Zadatak 501 (Katarina, maturantica)

Riješi jednadžbu $\frac{10^{2\cdot x}\cdot 100^{x}}{1000}=0.01.$

Rješenje 501

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m}\quad,\quad a^{n}\cdot a^{m}=a^{n+m}\quad,\quad a^{1}=a\quad,\quad a^{f(x)}=a^{g(x)}\Rightarrow f(x)=g(x).$$

$$100=10^{2}\quad,\quad 1000=10^{3}\quad,\quad 0.1=10^{-1}\quad,\quad 0.01=10^{-2}\quad,\quad a^{-n}=\frac{1}{a^{n}}.$$

$$\frac{a^{n}}{a^{m}}=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Decimalni broj množimo dekadskom jedinicom (10, 100, 1000, 10000, …) tako da mu decimalnu točku pomaknemo udesno za onoliko mjesta koliko dekadska jedinica ima nula.

1.inačica

$$\frac{10^{2\cdot x}\cdot 100^{x}}{1000}=0.01\Rightarrow\frac{10^{2\cdot x}\cdot 100^{x}}{1000}=0.01\ /\cdot 1000\Rightarrow 10^{2\cdot x}\cdot 100^{x}=10\Rightarrow$$

$$\Rightarrow 10^{2\cdot x}\cdot\left(10^{2}\right)^{x}=10\Rightarrow 10^{2\cdot x}\cdot 10^{2\cdot x}=10\Rightarrow 10^{2\cdot x+2\cdot x}=10\Rightarrow$$

$$\Rightarrow 10^{4\cdot x}=10^{1}\Rightarrow 4\cdot x=1\Rightarrow 4\cdot x=1\ /:4\Rightarrow x=\frac{1}{4}.$$

2.inačica

$$\frac{10^{2\cdot x}\cdot 100^{x}}{1000}=0.01\Rightarrow\frac{10^{2\cdot x}\cdot\left(10^{2}\right)^{x}}{10^{3}}=10^{-2}\Rightarrow\frac{10^{2\cdot x}\cdot 10^{2\cdot x}}{10^{3}}=10^{-2}\Rightarrow$$

$$\Rightarrow 10^{2\cdot x}\cdot 10^{2\cdot x}\cdot 10^{-3}=10^{-2}\Rightarrow 10^{2\cdot x+2\cdot x-3}=10^{-2}\Rightarrow 10^{4\cdot x-3}=10^{-2}\Rightarrow$$

$$\Rightarrow 4\cdot x-3=-2\Rightarrow 4\cdot x=-2+3\Rightarrow 4\cdot x=1\Rightarrow 4\cdot x=1\ /:4\Rightarrow x=\frac{1}{4}.$$

3.inačica

$$\frac{10^{2\cdot x}\cdot 100^{x}}{1000}=0.01\Rightarrow\frac{10^{2\cdot x}\cdot 100^{x}}{1000}=0.01\ /\cdot 10\Rightarrow\frac{10^{2\cdot x}\cdot 100^{x}}{100}=0.1\Rightarrow$$

$$\Rightarrow\frac{10^{2\cdot x}\cdot 100^{x}}{100^{1}}=0.1\Rightarrow 10^{2\cdot x}\cdot 100^{x-1}=10^{-1}\Rightarrow 10^{2\cdot x}\cdot\left(10^{2}\right)^{x-1}=10^{-1}\Rightarrow$$

$$\Rightarrow 10^{2\cdot x}\cdot 10^{2\cdot x-2}=10^{-1}\Rightarrow 10^{2\cdot x+2\cdot x-2}=10^{-1}\Rightarrow 10^{4\cdot x-2}=10^{-1}\Rightarrow$$

$$\Rightarrow 4\cdot x-2=-1\Rightarrow 4\cdot x=-1+2\Rightarrow 4\cdot x=1\Rightarrow 4\cdot x=1\ /:4\Rightarrow x=\frac{1}{4}.$$

Vježba 501

Riješi jednadžbu $\frac{10^{2\cdot x}\cdot 100^{x}}{100}=0.1.$

Rezultat: $x = \frac{1}{4}.$

Zadatak 502 (Milena, maturantica)

Ako je $\log_3 x \cdot \log_x z \cdot \log_z y = \log_x x^2$, onda je y jednak:

$A.\ 81 \qquad B.\ 27 \qquad C.\ 18 \qquad D.\ 9 \qquad E.\ 4.5$

Rješenje 502

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \quad \rightarrow \qquad a = b^c$$

$$\log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log_b a^n = n \cdot \log_b a \quad , \quad \log_v b = 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

$$\log_3 x \cdot \log_x z \cdot \log_z y = \log_x x^2 \Rightarrow \log_3 x \cdot \frac{\log_3 z}{\log_3 x} \cdot \frac{\log_3 y}{\log_3 z} = 2 \cdot \log_x x \Rightarrow$$

$$\Rightarrow \log_3 x \cdot \frac{\log_3 z}{\log_3 x} \cdot \frac{\log_3 y}{\log_3 z} = 2 \cdot 1 \Rightarrow \log_3 y = 2 \Rightarrow y = 3^2 \Rightarrow y = 9.$$

Odgovor je pod D.

Vježba 502

Ako je $\log_3 x \cdot \log_x z \cdot \log_z y = \log_x x^3$, onda je y jednak:

$A.\ 81 \qquad B.\ 27 \qquad C.\ 18 \qquad D.\ 9 \qquad E.\ 4.5$

Rezultat: B.

Zadatak 503 (Vinko, gimnazija)

Koliko rješenja ima jednadžba $\log(\cos x) = \log(\sin x)$ unutar intervala $\left[0, 2 \cdot \pi\right\rangle$?

$A.\ 0 \qquad B.\ 3 \qquad C.\ 1 \qquad D.\ 4 \qquad E.\ 2$

Rješenje 503

Ponovimo!

$$tg\ x = \frac{\sin x}{\cos x}.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \quad \rightarrow \qquad a = b^c$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

Logaritamska funkcija definirana je za pozitivne realne brojeve.

$$f(x) = \log x \quad \Rightarrow \quad x > 0.$$

Trigonometrijska jednadžba tg x = tg a, $a \in R$
Skup rješenja jednadžbe

$$tg\ x = tg\ a \quad , \quad a \in R$$

je

$$\{ a + k \cdot \pi \quad , \quad k \in Z \}.$$

Rasprava!
Funkcija kosinus pozitivna je u prvom i četvrtom kvadrantu.

$$\cos x > 0 \quad \Rightarrow \quad x \in \left\langle 0, \frac{\pi}{2} \right\rangle \text{ ili } x \in \left\langle \frac{3 \cdot \pi}{2}, 2 \cdot \pi \right\rangle.$$

Funkcija sinus pozitivna je u prvom i drugom kvadrantu.

$$\sin x > 0 \quad \Rightarrow \quad x \in \left\langle 0, \frac{\pi}{2} \right\rangle \text{ ili } x \in \left\langle \frac{\pi}{2}, \pi \right\rangle.$$

$$\left. \begin{array}{l} \cos x > 0 \\ \sin x > 0 \\ x \in [0, 2 \cdot \pi \rangle \end{array} \right\} \quad \Rightarrow \quad x \in \left\langle 0, \frac{\pi}{2} \right\rangle.$$

Riješimo jednadžbu!

$$\log(\cos x) = \log(\sin x) \quad \Rightarrow \quad \cos x = \sin x \quad \Rightarrow \quad \sin x = \cos x \quad \Rightarrow \quad \sin x = \cos x \ / \cdot \frac{1}{\cos x} \quad \Rightarrow$$

$$\Rightarrow \quad \frac{\sin x}{\cos x} = 1 \quad \Rightarrow \quad tg\ x = 1 \quad \Rightarrow \quad x = tg^{-1} 1 \quad \Rightarrow \quad x = \frac{\pi}{4} \in \left\langle 0, \frac{\pi}{2} \right\rangle.$$

Jednadžba ima jedno rješenje.

Odgovor je pod C.

Vježba 503

Koliko rješenja ima jednadžba $\log(\cos x) = \log(\sin x)$ unutar intervala $[0, \pi\rangle$?

A. 0   *B.* 3   *C.* 1   *D.* 4   *E.* 2

Rezultat: C.

Zadatak 504 (Dona, maturantica)

Čemu je jednak do kraja pojednostavljen izraz $\frac{\cos^2 x}{1 - \sin x} - 1$ za sve x za koje je definiran?

Rješenje 504

Ponovimo!

$$\cos^2 x + \sin^2 x = 1.$$

$$n = \frac{n}{1} \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d} \quad , \quad a^1 = a \quad , \quad a^n : a^m = a^{n-m}.$$

$$a^2 - b^2 = (a - b) \cdot (a + b).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

$$\frac{\cos^2 x}{1-\sin x} - 1 = \frac{\cos^2 x}{1-\sin x} - \frac{1}{1} = \frac{\cos^2 x - (1-\sin x)}{1-\sin x} = \frac{\cos^2 x - 1 + \sin x}{1-\sin x} = \frac{1-\sin^2 x - 1 + \sin x}{1-\sin x} =$$

$$= \frac{1-\sin^2 x - 1 + \sin x}{1-\sin x} = \frac{-\sin^2 x + \sin x}{1-\sin x} = \frac{\sin x - \sin^2 x}{1-\sin x} = \frac{\sin x \cdot (1-\sin x)}{1-\sin x} =$$

$$= \frac{\sin x \cdot (1-\sin x)}{1-\sin x} = \sin x.$$

2.inačica

$$\frac{\cos^2 x}{1-\sin x} - 1 = \frac{1-\sin^2 x}{1-\sin x} - 1 = \frac{(1-\sin x)\cdot(1+\sin x)}{1-\sin x} - 1 = \frac{(1-\sin x)\cdot(1+\sin x)}{1-\sin x} - 1 =$$

$$= 1 + \sin x - 1 = 1 + \sin x - 1 = \sin x.$$

Vježba 504

Čemu je jednak do kraja pojednostavljen izraz $\frac{\sin^2 x}{1-\cos x} - 1$ za sve x za koje je definiran?

Rezultat: $\cos x.$

Zadatak 505 (Valentina, ekonomska škola)

Koja od navedenih nejednadžba ima isti skup rješenja kao i nejednadžba $\left(\frac{4}{7}\right)^{5 \cdot x} > \frac{49}{16}$?

A. $5 \cdot x < -2$ *B.* $5 \cdot x < 2$ *C.* $5 \cdot x > -2$ *D.* $5 \cdot x > 2$

Rješenje 505

Ponovimo!

$$a^1 = a \quad , \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad , \quad \left(a^n\right)^m = a^{n \cdot m}.$$

$$a^{f(x)} > a^{g(x)} \ , \ 0 < a < 1 \ \Rightarrow \ f(x) < g(x).$$

$$\left(\frac{4}{7}\right)^{5 \cdot x} > \frac{49}{16} \Rightarrow \left(\frac{4}{7}\right)^{5 \cdot x} > \left(\frac{7}{4}\right)^2 \Rightarrow \left(\frac{4}{7}\right)^{5 \cdot x} > \left(\left(\frac{4}{7}\right)^{-1}\right)^2 \Rightarrow \left(\frac{4}{7}\right)^{5 \cdot x} > \left(\frac{4}{7}\right)^{-2} \Rightarrow$$

$$\Rightarrow \left[0 < \frac{4}{7} < 1\right] \Rightarrow 5 \cdot x < -2.$$

Odgovor je pod A.

Vježba 505

Koja od navedenih nejednadžba ima isti skup rješenja kao i nejednadžba $\left(\frac{3}{5}\right)^{5 \cdot x} > \frac{25}{9}$?

A. $5 \cdot x < -2$ *B.* $5 \cdot x < 2$ *C.* $5 \cdot x > -2$ *D.* $5 \cdot x > 2$

Rezultat: A.

Zadatak 506 (Branimir, ekonomska škola)

Ako je log x + log y = 3, 2 · log x – log y = 0, onda je zbroj x + y jednak:

$A.\ 10 \qquad B.\ 100 \qquad C.\ 110 \qquad D.\ 1000$

Rješenje 506

Ponovimo!

$$a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow a + c = b + d.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \underset{\rightarrow}{\log_b a = b^c} \qquad a = b^c$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log 1 = 0 \quad , \quad \log 10 = 1 \quad , \quad \log 100 = 2 \quad , \quad \log 1000 = 3 \quad , \quad \log a^n = n \cdot \log a.$$

$$\log(a \cdot b) = \log a + \log b \quad , \quad \log \frac{a}{b} = \log a - \log b.$$

1.inačica

$$\left.\begin{array}{l} \log x + \log y = 3 \\ 2 \cdot \log x - \log y = 0 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{zbrojimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow 3 \cdot \log x = 3 \Rightarrow 3 \cdot \log x = 3 \; /:3 \Rightarrow \log x = 1 \Rightarrow x = 10.$$

Računamo y.

$$\left.\begin{array}{l} x = 10 \\ \log x + \log y = 3 \end{array}\right\} \Rightarrow \log 10 + \log y = 3 \Rightarrow 1 + \log y = 3 \Rightarrow \log y = 3 - 1 \Rightarrow \log y = 2 \Rightarrow y = 100.$$

Tada je:

$$x + y = 10 + 100 = 110.$$

Odgovor je pod C.

2.inačica

$$\left.\begin{array}{l} \log x + \log y = 3 \\ 2 \cdot \log x - \log y = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log x + \log y = 3 \\ 2 \cdot \log x = \log y \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow \log x + 2 \cdot \log x = 3 \Rightarrow 3 \cdot \log x = 3 \Rightarrow$$

$$\Rightarrow 3 \cdot \log x = 3 \; /:3 \Rightarrow \log x = 1 \Rightarrow x = 10.$$

Računamo y.

$$\left.\begin{array}{l} x = 10 \\ \log x + \log y = 3 \end{array}\right\} \Rightarrow \log 10 + \log y = 3 \Rightarrow 1 + \log y = 3 \Rightarrow \log y = 3 - 1 \Rightarrow \log y = 2 \Rightarrow y = 100.$$

Tada je:

$$x + y = 10 + 100 = 110.$$

Odgovor je pod C.

3.inačica

$$\left.\begin{array}{l} \log x + \log y = 3 \\ 2 \cdot \log x - \log y = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log(x \cdot y) = 3 \\ \log x^2 - \log y = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x \cdot y = 1000 \\ \log \frac{x^2}{y} = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x \cdot y = 1000 \\ \frac{x^2}{y} = 1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x\cdot y=1000 \\ \frac{x^2}{y}=1 \;/\cdot y \end{array}\right\} \Rightarrow \left.\begin{array}{l} x\cdot y=1000 \\ x^2=y \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow x\cdot x^2=1000 \Rightarrow x^1\cdot x^2=1000 \Rightarrow$$

$$\Rightarrow x^3=1000 \Rightarrow x^3=1000 \;/\sqrt[3]{} \Rightarrow x=\sqrt[3]{1000} \Rightarrow x=10.$$

Računamo y.

$$\left.\begin{array}{l} x=10 \\ x^2=y \end{array}\right\} \Rightarrow 10^2=y \Rightarrow 100=y \Rightarrow y=100.$$

Tada je:

$$x+y=10+100=110.$$

Odgovor je pod C.

Vježba 506

Ako je log x + log y = 3, 2 · log x – log y = 0, onda je razlika y – x jednaka:

A. 90 *B.* 900 *C.* 20 *D.* 0

Rezultat: A.

Zadatak 507 (Dominik, srednja škola)

Riješi jednadžbu: $2^x+2^{x+1}+2^{x+2}+2^{x+3}=30.$

Rješenje 507

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

$$\frac{a}{b}\cdot\frac{b}{a}=1 \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

1.inačica

$$2^x+2^{x+1}+2^{x+2}+2^{x+3}=30 \Rightarrow 2^x+2^x\cdot 2^1+2^x\cdot 2^2+2^x\cdot 2^3=30 \Rightarrow$$

$$\Rightarrow 2^x\cdot\left(1+2^1+2^2+2^3\right)=30 \Rightarrow 2^x\cdot(1+2+4+8)=30 \Rightarrow 2^x\cdot 15=30 \Rightarrow$$

$$\Rightarrow 2^x\cdot 15=30 \;/:15 \Rightarrow 2^x=2 \Rightarrow 2^x=2^1 \Rightarrow x=1.$$

2.inačica

$$2^x+2^{x+1}+2^{x+2}+2^{x+3}=30 \Rightarrow 2^{x+3-3}+2^{x+3-2}+2^{x+3-1}+2^{x+3}=30 \Rightarrow$$

$$\Rightarrow 2^{x+3}\cdot 2^{-3}+2^{x+3}\cdot 2^{-2}+2^{x+3}\cdot 2^{-1}+2^{x+3}=30 \Rightarrow$$

$$\Rightarrow 2^{x+3}\cdot\left(2^{-3}+2^{-2}+2^{-1}+1\right)=30 \Rightarrow 2^{x+3}\cdot\left(\frac{1}{2^3}+\frac{1}{2^2}+\frac{1}{2^1}+1\right)=30 \Rightarrow$$

$$\Rightarrow\ 2^{x+3}\cdot\left(\frac{1}{8}+\frac{1}{4}+\frac{1}{2}+\frac{1}{1}\right)=30\ \Rightarrow\ 2^{x+3}\cdot\frac{1+2+4+8}{8}=30\ \Rightarrow\ 2^{x+3}\cdot\frac{15}{8}=30\ \Rightarrow$$

$$\Rightarrow\ 2^{x+3}\cdot\frac{15}{8}=30\ /\cdot\frac{8}{15}\ \Rightarrow\ 2^{x+3}=30\cdot\frac{8}{15}\ \Rightarrow\ 2^{x+3}=30\cdot\frac{8}{15}\ \Rightarrow\ 2^{x+3}=2\cdot 8\ \Rightarrow$$

$$\Rightarrow\ 2^{x+3}=16\ \Rightarrow\ 2^{x+3}=2^{4}\ \Rightarrow\ x+3=4\ \Rightarrow\ x=4-3\ \Rightarrow\ x=1.$$

3.inačica

$$2^{x}+2^{x+1}+2^{x+2}+2^{x+3}=30\ \Rightarrow\ 2^{x}+2^{x}\cdot 2^{1}+2^{x}\cdot 2^{2}+2^{x}\cdot 2^{3}=30\ \Rightarrow$$

$$\Rightarrow\ \left[\begin{array}{c}\text{zamjena}\\ 2^{x}=t\end{array}\right]\ \Rightarrow\ t+2\cdot t+4\cdot t+8\cdot t=30\ \Rightarrow\ 15\cdot t=30\ \Rightarrow\ 15\cdot t=30\ /:15\ \Rightarrow\ t=2.$$

Vraćamo se zamjeni.

$$\left.\begin{array}{l} t=2\\ 2^{x}=t\end{array}\right\}\ \Rightarrow\ 2^{x}=2\ \Rightarrow\ 2^{x}=2^{1}\ \Rightarrow\ x=1.$$

Vježba 507

Riješi jednadžbu: $2^{x}+2^{x+1}+2^{x+2}+2^{x+3}=60.$

Rezultat: $x=2.$

Zadatak 508 (Zlatko, gimnazija)

Riješi jednadžbu: $\log_{16}x+\log_{4}x+\log_{2}x=7.$

Rješenje 508

Ponovimo!

$$a^{1}=a\ ,\qquad a\cdot\frac{b}{c}=\frac{a\cdot b}{c}.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_{b}a=c\qquad \log_{b}a=b^{c}\ \rightarrow\qquad a=b^{c}$$

$$\log_{b^{n}}a=\frac{1}{n}\cdot\log_{b}a.$$

1.inačica

$$\log_{16}x+\log_{4}x+\log_{2}x=7\ \Rightarrow\ \log_{2^{4}}x+\log_{2^{2}}x+\log_{2}x=7\ \Rightarrow$$

$$\Rightarrow\ \frac{1}{4}\cdot\log_{2}x+\frac{1}{2}\cdot\log_{2}x+\log_{2}x=7\ \Rightarrow\ \frac{1}{4}\cdot\log_{2}x+\frac{1}{2}\cdot\log_{2}x+\log_{2}x=7\ /\cdot 4\ \Rightarrow$$

$$\Rightarrow\ \log_{2}x+2\cdot\log_{2}x+4\cdot\log_{2}x=28\ \Rightarrow\ 7\cdot\log_{2}x=28\ \Rightarrow\ 7\cdot\log_{2}x=28\ /:7\ \Rightarrow$$

$$\Rightarrow\ \log_{2}x=4\ \Rightarrow\ x=2^{4}\ \Rightarrow\ x=16.$$

2.inačica

$$\log_{16}x+\log_{4}x+\log_{2}x=7\ \Rightarrow\ \log_{16}x+\frac{2}{2}\cdot\log_{4}x+\frac{4}{4}\cdot\log_{2}x=7\ \Rightarrow$$

$$\Rightarrow \log_{16} x + 2 \cdot \frac{1}{2} \cdot \log_4 x + 4 \cdot \frac{1}{4} \cdot \log_2 x = 7 \Rightarrow \log_{16} x + 2 \cdot \log_{4^2} x + 4 \cdot \log_{2^4} x = 7 \Rightarrow$$

$$\Rightarrow \log_{16} x + 2 \cdot \log_{16} x + 4 \cdot \log_{16} x = 7 \Rightarrow 7 \cdot \log_{16} x = 7 \Rightarrow 7 \cdot \log_{16} x = 7 \ /:7 \Rightarrow$$

$$\Rightarrow \log_{16} x = 1 \Rightarrow x = 16^1 \Rightarrow x = 16.$$

Vježba 508

Riješi jednadžbu: $\log_{81} x + \log_9 x + \log_3 x = 7.$

Rezultat: $x = 81.$

Zadatak 509 (Nina, gimnazija)

Riješite nejednadžbu: $6^x - 16 \cdot 3^x < 0.$

Rješenje 509

Ponovimo!

$$a^n \cdot b^n = (a \cdot b)^n \quad , \quad a \cdot b < 0 \Rightarrow \left.\begin{matrix} a < 0 \\ b > 0 \end{matrix}\right\} \text{ ili } \left.\begin{matrix} a > 0 \\ b < 0 \end{matrix}\right\} \quad , \quad a^{f(x)} < a^{g(x)} \ , \ a > 1 \Rightarrow f(x) < g(x).$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

$$6^x - 16 \cdot 3^x < 0 \Rightarrow (2 \cdot 3)^x - 16 \cdot 3^x < 0 \Rightarrow 2^x \cdot 3^x - 16 \cdot 3^x < 0 \Rightarrow 3^x \cdot (2^x - 16) < 0 \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{broj } 3^x \text{ je pozitivan} \\ 3^x > 0 \end{bmatrix} \Rightarrow 2^x - 16 < 0 \Rightarrow 2^x < 16 \Rightarrow 2^x < 2^4 \Rightarrow x < 4 \Rightarrow x \in \langle -\infty, 4 \rangle.$$

Vježba 509

Riješite nejednadžbu: $6^x < 16 \cdot 3^x.$

Rezultat: $x \in \langle -\infty, 4 \rangle.$