Zadatak 421 (Antonio, gimnazija)

Riješi jednadžbu: $2\cdot 9^{\frac{1}{x}}+3\cdot 4^{\frac{1}{x}}=5\cdot 6^{\frac{1}{x}}.$

Rješenje 421

Ponovimo!

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad \left(a^n\right)^m=\left(a^m\right)^n=a^{n\cdot m} \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^1=a.$$

$$a^n\cdot a^m=a^{n+m} \quad , \quad a^0=1 \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$\frac{a}{b}=1 \Rightarrow a=b.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \quad , \quad n\neq 0 \quad , \quad n\neq 1.$$

$$2\cdot 9^{\frac{1}{x}}+3\cdot 4^{\frac{1}{x}}=5\cdot 6^{\frac{1}{x}} \Rightarrow 2\cdot 9^{\frac{1}{x}}+3\cdot 4^{\frac{1}{x}}=5\cdot 6^{\frac{1}{x}} \;/:6^{\frac{1}{x}} \Rightarrow 2\cdot\frac{9^{\frac{1}{x}}}{6^{\frac{1}{x}}}+3\cdot\frac{4^{\frac{1}{x}}}{6^{\frac{1}{x}}}=5 \Rightarrow$$

$$\Rightarrow 2\cdot\left(\frac{9}{6}\right)^{\frac{1}{x}}+3\cdot\left(\frac{4}{6}\right)^{\frac{1}{x}}=5 \Rightarrow 2\cdot\left(\frac{9}{6}\right)^{\frac{1}{x}}+3\cdot\left(\frac{4}{6}\right)^{\frac{1}{x}}=5 \Rightarrow 2\cdot\left(\frac{3}{2}\right)^{\frac{1}{x}}+3\cdot\left(\frac{2}{3}\right)^{\frac{1}{x}}=5 \Rightarrow$$

$$\Rightarrow 2\cdot\left(\frac{3}{2}\right)^{\frac{1}{x}}+3\cdot\left(\left(\frac{3}{2}\right)^{-1}\right)^{\frac{1}{x}}=5 \Rightarrow 2\cdot\left(\frac{3}{2}\right)^{\frac{1}{x}}+3\cdot\left(\left(\frac{3}{2}\right)^{\frac{1}{x}}\right)^{-1}=5 \Rightarrow \left[\begin{array}{c}\text{zamjena}\\ \left(\frac{3}{2}\right)^{\frac{1}{x}}=t\end{array}\right] \Rightarrow$$

$$\Rightarrow 2\cdot t+3\cdot t^{-1}=5 \Rightarrow 2\cdot t+\frac{3}{t}=5 \Rightarrow 2\cdot t+\frac{3}{t}=5 \;/\cdot t \Rightarrow 2\cdot t^2+3=5\cdot t \Rightarrow$$

$$\Rightarrow 2\cdot t^2+3-5\cdot t=0 \Rightarrow 2\cdot t^2-5\cdot t+3=0 \Rightarrow \left.\begin{array}{l}2\cdot t^2-5\cdot t+3=0\\ a=2 \;,\; b=-5 \;,\; c=3\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}a=2 \;,\; b=-5 \;,\; c=3\\ t_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\} \Rightarrow t_{1,2}=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 2\cdot 3}}{2\cdot 2} \Rightarrow t_{1,2}=\frac{5\pm\sqrt{25-24}}{4} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{5\pm\sqrt{1}}{4} \Rightarrow t_{1,2}=\frac{5\pm 1}{4} \Rightarrow \left.\begin{array}{l}t_1=\frac{5+1}{4}\\ t_2=\frac{5-1}{4}\end{array}\right\} \Rightarrow \left.\begin{array}{l}t_1=\frac{6}{4}\\ t_2=\frac{4}{4}\end{array}\right\} \Rightarrow \left.\begin{array}{l}t_1=\frac{6}{4}\\ t_2=\frac{4}{4}\end{array}\right\} \Rightarrow \left.\begin{array}{l}t_1=\frac{3}{2}\\ t_2=1\end{array}\right\}.$$

Vraćamo se na zamjenu.

- $$\left.\begin{array}{l}\left(\frac{3}{2}\right)^{\frac{1}{x}}=t\\ t=\frac{3}{2}\end{array}\right\} \Rightarrow \left(\frac{3}{2}\right)^{\frac{1}{x}}=\frac{3}{2} \Rightarrow \left(\frac{3}{2}\right)^{\frac{1}{x}}=\left(\frac{3}{2}\right)^1 \Rightarrow \frac{1}{x}=1 \Rightarrow x=1.$$

- $\left.\begin{matrix} \left(\frac{3}{2}\right)^{\frac{1}{x}} = t \\ t = 1 \end{matrix}\right\} \Rightarrow \left(\frac{3}{2}\right)^{\frac{1}{x}} = 1 \Rightarrow \left(\frac{3}{2}\right)^{\frac{1}{x}} = \left(\frac{3}{2}\right)^{0} \Rightarrow \frac{1}{x} = 0$ nema smisla.

Vježba 421

Riješi jednadžbu: $2 \cdot 9^{\frac{1}{x}} + 3 \cdot 4^{\frac{1}{x}} - 5 \cdot 6^{\frac{1}{x}} = 0.$

Rezultat: x = 3.

Zadatak 422 (Antonio, gimnazija)

Riješi jednadžbu: $9 \cdot 4^{\frac{1}{x}} + 5 \cdot 6^{\frac{1}{x}} = 4 \cdot 9^{\frac{1}{x}}.$

Rješenje 422

Ponovimo!

$$\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n \quad , \quad \left(a^n\right)^m = \left(a^m\right)^n = a^{n \cdot m} \quad , \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad , \quad a^1 = a.$$

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x) \quad , \quad n = \frac{n}{1}.$$

$$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$9 \cdot 4^{\frac{1}{x}} + 5 \cdot 6^{\frac{1}{x}} = 4 \cdot 9^{\frac{1}{x}} \Rightarrow 9 \cdot 4^{\frac{1}{x}} + 5 \cdot 6^{\frac{1}{x}} = 4 \cdot 9^{\frac{1}{x}} \;/ : 6^{\frac{1}{x}} \Rightarrow 9 \cdot \frac{4^{\frac{1}{x}}}{6^{\frac{1}{x}}} + 5 = 4 \cdot \frac{9^{\frac{1}{x}}}{6^{\frac{1}{x}}} \Rightarrow$$

$$\Rightarrow 9 \cdot \left(\frac{4}{6}\right)^{\frac{1}{x}} + 5 = 4 \cdot \left(\frac{9}{6}\right)^{\frac{1}{x}} \Rightarrow 9 \cdot \left(\frac{4}{6}\right)^{\frac{1}{x}} + 5 = 4 \cdot \left(\frac{9}{6}\right)^{\frac{1}{x}} \Rightarrow 9 \cdot \left(\frac{2}{3}\right)^{\frac{1}{x}} + 5 = 4 \cdot \left(\frac{3}{2}\right)^{\frac{1}{x}} \Rightarrow$$

$$\Rightarrow 9 \cdot \left(\frac{2}{3}\right)^{\frac{1}{x}} + 5 - 4 \cdot \left(\frac{3}{2}\right)^{\frac{1}{x}} = 0 \Rightarrow 9 \cdot \left(\frac{2}{3}\right)^{\frac{1}{x}} + 5 - 4 \cdot \left(\left(\frac{2}{3}\right)^{-1}\right)^{\frac{1}{x}} = 0 \Rightarrow$$

$$\Rightarrow 9 \cdot \left(\frac{2}{3}\right)^{\frac{1}{x}} + 5 - 4 \cdot \left(\left(\frac{2}{3}\right)^{\frac{1}{x}}\right)^{-1} = 0 \Rightarrow \left[\begin{matrix} \text{zamjena} \\ \left(\frac{2}{3}\right)^{\frac{1}{x}} = t \end{matrix}\right] \Rightarrow$$

$$\Rightarrow 9 \cdot t + 5 - 4 \cdot t^{-1} = 0 \Rightarrow 9 \cdot t + 5 - \frac{4}{t} = 0 \Rightarrow 9 \cdot t + 5 - \frac{4}{t} = 0 \;/ \cdot t \Rightarrow 9 \cdot t^2 + 5 \cdot t - 4 = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} 9 \cdot t^2 + 5 \cdot t - 4 = 0 \\ a = 9 \;,\; b = 5 \;,\; c = -4 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=9 \ , \ b=5 \ , \ c=-4 \\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow t_{1,2}=\frac{-5\pm\sqrt{5^2-4\cdot 9\cdot(-4)}}{2\cdot 9} \Rightarrow t_{1,2}=\frac{-5\pm\sqrt{25+144}}{18} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{-5\pm\sqrt{169}}{18} \Rightarrow t_{1,2}=\frac{-5\pm 13}{18} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{-5+13}{18} \\ t_2=\dfrac{-5-13}{18} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{8}{18} \\ t_2=-\dfrac{18}{18} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t_1=\dfrac{8}{18} \\ t_2=-\dfrac{18}{18} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{4}{9} \\ t_2=-1 \end{array}\right\}.$$

Vraćamo se na zamjenu.

- $$\left.\begin{array}{l} \left(\dfrac{2}{3}\right)^{\frac{1}{x}}=t \\ t=\dfrac{4}{9} \end{array}\right\} \Rightarrow \left(\frac{2}{3}\right)^{\frac{1}{x}}=\frac{4}{9} \Rightarrow \left(\frac{2}{3}\right)^{\frac{1}{x}}=\left(\frac{2}{3}\right)^2 \Rightarrow \frac{1}{x}=2 \Rightarrow \frac{1}{x}=\frac{2}{1} \Rightarrow x=\frac{1}{2}.$$
- $$\left.\begin{array}{l} \left(\dfrac{2}{3}\right)^{\frac{1}{x}}=t \\ t=-1 \end{array}\right\} \Rightarrow \left(\frac{2}{3}\right)^{\frac{1}{x}}=-1 \text{ nema smisla}.$$

Vježba 422

Riješi jednadžbu: $9\cdot 4^{\frac{1}{x}}+5\cdot 6^{\frac{1}{x}}-4\cdot 9^{\frac{1}{x}}=0.$

Rezultat: $x=\frac{1}{2}.$

Zadatak 423 (Roby, gimnazija)

Riješi jednadžbu: $2^{3\cdot x}\cdot 3^x-2^{3\cdot x-1}\cdot 3^{x+1}=-288.$

Rješenje 423

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^1=a.$$

$$a^n\cdot b^n=(a\cdot b)^n \quad , \quad a\cdot\frac{b}{c}=\frac{a\cdot b}{c} \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$2^{3\cdot x}\cdot 3^x-2^{3\cdot x-1}\cdot 3^{x+1}=-288 \Rightarrow \left(2^3\right)^x\cdot 3^x-2^{3\cdot x}\cdot 2^{-1}\cdot 3^x\cdot 3^1=-288 \Rightarrow$$

$$\Rightarrow 8^x\cdot 3^x-\left(2^3\right)^x\cdot\frac{1}{2}\cdot 3^x\cdot 3=-288 \Rightarrow (8\cdot 3)^x-8^x\cdot\frac{1}{2}\cdot 3^x\cdot 3=-288 \Rightarrow$$

$$\Rightarrow 24^x-8^x\cdot 3^x\cdot\frac{1}{2}\cdot 3=-288 \Rightarrow 24^x-(8\cdot 3)^x\cdot\frac{3}{2}=-288 \Rightarrow 24^x-24^x\cdot\frac{3}{2}=-288 \Rightarrow$$

$$\Rightarrow 24^x\cdot\left(1-\frac{3}{2}\right)=-288 \Rightarrow 24^x\cdot\left(\frac{1}{1}-\frac{3}{2}\right)=-288 \Rightarrow 24^x\cdot\frac{2-3}{2}=-288 \Rightarrow$$

$$\Rightarrow 24^x\cdot\left(-\frac{1}{2}\right)=-288 \Rightarrow 24^x\cdot\left(-\frac{1}{2}\right)=-288 \;/\cdot(-2) \Rightarrow 24^x=576 \Rightarrow$$

$$\Rightarrow 24^x=24^2 \Rightarrow x=2.$$

Vježba 423

Riješi jednadžbu: $2^{3\cdot x}\cdot 3^x-3\cdot 2^{3\cdot x-1}\cdot 3^x+288=0.$

Rezultat: $x=2.$

Zadatak 424 (Roby, gimnazija)

Riješi jednadžbu: $\left(11^x-11\right)^2=11^x+99.$

Rješenje 424

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2 \quad , \quad a^0=1 \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \; , \; n\neq 0 \; , \; n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=b^c \; \rightarrow \qquad a=b^c$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj $\log x$ zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x=\log x.$$

$$\log a^n=n\cdot\log a.$$

1.inačica

$$\left(11^x-11\right)^2=11^x+99 \Rightarrow \left(11^x\right)^2-2\cdot 11^x\cdot 11+11^2=11^x+99 \Rightarrow$$

$$\Rightarrow \left(11^x\right)^2-22\cdot 11^x+121=11^x+99 \Rightarrow \left(11^x\right)^2-22\cdot 11^x+121-11^x-99=0 \Rightarrow$$

$$\Rightarrow \left(11^{x}\right)^{2}-23\cdot 11^{x}+22=0 \Rightarrow \left[\begin{array}{c}\text{zamjena}\\ 11^{x}=t\end{array}\right] \Rightarrow t^{2}-23\cdot t+22=0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t^{2}-23\cdot t+22=0 \\ a=1\ ,\ b=-23\ ,\ c=22 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=1\ ,\ b=-23\ ,\ c=22 \\ t_{1,2}=\dfrac{-b\pm\sqrt{b^{2}-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow t_{1,2}=\frac{-(-23)\pm\sqrt{(-23)^{2}-4\cdot 1\cdot 22}}{2\cdot 1} \Rightarrow t_{1,2}=\frac{23\pm\sqrt{529-88}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{23\pm\sqrt{441}}{2} \Rightarrow t_{1,2}=\frac{23\pm 21}{2} \Rightarrow \left.\begin{array}{l} t_{1}=\dfrac{21+23}{2} \\ t_{2}=\dfrac{21-23}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_{1}=\dfrac{44}{2} \\ t_{2}=-\dfrac{2}{2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t_{1}=\dfrac{44}{2} \\ t_{2}=-\dfrac{2}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_{1}=22 \\ t_{2}=-1 \end{array}\right\}.$$

Vraćamo se na zamjenu.

- $$\left.\begin{array}{l} 11^{x}=t \\ t=22 \end{array}\right\} \Rightarrow 11^{x}=22 \Rightarrow \left[\begin{array}{l}\text{logaritmiramo}\\ \text{jednadžbu}\end{array}\right] \Rightarrow 11^{x}=22\ /\ \log \Rightarrow \log 11^{x}=\log 22 \Rightarrow$$

$$\Rightarrow x\cdot\log 11=\log 22 \Rightarrow x\cdot\log 11=\log 22\ /:\log 11 \Rightarrow x_{1}=\frac{\log 22}{\log 11}.$$

- $$\left.\begin{array}{l} 11^{x}=t \\ t=1 \end{array}\right\} \Rightarrow 11^{x}=1 \Rightarrow 11^{x}=11^{0} \Rightarrow x_{2}=0.$$

2.inačica

$$\left(11^{x}-11\right)^{2}=11^{x}+99 \Rightarrow \left[\begin{array}{c}\text{zamjena}\\ 11^{x}=t\end{array}\right] \Rightarrow (t-11)^{2}=t+99 \Rightarrow$$

$$\Rightarrow t^{2}-22\cdot t+121=t+99 \Rightarrow t^{2}-22\cdot t+121-t-99=0 \Rightarrow t^{2}-23\cdot t+22=0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t^{2}-23\cdot t+22=0 \\ a=1\ ,\ b=-23\ ,\ c=22 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=1\ ,\ b=-23\ ,\ c=22 \\ t_{1,2}=\dfrac{-b\pm\sqrt{b^{2}-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow t_{1,2}=\frac{-(-23)\pm\sqrt{(-23)^{2}-4\cdot 1\cdot 22}}{2\cdot 1} \Rightarrow t_{1,2}=\frac{23\pm\sqrt{529-88}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{23\pm\sqrt{441}}{2} \Rightarrow t_{1,2}=\frac{23\pm 21}{2} \Rightarrow \left.\begin{array}{l} t_{1}=\dfrac{21+23}{2} \\ t_{2}=\dfrac{23-21}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_{1}=\dfrac{44}{2} \\ t_{2}=\dfrac{2}{2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t_1 = \frac{44}{2} \\ t_2 = \frac{2}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = 22 \\ t_2 = 1 \end{array}\right\}.$$

Vraćamo se na zamjenu.

- $$\left.\begin{array}{l} 11^x = t \\ t = 22 \end{array}\right\} \Rightarrow 11^x = 22 \Rightarrow \left[\begin{array}{l} \text{logaritmiramo} \\ \text{jednadžbu} \end{array}\right] \Rightarrow 11^x = 22 \; / \log \Rightarrow \log 11^x = \log 22 \Rightarrow$$

$$\Rightarrow x \cdot \log 11 = \log 22 \Rightarrow x \cdot \log 11 = \log 22 \; /: \log 11 \Rightarrow x_1 = \frac{\log 22}{\log 11}.$$

- $$\left.\begin{array}{l} 11^x = t \\ t = 1 \end{array}\right\} \Rightarrow 11^x = 1 \Rightarrow 11^x = 11^\circ \Rightarrow x_2 = 0.$$

Vježba 424

Riješi jednadžbu: $\left(11^x - 11\right)^2 - 99 = 11^x.$

Rezultat: $x = 2.$

Zadatak 425 (Roby, gimnazija)

Riješi sustav jednadžbi: $\left\{\begin{array}{l} 2^x \cdot 3^y = 12 \\ 2^y \cdot 3^x = 18. \end{array}\right.$

Rješenje 425

Ponovimo!

$$\left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow \frac{a}{c} = \frac{b}{d} \quad , \quad \frac{a^n}{a^m} = a^{n-m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad \frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n.$$

$$a^1 = a \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x) \quad , \quad a^n \cdot a^m = a^{n+m}.$$

$$a^n \cdot b^n = (a \cdot b)^n \quad , \quad \left.\begin{array}{l} a = b \\ c = d \end{array}\right\} \Rightarrow a \cdot c = b \cdot d.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \; , \; n \neq 0 \; , \; n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$\left.\begin{array}{l} 2^x \cdot 3^y = 12 \\ 2^y \cdot 3^x = 18 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{podijelimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow \frac{2^x \cdot 3^y}{2^y \cdot 3^x} = \frac{12}{18} \Rightarrow \frac{2^x}{2^y} \cdot \frac{3^y}{3^x} = \frac{12}{18} \Rightarrow$$

$$\Rightarrow 2^{x-y} \cdot 3^{y-x} = \frac{2}{3} \Rightarrow 2^{x-y} \cdot 3^{-(x-y)} = \frac{2}{3} \Rightarrow \frac{2^{x-y}}{3^{x-y}} = \frac{2}{3} \Rightarrow \left(\frac{2}{3}\right)^{x-y} = \left(\frac{2}{3}\right)^1 \Rightarrow x - y = 1.$$

Sada riješimo sustav jednadžbi.

$$\left.\begin{array}{l} x-y=1 \\ 2^{x}\cdot 3^{y}=12 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=1+y \\ 2^{x}\cdot 3^{y}=12 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow 2^{1+y}\cdot 3^{y}=12 \Rightarrow 2^{1}\cdot 2^{y}\cdot 3^{y}=12 \Rightarrow$$

$$\Rightarrow 2\cdot 2^{y}\cdot 3^{y}=12 \Rightarrow 2\cdot(2\cdot 3)^{y}=12 \Rightarrow 2\cdot 6^{y}=12 \Rightarrow 2\cdot 6^{y}=12 \;/:2 \Rightarrow 6^{y}=6 \Rightarrow$$

$$\Rightarrow 6^{y}=6^{1} \Rightarrow y=1.$$

Računamo x.

$$\left.\begin{array}{l} x=1+y \\ y=1 \end{array}\right\} \Rightarrow x=1+1 \Rightarrow x=2.$$

Rješenje sustava je:

$$(x,\ y)=(2,\ 1).$$

2.inačica

$$\left.\begin{array}{l} 2^{x}\cdot 3^{y}=12 \\ 2^{y}\cdot 3^{x}=18 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{pomnožimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow 2^{x}\cdot 3^{y}\cdot 2^{y}\cdot 3^{x}=12\cdot 18 \Rightarrow 2^{x}\cdot 2^{y}\cdot 3^{x}\cdot 3^{y}=216 \Rightarrow$$

$$\Rightarrow 2^{x+y}\cdot 3^{x+y}=216 \Rightarrow (2\cdot 3)^{x+y}=6^{3} \Rightarrow 6^{x+y}=6^{3} \Rightarrow x+y=3.$$

Sada riješimo sustav jednadžbi.

$$\left.\begin{array}{l} x+y=3 \\ 2^{x}\cdot 3^{y}=12 \end{array}\right\} \Rightarrow \left.\begin{array}{l} y=3-x \\ 2^{x}\cdot 3^{y}=12 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{zamjene} \end{array}\right] \Rightarrow 2^{x}\cdot 3^{3-x}=12 \Rightarrow 2^{x}\cdot 3^{3}\cdot 3^{-x}=12 \Rightarrow$$

$$\Rightarrow 2^{x}\cdot 27\cdot\frac{1}{3^{x}}=12 \Rightarrow \frac{2^{x}}{3^{x}}\cdot 27=12 \Rightarrow \left(\frac{2}{3}\right)^{x}\cdot 27=12 \Rightarrow \left(\frac{2}{3}\right)^{x}\cdot 27=12 \;/\cdot\frac{1}{27} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{3}\right)^{x}=\frac{12}{27} \Rightarrow \left(\frac{2}{3}\right)^{x}=\frac{12}{27} \Rightarrow \left(\frac{2}{3}\right)^{x}=\frac{4}{9} \Rightarrow \left(\frac{2}{3}\right)^{x}=\left(\frac{2}{3}\right)^{2} \Rightarrow x=2.$$

Računamo y.

$$\left.\begin{array}{l} y=3-x \\ x=2 \end{array}\right\} \Rightarrow y=3-2 \Rightarrow y=1.$$

Rješenje sustava je:

$$(x,\ y)=(2,\ 1).$$

Vježba 425

Riješi sustav jednadžbi: $\begin{cases} 2^{y}\cdot 3^{x}=18 \\ 2^{x}\cdot 3^{y}=12. \end{cases}$

Rezultat: $(x,\ y)=(2,\ 1).$

Zadatak 426 (Ante, gimnazija)

Izračunati bez uporabe džepnog računala $\dfrac{\log 3-\log 2}{\log 225-2}$.

Rješenje 426

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_{b} a=c \qquad \log_{b} a=b^{c} \qquad a=b^{c}$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj $\log x$ zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log \frac{a}{b} = \log a - \log b \quad , \quad \log a^n = n \cdot \log a \quad , \quad \log 100 = 2 \quad , \quad \log(a \cdot b) = \log a + \log b.$$

$$\log 10 = 1.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$\frac{\log 3 - \log 2}{\log 225 - 2} = \frac{\log \frac{3}{2}}{\log 225 - \log 100} = \frac{\log \frac{3}{2}}{\log \frac{225}{100}} = \frac{\log \frac{3}{2}}{\log \frac{225}{100}} = \frac{\log \frac{3}{2}}{\log \frac{9}{4}} = \frac{\log \frac{3}{2}}{\log \left(\frac{3}{2}\right)^2} = \frac{\log \frac{3}{2}}{2 \cdot \log \frac{3}{2}} =$$

$$= \frac{\log \frac{3}{2}}{2 \cdot \log \frac{3}{2}} = \frac{1}{2}.$$

2.inačica

$$\frac{\log 3 - \log 2}{\log 225 - 2} = \frac{\log \frac{3}{2}}{\log 15^2 - 2} = \frac{\log \frac{3}{2}}{2 \cdot \log 15 - 2} = \frac{\log \frac{3}{2}}{2 \cdot (\log 15 - 1)} = \frac{\log \frac{3}{2}}{2 \cdot (\log 15 - \log 10)} = \frac{\log \frac{3}{2}}{2 \cdot \log \frac{15}{10}} =$$

$$= \frac{\log \frac{3}{2}}{2 \cdot \log \frac{15}{10}} = \frac{\log \frac{3}{2}}{2 \cdot \log \frac{3}{2}} = \frac{\log \frac{3}{2}}{2 \cdot \log \frac{3}{2}} = \frac{1}{2}.$$

3.inačica

$$\frac{\log 3 - \log 2}{\log 225 - 2} = \frac{\log 3 - \log 2}{\log 15^2 - 2} = \frac{\log 3 - \log 2}{2 \cdot \log 15 - 2} = \frac{\log 3 - \log 2}{2 \cdot (\log 15 - 1)} = \frac{\log 3 - \log 2}{2 \cdot (\log 15 - \log 10)} =$$

$$= \frac{\log 3 - \log 2}{2 \cdot (\log(3 \cdot 5) - \log(2 \cdot 5))} = \frac{\log 3 - \log 2}{2 \cdot (\log 3 + \log 5 - (\log 2 + \log 5))} =$$

$$= \frac{\log 3 - \log 2}{2 \cdot (\log 3 + \log 5 - \log 2 - \log 5)} = \frac{\log 3 - \log 2}{2 \cdot (\log 3 + \log 5 - \log 2 - \log 5)} = \frac{\log 3 - \log 2}{2 \cdot (\log 3 - \log 2)} =$$

$$= \frac{\log 3 - \log 2}{2 \cdot (\log 3 - \log 2)} = \frac{1}{2}.$$

Vježba 426

Izračunati bez uporabe džepnog računala $\frac{\log 225 - 2}{\log 3 - \log 2}$.

Rezultat: 2.

Zadatak 427 (Sonja, Bundeshandelsakademie)

Riješite jednadžbu: $8\cdot\left(4^{x}+4^{-x}\right)-54\cdot\left(2^{x}+2^{-x}\right)+101=0.$

Rješenje 427

Ponovimo!

$$(a+b)^2=a^2+2\cdot a\cdot b+b^2 \quad , \quad \left(a^n\right)^m=\left(a^m\right)^n=a^{n\cdot m} \quad , \quad a^n\cdot a^m=a^{n+m}.$$

$$a^{\circ}=1 \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^1=a \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \ , \ n\neq 0 \ , \ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Uočimo sljedeće:

$$\left(2^x+2^{-x}\right)^2=\left(2^x\right)^2+2\cdot 2^x\cdot 2^{-x}+\left(2^{-x}\right)^2 \Rightarrow$$

$$\Rightarrow \left(2^x+2^{-x}\right)^2=\left(2^2\right)^x+2\cdot 2^{x-x}+\left(2^2\right)^{-x} \Rightarrow$$

$$\Rightarrow \left(2^x+2^{-x}\right)^2=\left(2^2\right)^x+2\cdot 2^{x-x}+\left(2^2\right)^{-x} \Rightarrow$$

$$\Rightarrow \left(2^x+2^{-x}\right)^2=4^x+2\cdot 2^{\circ}+4^{-x} \Rightarrow$$

$$\Rightarrow \left(2^x+2^{-x}\right)^2=4^x+2\cdot 1+4^{-x} \Rightarrow$$

$$\Rightarrow \left(2^x+2^{-x}\right)^2=4^x+2+4^{-x} \Rightarrow$$

$$\Rightarrow 4^x+2+4^{-x}=\left(2^x+2^{-x}\right)^2 \Rightarrow$$

$$\Rightarrow 4^x+4^{-x}=\left(2^x+2^{-x}\right)^2-2.$$

Sada rješavamo jednadžbu.

$$8\cdot\left(4^x+4^{-x}\right)-54\cdot\left(2^x+2^{-x}\right)+101=0 \Rightarrow \left[4^x+4^{-x}=\left(2^x+2^{-x}\right)^2-2\right] \Rightarrow$$

$$\Rightarrow 8\cdot\left(\left(2^x+2^{-x}\right)^2-2\right)-54\cdot\left(2^x+2^{-x}\right)+101=0 \Rightarrow$$

$$\Rightarrow 8\cdot\left(2^x+2^{-x}\right)^2-16-54\cdot\left(2^x+2^{-x}\right)+101=0 \Rightarrow$$

$$\Rightarrow 8\cdot\left(2^x+2^{-x}\right)^2-54\cdot\left(2^x+2^{-x}\right)+85=0 \Rightarrow \left[\begin{array}{c}\text{zamjena}\\ 2^x+2^{-x}=t\end{array}\right] \Rightarrow$$

$$\Rightarrow\ 8\cdot t^2-54\cdot t+85=0\ \Rightarrow\ \left.\begin{array}{l}8\cdot t^2-54\cdot t+85=0\\ a=8\ ,\ b=-54\ ,\ c=85\end{array}\right\}\Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l}a=8\ ,\ b=-54\ ,\ c=85\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\}\Rightarrow\ t_{1,2}=\frac{-(-54)\pm\sqrt{(-54)^2-4\cdot 8\cdot 85}}{2\cdot 8}\ \Rightarrow$$

$$\Rightarrow\ t_{1,2}=\frac{54\pm\sqrt{2916-2720}}{16}\ \Rightarrow$$

$$\Rightarrow\ t_{1,2}=\frac{54\pm\sqrt{196}}{16}\ \Rightarrow\ t_{1,2}=\frac{54\pm 14}{16}\ \Rightarrow\ \left.\begin{array}{l}t_1=\dfrac{54+14}{16}\\ t_2=\dfrac{54-14}{16}\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}t_1=\dfrac{68}{16}\\ t_2=\dfrac{40}{16}\end{array}\right\}\Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l}t_1=\dfrac{68}{16}\\ t_2=\dfrac{40}{16}\end{array}\right\}\Rightarrow\ \left.\begin{array}{l}t_1=\dfrac{17}{4}\\ t_2=\dfrac{5}{2}\end{array}\right\}.$$

Vraćamo se na zamjenu.

$$\left.\begin{array}{l}2^x+2^{-x}=t\\ t=\dfrac{17}{4}\end{array}\right\}\Rightarrow\ 2^x+2^{-x}=\frac{17}{4}\ \Rightarrow\ 2^x+2^{-x}=\frac{17}{4}\ /\cdot 4\ \Rightarrow\ 4\cdot 2^x+4\cdot 2^{-x}=17\ \Rightarrow$$

$$\Rightarrow\ 4\cdot 2^x+4\cdot 2^{-x}-17=0\ \Rightarrow\ 4\cdot 2^x+4\cdot\frac{1}{2^x}-17=0\ \Rightarrow\ \left[\begin{array}{c}\text{zamjena}\\ 2^x=t\end{array}\right]\Rightarrow\ 4\cdot t+\frac{4}{t}-17=0\ \Rightarrow$$

$$\Rightarrow\ 4\cdot t+\frac{4}{t}-17=0\ /\cdot t\ \Rightarrow\ 4\cdot t^2+4-17\cdot t=0\ \Rightarrow$$

$$\Rightarrow\ 4\cdot t^2-17\cdot t+4=0\ \Rightarrow\ \left.\begin{array}{l}4\cdot t^2-17\cdot t+4=0\\ a=4\ ,\ b=-17\ ,\ c=4\end{array}\right\}\Rightarrow$$

$$\Rightarrow\ \left.\begin{array}{l}a=4\ ,\ b=-17\ ,\ c=4\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\}\Rightarrow\ t_{1,2}=\frac{-(-17)\pm\sqrt{(-17)^2-4\cdot 4\cdot 4}}{2\cdot 4}\ \Rightarrow$$

$$\Rightarrow \quad t_{1,2}=\frac{17\pm\sqrt{289-64}}{8} \quad \Rightarrow$$

$$\Rightarrow \quad t_{1,2}=\frac{17\pm\sqrt{225}}{8} \quad \Rightarrow \quad t_{1,2}=\frac{17\pm15}{8} \quad \Rightarrow \quad \left.\begin{array}{l} t_1=\dfrac{17+15}{8} \\ t_2=\dfrac{17-15}{8} \end{array}\right\} \Rightarrow \quad \left.\begin{array}{l} t_1=\dfrac{32}{8} \\ t_2=\dfrac{2}{8} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \quad \left.\begin{array}{l} t_1=\dfrac{32}{8} \\ t_2=\dfrac{2}{8} \end{array}\right\} \Rightarrow \quad \left.\begin{array}{l} t_1=4 \\ t_2=\dfrac{1}{4} \end{array}\right\}.$$

Vraćamo se na zamjenu.

- $$\left.\begin{array}{l} 2^x=t \\ t=4 \end{array}\right\} \Rightarrow \quad 2^x=4 \quad \Rightarrow \quad 2^x=2^2 \quad \Rightarrow \quad x_1=2$$
- $$\left.\begin{array}{l} 2^x=t \\ t=\dfrac{1}{4} \end{array}\right\} \Rightarrow \quad 2^x=\frac{1}{4} \quad \Rightarrow \quad 2^x=\frac{1}{2^2} \quad \Rightarrow \quad 2^x=2^{-2} \quad \Rightarrow \quad x_2=-2.$$

Vraćamo se na zamjenu.

$$\left.\begin{array}{l} 2^x+2^{-x}=t \\ t=\dfrac{5}{2} \end{array}\right\} \Rightarrow \quad 2^x+2^{-x}=\frac{5}{2} \quad \Rightarrow \quad 2^x+2^{-x}=\frac{5}{2} \;/\cdot 2 \quad \Rightarrow \quad 2\cdot2^x+2\cdot2^{-x}=5 \quad \Rightarrow$$

$$\Rightarrow \quad 2\cdot2^x+2\cdot2^{-x}-5=0 \quad \Rightarrow \quad 2\cdot2^x+2\cdot\frac{1}{2^x}-5=0 \quad \Rightarrow \quad \left[\begin{array}{c} \text{zamjena} \\ 2^x=t \end{array}\right] \Rightarrow \quad 2\cdot t+\frac{2}{t}-5=0 \quad \Rightarrow$$

$$\Rightarrow \quad 2\cdot t+\frac{2}{t}-5=0 \;/\cdot t \quad \Rightarrow \quad 2\cdot t^2+2-5\cdot t=0 \quad \Rightarrow$$

$$\Rightarrow \quad 2\cdot t^2-5\cdot t+2=0 \quad \Rightarrow \quad \left.\begin{array}{l} 2\cdot t^2-5\cdot t+2=0 \\ a=2 \;,\; b=-5 \;,\; c=2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \quad \left.\begin{array}{l} a=2 \;,\; b=-5 \;,\; c=2 \\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow \quad t_{1,2}=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot2\cdot2}}{2\cdot2} \quad \Rightarrow$$

$$\Rightarrow \quad t_{1,2}=\frac{5\pm\sqrt{25-16}}{4} \quad \Rightarrow\Rightarrow \quad t_{1,2}=\frac{5\pm\sqrt{9}}{4} \quad \Rightarrow \quad t_{1,2}=\frac{5\pm3}{4} \quad \Rightarrow \quad \left.\begin{array}{l} t_1=\dfrac{5+3}{4} \\ t_2=\dfrac{5-3}{4} \end{array}\right\} \Rightarrow \quad \left.\begin{array}{l} t_1=\dfrac{8}{4} \\ t_2=\dfrac{2}{4} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t_1 = \frac{8}{4} \\ t_2 = \frac{2}{4} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1 = 2 \\ t_2 = \frac{1}{2} \end{array}\right\}.$$

Vraćamo se na zamjenu.

- $\left.\begin{array}{l} 2^x = t \\ t = 2 \end{array}\right\} \Rightarrow 2^x = 2 \Rightarrow 2^x = 2^1 \Rightarrow x_3 = 1$
- $\left.\begin{array}{l} 2^x = t \\ t = \frac{1}{2} \end{array}\right\} \Rightarrow 2^x = \frac{1}{2} \Rightarrow 2^x = 2^{-1} \Rightarrow x_4 = -1.$

Vježba 427

Riješite jednadžbu: $4 \cdot \left(4^x + 4^{-x}\right) - 27 \cdot \left(2^x + 2^{-x}\right) + 50.5 = 0.$

Rezultat: $x_1 = 2,\ x_2 = -2,\ x_3 = 1,\ x_4 = -1.$

Zadatak 428 (Ane, ekonomska škola)

Izračunaj: $\log_9 \sqrt[4]{27}.$

Rješenje 428

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad \sqrt[n]{a^m} = a^{\frac{m}{n}} \quad , \quad b \cdot \frac{a}{b} = a \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = \underset{\rightarrow}{b^c} \qquad a = b^c$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \log_b a^n = n \cdot \log_b a \quad , \quad \log_b b = 1 \quad , \quad \log_b a = \frac{\log a}{\log b}.$$

$$\log a^n = n \cdot \log a \quad , \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n} = \frac{a}{b} \ , \ n \neq 0 \ , \ n \neq 1.$$

1.inačica

$$\log_9 \sqrt[4]{27} = x \Rightarrow 9^x = \sqrt[4]{27} \Rightarrow \left(3^2\right)^x = \sqrt[4]{3^3} \Rightarrow 3^{2 \cdot x} = 3^{\frac{3}{4}} \Rightarrow 2 \cdot x = \frac{3}{4} \Rightarrow$$

$$\Rightarrow \ 2 \cdot x = \frac{3}{4} \ / \cdot \frac{1}{2} \ \Rightarrow \ x = \frac{3}{8}.$$

2.inačica

$$\log_9 \sqrt[4]{27} = \log_{3^2} \sqrt[4]{3^3} = \frac{1}{2} \cdot \log_3 3^{\frac{3}{4}} = \frac{1}{2} \cdot \frac{3}{4} \cdot \log_3 3 = \frac{1}{2} \cdot \frac{3}{4} \cdot 1 = \frac{3}{8}.$$

3.inačica

$$\log_9 \sqrt[4]{27} = \frac{\log \sqrt[4]{27}}{\log 9} = \frac{\log \sqrt[4]{3^3}}{\log 3^2} = \frac{\log 3^{\frac{3}{4}}}{\log 3^2} = \frac{\frac{3}{4} \cdot \log 3}{2 \cdot \log 3} = \frac{\frac{3}{4} \cdot \log 3}{2 \cdot \log 3} = \frac{\frac{3}{4}}{2} = \frac{\frac{3}{4}}{\frac{2}{1}} = \frac{3}{8}.$$

Vježba 428

Izračunaj: $\log_9 \sqrt[5]{27}.$

Rezultat: $\frac{3}{10}.$

Zadatak 429 (4A, 4B, TUPŠ)

Riješi jednadžbu: $\log_3 (x-3) = -2.$

Rješenje 429

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \underset{\rightarrow}{\log_b a = b^c} \qquad a = b^c$$

Dekadski logaritam
Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$a^{-n} = \frac{1}{a^n} \quad , \quad n = \frac{n}{1} \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d} \quad , \quad \log_b b^n = n.$$

$$\log_b f(x) = \log_b g(x) \ \Rightarrow \ f(x) = g(x) \quad , \quad \log_b a = \frac{\log a}{\log b} \quad , \quad \log a^n = n \cdot \log a.$$

$$\log f(x) = \log g(x) \ \Rightarrow \ f(x) = g(x).$$

Neka su D i K dva neprazna skupa. Ako je svakom elementu $x \in D$ pridružen točno jedan element $f(x) \in K$, kažemo da je definirana funkcija f sa skupa D u skup K i pišemo

$$f : D \rightarrow K.$$

Skup D je domena funkcije ili područje definicije, a K kodomena funkcije ili područje vrijednosti funkcije. Ako je f funkcija iz R u R i ako je zadana formulom, podrazumijeva se da njezina domena obuhvaća sve realne brojeve za koje analitički izraz (formula) ima smisla. Ta domena zove se prirodno područje definicije ili prirodna domena funkcije f.
Područje definicije (domena) funkcije $f(x) = \log_b x$ je interval realnih brojeva

$$D(f) = \langle 0, +\infty \rangle.$$

Uočimo da iz $\log x$ slijedi $x > 0$.

1.inačica

$$\log_3(x-3)=-2 \Rightarrow \left[\begin{array}{c}\text{uvjet}\\ x-3> \ \Rightarrow\ x>3\end{array}\right] \Rightarrow 3^{-2}=x-3 \Rightarrow x-3=3^{-2} \Rightarrow$$

$$\Rightarrow x-3=\frac{1}{3^2} \Rightarrow x-3=\frac{1}{9} \Rightarrow x=\frac{1}{9}+3 \Rightarrow x=\frac{1}{9}+\frac{3}{1} \Rightarrow x=\frac{1+27}{9} \Rightarrow x=\frac{28}{9}.$$

2.inačica

$$\log_3(x-3)=-2 \Rightarrow \left[\begin{array}{c}\text{uvjet}\\ x-3>0\ \Rightarrow\ x>0\end{array}\right] \Rightarrow \log_3(x-3)=\log_3 3^{-2} \Rightarrow x-3=3^{-2} \Rightarrow$$

$$\Rightarrow x-3=\frac{1}{3^2} \Rightarrow x-3=\frac{1}{9} \Rightarrow x=\frac{1}{9}+3 \Rightarrow x=\frac{1}{9}+\frac{3}{1} \Rightarrow x=\frac{1+27}{9} \Rightarrow x=\frac{28}{9}.$$

3.inačica

$$\log_3(x-3)=-2 \Rightarrow \left[\begin{array}{c}\text{uvjet}\\ x-3>0\ \Rightarrow\ x>3\end{array}\right] \Rightarrow \frac{\log(x-3)}{\log 3}=-2 \Rightarrow \frac{\log(x-3)}{\log 3}=-2 \ /\cdot \log 3 \Rightarrow$$

$$\Rightarrow \log(x-3)=-2\cdot\log 3 \Rightarrow \log(x-3)=\log 3^{-2} \Rightarrow x-3=3^{-2} \Rightarrow$$

$$\Rightarrow x-3=\frac{1}{3^2} \Rightarrow x-3=\frac{1}{9} \Rightarrow x=\frac{1}{9}+3 \Rightarrow x=\frac{1}{9}+\frac{3}{1} \Rightarrow x=\frac{1+27}{9} \Rightarrow x=\frac{28}{9}.$$

Vježba 429

Riješi jednadžbu: $\log_3(x-3)=-1$.

Rezultat: $\frac{10}{3}$.

Zadatak 430 (4A, 4B, TUPŠ)

Riješi nejednadžbu: $32^{x+1}\le\frac{\sqrt{8}}{4}$.

Rješenje 430

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad \sqrt[n]{a^m}=a^{\frac{m}{n}} \quad , \quad b\cdot\frac{a}{b}=a.$$

$$\frac{a^n}{a^m}=a^{n-m} \quad , \quad a^{f(x)}\le a^{g(x)}\ ,\ a>1 \Rightarrow f(x)\le g(x).$$

$$a\le b\ ,\ c>0 \Rightarrow a\cdot c\le b\cdot c \quad , \quad a\le b\ ,\ c>0 \Rightarrow \frac{a}{c}\le\frac{b}{c} \quad , \quad a^n\cdot a^m=a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$32^{x+1}\le\frac{\sqrt{8}}{4} \Rightarrow \left(2^5\right)^{x+1}\le\frac{\sqrt{2^3}}{2^2} \Rightarrow 2^{5\cdot(x+1)}\le\frac{2^{\frac{3}{2}}}{2^2} \Rightarrow 2^{5\cdot x+5}\le 2^{\frac{3}{2}-2} \Rightarrow$$

$$\Rightarrow 5\cdot x+5\le\frac{3}{2}-2 \Rightarrow 5\cdot x+5\le\frac{3}{2}-2 \ /\cdot 2 \Rightarrow 10\cdot x+10\le 3-4 \Rightarrow 10\cdot x\le 3-4-10 \Rightarrow$$

$$\Rightarrow 10\cdot x\le -11 \Rightarrow 10\cdot x\le -11 \ /:10 \Rightarrow x\le-\frac{11}{10}.$$

2.inačica

$$32^{x+1} \le \frac{\sqrt{8}}{4} \Rightarrow 32^{x+1} \le \frac{\sqrt{8}}{4} \;/\cdot 4 \Rightarrow 4 \cdot 32^{x+1} \le \sqrt{8} \Rightarrow$$

$$\Rightarrow 2^2 \cdot \left(2^5\right)^{x+1} \le \sqrt{2^3} \Rightarrow 2^2 \cdot 2^{5\cdot(x+1)} \le 2^{\frac{3}{2}} \Rightarrow 2^2 \cdot 2^{5\cdot x+5} \le 2^{\frac{3}{2}} \Rightarrow$$

$$\Rightarrow 2^{2+5\cdot x+5} \le 2^{\frac{3}{2}} \Rightarrow 2+5\cdot x+5 \le \frac{3}{2} \Rightarrow 2+5\cdot x+5 \le \frac{3}{2} \;/\cdot 2 \Rightarrow$$

$$\Rightarrow 4+10\cdot x+10 \le 3 \Rightarrow 10\cdot x \le 3-4-10 \Rightarrow 10\cdot x \le -11 \Rightarrow 10\cdot x \le -11 \;/:10 \Rightarrow x \le -\frac{11}{10}.$$

Vježba 430

Riješi nejednadžbu: $32^{x+1} - \frac{\sqrt{8}}{4} \le 0.$

Rezultat: $x \le -\frac{11}{10}.$

Zadatak 431 (Jozo, hobby matematičar ☺)

Riješi jednadžbu: $\log_{\cos x}(\sin x) + \log_{\sin x}(\cos x) = 2.$

Rješenje 431

Ponovimo!

Cijelim brojevima zovemo skup brojeva $\{\dots -2, -1, 0, 1, 2, \dots\}$, tj. skup koji uključuje prirodne brojeve, nulu i negativne cijele brojeve. Skup cijelih brojeva u matematici označavamo velikim slovom Z.

$$Z = \{\dots -3, -2, -1, 0, 1, 2, 3, \dots\}.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \qquad a = b^c$$
$$\rightarrow$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj $\log x$ zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log_b a = \frac{1}{\log_a b} \quad , \quad \log_b b = 1 \quad , \quad a^1 = a \quad , \quad a^n \cdot a^m = a^{n+m}.$$

$$\frac{a}{b}\cdot b = a \quad , \quad a^2 - 2\cdot a\cdot b + b^2 = (a-b)^2 \quad , \quad tg\, x = \frac{\sin x}{\cos x} \quad , \quad \log_b a = \frac{\log a}{\log b}.$$

$$\log f(x) = \log g(x) \Rightarrow f(x) = g(x).$$

Trigonometrijska jednadžba $tg\, x = a$

Skup rješenja jednadžbe $tg\, x = a,\ a \in R,$ je $\{x_0 + k\cdot\pi : k \in Z\}$, gdje je $x_0 \in R$ jedno rješenje te jednadžbe.

$$tg\, x = a \Rightarrow tg\, x = tg\, x_0 \Rightarrow x = x_0 + k\cdot\pi \;,\; k \in Z.$$

1.inačica

$$\log_{\cos x}(\sin x)+\log_{\sin x}(\cos x)=2 \Rightarrow \log_{\cos x}(\sin x)+\frac{1}{\log_{\cos x}(\sin x)}=2 \Rightarrow$$

$$\Rightarrow \log_{\cos x}(\sin x)+\frac{1}{\log_{\cos x}(\sin x)}=2 \;/\cdot \log_{\cos x}(\sin x) \Rightarrow$$

$$\Rightarrow \left(\log_{\cos x}(\sin x)\right)^2+1=2\cdot\log_{\cos x}(\sin x) \Rightarrow$$

$$\Rightarrow \left(\log_{\cos x}(\sin x)\right)^2-2\cdot\log_{\cos x}(\sin x)+1=0 \Rightarrow \left(\log_{\cos x}(\sin x)-1\right)^2=0 \Rightarrow$$

$$\Rightarrow \left(\log_{\cos x}(\sin x)-1\right)^2=0 \;/\sqrt{\;} \Rightarrow \log_{\cos x}(\sin x)-1=0 \Rightarrow \log_{\cos x}(\sin x)=1 \Rightarrow$$

$$\Rightarrow \cos x=\sin x \Rightarrow \sin x=\cos x \Rightarrow \sin x=\cos x \;/\cdot\frac{1}{\cos x} \Rightarrow \frac{\sin x}{\cos x}=1 \Rightarrow tg\, x=1 \Rightarrow$$

$$\Rightarrow x=tg^{-1}1 \Rightarrow x=\frac{\pi}{4}+k\cdot\pi \;,\; k\in Z.$$

2.inačica

$$\log_{\cos x}(\sin x)+\log_{\sin x}(\cos x)=2 \Rightarrow \frac{\log\sin x}{\log\cos x}+\frac{\log\cos x}{\log\sin x}=2 \Rightarrow$$

$$\Rightarrow \frac{\log\sin x}{\log\cos x}+\frac{\log\cos x}{\log\sin x}=2 \;/\cdot \log\cos x\cdot\log\sin x \Rightarrow$$

$$\Rightarrow (\log\sin x)^2+(\log\cos x)^2=2\cdot\log\cos x\cdot\log\sin x \Rightarrow$$

$$\Rightarrow (\log\sin x)^2+(\log\cos x)^2-2\cdot\log\cos x\cdot\log\sin x=0 \Rightarrow$$

$$\Rightarrow (\log\sin x)^2-2\cdot\log\cos x\cdot\log\sin x+(\log\cos x)^2=0 \Rightarrow$$

$$\Rightarrow (\log\sin x-\log\cos x)^2=0 \Rightarrow (\log\sin x-\log\cos x)^2=0 \;/\sqrt{\;} \Rightarrow$$

$$\Rightarrow \log\sin x-\log\cos x=0 \Rightarrow \log\sin x=\log\cos x \Rightarrow \sin x=\cos x \Rightarrow$$

$$\Rightarrow \sin x=\cos x \;/\cdot\frac{1}{\cos x} \Rightarrow \frac{\sin x}{\cos x}=1 \Rightarrow tg\, x=1 \Rightarrow x=tg^{-1}1 \Rightarrow x=\frac{\pi}{4}+k\cdot\pi \;,\; k\in Z.$$

Vježba 431

Riješi jednadžbu: $\log_{\cos x}(\sin x)+\log_{\sin x}(\cos x)-2=0.$

Rezultat: $x=\frac{\pi}{4}+k\cdot\pi \;,\; k\in Z.$

Zadatak 432 (Marko, srednja škola)

Riješi jednadžbu: $\log_2\log_2 x=\log_2 3+\log_2 4.$

Rješenje 432

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=\overset{\rightarrow}{b^c} \qquad a=b^c$$

$$\log_b(x \cdot y) = \log_b x + \log_b y \quad , \quad \log_b f(x) = \log_b g(x) \;\Rightarrow\; f(x) = g(x).$$

Neka su D i K dva neprazna skupa. Ako je svakom elementu $x \in D$ pridružen točno jedan element $f(x) \in K$, kažemo da je definirana funkcija f sa skupa D u skup K i pišemo

$$f : D \rightarrow K.$$

Skup D je domena funkcije ili područje definicije, a K kodomena funkcije ili područje vrijednosti funkcije. Ako je f funkcija iz R u R i ako je zadana formulom, podrazumijeva se da njezina domena obuhvaća sve realne brojeve za koje analitički izraz (formula) ima smisla. Ta domena zove se prirodno područje definicije ili prirodna domena funkcije f.

Područje definicije (domena) funkcije $f(x) = \log_b x$ je interval realnih brojeva

$$D(f) = \langle 0, +\infty \rangle.$$

Uočimo da iz $\log x$ slijedi x > 0.

$$\log_2 \log_2 x = \log_2 3 + \log_2 4 \;\Rightarrow\; \left[\begin{array}{l} \text{uvjet} \\ \log_2 x > 0 \\ x > 1 \end{array}\right] \;\Rightarrow\; \log_2 \log_2 x = \log_2 (3 \cdot 4) \;\Rightarrow$$

$$\Rightarrow\; \log_2 \log_2 x = \log_2 12 \;\Rightarrow\; \log_2 x = 12 \;\Rightarrow\; 2^{12} = x \;\Rightarrow\; x = 2^{12} \;\Rightarrow\; x = 4\,096.$$

Vježba 432

Riješi jednadžbu: $\log_2 \log_2 x - \log_2 3 - \log_2 4 = 0.$

Rezultat: 4096.

Zadatak 433 (Marko, srednja škola)

Riješi jednadžbu: $\log_2 x + \log_4 x + \log_{16} x = 7.$

Rješenje 433

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \qquad a = b^c$$
$$\rightarrow$$

$$\log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \log_b a = \frac{\log a}{\log b}.$$

Neka su D i K dva neprazna skupa. Ako je svakom elementu $x \in D$ pridružen točno jedan element $f(x) \in K$, kažemo da je definirana funkcija f sa skupa D u skup K i pišemo

$$f : D \rightarrow K.$$

Skup D je domena funkcije ili područje definicije, a K kodomena funkcije ili područje vrijednosti funkcije. Ako je f funkcija iz R u R i ako je zadana formulom, podrazumijeva se da njezina domena obuhvaća sve realne brojeve za koje analitički izraz (formula) ima smisla. Ta domena zove se prirodno područje definicije ili prirodna domena funkcije f.

Područje definicije (domena) funkcije $f(x) = \log_b x$ je interval realnih brojeva

$$D(f) = \langle 0, +\infty \rangle.$$

Uočimo da iz $\log x$ slijedi x > 0.

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log a^n = n \cdot \log a \quad , \quad \log f(x) = \log g(x) \Rightarrow f(x) = g(x).$$

1.inačica

$$\log_2 x + \log_4 x + \log_{16} x = 7 \Rightarrow \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \Rightarrow \log_2 x + \log_{2^2} x + \log_{2^4} x = 7 \Rightarrow$$

$$\Rightarrow \log_2 x + \frac{1}{2} \cdot \log_2 x + \frac{1}{4} \cdot \log_2 x = 7 \Rightarrow \log_2 x + \frac{1}{2} \cdot \log_2 x + \frac{1}{4} \cdot \log_2 x = 7 \ / \cdot 4 \Rightarrow$$

$$\Rightarrow 4 \cdot \log_2 x + 2 \cdot \log_2 x + \log_2 x = 28 \Rightarrow 7 \cdot \log_2 x = 28 \Rightarrow 7 \cdot \log_2 x = 28 \ /: 7 \Rightarrow$$

$$\Rightarrow \log_2 x = 4 \Rightarrow 2^4 = x \Rightarrow x = 2^4 \Rightarrow x = 16.$$

2.inačica

$$\log_2 x + \log_4 x + \log_{16} x = 7 \Rightarrow \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \Rightarrow \frac{\log x}{\log 2} + \frac{\log x}{\log 4} + \frac{\log x}{\log 16} = 7 \Rightarrow$$

$$\Rightarrow \frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} + \frac{\log x}{\log 2^4} = 7 \Rightarrow \frac{\log x}{\log 2} + \frac{\log x}{2 \cdot \log 2} + \frac{\log x}{4 \cdot \log 2} = 7 \Rightarrow$$

$$\Rightarrow \frac{\log x}{\log 2} + \frac{\log x}{2 \cdot \log 2} + \frac{\log x}{4 \cdot \log 2} = 7 \ / \cdot 4 \cdot \log 2 \Rightarrow 4 \cdot \log x + 2 \cdot \log x + \log x = 7 \cdot 4 \cdot \log 2 \Rightarrow$$

$$\Rightarrow 7 \cdot \log x = 28 \cdot \log 2 \Rightarrow 7 \cdot \log x = 28 \cdot \log 2 \ /: 7 \Rightarrow \log x = 4 \cdot \log 2 \Rightarrow$$

$$\Rightarrow \log x = \log 2^4 \Rightarrow x = 2^4 \Rightarrow x = 16.$$

Vježba 433

Riješi jednadžbu: $\log_2 x + \log_4 x + \log_{16} x - 7 = 0$

Rezultat: 16.

Zadatak 434 (Marina, građevinska škola)

Riješi jednadžbu: $\log_2^2 x + 2 \cdot \log_2 \sqrt{x} - 2 = 0.$

Rješenje 434

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \qquad a = b^c$$

$$\rightarrow$$

$$\log_b \sqrt[n]{a} = \frac{1}{n} \cdot \log_b a \quad , \quad b \cdot \frac{a}{b} = a \quad , \quad a^1 = a \quad , \quad a^{-n} = \frac{1}{a^n}.$$

Neka su D i K dva neprazna skupa. Ako je svakom elementu $x \in D$ pridružen točno jedan element $f(x) \in K$, kažemo da je definirana funkcija f sa skupa D u skup K i pišemo

$$f : D \rightarrow K.$$

Skup D je domena funkcije ili područje definicije, a K kodomena funkcije ili područje vrijednosti funkcije. Ako je f funkcija iz R u R i ako je zadana formulom, podrazumijeva se da njezina domena obuhvaća sve realne brojeve za koje analitički izraz (formula) ima smisla. Ta domena zove se prirodno

područje definicije ili prirodna domena funkcije f.
Područje definicije (domena) funkcije $f(x)=\log_b x$ je interval realnih brojeva

$$D(f)=\langle 0, +\infty \rangle.$$

Uočimo da iz $\log x$ slijedi x > 0.
Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a \cdot n}{b \cdot n}=\frac{a}{b} \quad , \quad n \neq 0 \quad , \quad n \neq 1.$$

$$\log_2^2 x+2 \cdot \log_2 \sqrt{x}-2=0 \Rightarrow \log_2^2 x+2 \cdot \frac{1}{2} \cdot \log_2 x-2=0 \Rightarrow \log_2^2 x+2 \cdot \frac{1}{2} \cdot \log_2 x-2=0 \Rightarrow$$

$$\Rightarrow \log_2^2 x+\log_2 x-2=0 \Rightarrow \left[\begin{array}{c} \text{zamjena} \\ \log_2 x=t \end{array}\right] \Rightarrow t^2+t-2=0 \Rightarrow \left.\begin{array}{c} t^2+t-2=0 \\ a=1 \ , \ b=1 \ , \ c=-2 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{c} a=1 \ , \ b=1 \ , \ c=-2 \\ t_{1,2}=\dfrac{-b \pm \sqrt{b^2-4 \cdot a \cdot c}}{2 \cdot a} \end{array}\right\} \Rightarrow t_{1,2}=\frac{-1 \pm \sqrt{1^2-4 \cdot 1 \cdot (-2)}}{2 \cdot 1} \Rightarrow t_{1,2}=\frac{-1 \pm \sqrt{1+8}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{-1 \pm \sqrt{9}}{2} \Rightarrow t_{1,2}=\frac{-1 \pm 3}{2} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{-1+3}{2} \\ t_2=\dfrac{-1-3}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{2}{2} \\ t_2=-\dfrac{4}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{2}{2} \\ t_2=-\dfrac{4}{2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} t_1=1 \\ t_2=-2 \end{array}\right\}.$$

Vraćamo se na zamjenu.

- $\left.\begin{array}{l} \log_2 x=t \\ t=1 \end{array}\right\} \Rightarrow \log_2 x=1 \Rightarrow 2^1=x \Rightarrow x_1=2.$
- $\left.\begin{array}{l} \log_2 x=t \\ t=-2 \end{array}\right\} \Rightarrow \log_2 x=-2 \Rightarrow 2^{-2}=x \Rightarrow x=2^{-2} \Rightarrow x=\frac{1}{2^2} \Rightarrow x_2=\frac{1}{4}.$

Vježba 434

Riješi jednadžbu: $\log_2^2 x+2 \cdot \log_2 \sqrt{x}=2.$

Rezultat: $x_1=2, \ x_2=\frac{1}{4}.$

Zadatak 435 (4A, 4B, TUPŠ)

Riješi jednadžbu: $\log_3 x+\log_9 x=27.$

Rješenje 435

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=b^c \qquad a=b^c$$

Dekadski logaritam
Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \frac{a}{b} \cdot b = a \quad , \quad \log_b a = \frac{\log a}{\log b} \quad , \quad \log a^n = n \cdot \log a.$$

$$\log f(x) = \log g(x) \Rightarrow f(x) = g(x).$$

Neka su D i K dva neprazna skupa. Ako je svakom elementu $x \in D$ pridružen točno jedan element $f(x) \in K$, kažemo da je definirana funkcija f sa skupa D u skup K i pišemo

$$f : D \to K.$$

Skup D je domena funkcije ili područje definicije, a K kodomena funkcije ili područje vrijednosti funkcije. Ako je f funkcija iz R u R i ako je zadana formulom, podrazumijeva se da njezina domena obuhvaća sve realne brojeve za koje analitički izraz (formula) ima smisla. Ta domena zove se prirodno područje definicije ili prirodna domena funkcije f.
Područje definicije (domena) funkcije $f(x) = \log_b x$ je interval realnih brojeva

$$D(f) = \langle 0, +\infty \rangle.$$

Uočimo da iz $\log x$ slijedi $x > 0$.

1.inačica

$$\log_3 x + \log_9 x = 27 \Rightarrow \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \Rightarrow \log_3 x + \log_{3^2} x = 27 \Rightarrow \log_3 x + \frac{1}{2} \cdot \log_3 x = 27 \Rightarrow$$

$$\Rightarrow \log_3 x + \frac{1}{2} \cdot \log_3 x = 27 \ / \cdot 2 \Rightarrow 2 \cdot \log_3 x + \log_3 x = 54 \Rightarrow 3 \cdot \log_3 x = 54 \Rightarrow$$

$$\Rightarrow 3 \cdot \log_3 x = 54 \ /:3 \Rightarrow \log_3 x = 18 \Rightarrow 3^{18} = x \Rightarrow x = 3^{18}.$$

2.inačica

$$\log_3 x + \log_9 x = 27 \Rightarrow \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \Rightarrow \frac{\log x}{\log 3} + \frac{\log x}{\log 9} = 27 \Rightarrow \frac{\log x}{\log 3} + \frac{\log x}{\log 3^2} = 27 \Rightarrow$$

$$\Rightarrow \frac{\log x}{\log 3} + \frac{\log x}{2 \cdot \log 3} = 27 \Rightarrow \frac{\log x}{\log 3} + \frac{\log x}{2 \cdot \log 3} = 27 \ / \cdot 2 \cdot \log 3 \Rightarrow 2 \cdot \log x + \log x = 27 \cdot 2 \cdot \log 3 \Rightarrow$$

$$\Rightarrow 3 \cdot \log x = 54 \cdot \log 3 \Rightarrow 3 \cdot \log x = 54 \cdot \log 3 \ /:3 \Rightarrow \log x = 18 \cdot \log 3 \Rightarrow$$

$$\Rightarrow \log x = \log 3^{18} \Rightarrow x = 3^{18}.$$

Vježba 435

Riješi jednadžbu: $\log_3 x + \log_9 x - 27 = 0.$

Rezultat: 3^{18}.

Zadatak 436 (4A, 4B, TUPŠ)

Riješi jednadžbu: $\log_2 x - \log_{16} x = 3.$

Rješenje 436

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \underset{\rightarrow}{} \qquad a = b^c$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \frac{a}{b} \cdot b = a \quad , \quad \log_b a = \frac{\log a}{\log b} \quad , \quad \log a^n = n \cdot \log a.$$

$$\log f(x) = \log g(x) \;\Rightarrow\; f(x) = g(x).$$

Neka su D i K dva neprazna skupa. Ako je svakom elementu $x \in D$ pridružen točno jedan element $f(x) \in K$, kažemo da je definirana funkcija f sa skupa D u skup K i pišemo

$$f : D \rightarrow K.$$

Skup D je domena funkcije ili područje definicije, a K kodomena funkcije ili područje vrijednosti funkcije. Ako je f funkcija iz R u R i ako je zadana formulom, podrazumijeva se da njezina domena obuhvaća sve realne brojeve za koje analitički izraz (formula) ima smisla. Ta domena zove se prirodno područje definicije ili prirodna domena funkcije f.

Područje definicije (domena) funkcije $f(x) = \log_b x$ je interval realnih brojeva

$$D(f) = \langle 0, +\infty \rangle.$$

Uočimo da iz $\log x$ slijedi x > 0.

1.inačica

$$\log_2 x - \log_{16} x = 3 \;\Rightarrow\; \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \;\Rightarrow\; \log_2 x - \log_{2^4} x = 3 \;\Rightarrow\; \log_2 x - \frac{1}{4} \cdot \log_2 x = 3 \;\Rightarrow$$

$$\Rightarrow\; \log_2 x - \frac{1}{4} \cdot \log_2 x = 3 \;/\cdot 4 \;\Rightarrow\; 4 \cdot \log_2 x - \log_2 x = 12 \;\Rightarrow\; 3 \cdot \log_2 x = 12 \;\Rightarrow$$

$$\Rightarrow\; 3 \cdot \log_2 x = 12 \;/:3 \;\Rightarrow\; \log_2 x = 4 \;\Rightarrow\; 2^4 = x \;\Rightarrow\; x = 2^4 \;\Rightarrow\; x = 16.$$

2.inačica

$$\log_2 x - \log_{16} x = 3 \;\Rightarrow\; \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \;\Rightarrow\; \frac{\log x}{\log 2} - \frac{\log x}{\log 16} = 3 \;\Rightarrow\; \frac{\log x}{\log 2} - \frac{\log x}{\log 2^4} = 3 \;\Rightarrow$$

$$\Rightarrow\; \frac{\log x}{\log 2} - \frac{\log x}{4 \cdot \log 2} = 3 \;\Rightarrow\; \frac{\log x}{\log 2} - \frac{\log x}{4 \cdot \log 2} = 3 \;/\cdot 4 \cdot \log 2 \;\Rightarrow\; 4 \cdot \log x - \log x = 3 \cdot 4 \cdot \log 2 \;\Rightarrow$$

$$\Rightarrow\; 3 \cdot \log x = 12 \cdot \log 2 \;\Rightarrow\; 3 \cdot \log x = 12 \cdot \log 2 \;/:3 \;\Rightarrow\; \log x = 4 \cdot \log 2 \;\Rightarrow\; \log x = \log 2^4 \;\Rightarrow$$

$$\Rightarrow\; x = 2^4 \;\Rightarrow\; x = 16.$$

Vježba 436

Riješi jednadžbu: $\log_2 x - 3 = \log_{16} x.$

Rezultat: 16.

Zadatak 437 (4A, 4B, TUPŠ)

Riješi jednadžbu: $\log_4 x - \log_{32} x = \frac{3}{10}.$

Rješenje 437

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \qquad a = b^c$$
$$\rightarrow$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$\log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad a^1 = a \quad , \quad \log_b a = \frac{\log a}{\log b} \quad , \quad \log a^n = n \cdot \log a.$$

$$\log f(x) = \log g(x) \Rightarrow f(x) = g(x).$$

Neka su D i K dva neprazna skupa. Ako je svakom elementu $x \in D$ pridružen točno jedan element $f(x) \in K$, kažemo da je definirana funkcija f sa skupa D u skup K i pišemo

$$f : D \rightarrow K.$$

Skup D je domena funkcije ili područje definicije, a K kodomena funkcije ili područje vrijednosti funkcije. Ako je f funkcija iz R u R i ako je zadana formulom, podrazumijeva se da njezina domena obuhvaća sve realne brojeve za koje analitički izraz (formula) ima smisla. Ta domena zove se prirodno područje definicije ili prirodna domena funkcije f.
Područje definicije (domena) funkcije $f(x) = \log_b x$ je interval realnih brojeva

$$D(f) = \langle 0, +\infty \rangle.$$

Uočimo da iz $\log x$ slijedi x > 0.

1.inačica

$$\log_4 x - \log_{32} x = \frac{3}{10} \Rightarrow \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \Rightarrow \log_{2^2} x - \log_{2^5} x = \frac{3}{10} \Rightarrow \frac{1}{2} \cdot \log_2 x - \frac{1}{5} \cdot \log_2 x = \frac{3}{10} \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot \log_2 x - \frac{1}{5} \cdot \log_2 x = \frac{3}{10} \; / \cdot 10 \Rightarrow 5 \cdot \log_2 x - 2 \cdot \log_2 x = 3 \Rightarrow 3 \cdot \log_2 x = 3 \Rightarrow$$

$$\Rightarrow 3 \cdot \log_2 x = 3 \; / : 3 \Rightarrow \log_2 x = 1 \Rightarrow 2^1 = x \Rightarrow x = 2^1 \Rightarrow x = 2.$$

2.inačica

$$\log_4 x - \log_{32} x = \frac{3}{10} \Rightarrow \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \Rightarrow \frac{\log x}{\log 4} - \frac{\log x}{\log 32} = \frac{3}{10} \Rightarrow \frac{\log x}{\log 2^2} - \frac{\log x}{\log 2^5} = \frac{3}{10} \Rightarrow$$

$$\Rightarrow \frac{\log x}{2 \cdot \log 2} - \frac{\log x}{5 \cdot \log 2} = \frac{3}{10} \Rightarrow \frac{\log x}{2 \cdot \log 2} - \frac{\log x}{5 \cdot \log 2} = \frac{3}{10} \; / \cdot 10 \cdot \log 2 \Rightarrow$$

$$\Rightarrow 5 \cdot \log x - 2 \cdot \log x = 3 \cdot \log 2 \Rightarrow 3 \cdot \log x = 3 \cdot \log 2 \; / : 3 \Rightarrow \log x = \log 2 \Rightarrow x = 2.$$

Vježba 437

Riješi jednadžbu: $\log_4 x - \frac{3}{10} = \log_{32} x.$

Rezultat: 2.

Zadatak 438 (4A, 4B, TUPŠ)

Riješi jednadžbu: $\log_{\frac{1}{3}} x - \log_3 x = 4.$

Rješenje 438

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \qquad a = b^c$$
$$\rightarrow$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log x zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10} x = \log x.$$

$$a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a \quad , \quad \log_b a = \frac{\log a}{\log b} \quad , \quad \log a^n = n \cdot \log a.$$

$$\log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \log f(x) = \log g(x) \;\Rightarrow\; f(x) = g(x).$$

Neka su D i K dva neprazna skupa. Ako je svakom elementu $x \in D$ pridružen točno jedan element $f(x) \in K$, kažemo da je definirana funkcija f sa skupa D u skup K i pišemo

$$f : D \to K.$$

Skup D je domena funkcije ili područje definicije, a K kodomena funkcije ili područje vrijednosti funkcije. Ako je f funkcija iz R u R i ako je zadana formulom, podrazumijeva se da njezina domena obuhvaća sve realne brojeve za koje analitički izraz (formula) ima smisla. Ta domena zove se prirodno područje definicije ili prirodna domena funkcije f.

Područje definicije (domena) funkcije $f(x) = \log_b x$ je interval realnih brojeva

$$D(f) = \langle 0, +\infty \rangle.$$

Uočimo da iz $\log x$ slijedi $x > 0$.

1.inačica

$$\log_{\frac{1}{3}} x - \log_3 x = 4 \;\Rightarrow\; \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \;\Rightarrow\; \log_{3^{-1}} x - \log_3 x = 4 \;\Rightarrow\; \frac{1}{-1} \cdot \log_3 x - \log_3 x = 4 \;\Rightarrow$$

$$\Rightarrow\; -\log_3 x - \log_3 x = 4 \;\Rightarrow\; -2 \cdot \log_3 x = 4 \;\Rightarrow\; -2 \cdot \log_3 x = 4 \; /:(-2) \;\Rightarrow\; \log_3 x = -2 \;\Rightarrow$$

$$\Rightarrow\; 3^{-2} = x \;\Rightarrow\; x = 3^{-2} \;\Rightarrow\; x = \frac{1}{3^2} \;\Rightarrow\; x = \frac{1}{9}.$$

2.inačica

$$\log_{\frac{1}{3}} x - \log_3 x = 4 \;\Rightarrow\; \begin{bmatrix} \text{uvjet} \\ x > 0 \end{bmatrix} \;\Rightarrow\; \frac{\log x}{\log \frac{1}{3}} - \frac{\log x}{\log 3} = 4 \;\Rightarrow\; \frac{\log x}{\log 3^{-1}} - \frac{\log x}{\log 3} = 4 \;\Rightarrow$$

$$\Rightarrow\; \frac{\log x}{-\log 3} - \frac{\log x}{\log 3} = 4 \;\Rightarrow\; \frac{\log x}{-\log 3} - \frac{\log x}{\log 3} = 4 \; / \cdot (-\log 3) \;\Rightarrow\; \log x + \log x = -4 \cdot \log 3 \;\Rightarrow$$

$$\Rightarrow\; 2 \cdot \log x = -4 \cdot \log 3 \;\Rightarrow\; 2 \cdot \log x = -4 \cdot \log 3 \; /: 2 \;\Rightarrow\; \log x = -2 \cdot \log 3 \;\Rightarrow$$

$$\Rightarrow\; \log x = \log 3^{-2} \;\Rightarrow\; x = 3^{-2} \;\Rightarrow\; x = \frac{1}{3^2} \;\Rightarrow\; x = \frac{1}{9}.$$

Vježba 438

Riješi jednadžbu: $\log_{\frac{1}{3}} x - 4 = \log_3 x.$

Rezultat: $\frac{1}{9}$.

Zadatak 439 (Ana, gimnazija)

Riješi nejednadžbu: $5^{x}-3^{x+1}>2\cdot\left(5^{x-1}-3^{x-2}\right)$.

Rješenje 439

Ponovimo!

$$a^{n}\cdot a^{m}=a^{n+m}\quad,\quad a^{-n}=\frac{1}{a^{n}}\quad,\quad a^{1}=a\quad,\quad n=\frac{n}{1}\quad,\quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

$$\frac{a}{b}\cdot\frac{b}{a}=1\quad,\quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}\quad,\quad \frac{a^{n}}{b^{n}}=\left(\frac{a}{b}\right)^{n}.$$

$$a>b\ ,\ c>0\ \Rightarrow\ a\cdot c>b\cdot c\quad,\quad a^{f(x)}>a^{g(x)}\ ,\ a>1\ \Rightarrow\ f(x)>g(x).$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b}\ ,\ n\neq 0\ ,\ n\neq 1.$$

Zakon distribucije množenja prema zbrajanju.

$$a\cdot(b+c)=a\cdot b+a\cdot c\quad,\quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$5^{x}-3^{x+1}>2\cdot\left(5^{x-1}-3^{x-2}\right)\ \Rightarrow\ 5^{x}-3^{x+1}>2\cdot 5^{x-1}-2\cdot 3^{x-2}\ \Rightarrow$$

$$\Rightarrow\ 5^{x}-2\cdot 5^{x-1}>3^{x+1}-2\cdot 3^{x-2}\ \Rightarrow\ 5^{x}-2\cdot 5^{x}\cdot 5^{-1}>3^{x}\cdot 3^{1}-2\cdot 3^{x}\cdot 3^{-2}\ \Rightarrow$$

$$\Rightarrow\ 5^{x}-2\cdot 5^{x}\cdot\frac{1}{5}>3^{x}\cdot 3-2\cdot 3^{x}\cdot\frac{1}{3^{2}}\ \Rightarrow\ 5^{x}-\frac{2}{5}\cdot 5^{x}>3^{x}\cdot 3-2\cdot 3^{x}\cdot\frac{1}{9}\ \Rightarrow$$

$$\Rightarrow\ 5^{x}-\frac{2}{5}\cdot 5^{x}>3^{x}\cdot 3-\frac{2}{9}\cdot 3^{x}\ \Rightarrow\ 5^{x}\cdot\left(1-\frac{2}{5}\right)>3^{x}\cdot\left(3-\frac{2}{9}\right)\ \Rightarrow$$

$$\Rightarrow\ 5^{x}\cdot\left(\frac{1}{1}-\frac{2}{5}\right)>3^{x}\cdot\left(\frac{3}{1}-\frac{2}{9}\right)\ \Rightarrow\ 5^{x}\cdot\frac{5-2}{5}>3^{x}\cdot\frac{27-2}{9}\ \Rightarrow\ 5^{x}\cdot\frac{3}{5}>3^{x}\cdot\frac{25}{9}\ \Rightarrow$$

$$\Rightarrow\ 5^{x}\cdot\frac{3}{5}>3^{x}\cdot\frac{25}{9}\ /\cdot\frac{5}{3\cdot 3^{x}}\ \Rightarrow\ \frac{5^{x}}{3^{x}}>\frac{25}{9}\cdot\frac{5}{3}\ \Rightarrow\ \left(\frac{5}{3}\right)^{x}>\frac{125}{27}\ \Rightarrow$$

$$\Rightarrow\ \left(\frac{5}{3}\right)^{x}>\left(\frac{5}{3}\right)^{3}\ \Rightarrow\ x>3.$$

2.inačica

$$5^{x}-3^{x+1}>2\cdot\left(5^{x-1}-3^{x-2}\right)\ \Rightarrow\ 5^{x}-3^{x+1}>2\cdot 5^{x-1}-2\cdot 3^{x-2}\ \Rightarrow$$

$$\Rightarrow\ 5^{x}-2\cdot 5^{x-1}>3^{x+1}-2\cdot 3^{x-2}\ \Rightarrow$$

$$\Rightarrow\ 5^{x-2}\cdot 5^{2}-2\cdot 5^{x-2}\cdot 5^{1}>3^{x-2}\cdot 3^{3}-2\cdot 3^{x-2}\ \Rightarrow$$

$$\Rightarrow\ 5^{x-2}\cdot 5^{2}-2\cdot 5^{x-2}\cdot 5>3^{x-2}\cdot 3^{3}-2\cdot 3^{x-2}\ \Rightarrow$$

$$\Rightarrow 5^{x-2}\cdot\left(5^2-2\cdot 5\right)>3^{x-2}\cdot\left(3^3-2\right) \Rightarrow 5^{x-2}\cdot(25-10)>3^{x-2}\cdot(27-2) \Rightarrow$$

$$\Rightarrow 5^{x-2}\cdot 15>3^{x-2}\cdot 25 \Rightarrow 5^{x-2}\cdot 15>3^{x-2}\cdot 25 \;/\cdot\frac{1}{15\cdot 3^{x-2}} \Rightarrow$$

$$\Rightarrow \frac{5^{x-2}}{3^{x-2}}>\frac{25}{15} \Rightarrow \left(\frac{5}{3}\right)^{x-2}>\frac{25}{15} \Rightarrow \left(\frac{5}{3}\right)^{x-2}>\frac{5}{3} \Rightarrow \left(\frac{5}{3}\right)^{x-2}>\left(\frac{5}{3}\right)^{1} \Rightarrow$$

$$\Rightarrow x-2>1 \Rightarrow x>1+2 \Rightarrow x>3.$$

Vježba 439

Riješi nejednadžbu: $5^x-3\cdot 3^x>2\cdot\left(5^{x-1}-3^{x-2}\right)$.

Rezultat: x > 3.

Zadatak 440 (Sonja, strojarska škola)

Riješi jednadžbu: $4^{-\frac{1}{x}}+6^{-\frac{1}{x}}=9^{-\frac{1}{x}}$.

Rješenje 440

Ponovimo!

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^1=a \quad , \quad n=\frac{n}{1}.$$

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad \frac{a}{b}=\frac{c}{d} \Rightarrow \frac{b}{a}=\frac{d}{c}.$$

Skratiti razlomak znači brojnik i nazivnik tog razlomka podijeliti istim brojem različitim od nule i jedinice

$$\frac{a\cdot n}{b\cdot n}=\frac{a}{b} \;,\; n\neq 0 \;,\; n\neq 1.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=b^c \qquad a=b^c$$

Dekadski logaritam

Logaritamska funkcija $\log_{10}$ označava se simbolom log. Broj log *x* zovemo dekadski, Briggsov ili obični logaritam.

$$\log_{10}x=\log x.$$

$$\log a^n=n\cdot\log a \quad , \quad \log\frac{a}{b}=\log a-\log b.$$

$$4^{-\frac{1}{x}}+6^{-\frac{1}{x}}=9^{-\frac{1}{x}} \Rightarrow \left[\begin{array}{c}\text{zamjena}\\ -\frac{1}{x}=y\end{array}\right] \Rightarrow 4^y+6^y=9^y \Rightarrow 4^y+6^y=9^y \;/\cdot\frac{1}{6^y} \Rightarrow$$

$$\Rightarrow \frac{4^y}{6^y}+1=\frac{9^y}{6^y} \Rightarrow \left(\frac{4}{6}\right)^y+1=\left(\frac{9}{6}\right)^y \Rightarrow \left(\frac{4}{6}\right)^y+1=\left(\frac{9}{6}\right)^y \Rightarrow \left(\frac{2}{3}\right)^y+1=\left(\frac{3}{2}\right)^y \Rightarrow$$

$$\Rightarrow \left(\frac{2}{3}\right)^y+1=\left(\left(\frac{2}{3}\right)^{-1}\right)^y \Rightarrow \left(\frac{2}{3}\right)^y+1=\left(\frac{2}{3}\right)^{-y} \Rightarrow \left(\frac{2}{3}\right)^y+1=\frac{1}{\left(\frac{2}{3}\right)^y} \Rightarrow \left[\begin{array}{l}\text{zamjena}\\ \left(\frac{2}{3}\right)^y=t\end{array}\right] \Rightarrow$$

$$\Rightarrow t+1=\frac{1}{t} \Rightarrow t+1=\frac{1}{t}\ /\cdot t \Rightarrow t^2+t=1 \Rightarrow t^2+t-1=0 \Rightarrow \left.\begin{array}{l} t^2+t-1=0\\ a=1\ ,\ b=1\ ,\ c=-1\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=1\ ,\ b=1\ ,\ c=-1\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\} \Rightarrow t_{1,2}=\frac{-1\pm\sqrt{1^2-4\cdot 1\cdot(-1)}}{2\cdot 1} \Rightarrow t_{1,2}=\frac{-1\pm\sqrt{1+4}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{-1\pm\sqrt{5}}{2} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{-1+\sqrt{5}}{2}\\ t_2=\dfrac{-1-\sqrt{5}}{2}\end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{\sqrt{5}-1}{2}\\ t_2=-\dfrac{\sqrt{5}+1}{2}\end{array}\right\}.$$

Vraćamo se na posljednju zamjenu.

- $$\left.\begin{array}{l} \left(\frac{2}{3}\right)^y=t\\ t=\dfrac{\sqrt{5}-1}{2}\end{array}\right\} \Rightarrow \left(\frac{2}{3}\right)^y=\frac{\sqrt{5}-1}{2} \Rightarrow \left[\begin{array}{l}\text{logaritmiramo}\\ \text{jednadžbu}\end{array}\right] \Rightarrow \left(\frac{2}{3}\right)^y=\frac{\sqrt{5}-1}{2}\ /\log \Rightarrow$$

$$\Rightarrow \log\left(\frac{2}{3}\right)^y=\log\frac{\sqrt{5}-1}{2} \Rightarrow y\cdot\log\left(\frac{2}{3}\right)=\log\frac{\sqrt{5}-1}{2} \Rightarrow y\cdot\log\left(\frac{2}{3}\right)=\log\frac{\sqrt{5}-1}{2}\ /\cdot\frac{1}{\log\left(\frac{2}{3}\right)} \Rightarrow$$

$$\Rightarrow y=\frac{\log\frac{\sqrt{5}-1}{2}}{\log\left(\frac{2}{3}\right)} \Rightarrow y=\frac{\log\left(\sqrt{5}-1\right)-\log 2}{\log 2-\log 3}$$

- $$\left.\begin{array}{l} \left(\frac{2}{3}\right)^y=t\\ t=-\dfrac{\sqrt{5}+1}{2}\end{array}\right\} \Rightarrow \left(\frac{2}{3}\right)^y=-\frac{\sqrt{5}+1}{2}\quad \text{nema smisla}.$$

Sada se vraćamo na prvu zamjenu.

$$\left.\begin{array}{l} -\dfrac{1}{x}=y\\ y=\dfrac{\log\left(\sqrt{5}-1\right)-\log 2}{\log 2-\log 3}\end{array}\right\} \Rightarrow -\frac{1}{x}=\frac{\log\left(\sqrt{5}-1\right)-\log 2}{\log 2-\log 3} \Rightarrow$$

$$\Rightarrow -\frac{1}{x}=\frac{\log\left(\sqrt{5}-1\right)-\log 2}{\log 2-\log 3}\ /\cdot(-1) \Rightarrow \frac{1}{x}=\frac{\log\left(\sqrt{5}-1\right)-\log 2}{-\left(\log 2-\log 3\right)} \Rightarrow$$

$$\Rightarrow \frac{1}{x}=\frac{\log\left(\sqrt{5}-1\right)-\log 2}{\log 3-\log 2} \Rightarrow \frac{x}{1}=\frac{\log 3-\log 2}{\log\left(\sqrt{5}-1\right)-\log 2} \Rightarrow x=\frac{\log 3-\log 2}{\log\left(\sqrt{5}-1\right)-\log 2}.$$

Vježba 440

Riješi jednadžbu: $9^{-\frac{1}{x}}-6^{-\frac{1}{x}}-4^{-\frac{1}{x}}=0.$

Rezultat: $x=\frac{\log 3-\log 2}{\log\left(\sqrt{5}-1\right)-\log 2}.$