a = b = c \Rightarrow \alpha = \beta = \gamma. \end{cases}$$

Nasuprot jednakim kutovima nalaze se jednake stranice.

$$\begin{cases} \alpha = \beta \Rightarrow a = b \\ \alpha = \gamma \Rightarrow a = c \\ \beta = \gamma \Rightarrow b = c \\ \alpha = \beta = \gamma \Rightarrow a = b = c. \end{cases}$$

Zbroj svih kutova u trokutu je 180º ili π rad.

Trokute dijelimo:

- prema odnosu među duljinama stranica

$$\begin{cases} \text{raznostraničan} \\ \text{jednakokračan} \\ \text{jednakostraničan} \end{cases}$$

- prema kutovima

$$\begin{cases} \text{šiljastokutan} \\ \text{tupokutan} \\ \text{pravokutan.} \end{cases}$$

Kod jednakokračnog trokuta duljine dviju stranica su jednake. Stranice jednakih duljina zovemo kraci trokuta. Pravokutni trokuti imaju jedan pravi kut (90º).

$$a \in R \setminus \{0\} \Rightarrow a^2 > 0 \quad , \quad tg\, x = \frac{\sin x}{\cos x} \quad , \quad a \cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad 2 \cdot \sin x \cdot \cos x = \sin 2x.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Trigonometrijska jednadžba sin x = a , $|a| \le 1$

Skup rješenja jednadžbe $\sin x = a$, $|a| \le 1$ je

$$\{x_0 + k \cdot 2 \cdot \pi : k \in Z\} \cup \{\pi - x_0 + k \cdot 2 \cdot \pi : k \in Z\},$$

gdje je $x_0 \in R$ jedno rješenje te jednadžbe.
Budući da se na lijevoj strani jednakosti

$$\frac{\sin^2\alpha}{\sin^2\beta}=\frac{tg\,\alpha}{tg\,\beta}$$

pojavljuju kvadrati, slijedi:

$$\frac{tg\,\alpha}{tg\,\beta}>0.$$

To je moguće ako su α i β šiljasti kutovi trokuta, tj.

$$\alpha,\ \beta\in\left\langle 0,\ \frac{\pi}{2}\right\rangle.$$

Transformiramo zadanu jednakost.

$$\frac{\sin^2\alpha}{\sin^2\beta}=\frac{tg\,\alpha}{tg\,\beta}\ \Rightarrow\ \frac{\sin^2\alpha}{\sin^2\beta}=\frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin\beta}{\cos\beta}}\ \Rightarrow\ \frac{\sin^2\alpha}{\sin^2\beta}=\frac{\sin\alpha\cdot\cos\beta}{\sin\beta\cdot\cos\alpha}\ \Rightarrow\ \frac{\sin^2\alpha}{\sin^2\beta}-\frac{\sin\alpha\cdot\cos\beta}{\sin\beta\cdot\cos\alpha}=0\ \Rightarrow$$

$$\Rightarrow\ \frac{\sin\alpha}{\sin\beta}\cdot\left(\frac{\sin\alpha}{\sin\beta}-\frac{\cos\beta}{\cos\alpha}\right)=0\ \Rightarrow\ \left.\begin{array}{l}\frac{\sin\alpha}{\sin\beta}=0\ \text{nije moguće zbog}\ \alpha,\ \beta\in\left\langle 0,\ \frac{\pi}{2}\right\rangle\\ \frac{\sin\alpha}{\sin\beta}-\frac{\cos\beta}{\cos\alpha}=0\end{array}\right\}\ \Rightarrow$$

$$\Rightarrow\ \frac{\sin\alpha}{\sin\beta}-\frac{\cos\beta}{\cos\alpha}=0\ \Rightarrow\ \frac{\sin\alpha}{\sin\beta}=\frac{\cos\beta}{\cos\alpha}\ \Rightarrow\ \frac{\sin\alpha}{\sin\beta}=\frac{\cos\beta}{\cos\alpha}\ /\cdot\sin\beta\cdot\cos\alpha\ \Rightarrow\ \sin\alpha\cdot\cos\alpha=\sin\beta\cdot\cos\beta\ \Rightarrow$$

$$\Rightarrow\ \sin\alpha\cdot\cos\alpha=\sin\beta\cdot\cos\beta\ /\cdot 2\ \Rightarrow\ 2\cdot\sin\alpha\cdot\cos\alpha=2\cdot\sin\beta\cdot\cos\beta\ \Rightarrow\ \sin 2\alpha=\sin 2\beta.$$

Rješenja jednadžbe

$$\sin 2\alpha=\sin 2\beta$$

su:

1.rješenje

$$\sin 2\alpha=\sin 2\beta\ \Rightarrow\ 2\cdot\alpha=2\cdot\beta\ \Rightarrow\ 2\cdot\alpha=2\cdot\beta\ /:2\ \Rightarrow\ \alpha=\beta.$$

Trokut je jednakokračan.

2.rješenje

$$\sin 2\alpha=\sin 2\beta\ \Rightarrow\ \pi-2\cdot\alpha=2\cdot\beta\ \Rightarrow\ \pi=2\cdot\alpha+2\cdot\beta\ \Rightarrow\ 2\cdot\alpha+2\cdot\beta=\pi\ \Rightarrow$$

$$\Rightarrow\ 2\cdot\alpha+2\cdot\beta=\pi\ /:2\ \Rightarrow\ \alpha+\beta=\frac{\pi}{2}.$$

Budući da je zbroj svih kutova u trokutu π rad (180º), slijedi:

$$\left.\begin{array}{l}\alpha+\beta=\frac{\pi}{2}\\ \alpha+\beta+\gamma=\pi\end{array}\right\}\ \Rightarrow\ \left[\begin{array}{l}\text{metoda}\\ \text{supstitucije}\end{array}\right]\ \Rightarrow\ \frac{\pi}{2}+\gamma=\pi\ \Rightarrow\ \gamma=\pi-\frac{\pi}{2}\ \Rightarrow\ \gamma=\frac{\pi}{1}-\frac{\pi}{2}\ \Rightarrow$$

$$\Rightarrow\ \gamma=\frac{2\cdot\pi-\pi}{2}\ \Rightarrow\ \gamma=\frac{\pi}{2}.$$

Trokut je pravokutan.

Vježba 187

Ako za kutove α i β trokuta ABC vrijedi

$$\frac{\sin^2\alpha}{\sin^2\beta}=\frac{ctg\,\beta}{ctg\,\alpha},$$

tada je trokut jednakokračan ili pravokutan. Dokaži!

Rezultat: Dokaz analogan.

Zadatak 188 (Tomy, gimnazija)

Neka je $\cos x+\cos y=A$ te $\sin x+\sin y=B,$ pri čemu A i B nisu istodobno nula. Izračunaj $\sin(x+y).$

Rješenje 188

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \cos^2\alpha+\sin^2\alpha=1 \quad , \quad \cos(\alpha-\beta)=\cos\alpha\cdot\cos\beta+\sin\alpha\cdot\sin\beta.$$

$$\sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha \quad , \quad \sin\alpha+\sin\beta=2\cdot\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2} \quad , \quad \frac{a}{b}=\frac{a\cdot n}{b\cdot n} \ , \ n\neq 0.$$

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Množenje zagrada.

$$(a+b)\cdot(c+d)=a\cdot c+a\cdot d+b\cdot c+b\cdot d.$$

Zadane jednakosti kvadriramo i zbrojimo.

$$\left.\begin{array}{l}\cos x+\cos y=A\\ \sin x+\sin y=B\end{array}\right\}\Rightarrow\left.\begin{array}{l}\cos x+\cos y=A\ /^2\\ \sin x+\sin y=B\ /^2\end{array}\right\}\Rightarrow\left.\begin{array}{l}(\cos x+\cos y)^2=A^2\\ (\sin x+\sin y)^2=B^2\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}\cos^2 x+2\cdot\cos x\cdot\cos y+\cos^2 y=A^2\\ \sin^2 x+2\cdot\sin x\cdot\sin y+\sin^2 y=B^2\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{zbrojimo}\\ \text{jednakosti}\end{array}\right]\Rightarrow$$

$$\Rightarrow\cos^2 x+2\cdot\cos x\cdot\cos y+\cos^2 y+\sin^2 x+2\cdot\sin x\cdot\sin y+\sin^2 y=A^2+B^2\Rightarrow$$

$$\Rightarrow\cos^2 x+\sin^2 x+2\cdot\cos x\cdot\cos y+2\cdot\sin x\cdot\sin y+\cos^2 y+\sin^2 y=A^2+B^2\Rightarrow$$

$$\Rightarrow\left(\cos^2 x+\sin^2 x\right)+2\cdot\cos x\cdot\cos y+2\cdot\sin x\cdot\sin y+\left(\cos^2 y+\sin^2 y\right)=A^2+B^2\Rightarrow$$

$$\Rightarrow 1+2\cdot\cos x\cdot\cos y+2\cdot\sin x\cdot\sin y+1=A^2+B^2\Rightarrow 2\cdot\cos x\cdot\cos y+2\cdot\sin x\cdot\sin y+2=A^2+B^2\Rightarrow$$

$$\Rightarrow 2\cdot\cos x\cdot\cos y+2\cdot\sin x\cdot\sin y=A^2+B^2-2\Rightarrow 2\cdot\cos x\cdot\cos y+2\cdot\sin x\cdot\sin y=A^2+B^2-2\ /:2\Rightarrow$$

$$\Rightarrow\cos x\cdot\cos y+\sin x\cdot\sin y=\frac{A^2+B^2-2}{2}\Rightarrow\cos(x-y)=\frac{A^2+B^2-2}{2}.$$

Zadane jednakosti međusobno pomnožimo.

$$\left.\begin{array}{l}\cos x+\cos y=A\\ \sin x+\sin y=B\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{pomnožimo}\\ \text{jednakosti}\end{array}\right]\Rightarrow(\cos x+\cos y)\cdot(\sin x+\sin y)=A\cdot B\Rightarrow$$

$$\Rightarrow \cos x\cdot\sin x+\cos x\cdot\sin y+\cos y\cdot\sin x+\cos y\cdot\sin y=A\cdot B \Rightarrow$$

$$\Rightarrow \cos x\cdot\sin x+\cos y\cdot\sin y+\cos x\cdot\sin y+\cos y\cdot\sin x=A\cdot B \Rightarrow$$

$$\Rightarrow \frac{2\cdot\cos x\cdot\sin x}{2}+\frac{2\cdot\cos y\cdot\sin y}{2}+\left(\cos x\cdot\sin y+\cos y\cdot\sin x\right)=A\cdot B \Rightarrow$$

$$\Rightarrow \frac{\sin 2x}{2}+\frac{\sin 2y}{2}+\sin\left(x+y\right)=A\cdot B \Rightarrow \frac{1}{2}\cdot\sin 2x+\frac{1}{2}\cdot\sin 2y+\sin\left(x+y\right)=A\cdot B \Rightarrow$$

$$\Rightarrow \frac{1}{2}\cdot\left(\sin 2x+\sin 2y\right)+\sin\left(x+y\right)=A\cdot B \Rightarrow$$

$$\Rightarrow \frac{1}{2}\cdot 2\cdot\sin\frac{2\cdot x+2\cdot y}{2}\cdot\cos\frac{2\cdot x-2\cdot y}{2}+\sin\left(x+y\right)=A\cdot B \Rightarrow$$

$$\Rightarrow \frac{1}{2}\cdot 2\cdot\sin\frac{2\cdot\left(x+y\right)}{2}\cdot\cos\frac{2\cdot\left(x-y\right)}{2}+\sin\left(x+y\right)=A\cdot B \Rightarrow$$

$$\Rightarrow \frac{1}{2}\cdot 2\cdot\sin\frac{2\cdot\left(x+y\right)}{2}\cdot\cos\frac{2\cdot\left(x-y\right)}{2}+\sin\left(x+y\right)=A\cdot B \Rightarrow$$

$$\Rightarrow \sin\left(x+y\right)\cdot\cos\left(x-y\right)+\sin\left(x+y\right)=A\cdot B \Rightarrow \sin\left(x+y\right)\cdot\left[\cos\left(x-y\right)+1\right]=A\cdot B \Rightarrow$$

$$\Rightarrow \sin\left(x+y\right)\cdot\left[\cos\left(x-y\right)+1\right]=A\cdot B \;/\cdot\frac{1}{\cos\left(x-y\right)+1} \Rightarrow \sin\left(x+y\right)=\frac{A\cdot B}{\cos\left(x-y\right)+1}.$$

Iz sustava jednakosti dobije se traženi rezultat.

$$\left.\begin{array}{l}\cos\left(x-y\right)=\dfrac{A^2+B^2-2}{2}\\ \sin\left(x+y\right)=\dfrac{A\cdot B}{\cos\left(x-y\right)+1}\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{metoda}\\ \text{supstitucije}\end{array}\right] \Rightarrow \sin\left(x+y\right)=\frac{A\cdot B}{\dfrac{A^2+B^2-2}{2}+1} \Rightarrow$$

$$\Rightarrow \sin\left(x+y\right)=\frac{A\cdot B}{\dfrac{A^2+B^2-2}{2}+\dfrac{1}{1}} \Rightarrow \sin\left(x+y\right)=\frac{A\cdot B}{\dfrac{A^2+B^2-2+2}{2}} \Rightarrow \sin\left(x+y\right)=\frac{A\cdot B}{\dfrac{A^2+B^2-2+2}{2}} \Rightarrow$$

$$\Rightarrow \sin\left(x+y\right)=\frac{A\cdot B}{\dfrac{A^2+B^2}{2}} \Rightarrow \sin\left(x+y\right)=\frac{\dfrac{A\cdot B}{1}}{\dfrac{A^2+B^2}{2}} \Rightarrow \sin\left(x+y\right)=\frac{2\cdot A\cdot B}{A^2+B^2}.$$

Vježba 188

Neka je $\cos x+\cos y=A$ te $\sin x+\sin y=B$, pri čemu A i B nisu istodobno nula. Izračunaj $\cos\left(x+y\right)$.

Rezultat: $\cos\left(x+y\right)=\dfrac{A^2-B^2}{A^2+B^2}.$

Zadatak 189 (Mira, gimnazija)

Ako je $a=\cos\frac{\pi}{5}\cdot\cos\frac{2\cdot\pi}{5}$, onda je a^{-1} jednako:

$A)\ 3 \qquad B)\ 4 \qquad C)\ 2 \qquad D)\ 6 \qquad E)\ 8$

Rješenje 189

Ponovimo!

$$\frac{x}{y}=\frac{x\cdot n}{y\cdot n}\ ,\ n\neq 0\quad ,\quad \sin 2x=2\cdot\sin x\cdot\cos x\quad ,\quad \sin\left(\pi-x\right)=\sin x\quad ,\quad \left(\frac{x}{y}\right)^{-n}=\left(\frac{y}{x}\right)^{n}.$$

$$n=\frac{n}{1}.$$

$$a=\cos\frac{\pi}{5}\cdot\cos\frac{2\cdot\pi}{5}\ \Rightarrow\ a=\frac{2\cdot\sin\frac{\pi}{5}\cdot\cos\frac{\pi}{5}\cdot\cos\frac{2\cdot\pi}{5}}{2\cdot\sin\frac{\pi}{5}}\ \Rightarrow\ a=\frac{\left(2\cdot\sin\frac{\pi}{5}\cdot\cos\frac{\pi}{5}\right)\cdot\cos\frac{2\cdot\pi}{5}}{2\cdot\sin\frac{\pi}{5}}\ \Rightarrow$$

$$\Rightarrow\ a=\frac{\sin\frac{2\cdot\pi}{5}\cdot\cos\frac{2\cdot\pi}{5}}{2\cdot\sin\frac{\pi}{5}}\ \Rightarrow\ a=\frac{2\cdot\sin\frac{2\cdot\pi}{5}\cdot\cos\frac{2\cdot\pi}{5}}{2\cdot 2\cdot\sin\frac{\pi}{5}}\ \Rightarrow\ a=\frac{\sin\frac{4\cdot\pi}{5}}{4\cdot\sin\frac{\pi}{5}}\ \Rightarrow\ a=\frac{\sin\left(\pi-\frac{\pi}{5}\right)}{4\cdot\sin\frac{\pi}{5}}\ \Rightarrow$$

$$\Rightarrow\ a=\frac{\sin\frac{\pi}{5}}{4\cdot\sin\frac{\pi}{5}}\ \Rightarrow\ a=\frac{\sin\frac{\pi}{5}}{4\cdot\sin\frac{\pi}{5}}\ \Rightarrow\ a=\frac{1}{4}.$$

Tada je:

$$a^{-1}=\left(\frac{1}{4}\right)^{-1}\ \Rightarrow\ a^{-1}=4.$$

Odgovor je pod B.

Vježba 189

Ako je $a=\cos 36^{0}\cdot\cos 72^{0}$, onda je a^{-2} jednako:

$A)$ 9 $\quad B)$ 16 $\quad C)$ 4 $\quad D)$ 36 $\quad E)$ 25

Rezultat: B.

Zadatak 190 (Ivona, gimnazija)

Ako je $tg\ x+ctg\ x=4$, koliko je $\sin x\cdot\cos x$?

Rješenje 190

Ponovimo!

$$tg\ x=\frac{\sin x}{\cos x}\quad ,\quad ctg\ x=\frac{\cos x}{\sin x}\quad ,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad ,\quad \sin^{2}x+\cos^{2}x=1.$$

$$\frac{a}{b}=\frac{c}{d}\ \Rightarrow\ \frac{b}{a}=\frac{d}{c}\quad ,\quad n=\frac{n}{1}\quad ,\quad a=\frac{1}{\frac{1}{a}}\quad ,\quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}.$$

1.inačica

$$tg\ x+ctg\ x=4\ \Rightarrow\ \frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=4\ \Rightarrow\ \frac{\sin^{2}x+\cos^{2}x}{\cos x\cdot\sin x}=4\ \Rightarrow\ \frac{1}{\cos x\cdot\sin x}=4\ \Rightarrow$$

$$\Rightarrow\ \frac{1}{\sin x\cdot\cos x}=4\ \Rightarrow\ \frac{1}{\sin x\cdot\cos x}=\frac{4}{1}\ \Rightarrow\ \sin x\cdot\cos x=\frac{1}{4}.$$

2.inačica

$$\sin x\cdot\cos x=\frac{1}{\frac{1}{\sin x\cdot\cos x}}=\frac{1}{\frac{\sin^2 x+\cos^2 x}{\sin x\cdot\cos x}}=\frac{1}{\frac{\sin^2 x}{\sin x\cdot\cos x}+\frac{\cos^2 x}{\sin x\cdot\cos x}}=$$

$$=\frac{1}{\frac{\sin^2 x}{\sin x\cdot\cos x}+\frac{\cos^2 x}{\sin x\cdot\cos x}}=\frac{1}{\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}}=\frac{1}{tg\ x+ctg\ x}=\frac{1}{4}.$$

Vježba 190

Ako je $tg\ x+ctg\ x=2$, koliko je $\sin x\cdot\cos x$?

Rezultat: $\frac{1}{2}$.

Zadatak 191 (Nina, gimnazija)

Dokaži identitet $tg\ x+ctg\ x=\frac{2}{\sin 2x}$.

Rješenje 191

Ponovimo!

$$tg\ x=\frac{\sin x}{\cos x}\quad,\quad ctg\ x=\frac{\cos x}{\sin x}\quad,\quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}\quad,\quad \sin^2 x+\cos^2 x=1.$$

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n}\quad,\quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha\quad,\quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n}.$$

1.inačica

$$tg\ x+ctg\ x=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{\sin^2 x+\cos^2 x}{\cos x\cdot\sin x}=\frac{1}{\cos x\cdot\sin x}=\left[\begin{matrix}\text{proširimo}\\ \text{razlomak sa } 2\end{matrix}\right]=$$

$$=\frac{1\cdot 2}{\sin x\cdot\cos x\cdot 2}=\frac{2}{2\cdot\sin x\cdot\cos x}=\frac{2}{\sin 2x}.$$

2.inačica

$$\frac{2}{\sin 2x}=\frac{2}{2\cdot\sin x\cdot\cos x}=\frac{2}{2\cdot\sin x\cdot\cos x}=\frac{1}{\sin x\cdot\cos x}=\frac{\sin^2 x+\cos^2 x}{\sin x\cdot\cos x}=\frac{\sin^2 x}{\sin x\cdot\cos x}+\frac{\cos^2 x}{\sin x\cdot\cos x}=$$

$$=\frac{\sin^2 x}{\sin x\cdot\cos x}+\frac{\cos^2 x}{\sin x\cdot\cos x}=\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=tg\ x+ctg\ x.$$

3.inačica

$$tg\ x+ctg\ x=\frac{2}{\sin 2x}\ \Rightarrow\ \frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{2}{2\cdot\sin x\cdot\cos x}\ \Rightarrow\ \frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{2}{2\cdot\sin x\cdot\cos x}\ \Rightarrow$$

$$\Rightarrow\ \frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{1}{\sin x\cdot\cos x}\ \Rightarrow\ \frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}=\frac{1}{\sin x\cdot\cos x}\ /\cdot\sin x\cdot\cos x\ \Rightarrow$$

$$\Rightarrow\ \sin^2 x+\cos^2 x=1\ \Rightarrow\ 1=1.$$

Vježba 191

Dokaži identitet $(\sin x+\cos x)^2=1+\sin 2x$.

Rezultat: Točno je.

Zadatak 192 (Ivana, srednja škola)

Ako je $tg\, x = 2$, izračunajte $(tg\, x + ctg\, x)^2$.

Rješenje 192

Ponovimo!

$$tg\, x \cdot ctg\, x = 1 \Rightarrow ctg\, x = \frac{1}{tg\, x} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad n = \frac{n}{1}.$$

$$(a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

1.inačica

$$(tg\, x + ctg\, x)^2 = \left(tg\, x + \frac{1}{tg\, x}\right)^2 = [tg\, x = 2] = \left(2 + \frac{1}{2}\right)^2 = \left(\frac{2}{1} + \frac{1}{2}\right)^2 = \left(\frac{4+1}{2}\right)^2 = \left(\frac{5}{2}\right)^2 = \frac{25}{4}.$$

2.inačica

$$(tg\, x + ctg\, x)^2 = tg^2\, x + 2 \cdot tg\, x \cdot ctg\, x + ctg^2\, x = tg^2\, x + 2 \cdot 1 + \frac{1}{tg^2\, x} = tg^2\, x + 2 + \frac{1}{tg^2\, x} =$$

$$= 2^2 + 2 + \frac{1}{2^2} = 4 + 2 + \frac{1}{4} = 6 + \frac{1}{4} = \frac{6}{1} + \frac{1}{4} = \frac{24+1}{4} = \frac{25}{4}.$$

Vježba 192

Ako je $tg\, x = 2$, izračunajte $(tg\, x - ctg\, x)^2$.

Rezultat: $\frac{9}{4}$.

Zadatak 193 (Mario, srednja škola)

Riješi jednadžbu $\sin\left(\frac{x-\pi}{4}\right) \cdot \cos\left(\frac{x-\pi}{4}\right) = \frac{\sqrt{2}}{4}$.

Rješenje 193

Ponovimo!

Trigonometrijska jednadžba $\sin x = a, \; |a| \le 1$

Skup rješenja jednadžbe $\sin x = a, \; |a| \le 1$, je $\{x_0 + k \cdot 2 \cdot \pi : k \in Z\} \cup \{\pi - x_0 + k \cdot 2 \cdot \pi : k \in Z\}$

gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$\sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha \quad , \quad a = \frac{a}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$\sin\left(\frac{x-\pi}{4}\right) \cdot \cos\left(\frac{x-\pi}{4}\right) = \frac{\sqrt{2}}{4} \Rightarrow \sin\left(\frac{x-\pi}{4}\right) \cdot \cos\left(\frac{x-\pi}{4}\right) = \frac{\sqrt{2}}{4} \; / \cdot 2 \Rightarrow$$

$$\Rightarrow 2 \cdot \sin\left(\frac{x-\pi}{4}\right) \cdot \cos\left(\frac{x-\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{4} \Rightarrow 2 \cdot \sin\left(\frac{x-\pi}{4}\right) \cdot \cos\left(\frac{x-\pi}{4}\right) = 2 \cdot \frac{\sqrt{2}}{4} \Rightarrow$$

$$\Rightarrow \sin 2 \cdot \left(\frac{x-\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow \sin 2 \cdot \left(\frac{x-\pi}{4}\right) = \frac{\sqrt{2}}{2} \Rightarrow \sin\left(\frac{x-\pi}{2}\right) = \frac{\sqrt{2}}{2}.$$

Uvodimo supstituciju:

$$t = \frac{x-\pi}{2}.$$

Tada jednadžba glasi:

$$\sin t=\frac{\sqrt{2}}{2}\Rightarrow t=\sin^{-1}\left(\frac{\sqrt{2}}{2}\right)\Rightarrow\left.\begin{matrix}t_1=\frac{\pi}{4}+k\cdot 2\cdot\pi\ ,\ k\in Z\\ t_2=\pi-\frac{\pi}{4}+k\cdot 2\cdot\pi\ ,\ k\in Z\end{matrix}\right\}\Rightarrow\left.\begin{matrix}t_1=\frac{\pi}{4}+k\cdot 2\cdot\pi\ ,\ k\in Z\\ t_2=\frac{3\cdot\pi}{4}+k\cdot 2\cdot\pi\ ,\ k\in Z\end{matrix}\right\}.$$

Vraćamo se supstituciji.

$$\left.\begin{matrix}t_1=\frac{\pi}{4}+k\cdot 2\cdot\pi\ ,\ k\in Z\\ t_2=\frac{3\cdot\pi}{4}+k\cdot 2\cdot\pi\ ,\ k\in Z\end{matrix}\right\}\Rightarrow\left[\begin{matrix}\text{supstitucija}\\ \frac{x-\pi}{2}=t\end{matrix}\right]\Rightarrow\left.\begin{matrix}\frac{x-\pi}{2}=\frac{\pi}{4}+k\cdot 2\cdot\pi\ ,\ k\in Z\\ \frac{x-\pi}{2}=\frac{3\cdot\pi}{4}+k\cdot 2\cdot\pi\ ,\ k\in Z\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{matrix}\frac{x-\pi}{2}=\frac{\pi}{4}+k\cdot 2\cdot\pi\ /\cdot 2\\ \frac{x-\pi}{2}=\frac{3\cdot\pi}{4}+k\cdot 2\cdot\pi\ /\cdot 2\end{matrix}\right\}\Rightarrow\left.\begin{matrix}x-\pi=\frac{\pi}{2}+k\cdot 4\cdot\pi\\ x-\pi=\frac{3\cdot\pi}{2}+k\cdot 4\cdot\pi\end{matrix}\right\}\Rightarrow\left.\begin{matrix}x=\frac{\pi}{2}+\pi+k\cdot 4\cdot\pi\\ x=\frac{3\cdot\pi}{2}+\pi+k\cdot 4\cdot\pi\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{matrix}x_1=\frac{3\cdot\pi}{2}+k\cdot 4\cdot\pi\ ,\ k\in Z\\ x_2=\frac{5\cdot\pi}{2}+k\cdot 4\cdot\pi\ ,\ k\in Z\end{matrix}\right\}.$$

Vježba 193

Riješi jednadžbu $4\cdot\sin\left(\frac{x-\pi}{4}\right)\cdot\cos\left(\frac{x-\pi}{4}\right)=\sqrt{2}.$

Rezultat: $x_1=\frac{3\cdot\pi}{2}+k\cdot 4\cdot\pi\ ,\ x_2=\frac{5\cdot\pi}{2}+k\cdot 4\cdot\pi\ ,\ k\in Z.$

Zadatak 194 (Ivana, srednja škola)

Dokaži: $\sin(x+y)+\sin(x-y)=2\cdot\sin x\cdot\cos y.$

Rješenje 194

Ponovimo!

Za svaka dva realna broja x i y vrijedi:

$$\sin(x+y)=\sin x\cdot\cos y+\cos x\cdot\sin y.$$

$$\sin(x-y)=\sin x\cdot\cos y-\cos x\cdot\sin y.$$

$$\sin x\cdot\cos y=\frac{1}{2}\cdot\left[\sin(x+y)+\sin(x-y)\right].$$

1.inačica

$$\sin(x+y)+\sin(x-y)=\sin x\cdot\cos y+\cos x\cdot\sin y+\sin x\cdot\cos y-\cos x\cdot\sin y=$$

$$=\sin x\cdot\cos y+\cos x\cdot\sin y+\sin x\cdot\cos y-\cos x\cdot\sin y=2\cdot\sin x\cdot\cos y.$$

2.inačica

$$2\cdot\sin x\cdot\cos y=2\cdot\frac{1}{2}\cdot\left[\sin(x+y)+\sin(x-y)\right]=2\cdot\frac{1}{2}\cdot\left[\sin(x+y)+\sin(x-y)\right]=\sin(x+y)+\sin(x-y).$$

Vježba 194

Dokaži: $\sin(x+y)-\sin(x-y)=2\cdot\cos x\cdot\sin y.$

Rezultat: Dokaz analogan.

Zadatak 195 (Izzy, medicinska škola)

Pojednostavnite: $\sin\left(3960^{0}+\alpha\right)$.

Rješenje 195

Ponovimo!

Za funkciju f kažemo da je periodična ako postoji realan broj P > 0 takav da za svaki realni broj x iz domene te funkcije vrijedi:

$$f\left(x+P\right)=f\left(x\right).$$

Broj P zove se period funkcije f. Najmanji takav pozitivan broj zove se temeljni period funkcije f. Funkcija sinus je periodična s temeljnim periodom 2π ili 360º:

$$\sin\left(\alpha+360^{0}\right)=\sin\alpha \quad , \quad \sin\left(\alpha+k\cdot 360^{0}\right)=\sin\alpha \ , \ k\in Z.$$

$$\sin\left(3960^{0}+\alpha\right)=\sin\left(\alpha+3960^{0}\right)=\begin{bmatrix} 3960:360=11\rightarrow k \\ =360 \\ = \end{bmatrix}=\sin\left(\alpha+11\cdot 360^{0}\right)=\sin\alpha.$$

Vježba 195

Pojednostavnite: $\sin\left(3600^{0}+\alpha\right)$.

Rezultat: $\sin\alpha.$

Zadatak 196 (Jasna, maturantica)

Koliko rješenja ima jednadžba $\sin x=\frac{1}{2}\cdot x$?

A. jedno *B.* tri *C.* pet *D.* sedam

Rješenje 196

U zadanoj jednadžbi

$$\sin x=\frac{1}{2}\cdot x$$

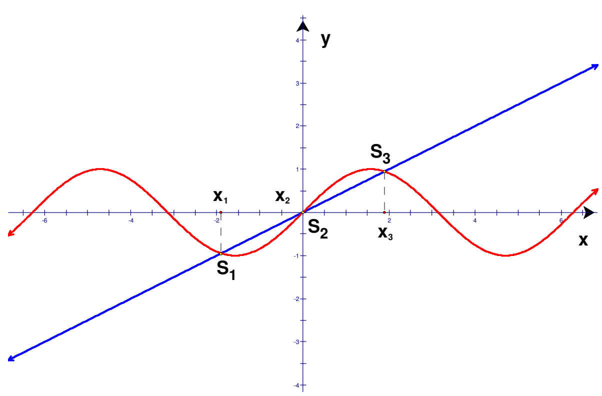
uočimo funkcije

$$f\left(x\right)=\sin x \ \text{i} \ g\left(x\right)=\frac{1}{2}\cdot x.$$

Skicirajmo grafove funkcija

$$f\left(x\right)=\sin x \ \text{i} \ g\left(x\right)=\frac{1}{2}\cdot x.$$

Graf funkcije $f\left(x\right)=\sin x$ je sinusoida, a graf funkcije $g\left(x\right)=\frac{1}{2}\cdot x$ je pravac koji prolazi ishodištem koordinatnog sustava.



Uočimo da se grafovi tih funkcija sijeku u tri točke S_1, S_2 i S_3 pa zadana jednadžba ima tri rješenja x_1, x_2 i x_3. Odgovor je pod B.

Vježba 196

Koliko rješenja ima jednadžba $\sin x = \frac{1}{3} \cdot x$?

A. jedno *B.* tri *C.* pet *D.* sedam

Rezultat: B.

Zadatak 197 (Jasna, maturantica)

Koliko rješenja ima jednadžba $\cos x - x^2 + 1 = 0$?

A. jedno *B.* dva *C.* tri *D.* sedam

Rješenje 197

Transformiramo zadanu jednadžbu

$$\cos x - x^2 + 1 = 0 \Rightarrow \cos x = x^2 - 1.$$

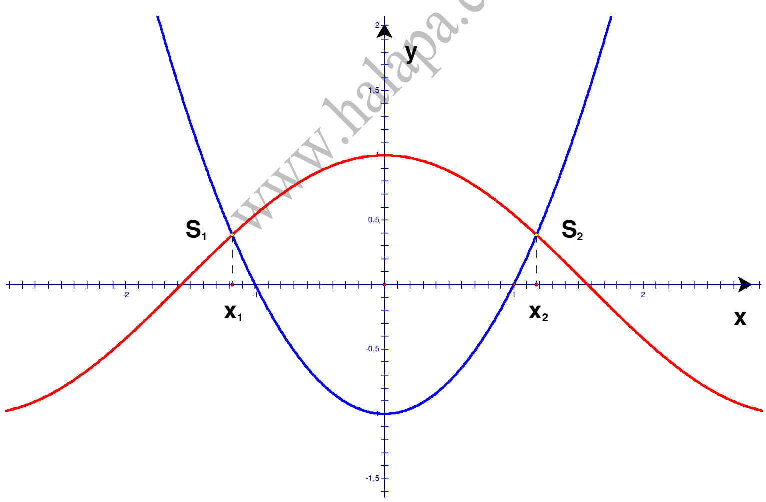
Uočimo funkcije

$$f(x) = \cos x \text{ i } g(x) = x^2 - 1.$$

Skicirajmo grafove funkcija

$$f(x) = \cos x \text{ i } g(x) = x^2 - 1.$$

Graf funkcije $f(x) = \cos x$ je kosinusoida, a graf funkcije $g(x) = x^2 - 1$ je parabola s tjemenom u točki T(0, – 1).



Uočimo da se grafovi tih funkcija sijeku u dvije točke S_1 i S_2 pa zadana jednadžba ima dva rješenja x_1 i x_2. Odgovor je pod B.

Vježba 197

Koliko rješenja ima jednadžba $\cos x - x^2 = 0$?

A. jedno *B.* dva *C.* tri *D.* sedam

Rezultat: B.

Zadatak 198 (Biba, maturantica)

Ako je $\sin \frac{x}{2} + \cos \frac{x}{2} = a$, $|a| \le \sqrt{2}$, nađi $\sin x$.

A. a^2 *B.* $1 - a^2$ *C.* $a + 1$ *D.* $a^2 - 1$

Rješenje 198

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \cos^2\alpha+\sin^2\alpha=1 \quad , \quad \sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha.$$

$$\sin\frac{x}{2}+\cos\frac{x}{2}=a \;\Rightarrow\; \sin\frac{x}{2}+\cos\frac{x}{2}=a \;/^2 \;\Rightarrow\; \left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2=a^2 \;\Rightarrow$$

$$\Rightarrow\; \sin^2\frac{x}{2}+2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}+\cos^2\frac{x}{2}=a^2 \;\Rightarrow\; \left(\sin^2\frac{x}{2}+\cos^2\frac{x}{2}\right)+2\cdot\sin\frac{x}{2}\cdot\cos\frac{x}{2}=a^2 \;\Rightarrow$$

$$\Rightarrow\; 1+\sin 2\cdot\frac{x}{2}=a^2 \;\Rightarrow\; 1+\sin 2\cdot\frac{x}{2}=a^2 \;\Rightarrow\; 1+\sin x=a^2 \;\Rightarrow\; \sin x=a^2-1.$$

Odgovor je pod D.

Vježba 198

Ako je $\sin\frac{x}{2}+\cos\frac{x}{2}=1$, nađi $\sin x$.

$A.\ 1 \qquad B.\ 0 \qquad C.\ -1 \qquad D.\ 2$

Rezultat: B.

Zadatak 199 (Milly, maturantica)

Izračunaj bez uporabe računala vrijednost sinusa od 15°.

Rješenje 199

Ponovimo!

$$\sin 30^0=\frac{1}{2} \quad , \quad \cos 30^0=\frac{\sqrt{3}}{2} \quad , \quad n=\frac{n}{1} \quad , \quad \sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}} \quad , \quad \left(\sqrt{a}\right)^2=a.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\sqrt{A\pm\sqrt{B}}=\sqrt{\frac{A+\sqrt{A^2-B}}{2}}\pm\sqrt{\frac{A-\sqrt{A^2-B}}{2}} \;,\; A\ge 0 \;,\; B\ge 0 \;,\; A^2>B.$$

Sinus i kosinus polovičnog kuta
Za svaki realni broj x vrijede sljedeći identiteti:

$$\sin\frac{x}{2}=\pm\sqrt{\frac{1-\cos x}{2}} \quad , \quad \cos\frac{x}{2}=\pm\sqrt{\frac{1+\cos x}{2}}.$$

Predznak se uzima prema kvadrantu u kojem se nalazi kut $\frac{x}{2}$.

Predznaci trigonometrijskih funkcija sinus i kosinus
U prvom kvadrantu (0° – 90°) sinus i kosinus su pozitivni.

$$\sin 15^0=\sin\frac{30^0}{2}=\sqrt{\frac{1-\cos 30^0}{2}}=\sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}}=\sqrt{\frac{\frac{1}{1}-\frac{\sqrt{3}}{2}}{2}}=\sqrt{\frac{\frac{2-\sqrt{3}}{2}}{\frac{2}{1}}}=\sqrt{\frac{2-\sqrt{3}}{4}}=$$

$$=\frac{\sqrt{2-\sqrt{3}}}{\sqrt{4}}=\frac{\sqrt{2-\sqrt{3}}}{2}=\frac{1}{2}\cdot\sqrt{2-\sqrt{3}}=\frac{1}{2}\cdot\left(\sqrt{\frac{2+\sqrt{2^2-3}}{2}}-\sqrt{\frac{2-\sqrt{2^2-3}}{2}}\right)=$$

$$=\frac{1}{2}\cdot\left(\sqrt{\frac{2+\sqrt{4-3}}{2}}-\sqrt{\frac{2-\sqrt{4-3}}{2}}\right)=\frac{1}{2}\cdot\left(\sqrt{\frac{2+\sqrt{1}}{2}}-\sqrt{\frac{2-\sqrt{1}}{2}}\right)=\frac{1}{2}\cdot\left(\sqrt{\frac{2+1}{2}}-\sqrt{\frac{2-1}{2}}\right)=$$

$$=\frac{1}{2}\cdot\left(\sqrt{\frac{3}{2}}-\sqrt{\frac{1}{2}}\right)=\frac{1}{2}\cdot\left(\frac{\sqrt{3}}{\sqrt{2}}-\frac{\sqrt{1}}{\sqrt{2}}\right)=\frac{1}{2}\cdot\left(\frac{\sqrt{3}}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)=\frac{1}{2\cdot\sqrt{2}}\cdot\left(\sqrt{3}-1\right)=\left[\begin{matrix}\text{racionalizacija}\\ \text{nazivnika}\end{matrix}\right]=$$

$$=\frac{1}{2\cdot\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}\cdot\left(\sqrt{3}-1\right)=\frac{\sqrt{2}}{2\cdot\left(\sqrt{2}\right)^2}\cdot\left(\sqrt{3}-1\right)=\frac{\sqrt{2}}{2\cdot 2}\cdot\left(\sqrt{3}-1\right)=\frac{\sqrt{2}}{4}\cdot\left(\sqrt{3}-1\right).$$

Vježba 199

Izračunaj bez uporabe računala vrijednost kosinusa od 15°.

Rezultat: $\frac{\sqrt{2}}{4}\cdot\left(\sqrt{3}+1\right).$

Zadatak 200 (Josipa, srednja škola)

Pojednostavni izraz: $\frac{2\cdot\sin x+\sin 2x}{\left(2-\cos^2 x\right)^2-\cos^4 x}.$

Rješenje 200

Ponovimo!

$$\sin 2\alpha=2\cdot\sin\alpha\cdot\cos\alpha\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad(a-b)^2=a^2-2\cdot a\cdot b+b^2.$$

$$a^2-b^2=(a-b)\cdot(a+b).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{2\cdot\sin x+\sin 2x}{\left(2-\cos^2 x\right)^2-\cos^4 x}=\frac{2\cdot\sin x+2\cdot\sin x\cdot\cos x}{4-4\cdot\cos^2 x+\cos^4 x-\cos^4 x}=\frac{2\cdot\sin x+2\cdot\sin x\cdot\cos x}{4-4\cdot\cos^2 x+\cos^4 x-\cos^4 x}=$$

$$=\frac{2\cdot\sin x+2\cdot\sin x\cdot\cos x}{4-4\cdot\cos^2 x}=\frac{2\cdot\sin x\cdot(1+\cos x)}{4\cdot\left(1-\cos^2 x\right)}=\frac{2\cdot\sin x\cdot(1+\cos x)}{4\cdot(1-\cos x)\cdot(1+\cos x)}=\frac{2\cdot\sin x\cdot(1+\cos x)}{4\cdot(1-\cos x)\cdot(1+\cos x)}=$$

$$=\frac{\sin x}{2\cdot(1-\cos x)}.$$

2.inačica

$$\frac{2\cdot\sin x+\sin 2x}{\left(2-\cos^2 x\right)^2-\cos^4 x}=\frac{2\cdot\sin x+2\cdot\sin x\cdot\cos x}{\left(2-\cos^2 x\right)^2-\left(\cos^2 x\right)^2}=\frac{2\cdot\sin x\cdot(1+\cos x)}{\left(2-\cos^2 x-\cos^2 x\right)\cdot\left(2-\cos^2 x+\cos^2 x\right)}=$$

$$=\frac{2\cdot\sin x\cdot(1+\cos x)}{\left(2-\cos^2 x-\cos^2 x\right)\cdot\left(2-\cos^2 x+\cos^2 x\right)}=\frac{2\cdot\sin x\cdot(1+\cos x)}{\left(2-\cos^2 x-\cos^2 x\right)\cdot 2}=\frac{2\cdot\sin x\cdot(1+\cos x)}{\left(2-\cos^2 x-\cos^2 x\right)\cdot 2}=$$

$$=\frac{\sin x\cdot(1+\cos x)}{2-\cos^2 x-\cos^2 x}=\frac{\sin x\cdot(1+\cos x)}{2-2\cdot\cos^2 x}=\frac{\sin x\cdot(1+\cos x)}{2\cdot\left(1-\cos^2 x\right)}=\frac{\sin x\cdot(1+\cos x)}{2\cdot(1-\cos x)\cdot(1+\cos x)}=$$

$$=\frac{\sin x\cdot(1+\cos x)}{2\cdot(1-\cos x)\cdot(1+\cos x)}=\frac{\sin x}{2\cdot(1-\cos x)}.$$

Vježba 200

Pojednostavni izraz: $\frac{\left(2-\cos^2 x\right)^2-\cos^4 x}{2\cdot\sin x+\sin 2x}$.

Rezultat: $\frac{2\cdot(1-\cos x)}{\sin x}$.