Zadatak 161 (Ana, gimnazija)

Ako je $\frac{5\cdot\sin\alpha-4\cdot\cos\alpha}{7\cdot\sin\alpha-5\cdot\cos\alpha}=1,$ nađi $tg\,\alpha.$

Rješenje 161

Ponovimo!

$$tg\,x=\frac{\sin x}{\cos x}\quad,\quad\frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}.$$

$$\frac{5\cdot\sin\alpha-4\cdot\cos\alpha}{7\cdot\sin\alpha-5\cdot\cos\alpha}=1\ \Rightarrow\ \frac{\frac{5\cdot\sin\alpha-4\cdot\cos\alpha}{\cos\alpha}}{\frac{7\cdot\sin\alpha-5\cdot\cos\alpha}{\cos\alpha}}=1\ \Rightarrow\ \frac{\frac{5\cdot\sin\alpha-4\cdot\cos\alpha}{\cos\alpha}}{\frac{7\cdot\sin\alpha-5\cdot\cos\alpha}{\cos\alpha}}=1\ \Rightarrow$$

$$\Rightarrow\ \frac{\frac{5\cdot\sin\alpha}{\cos\alpha}-\frac{4\cdot\cos\alpha}{\cos\alpha}}{\frac{7\cdot\sin\alpha}{\cos\alpha}-\frac{5\cdot\cos\alpha}{\cos\alpha}}=1\ \Rightarrow\ \frac{\frac{5\cdot\sin\alpha}{\cos\alpha}-\frac{4\cdot\cos\alpha}{\cos\alpha}}{\frac{7\cdot\sin\alpha}{\cos\alpha}-\frac{5\cdot\cos\alpha}{\cos\alpha}}=1\ \Rightarrow\ \frac{5\cdot tg\,\alpha-4}{7\cdot tg\,\alpha-5}=1\ \Rightarrow\ 5\cdot tg\,\alpha-4=7\cdot tg\,\alpha-5\ \Rightarrow$$

$$\Rightarrow\ 5\cdot tg\,\alpha-7\cdot tg\,\alpha=-5+4\ \Rightarrow\ -2\cdot tg\,\alpha=-1\ /:(-2)\ \Rightarrow\ tg\,\alpha=\frac{1}{2}\ \Rightarrow\ tg\,\alpha=0.5.$$

Vježba 161

Ako je $\frac{7\cdot\sin\alpha-5\cdot\cos\alpha}{5\cdot\sin\alpha-4\cdot\cos\alpha}=1,$ nađi $tg\,\alpha.$

Rezultat: 0.5.

Zadatak 162 (Ana, gimnazija)

Ako je $\frac{1}{\sin^2\alpha}+\frac{1}{\cos^2\alpha}+\frac{1}{tg^2\alpha}+\frac{1}{ctg^2\alpha}=6,$ nađi $\sin^2 2\alpha.$

Rješenje 162

Ponovimo!

$$tg\,x=\frac{\sin x}{\cos x}\quad,\quad ctg\,x=\frac{\cos x}{\sin x}\quad,\quad\cos^2x+\sin^2x=1\quad,\quad\sin 2x=2\cdot\sin x\cdot\cos x.$$

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2\quad,\quad\left(a^2+b^2\right)^2=a^4+2\cdot a^2\cdot b^2+b^4\quad,\quad\frac{a-b}{n}=\frac{a}{n}-\frac{b}{n}.$$

$$\frac{1}{\sin^2\alpha}+\frac{1}{\cos^2\alpha}+\frac{1}{tg^2\alpha}+\frac{1}{ctg^2\alpha}=6\ \Rightarrow\ \frac{1}{\sin^2\alpha}+\frac{1}{\cos^2\alpha}+\frac{1}{\frac{\sin^2\alpha}{\cos^2\alpha}}+\frac{1}{\frac{\cos^2\alpha}{\sin^2\alpha}}=6\ \Rightarrow$$

$$\Rightarrow\ \frac{1}{\sin^2\alpha}+\frac{1}{\cos^2\alpha}+\frac{\cos^2\alpha}{\sin^2\alpha}+\frac{\sin^2\alpha}{\cos^2\alpha}=6\ \Rightarrow\ \Rightarrow\ \left(\frac{1}{\sin^2\alpha}+\frac{1}{\cos^2\alpha}\right)+\left(\frac{\cos^2\alpha}{\sin^2\alpha}+\frac{\sin^2\alpha}{\cos^2\alpha}\right)=6\ \Rightarrow$$

$$\Rightarrow\ \frac{\cos^2\alpha+\sin^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{\cos^4\alpha+\sin^4x}{\sin^2\alpha\cdot\cos^2\alpha}=6\ \Rightarrow$$

$$\Rightarrow\ \frac{1}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{\cos^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\sin^4x-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6\ \Rightarrow$$

$$\Rightarrow \frac{1}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{\left(\cos^4\alpha+2\cdot\sin^2\alpha\cdot\cos^2\alpha+\sin^4 x\right)-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6 \Rightarrow$$

$$\Rightarrow \frac{1}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{\left(\cos^2\alpha+\sin^2 x\right)^2-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6 \Rightarrow$$

$$\Rightarrow \frac{1}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{1^2-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6 \Rightarrow \frac{1}{\sin^2\alpha\cdot\cos^2\alpha}+\frac{1-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6 \Rightarrow$$

$$\Rightarrow \frac{1+1-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6 \Rightarrow \frac{2-2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6 \Rightarrow$$

$$\Rightarrow \frac{2}{\sin^2\alpha\cdot\cos^2\alpha}-\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6 \Rightarrow \frac{2}{\sin^2\alpha\cdot\cos^2\alpha}-\frac{2\cdot\sin^2\alpha\cdot\cos^2\alpha}{\sin^2\alpha\cdot\cos^2\alpha}=6 \Rightarrow$$

$$\Rightarrow \frac{2}{\sin^2\alpha\cdot\cos^2\alpha}-2=6 \Rightarrow \frac{2}{\sin^2\alpha\cdot\cos^2\alpha}=6+2 \Rightarrow \frac{2}{\sin^2\alpha\cdot\cos^2\alpha}=8 \Rightarrow$$

$$\Rightarrow \frac{4\cdot 2}{4\cdot\sin^2\alpha\cdot\cos^2\alpha}=8 \Rightarrow \frac{4\cdot 2}{4\cdot\sin^2\alpha\cdot\cos^2\alpha}=8 \Rightarrow \frac{8}{\left(2\cdot\sin\alpha\cdot\cos\alpha\right)^2}=8 \Rightarrow \frac{8}{\sin^2 2\alpha}=8 \Rightarrow$$

$$\Rightarrow \frac{8}{\sin^2 2\alpha}=8 \;/:8 \Rightarrow \frac{1}{\sin^2 2\alpha}=1 \Rightarrow \sin^2 2\alpha=1.$$

Vježba 162

Ako je $\frac{1}{tg^2\alpha}+\frac{1}{ctg^2\alpha}=6-\frac{1}{\sin^2\alpha\cdot\cos^2\alpha}$, nađi $\sin^2 2\alpha$.

Rezultat: 1.

Zadatak 163 (Ana, gimnazija)

Pojednostavni: $\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}$.

Rješenje 163

Ponovimo!

$$a^3+b^3=(a+b)\cdot\left(a^2-a\cdot b+b^2\right) \quad , \quad a^3-b^3=(a-b)\cdot\left(a^2+a\cdot b+b^2\right).$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad a^4-b^4=\left(a^2-b^2\right)\cdot\left(a^2+b^2\right) \quad , \quad \cos^2 x+\sin^2 x=1.$$

$$\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=\frac{(\sin x+\cos x)\cdot\left(\sin^2 x-\sin x\cdot\cos x+\cos^2 x\right)}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=\frac{(\sin x+\cos x)\cdot\left(\sin^2 x-\sin x\cdot\cos x+\cos^2 x\right)}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=\sin^2 x-\sin x\cdot\cos x+\cos^2 x+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=\left(\sin^2 x+\cos^2 x\right)-\sin x\cdot\cos x+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=1-\sin x\cdot\cos x+\frac{(\sin x-\cos x)\cdot\left(\sin^2 x+\sin x\cdot\cos x+\cos^2 x\right)}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=1-\sin x\cdot\cos x+\frac{(\sin x-\cos x)\cdot\left(\sin^2 x+\sin x\cdot\cos x+\cos^2 x\right)}{\sin x-\cos x}+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=1-\sin x\cdot\cos x+\sin^2 x+\sin x\cdot\cos x+\cos^2 x+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=1-\sin x\cdot\cos x+\sin^2 x+\sin x\cdot\cos x+\cos^2 x+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=1+\sin^2 x+\cos^2 x+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=1+1+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=2+\frac{\sin^4 x-\cos^4 x}{\sin^2 x-\cos^2 x}=$$

$$=2+\frac{\left(\sin^2 x-\cos^2 x\right)\cdot\left(\sin^2 x+\cos^2 x\right)}{\sin^2 x-\cos^2 x}=2+\frac{\left(\sin^2 x-\cos^2 x\right)\cdot\left(\sin^2 x+\cos^2 x\right)}{\sin^2 x-\cos^2 x}=$$

$$=2+\sin^2 x+\cos^2 x=2+1=3.$$

Vježba 163

Pojednostavni: $\frac{\sin^3 x+\cos^3 x}{\sin x+\cos x}+\frac{\sin^3 x-\cos^3 x}{\sin x-\cos x}+1.$

Rezultat: 3.

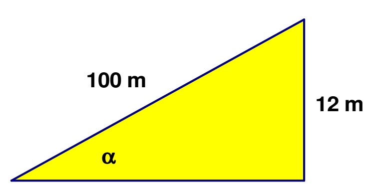
Zadatak 164 (Maturanti, HTT)

Uz cestu stoji prometni znak s oznakom uspona 12%. Pod kojim se kutom prema horizontalnoj ravnini uspinje cesta?

Rješenje 164

Ponovimo!

Sinus šiljastog kuta pravokutnog trokuta jednak je omjeru duljine katete nasuprot toga kuta i duljine hipotenuze.



Što znači oznaka uspona 12%?

Pri prirastu puta od 100 m okomiti (vertikalni) prirast je 12 m.

Dakle, prirast visinske razlike puta po jednom metru duljine iznosi 0.12 m.

$$\sin\alpha = \frac{12}{100} \Rightarrow \alpha = \sin^{-1}\left(\frac{12}{100}\right) \Rightarrow \alpha = 6^0 53'32''.$$

Vježba 164

Uz cestu stoji prometni znak s oznakom uspona 8%. Pod kojim se kutom prema horizontalnoj ravnini uspinje cesta?

Rezultat: 4° 35' 19".

Zadatak 165 (Josipa, srednja škola)

Pojednostavni : $\cos^2 x + \sin^2 x \cdot \cos^2 x + \sin^4 x - 1.$

Rješenje 165

Ponovimo!

$$\cos^2\alpha + \sin^2\alpha = 1.$$

Zakon distribucije množenja prema zbrajanju:

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

1.inačica

$$\cos^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - 1 = \cos^2 x + \sin^2 x\cdot\left(\cos^2 x + \sin^2 x\right) - 1 = \cos^2 x + \sin^2 x\cdot 1 - 1 =$$

$$= \cos^2 x + \sin^2 x - 1 = 1 - 1 = 0.$$

2.inačica

$$\cos^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - 1 = \cos^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - \left(\cos^2 x + \sin^2 x\right) =$$

$$= \cos^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - \cos^2 x - \sin^2 x = \cos^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - \cos^2 x - \sin^2 x =$$

$$= \sin^2 x\cdot\cos^2 x + \sin^4 x - \sin^2 x = \sin^2 x\cdot\left(\cos^2 x + \sin^2 x - 1\right) = \sin^2 x\cdot(1-1) = \sin^2 x\cdot 0 = 0.$$

3.inačica

$$\cos^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - 1 = \cos^2 x + \sin^2 x\cdot\left(1 - \sin^2 x\right) + \sin^4 x - 1 =$$

$$= \cos^2 x + \sin^2 x - \sin^4 x + \sin^4 x - 1 = \cos^2 x + \sin^2 x - \sin^4 x + \sin^4 x - 1 = \cos^2 x + \sin^2 x - 1 =$$

$$= 1 - 1 = 0.$$

4.inačica

$$\cos^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - 1 = 1 - \sin^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - 1 =$$

$$= 1 - \sin^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x - 1 = -\sin^2 x + \sin^2 x\cdot\cos^2 x + \sin^4 x =$$

$$= \sin^2 x\cdot\left(-1 + \cos^2 x + \sin^2 x\right) = \sin^2 x\cdot(-1+1) = \sin^2 x\cdot 0 = 0.$$

Vježba 165

Pojednostavni : $\cos^2 x + \sin^2 x \cdot \cos^2 x + \sin^4 x - 1.$

Rezultat: 0.

Zadatak 166 (Borut, srednja škola)

Pojednostavni : $\sin^3\alpha\cdot(1 + ctg\,\alpha) + \cos^3\alpha\cdot(1 + tg\,\alpha).$

Rješenje 166

Ponovimo!

$$ctg\,x = \frac{\cos x}{\sin x} \quad , \quad tg\,x = \frac{\sin x}{\cos x} \quad , \quad \cos^2\alpha + \sin^2\alpha = 1.$$

Zakon distribucije množenja prema zbrajanju:

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

$$\sin^3 \alpha \cdot (1 + ctg\,\alpha) + \cos^3 \alpha \cdot (1 + tg\,\alpha) = \sin^3 \alpha \cdot \left(1 + \frac{\cos \alpha}{\sin \alpha}\right) + \cos^3 \alpha \cdot \left(1 + \frac{\sin \alpha}{\cos \alpha}\right) =$$

$$= \sin^3 \alpha \cdot \frac{\sin \alpha + \cos \alpha}{\sin \alpha} + \cos^3 \alpha \cdot \frac{\cos \alpha + \sin \alpha}{\cos \alpha} = \sin^3 \alpha \cdot \frac{\sin \alpha + \cos \alpha}{\sin \alpha} + \cos^3 \alpha \cdot \frac{\cos \alpha + \sin \alpha}{\cos \alpha} =$$

$$= \sin^2 \alpha \cdot (\sin \alpha + \cos \alpha) + \cos^2 \alpha \cdot (\cos \alpha + \sin \alpha) = \sin^2 \alpha \cdot (\sin \alpha + \cos \alpha) + \cos^2 \alpha \cdot (\sin \alpha + \cos \alpha) =$$

$$= (\sin \alpha + \cos \alpha) \cdot \left(\sin^2 \alpha + \cos^2 \alpha\right) = (\sin \alpha + \cos \alpha) \cdot 1 = \sin \alpha + \cos \alpha.$$

Vježba 166

Pojednostavni : $\left(1 - \sin^2 x\right) \cdot \left(1 + tg^2 x\right).$

Rezultat: 1.

Zadatak 167 (Borut, srednja škola)

Dokaži : $\dfrac{1 - 2 \cdot \cos^2 x}{\sin x \cdot \cos x} = tg\,x - ctg\,x.$

Rješenje 167

Ponovimo!

$$ctg\,x = \frac{\cos x}{\sin x} \quad , \quad tg\,x = \frac{\sin x}{\cos x} \quad , \quad \cos^2 \alpha + \sin^2 \alpha = 1 \quad , \quad \frac{a-b}{n} = \frac{a}{n} - \frac{b}{n}.$$

$$\frac{1 - 2 \cdot \cos^2 x}{\sin x \cdot \cos x} = \frac{\cos^2 x + \sin^2 x - 2 \cdot \cos^2 x}{\sin x \cdot \cos x} = \frac{\sin^2 x - \cos^2 x}{\sin x \cdot \cos x} = \frac{\sin^2 x}{\sin x \cdot \cos x} - \frac{\cos^2 x}{\sin x \cdot \cos x} =$$

$$= \frac{\sin^2 x}{\sin x \cdot \cos x} - \frac{\cos^2 x}{\sin x \cdot \cos x} = \frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} = tg\,x - ctg\,x.$$

Vježba 167

Dokaži : $\dfrac{1 - 2 \cdot \sin^2 x}{\sin x \cdot \cos x} = ctg\,x - tg\,x.$

Rezultat: Dokaz analogan.

Zadatak 168 (Tiny, gimnazija)

Pojednostavni : $\cos 2x + \sin 2x \cdot tg\,x.$

Rješenje 168

Ponovimo!

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha \quad , \quad \sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha \quad , \quad tg\,\alpha = \frac{\sin \alpha}{\cos \alpha} \quad , \quad \cos^2 \alpha + \sin^2 \alpha = 1.$$

$$\cos 2x + \sin 2x \cdot tg\,x = \cos^2 x - \sin^2 x + 2 \cdot \sin x \cdot \cos x \cdot \frac{\sin x}{\cos x} = \cos^2 x - \sin^2 x + 2 \cdot \sin x \cdot \cos x \cdot \frac{\sin x}{\cos x} =$$

$$= \cos^2 x - \sin^2 x + 2 \cdot \sin x \cdot \sin x = \cos^2 x - \sin^2 x + 2 \cdot \sin^2 x = \cos^2 x + \sin^2 x = 1.$$

Vježba 168

Pojednostavni : $1 - \sin 2x \cdot tg\,x.$

Rezultat: cos2x.

Zadatak 169 (Tiny, gimnazija)

$$\text{Dokaži}: \cos\left(\frac{\pi}{4}+\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=\frac{\cos 2\alpha}{2}.$$

Rješenje 169

Ponovimo!

$$\cos(x+y)=\cos x\cdot\cos y-\sin x\cdot\sin y \quad , \quad \cos(x-y)=\cos x\cdot\cos y+\sin x\cdot\sin y.$$

$$\cos\frac{\pi}{4}=\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2} \quad , \quad a^2-b^2=(a-b)\cdot(a+b) \quad , \quad \cos 2x=\cos^2 x-\sin^2 x.$$

$$\sin\left(\frac{\pi}{2}-x\right)=\cos x \quad , \quad \sin 2x=2\cdot\sin x\cdot\cos x \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad \left(\sqrt{a}\right)^2=a.$$

1.inačica

$$\cos\left(\frac{\pi}{4}+\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=\sin\left(\frac{\pi}{2}-\left(\frac{\pi}{4}+\alpha\right)\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=\sin\left(\frac{\pi}{2}-\frac{\pi}{4}-\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=$$

$$=\sin\left(\frac{\pi}{4}-\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=\frac{1}{2}\cdot 2\cdot\sin\left(\frac{\pi}{4}-\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=\frac{1}{2}\cdot 2\cdot\sin\left(\frac{\pi}{4}-\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=$$

$$=\frac{1}{2}\cdot\sin 2\cdot\left(\frac{\pi}{4}-\alpha\right)=\frac{1}{2}\cdot\sin\left(\frac{\pi}{2}-2\cdot\alpha\right)=\frac{1}{2}\cdot\cos 2\alpha=\frac{\cos 2\alpha}{2}.$$

2.inačica

$$\cos\left(\frac{\pi}{4}+\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=\left(\cos\frac{\pi}{4}\cdot\cos\alpha-\sin\frac{\pi}{4}\cdot\sin\alpha\right)\cdot\left(\cos\frac{\pi}{4}\cdot\cos\alpha+\sin\frac{\pi}{4}\cdot\sin\alpha\right)=$$

$$=\left(\frac{\sqrt{2}}{2}\cdot\cos\alpha-\frac{\sqrt{2}}{2}\cdot\sin\alpha\right)\cdot\left(\frac{\sqrt{2}}{2}\cdot\cos\alpha+\frac{\sqrt{2}}{2}\cdot\sin\alpha\right)=\frac{\sqrt{2}}{2}\cdot(\cos\alpha-\sin\alpha)\cdot\frac{\sqrt{2}}{2}\cdot(\cos\alpha+\sin\alpha)=$$

$$=\left(\frac{\sqrt{2}}{2}\right)^2\cdot(\cos\alpha-\sin\alpha)\cdot(\cos\alpha+\sin\alpha)=\frac{2}{4}\cdot\left(\cos^2\alpha-\sin^2\alpha\right)=\frac{2}{4}\cdot\cos 2\alpha=\frac{1}{2}\cdot\cos 2\alpha=\frac{\cos 2\alpha}{2}.$$

Vježba 169

$$\text{Dokaži}: 2\cdot\cos\left(\frac{\pi}{4}+\alpha\right)\cdot\cos\left(\frac{\pi}{4}-\alpha\right)=\cos^2\alpha-\sin^2\alpha.$$

Rezultat: Dokaz analogan.

Zadatak 170 (Tiny, gimnazija)

$$\text{Dokaži}: \sin 18^0=\frac{\sqrt{5}-1}{4}.$$

Rješenje 170

Ponovimo!

$$\sin\left(90^0-x\right)=\cos x \quad , \quad \sin 2x=2\cdot\sin x\cdot\cos x \quad , \quad \cos 3x=4\cdot\cos^3 x-3\cdot\cos x.$$

$$\cos^2 x+\sin^2 x=1.$$

Postavimo identitet:

$$\sin\left(2\cdot 18^0\right)=\sin 36^0=\sin\left(90^0-54^0\right)=\cos 54^0=\cos\left(3\cdot 18^0\right).$$

Dobili smo

$$\sin\left(2\cdot 18^0\right)=\cos\left(3\cdot 18^0\right).$$

Zbog jednostavnosti zapisivanja pišemo

$$\alpha=18^0$$

pa je

$$\sin 2\alpha=\cos 3\alpha.$$

Uočimo da vrijedi:

$$\sin\alpha>0 \qquad (1)$$

$$\cos\alpha\neq 0. \qquad (2)$$

Sada rješavamo jednadžbu:

$$\sin 2\alpha=\cos 3\alpha \;\Rightarrow\; 2\cdot\sin\alpha\cdot\cos\alpha=4\cdot\cos^3\alpha-3\cdot\cos\alpha \;\Rightarrow$$

$$\Rightarrow\; 2\cdot\sin\alpha\cdot\cos\alpha=4\cdot\cos^3\alpha-3\cdot\cos\alpha \;/\cdot\frac{1}{\cos\alpha} \;\Rightarrow\; 2\cdot\sin\alpha=4\cdot\cos^2\alpha-3 \;\Rightarrow$$

$$\Rightarrow\; 2\cdot\sin\alpha=4\cdot\left(1-\sin^2\alpha\right)-3 \;\Rightarrow\; 2\cdot\sin\alpha=4-4\cdot\sin^2\alpha-3 \;\Rightarrow\; 4\cdot\sin^2\alpha+2\cdot\sin\alpha-4+3=0 \;\Rightarrow$$

$$\Rightarrow\; 4\cdot\sin^2\alpha+2\cdot\sin\alpha-1=0 \;\Rightarrow\; \left.\begin{matrix}4\cdot\sin^2\alpha+2\cdot\sin\alpha-1=0\\ a=4\;,\;b=2\;,\;c=-1\end{matrix}\right\}\Rightarrow \left.\begin{matrix}a=4\;,\;b=2\;,\;c=-1\\ (\sin\alpha)_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\; (\sin\alpha)_{1,2}=\frac{-2\pm\sqrt{4-4\cdot 4\cdot(-1)}}{2\cdot 4} \;\Rightarrow\; (\sin\alpha)_{1,2}=\frac{-2\pm\sqrt{4+16}}{8} \;\Rightarrow\; (\sin\alpha)_{1,2}=\frac{-2\pm\sqrt{20}}{8} \;\Rightarrow$$

$$\Rightarrow\; \begin{bmatrix}\text{djelomično}\\ \text{korjenovanje}\end{bmatrix} \;\Rightarrow\; (\sin\alpha)_{1,2}=\frac{-2\pm\sqrt{4\cdot 5}}{8} \;\Rightarrow\; (\sin\alpha)_{1,2}=\frac{-2\pm 2\cdot\sqrt{5}}{8} \;\Rightarrow$$

$$\Rightarrow\; \left.\begin{matrix}(\sin\alpha)_1=\dfrac{-2+2\cdot\sqrt{5}}{8}\\ (\sin\alpha)_2=\dfrac{-2-2\cdot\sqrt{5}}{8}\end{matrix}\right\}\Rightarrow \left.\begin{matrix}(\sin\alpha)_1=\dfrac{2\cdot\left(-1+\sqrt{5}\right)}{8}\\ (\sin\alpha)_2=\dfrac{2\cdot\left(-1-\sqrt{5}\right)}{8}\end{matrix}\right\}\Rightarrow \left.\begin{matrix}(\sin\alpha)_1=\dfrac{2\cdot\left(-1+\sqrt{5}\right)}{8}\\ (\sin\alpha)_2=\dfrac{2\cdot\left(-1-\sqrt{5}\right)}{8}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow\; \left.\begin{matrix}(\sin\alpha)_1=\dfrac{-1+\sqrt{5}}{4}\\ (\sin\alpha)_2=\dfrac{-1-\sqrt{5}}{4}\end{matrix}\right\}\Rightarrow \left.\begin{matrix}(\sin\alpha)_1=\dfrac{\sqrt{5}-1}{4}\\ (\sin\alpha)_2=-\dfrac{\sqrt{5}+1}{4}\quad \text{nema smisla zbog (1)}\end{matrix}\right\}\Rightarrow \sin\alpha=\frac{\sqrt{5}-1}{4} \;\Rightarrow$$

$$\Rightarrow\; \sin 18^0=\frac{\sqrt{5}-1}{4}.$$

Vježba 170

Dokaži : $\cos 72^0=\dfrac{\sqrt{5}-1}{4}.$

Rezultat: Dokaz analogan.

Zadatak 171 (Helena, gimnazija)

$$\text{Izračunaj: } \sin\frac{\pi}{18}\cdot\sin\frac{3\cdot\pi}{18}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}\cdot\sin\frac{9\cdot\pi}{18}.$$

Rješenje 171

Ponovimo!

$$\cos\left(\frac{\pi}{2}-x\right)=\sin x \quad , \quad \sin 2x=2\cdot\cos x\cdot\sin x \quad , \quad \sin\frac{\pi}{6}=\frac{1}{2} \quad , \quad \sin\frac{\pi}{2}=1.$$

$$\sin\frac{\pi}{18}\cdot\sin\frac{3\cdot\pi}{18}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}\cdot\sin\frac{9\cdot\pi}{18}=\sin\frac{\pi}{18}\cdot\sin\frac{3\cdot\pi}{18}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}\cdot\sin\frac{9\cdot\pi}{18}=$$

$$=\sin\frac{\pi}{18}\cdot\sin\frac{\pi}{6}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}\cdot\sin\frac{\pi}{2}=\sin\frac{\pi}{18}\cdot\frac{1}{2}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}\cdot1=\frac{1}{2}\cdot\sin\frac{\pi}{18}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}=$$

$$=\left[\begin{matrix}\text{proširimo sa}\\ 2\cdot\cos\frac{\pi}{18}\end{matrix}\right]=\frac{1}{2}\cdot\frac{2\cdot\cos\frac{\pi}{18}\cdot\sin\frac{\pi}{18}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}}{2\cdot\cos\frac{\pi}{18}}=\frac{1}{2}\cdot\frac{\sin\frac{2\cdot\pi}{18}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}}{2\cdot\cos\frac{\pi}{18}}=$$

$$=\frac{1}{4}\cdot\frac{\sin\frac{2\cdot\pi}{18}\cdot\cos\left(\frac{\pi}{2}-\frac{5\cdot\pi}{18}\right)\cdot\cos\left(\frac{\pi}{2}-\frac{7\cdot\pi}{18}\right)}{\cos\frac{\pi}{18}}=\frac{1}{4}\cdot\frac{\sin\frac{2\cdot\pi}{18}\cdot\cos\frac{9\cdot\pi-5\cdot\pi}{18}\cdot\cos\frac{9\cdot\pi-7\cdot\pi}{18}}{\cos\frac{\pi}{18}}=$$

$$=\frac{1}{4}\cdot\frac{\sin\frac{2\cdot\pi}{18}\cdot\cos\frac{4\cdot\pi}{18}\cdot\cos\frac{2\cdot\pi}{18}}{\cos\frac{\pi}{18}}=\frac{1}{4}\cdot\frac{\cos\frac{2\cdot\pi}{18}\cdot\sin\frac{2\cdot\pi}{18}\cdot\cos\frac{4\cdot\pi}{18}}{\cos\frac{\pi}{18}}=\left[\begin{matrix}\text{proširimo}\\ \text{sa } 2\end{matrix}\right]=$$

$$=\frac{1}{4}\cdot\frac{2\cdot\cos\frac{2\cdot\pi}{18}\cdot\sin\frac{2\cdot\pi}{18}\cdot\cos\frac{4\cdot\pi}{18}}{2\cdot\cos\frac{\pi}{18}}=\frac{1}{4}\cdot\frac{\sin\frac{4\cdot\pi}{18}\cdot\cos\frac{4\cdot\pi}{18}}{2\cdot\cos\frac{\pi}{18}}=\frac{1}{8}\cdot\frac{\sin\frac{4\cdot\pi}{18}\cdot\cos\frac{4\cdot\pi}{18}}{\cos\frac{\pi}{18}}=\left[\begin{matrix}\text{proširimo}\\ \text{sa } 2\end{matrix}\right]=$$

$$=\frac{1}{8}\cdot\frac{2\cdot\sin\frac{4\cdot\pi}{18}\cdot\cos\frac{4\cdot\pi}{18}}{2\cdot\cos\frac{\pi}{18}}=\frac{1}{8}\cdot\frac{\sin\frac{8\cdot\pi}{18}}{2\cdot\cos\frac{\pi}{18}}=\frac{1}{16}\cdot\frac{\sin\frac{8\cdot\pi}{18}}{\cos\frac{\pi}{18}}=\frac{1}{16}\cdot\frac{\cos\left(\frac{\pi}{2}-\frac{8\cdot\pi}{18}\right)}{\cos\frac{\pi}{18}}=\frac{1}{16}\cdot\frac{\cos\frac{9\cdot\pi-8\cdot\pi}{18}}{\cos\frac{\pi}{18}}=$$

$$=\frac{1}{16}\cdot\frac{\cos\frac{\pi}{18}}{\cos\frac{\pi}{18}}=\frac{1}{16}\cdot\frac{\cos\frac{\pi}{18}}{\cos\frac{\pi}{18}}=\frac{1}{16}.$$

Vježba 171

$$\text{Izračunaj: } \frac{1}{\sin\frac{\pi}{18}\cdot\sin\frac{3\cdot\pi}{18}\cdot\sin\frac{5\cdot\pi}{18}\cdot\sin\frac{7\cdot\pi}{18}\cdot\sin\frac{9\cdot\pi}{18}}.$$

Rezultat: 16.

Zadatak 172 (Sara, srednja škola)

Koji je rezultat sređivanja izraza: $\frac{\sin(25\cdot\pi+x)}{\cos(32\cdot\pi+x)}+tg(17\cdot\pi-x)$ za $x\neq\frac{\pi}{2}+k\cdot\pi \; , \; k\in Z$?

$$A)\ 0 \qquad B)\ 1 \qquad C)\ -2\cdot tg\,x \qquad D)\ tg\,x+1$$

Rješenje 172

Ponovimo!

$$tg\,x=\frac{\sin x}{\cos x} \quad , \quad \sin(\pi+x)=-\sin x.$$

Parnost tangensa

Za svaki realni broj x vrijedi

$$tg(-x)=-tg\,x,$$

tj. tangens je neparna funkcija.

Periodičnost sinusa, kosinusa i tangensa

Sinus je periodična funkcija s temeljnim periodom 2 · π ili 360°.

$$\sin(\alpha+k\cdot 2\cdot\pi)=\sin\alpha,\ k\in Z \quad , \quad \sin\left(\alpha+k\cdot 360^0\right)=\sin\alpha,\ k\in Z.$$

Kosinus je periodična funkcija s temeljnim periodom 2 · π ili 360°.

$$\cos(\alpha+k\cdot 2\cdot\pi)=\cos\alpha,\ k\in Z \quad , \quad \cos\left(\alpha+k\cdot 360^0\right)=\cos\alpha,\ k\in Z.$$

Tangens je periodična funkcija s temeljnim periodom π ili 180°.

$$tg(\alpha+k\cdot\pi)=tg\,\alpha,\ k\in Z \quad , \quad tg\left(\alpha+k\cdot 180^0\right)=tg\,\alpha,\ k\in Z.$$

$$\frac{\sin(25\cdot\pi+x)}{\cos(32\cdot\pi+x)}+tg(17\cdot\pi-x)=\frac{\sin(\pi+x+12\cdot 2\cdot\pi)}{\cos(x+16\cdot 2\cdot\pi)}+tg(-x+17\cdot\pi)=\frac{\sin(\pi+x)}{\cos x}+tg(-x)=$$

$$=\frac{-\sin x}{\cos x}-tg\,x=-tg\,x-tg\,x=-2\cdot tg\,x.$$

Odgovor je pod C.

Vježba 172

Koji je rezultat sređivanja izraza: $\frac{\sin(23\cdot\pi+x)}{\cos(30\cdot\pi+x)}+tg(15\cdot\pi-x)$ za $x\neq\frac{\pi}{2}+k\cdot\pi \ ,\ k\in Z$?

$$A)\ 0 \qquad B)\ 1 \qquad C)\ -2\cdot tg\,x \qquad D)\ tg\,x+1$$

Rezultat: C.

Zadatak 173 (Biba, srednja škola)

Odredite opće rješenje trigonometrijske jednadžbe: $\cos^2 x=\sin^4 x+\cos^2 x\cdot\sin^2 x.$

Rješenje 173

Ponovimo!

$$\cos^2\alpha+\sin^2\alpha=1 \quad , \quad tg\,x=\frac{\sin x}{\cos x}.$$

Trigonometrijska jednadžba tg x = a

Skup rješenja jednadžbe $tg\,x=a,\ a\in R,$ je $\{x_0+k\cdot\pi : k\in Z\}$, gdje je $x_0\in R$ jedno rješenje te jednadžbe.

$$\cos^2 x=\sin^4 x+\cos^2 x\cdot\sin^2 x \Rightarrow \cos^2 x=\sin^2 x\cdot\left(\sin^2 x+\cos^2 x\right) \Rightarrow \cos^2 x=\sin^2 x\cdot 1 \Rightarrow$$

$$\Rightarrow \cos^2 x=\sin^2 x \Rightarrow \cos^2 x=\sin^2 x \ /\cdot\frac{1}{\cos^2 x} \Rightarrow 1=\frac{\sin^2 x}{\cos^2 x} \Rightarrow 1=tg^2 x \Rightarrow tg^2 x=1 \Rightarrow$$

$$\Rightarrow\ tg^2 x = 1\ /\sqrt{\ } \Rightarrow\ tg\,x = \pm\sqrt{1} \Rightarrow \left.\begin{array}{l} tg\,x = 1 \\ tg\,x = -1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = tg^{-1}1 \\ x_2 = tg^{-1}(-1) \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = \frac{\pi}{4} + k\cdot\pi,\ k\in Z \\ x_2 = -\frac{\pi}{4} + k\cdot\pi,\ k\in Z \end{array}\right\}.$$

Vježba 173

Odredite opće rješenje trigonometrijske jednadžbe: $\cos^2 x - \cos^2 x\cdot\sin^2 x = \sin^4 x.$

Rezultat: $x_1 = \frac{\pi}{4} + k\cdot\pi,\ k\in Z\ ,\ x_2 = -\frac{\pi}{4} + k\cdot\pi,\ k\in Z.$

Zadatak 174 (Ivan, tehnička škola)

Ako je $\alpha + \beta = \frac{3\cdot\pi}{4},\ 0 < \beta \le \frac{\pi}{2}$, i $\sin\beta = \frac{1}{3}$, koliko je $\cos\alpha$?

Rješenje 174

Ponovimo!

$$\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}\quad,\quad \left(\sqrt{x}\right)^2 = x\quad,\quad \cos(x-y) = \cos x\cdot\cos y + \sin x\cdot\sin y\quad,\quad \cos(\pi - x) = -\cos x.$$

$$\sin(\pi - x) = \sin x\quad,\quad \cos^2 x + \sin^2 x = 1\quad,\quad \cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}.$$

Najprije odredimo cos β.

$$\cos^2\beta + \sin^2\beta = 1 \Rightarrow \cos^2\beta = 1 - \sin^2\beta \Rightarrow \cos^2\beta = 1 - \left(\frac{1}{3}\right)^2 \Rightarrow \cos^2\beta = 1 - \frac{1}{9} \Rightarrow$$

$$\Rightarrow \cos^2\beta = \frac{1}{1} - \frac{1}{9} \Rightarrow \cos^2\beta = \frac{9-1}{9} \Rightarrow \cos^2\beta = \frac{8}{9} \Rightarrow \cos^2\beta = \frac{8}{9}\ /\sqrt{\ } \Rightarrow \cos\beta = \sqrt{\frac{8}{9}} \Rightarrow$$

$$\Rightarrow \cos\beta = \frac{\sqrt{8}}{\sqrt{9}} \Rightarrow \cos\beta = \frac{\sqrt{8}}{3} \Rightarrow \left[\begin{array}{l}\text{djelomično}\\ \text{korjenovanje}\end{array}\right] \Rightarrow \cos\beta = \frac{\sqrt{4\cdot 2}}{3} \Rightarrow \cos\beta = \frac{2\cdot\sqrt{2}}{3}.$$

Računamo cos α.

$$\alpha + \beta = \frac{3\cdot\pi}{4} \Rightarrow \alpha = \frac{3\cdot\pi}{4} - \beta.$$

$$\cos\alpha = \cos\left(\frac{3\cdot\pi}{4} - \beta\right) = \cos\frac{3\cdot\pi}{4}\cdot\cos\beta + \sin\frac{3\cdot\pi}{4}\cdot\sin\beta = \cos\left(\pi - \frac{\pi}{4}\right)\cdot\cos\beta + \sin\left(\pi - \frac{\pi}{4}\right)\cdot\sin\beta =$$

$$= -\cos\frac{\pi}{4}\cdot\cos\beta + \sin\frac{\pi}{4}\cdot\sin\beta = -\frac{\sqrt{2}}{2}\cdot\frac{2\cdot\sqrt{2}}{3} + \frac{\sqrt{2}}{2}\cdot\frac{1}{3} = -\frac{\sqrt{2}}{2}\cdot\frac{2\cdot\sqrt{2}}{3} + \frac{\sqrt{2}}{2}\cdot\frac{1}{3} =$$

$$= -\frac{\left(\sqrt{2}\right)^2}{3} + \frac{\sqrt{2}}{6} = -\frac{2}{3} + \frac{\sqrt{2}}{6} = \frac{-4 + \sqrt{2}}{6} = \frac{\sqrt{2} - 4}{6}.$$

Vježba 174

Ako je $\alpha + \beta = \frac{11\cdot\pi}{4},\ 0 < \beta \le \frac{\pi}{2}$, i $\sin\beta = \frac{1}{3}$, koliko je $\cos\alpha$?

Rezultat: $\frac{\sqrt{2} - 4}{6}.$

Zadatak 175 (Jasmin, gimnazija)

Eliminiraj x iz jednadžbi: $\cos x - \sin x = m \;\; , \;\; \sin 2x = n.$

Rješenje 175

Ponovimo!

$$(a-b)^2 = a^2 - 2\cdot a\cdot b + b^2 \quad , \quad \cos^2\alpha + \sin^2\alpha = 1 \quad , \quad \sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha.$$

Kvadriramo prvu jednakost.

$$\cos x - \sin x = m \Rightarrow \cos x - \sin x = m \;/^2 \Rightarrow (\cos x - \sin x)^2 = m^2 \Rightarrow$$

$$\Rightarrow \cos^2 x - 2\cdot\cos x\cdot\sin x + \sin^2 x = m^2 \Rightarrow 1 - 2\cdot\cos x\cdot\sin x = m^2 \Rightarrow 1 - \sin 2x = m^2 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} 1-\sin 2x = m^2 \\ \sin 2x = n \end{matrix}\right\} \Rightarrow 1 - n = m^2.$$

Vježba 175

Eliminiraj x iz jednadžbi: $\cos x + \sin x = m \;\; , \;\; \sin 2x = n.$

Rezultat: $1 + n = m^2.$

Zadatak 176 (Igor, gimnazija)

Dokazati da je $\cos^2 18^0 + \cos^2 36^0 + \cos^2 54^0 + \cos^2 72^0 = 2.$

Rješenje 176

Ponovimo!

$$\cos\left(90^0 - x\right) = \sin x \quad , \quad \cos^2 x + \sin^2 x = 1.$$

$$\cos^2 18^0 + \cos^2 36^0 + \cos^2 54^0 + \cos^2 72^0 = \begin{bmatrix} \cos^2 18^0 = \sin^2 72^0 \\ \cos^2 36^0 = \sin^2 54^0 \end{bmatrix} =$$

$$= \sin^2 72^0 + \sin^2 54^0 + \cos^2 54^0 + \cos^2 72^0 = \left(\sin^2 72^0 + \cos^2 72^0\right) + \left(\sin^2 54^0 + \cos^2 54^0\right) =$$

$$= 1 + 1 = 2.$$

Vježba 176

Dokazati da je $\sin^2 18^0 + \sin^2 36^0 + \sin^2 54^0 + \sin^2 72^0 = 2.$

Rezultat: Dokaz analogan.

Zadatak 177 (Antun, maturant)

Riješi jednadžbu : $tg\, x - \dfrac{8\cdot\sin^2 x + 3\cdot\sin 2x + 1}{8\cdot\cos^2 x + 3\cdot\sin 2x + 1} = 0.$

Rješenje 177

Ponovimo!

Trigonometrijska jednadžba tg x = a

Skup rješenja jednadžbe $tg\, x = a,\; a \in R$ je $\left\{ x_0 + k\cdot\pi \right\}$ gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$\sin 2\alpha = 2\cdot\sin\alpha\cdot\cos\alpha \quad , \quad \cos^2\alpha + \sin^2\alpha = 1 \quad , \quad tg\,\alpha = \frac{\sin\alpha}{\cos\alpha} \quad , \quad \sqrt[3]{a^3} = a.$$

$$\frac{a+b}{n} = \frac{a}{n} + \frac{b}{n} \quad , \quad (a-b)^3 = a^3 - 3\cdot a^2\cdot b + 3\cdot a\cdot b^2 - b^3.$$

$$tg\,x-\frac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}=0\;\Rightarrow\; tg\,x-\frac{8\cdot\sin^2 x+3\cdot 2\cdot\sin x\cdot\cos x+\cos^2 x+\sin^2 x}{8\cdot\cos^2 x+3\cdot 2\cdot\sin x\cdot\cos x+\cos^2 x+\sin^2 x}=0\;\Rightarrow$$

$$\Rightarrow\; tg\,x-\frac{9\cdot\sin^2 x+6\cdot\sin x\cdot\cos x+\cos^2 x}{9\cdot\cos^2 x+6\cdot\sin x\cdot\cos x+\sin^2 x}=0\;\Rightarrow\; tg\,x-\frac{\dfrac{9\cdot\sin^2 x+6\cdot\sin x\cdot\cos x+\cos^2 x}{\cos^2 x}}{\dfrac{9\cdot\cos^2 x+6\cdot\sin x\cdot\cos x+\sin^2 x}{\cos^2 x}}=0\;\Rightarrow$$

$$\Rightarrow\; tg\,x-\frac{\dfrac{9\cdot\sin^2 x}{\cos^2 x}+\dfrac{6\cdot\sin x\cdot\cos x}{\cos^2 x}+\dfrac{\cos^2 x}{\cos^2 x}}{\dfrac{9\cdot\cos^2 x}{\cos^2 x}+\dfrac{6\cdot\sin x\cdot\cos x}{\cos^2 x}+\dfrac{\sin^2 x}{\cos^2 x}}=0\;\Rightarrow\; tg\,x-\frac{9\cdot\dfrac{\sin^2 x}{\cos^2 x}+6\cdot\dfrac{\sin x\cdot\cos x}{\cos^2 x}+\dfrac{\cos^2 x}{\cos^2 x}}{9\cdot\dfrac{\cos^2 x}{\cos^2 x}+6\cdot\dfrac{\sin x\cdot\cos x}{\cos^2 x}+\dfrac{\sin^2 x}{\cos^2 x}}=0\;\Rightarrow$$

$$\Rightarrow\; tg\,x-\frac{9\cdot\dfrac{\sin^2 x}{\cos^2 x}+6\cdot\dfrac{\sin x\cdot\cos x}{\cos^2 x}+\dfrac{\cos^2 x}{\cos^2 x}}{9\cdot\dfrac{\cos^2 x}{\cos^2 x}+6\cdot\dfrac{\sin x\cdot\cos x}{\cos^2 x}+\dfrac{\sin^2 x}{\cos^2 x}}=0\;\Rightarrow\; tg\,x-\frac{9\cdot tg^2 x+6\cdot tg\,x+1}{9+6\cdot tg\,x+tg^2 x}=0\;\Rightarrow$$

$$\Rightarrow\; tg\,x-\frac{9\cdot tg^2 x+6\cdot tg\,x+1}{tg^2 x+6\cdot tg\,x+9}=0\;\Rightarrow\; tg\,x-\frac{9\cdot tg^2 x+6\cdot tg\,x+1}{tg^2 x+6\cdot tg\,x+9}=0\;/\cdot\left(tg^2 x+6\cdot tg\,x+9\right)\;\Rightarrow$$

$$\Rightarrow\; tg\,x\cdot\left(tg^2 x+6\cdot tg\,x+9\right)-\left(9\cdot tg^2 x+6\cdot tg\,x+1\right)=0\;\Rightarrow$$

$$\Rightarrow\; tg^3 x+6\cdot tg^2 x+9\cdot tg\,x-9\cdot tg^2 x-6\cdot tg\,x-1=0\;\Rightarrow$$

$$\Rightarrow\; tg^3 x-3\cdot tg^2 x+3\cdot tg\,x-1=0\;\Rightarrow\;(tg\,x-1)^3=0\;\Rightarrow\;(tg\,x-1)^3=0\;/\sqrt[3]{\;}\;\Rightarrow$$

$$\Rightarrow\;\sqrt[3]{(tg\,x-1)^3}=\sqrt[3]{0}\;\Rightarrow\; tg\,x-1=0\;\Rightarrow\; tg\,x=1\;\Rightarrow\; x=tg^{-1}1\;\Rightarrow\; x=\frac{\pi}{4}+k\cdot\pi\;,\;k\in Z.$$

Vježba 177

Riješi jednadžbu : $\dfrac{\sin x}{\cos x}=\dfrac{8\cdot\sin^2 x+3\cdot\sin 2x+1}{8\cdot\cos^2 x+3\cdot\sin 2x+1}$.

Rezultat: $x=\dfrac{\pi}{4}+k\cdot\pi\;,\;k\in Z.$

Zadatak 178 (Vlado, gimnazija)

Izračunaj: $\dfrac{\sin 1}{\cos 0\cdot\cos 1}+\dfrac{\sin 1}{\cos 1\cdot\cos 2}+\dfrac{\sin 1}{\cos 2\cdot\cos 3}+\dfrac{\sin 1}{\cos 3\cdot\cos 4}+\;...\;+\dfrac{\sin 1}{\cos(n-1)\cdot\cos n}$.

Rješenje 178

Ponovimo!

$\sin(x-y)=\sin x\cdot\cos y-\cos x\cdot\sin y\quad,\quad\dfrac{a-b}{n}=\dfrac{a}{n}-\dfrac{b}{n}\quad,\quad tg\,x=\dfrac{\sin x}{\cos x}\quad,\quad tg\,0=0.$

Uočimo da se svaki pribrojnik zadanog zbroja može transformirati u razliku dva tangensa.

$$\frac{\sin 1}{\cos(k-1)\cdot\cos k}=\frac{\sin(k-k+1)}{\cos(k-1)\cdot\cos k}=\frac{\sin(k-(k-1))}{\cos(k-1)\cdot\cos k}=\frac{\sin k\cdot\cos(k-1)-\cos k\cdot\sin(k-1)}{\cos(k-1)\cdot\cos k}=$$

$$=\frac{\sin k\cdot\cos(k-1)}{\cos(k-1)\cdot\cos k}-\frac{\cos k\cdot\sin(k-1)}{\cos(k-1)\cdot\cos k}=\frac{\sin k\cdot\cos(k-1)}{\cos(k-1)\cdot\cos k}-\frac{\cos k\cdot\sin(k-1)}{\cos(k-1)\cdot\cos k}=$$

$$=\frac{\sin k}{\cos k}-\frac{\sin(k-1)}{\cos(k-1)}=tg\,k-tg\,(k-1).$$

Zadani zbroj transformiramo na sljedeći način:

$$\frac{\sin 1}{\cos 0\cdot\cos 1}+\frac{\sin 1}{\cos 1\cdot\cos 2}+\frac{\sin 1}{\cos 2\cdot\cos 3}+\frac{\sin 1}{\cos 3\cdot\cos 4}+\ \dots\ +\frac{\sin 1}{\cos(n-1)\cdot\cos n}=$$

$$=(tg\,1-tg\,0)+(tg\,2-tg\,1)+(tg\,3-tg\,2)+(tg\,4-tg\,3)+\ \dots\ +(tg\,n-tg\,(n-1))=$$

$$=tg\,1-tg\,0+tg\,2-tg\,1+tg\,3-tg\,2+tg\,4-tg\,3+\ \dots\ +tg\,n-tg\,(n-1)=$$

$$=tg\,1-tg\,0+tg\,2-tg\,1+tg\,3-tg\,2+tg\,4-tg\,3+\ \dots\ +tg\,n-tg\,(n-1)=-tg\,0+tg\,n=$$

$$=0+tg\,n=tg\,n.$$

Vježba 178

Izračunaj: $\frac{\sin 1}{\cos 0\cdot\cos 1}+\frac{\sin 1}{\cos 1\cdot\cos 2}+\frac{\sin 1}{\cos 2\cdot\cos 3}+\frac{\sin 1}{\cos 3\cdot\cos 4}+\ \dots\ +\frac{\sin 1}{\cos 9\cdot\cos 10}.$

Rezultat: $tg\,10.$

Zadatak 179 (Goga, gimnazija)

Izračunajte bez uporabe tablica ili računala: $\sin 75^0\cdot\cos 75^0.$

Rješenje 179

Ponovimo!

$$\frac{a}{b}=\frac{a\cdot n}{b\cdot n},\quad n\neq 0.$$

$$\sin 2x=2\cdot\sin x\cdot\cos x\quad,\quad\sin\left(180^0-x\right)=\sin x\quad,\quad\sin 30^0=\frac{1}{2}\quad,\quad\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

$$\sin 75^0\cdot\cos 75^0=\frac{2\cdot\sin 75^0\cdot\cos 75^0}{2}=\frac{2\cdot\sin 75^0\cdot\cos 75^0}{2}=\frac{\sin 150^0}{2}=\frac{\sin\left(180^0-30^0\right)}{2}=$$

$$=\frac{\sin 30^0}{2}=\frac{\frac{1}{2}}{2}=\frac{\frac{1}{2}}{\frac{2}{1}}=\frac{1}{4}=0.25.$$

Vježba 179

Izračunajte bez uporabe tablica ili računala: $\sin 15^0\cdot\cos 15^0.$

Rezultat: 0.25.

Zadatak 180 (Goga, gimnazija)

Ako je $tg\,x+ctg\,x=m,\ m\neq 0,$ koliko je $tg^3\,x+ctg^3\,x$?

Rješenje 180

Ponovimo!

$$(a+b)^3 = a^3 + 3\cdot a^2\cdot b + 3\cdot a\cdot b^2 + b^3 \quad , \quad a^3 + b^3 = (a+b)^3 - 3\cdot a\cdot b\cdot(a+b).$$

$$tg\, x\cdot ctg\, x = 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

1.inačica

Kubiramo zadanu jednakost.

$$tg\, x + ctg\, x = m \Rightarrow tg\, x + ctg\, x = m \;/^3 \Rightarrow (tg\, x + ctg\, x)^3 = m^3 \Rightarrow$$

$$\Rightarrow tg^3 x + 3\cdot tg^2 x\cdot ctg\, x + 3\cdot tg\, x\cdot ctg^2 x + ctg^3 x = m^3 \Rightarrow$$

$$\Rightarrow tg^3 x + 3\cdot tg\, x\cdot ctg\, x\cdot(tg\, x + ctg\, x) + ctg^3 x = m^3 \Rightarrow tg^3 x + 3\cdot 1\cdot(tg\, x + ctg\, x) + ctg^3 x = m^3 \Rightarrow$$

$$\Rightarrow tg^3 x + 3\cdot(tg\, x + ctg\, x) + ctg^3 x = m^3 \Rightarrow tg^3 x + 3\cdot\underbrace{(tg\, x + ctg\, x)}_{=m} + ctg^3 x = m^3 \Rightarrow$$

$$\Rightarrow tg^3 x + 3\cdot m + ctg^3 x = m^3 \Rightarrow tg^3 x + ctg^3 x = m^3 - 3\cdot m.$$

2.inačica

Uporabit ćemo identitet:

$$a^3 + b^3 = (a+b)^3 - 3\cdot a\cdot b\cdot(a+b).$$

$$tg^3 x + ctg^3 x = (tg\, x + ctg\, x)^3 - 3\cdot tg\, x\cdot ctg\, x\cdot(tg\, x + ctg\, x) = (tg\, x + ctg\, x)^3 - 3\cdot 1\cdot(tg\, x + ctg\, x) =$$

$$= (tg\, x + ctg\, x)^3 - 3\cdot(tg\, x + ctg\, x) = \underbrace{(tg\, x + ctg\, x)}_{=m}^3 - 3\cdot\underbrace{(tg\, x + ctg\, x)}_{=m} = m^3 - 3\cdot m.$$

Vježba 180

Ako je $tg\, x + ctg\, x = 1$, $m \neq 0$, koliko je $tg^3 x + ctg^3 x$?

Rezultat: – 2.