Zadatak 241 (Ivan, gimnazija)

Skrati razlomak: $\frac{1-\log^2 x}{\log(10\cdot x)}$.

$A.\ 1+\log x \qquad B.\ \log x-1 \qquad C.\ \log\frac{x}{10} \qquad D.\ \log\frac{10}{x}$

Rješenje 241

Ponovimo!

$$\log_{10} a=\log a \quad , \quad \log 10=1 \quad , \quad \log(a\cdot b)=\log a+\log b \quad , \quad \log\frac{a}{b}=\log a-\log b.$$

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad n=\frac{n}{1}.$$

$$\frac{1-\log^2 x}{\log(10\cdot x)}=\frac{(1-\log x)\cdot(1+\log x)}{\log 10+\log x}=\frac{(1-\log x)\cdot(1+\log x)}{1+\log x}=\frac{(1-\log x)\cdot(1+\log x)}{1+\log x}=\frac{1-\log x}{1}=1-\log=$$

$$=\log 10-\log x=\log\frac{10}{x}.$$

Odgovor je pod D.

Vježba 241

Skrati razlomak: $\frac{4-\log^2 x}{\log(100\cdot x)}$.

$A.\ 2+\log x \qquad B.\ \log x-2 \qquad C.\ \log\frac{x}{100} \qquad D.\ \log\frac{100}{x}$

Rezultat: D.

Zadatak 242 (Maturantica, gimnazija)

Za koji prirodni broj n vrijedi jednakost $3^n+3^n+3^n=3^4$?

Rješenje 242

Ponovimo!

$$\underbrace{a+a+a+\ ...\ +a}_{\text{n-puta}}=n\cdot a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^1=a \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$3^n+3^n+3^n=3^4 \Rightarrow 3\cdot 3^n=3^4 \Rightarrow 3^1\cdot 3^n=3^4 \Rightarrow 3^{1+n}=3^4 \Rightarrow 1+n=4 \Rightarrow n=4-1 \Rightarrow n=3.$$

Vježba 242

Za koji prirodni broj n vrijedi jednakost $4^n+4^n+4^n+4^n=4^4$?

Rezultat: n = 3.

Zadatak 243 (Maturantica, gimnazija)

Provjeri da vrijednost sljedećeg razlomka ne ovisi o vrijednosti prirodnog broja n.

$$\frac{27^{n+3}\cdot 9^{n+4}}{3^{n+1}\cdot 81^{n+2}}.$$

Rješenje 243

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^n\cdot b^n=(a\cdot b)^n \quad , \quad \frac{a^n}{a^m}=a^{n-m}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\frac{27^{n+3}\cdot 9^{n+4}}{3^{n+1}\cdot 81^{n+2}}=\frac{\left(3^3\right)^{n+3}\cdot\left(3^2\right)^{n+4}}{3^{n+1}\cdot\left(3^4\right)^{n+2}}=\frac{3^{3\cdot(n+3)}\cdot 3^{2\cdot(n+4)}}{3^{n+1}\cdot 3^{4\cdot(n+2)}}=\frac{3^{3\cdot n+9}\cdot 3^{2\cdot n+8}}{3^{n+1}\cdot 3^{4\cdot n+8}}=\frac{3^{3\cdot n+9+2\cdot n+8}}{3^{n+1+4\cdot n+8}}=$$

$$=\frac{3^{5\cdot n+17}}{3^{5\cdot n+9}}=\frac{3^{5\cdot n+9}\cdot 3^8}{3^{5\cdot n+9}}=\frac{3^{5\cdot n+9}\cdot 3^8}{3^{5\cdot n+9}}=3^8.$$

2.inačica

$$\frac{27^{n+3}\cdot 9^{n+4}}{3^{n+1}\cdot 81^{n+2}}=\frac{27^n\cdot 27^3\cdot 9^n\cdot 9^4}{3^n\cdot 3^1\cdot 81^n\cdot 81^2}=\frac{(27\cdot 9)^n\cdot 27^3\cdot 9^4}{(3\cdot 81)^n\cdot 3^1\cdot 81^2}=\frac{243^n\cdot 27^3\cdot 9^4}{243^n\cdot 3^1\cdot 81^2}=\frac{243^n\cdot 27^3\cdot 9^4}{243^n\cdot 3^1\cdot 81^2}=$$

$$=\frac{27^3\cdot 9^4}{3^1\cdot 81^2}=\frac{\left(3^3\right)^3\cdot\left(3^2\right)^4}{3^1\cdot\left(3^4\right)^2}=\frac{3^9\cdot 3^8}{3^1\cdot 3^8}=\frac{3^9\cdot 3^8}{3^1\cdot 3^8}=\frac{3^9}{3^1}=3^{9-1}=3^8.$$

Vježba 243

Provjeri da vrijednost sljedećeg razlomka ne ovisi o vrijednosti prirodnog broja n.

$$\frac{8^{n+3}\cdot 4^{n+4}}{2^{n+1}\cdot 16^{n+2}}.$$

Rezultat: 2^8.

Zadatak 244 (Bole, gimnazija)

Koliki je zbroj rješenja jednadžbe $3\cdot 4^x+2\cdot 9^x=5\cdot 6^x$.

Rješenje 244

Ponovimo!

$$\frac{a^n}{b^n}=\left(\frac{a}{b}\right)^n\quad,\quad\left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n\quad,\quad a^{-n}=\frac{1}{a^n}\quad,\quad a^0=1\quad,\quad a^1=a.$$

$$a^{f(x)}=a^{g(x)}\;\Rightarrow\; f(x)=g(x)\quad,\quad\left(a^n\right)^m=a^{n\cdot m}.$$

$$3\cdot 4^x+2\cdot 9^x=5\cdot 6^x\;\Rightarrow\;3\cdot 4^x+2\cdot 9^x=5\cdot 6^x\;/:6^x\;\Rightarrow\;3\cdot\frac{4^x}{6^x}+2\cdot\frac{9^x}{6^x}=5\;\Rightarrow$$

$$\Rightarrow\;3\cdot\left(\frac{4}{6}\right)^x+2\cdot\left(\frac{9}{6}\right)^x=5\;\Rightarrow\;3\cdot\left(\frac{4}{6}\right)^x+2\cdot\left(\frac{9}{6}\right)^x=5\;\Rightarrow\;3\cdot\left(\frac{2}{3}\right)^x+2\cdot\left(\frac{3}{2}\right)^x=5\;\Rightarrow$$

$$\Rightarrow\;3\cdot\left(\frac{2}{3}\right)^x+2\cdot\left(\frac{2}{3}\right)^{-x}=5\;\Rightarrow\;\left[\begin{array}{c}\text{supstitucija}\\ t=\left(\frac{2}{3}\right)^x\\ t^{-1}=\left(\frac{2}{3}\right)^{-x}\end{array}\right]\;\Rightarrow\;3\cdot t+2\cdot t^{-1}=5\;\Rightarrow\;3\cdot t+2\cdot\frac{1}{t}=5\;\Rightarrow$$

$$\Rightarrow\;3\cdot t+2\cdot\frac{1}{t}=5\;/\cdot t\;\Rightarrow\;3\cdot t^2+2=5\cdot t\;\Rightarrow\;3\cdot t^2-5\cdot t+2=0\;\Rightarrow\;\left.\begin{array}{l}3\cdot t^2-5\cdot t+2=0\\ a=3\;,\;b=-5\;,\;c=2\end{array}\right\}\;\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=3 \;,\; b=-5 \;,\; c=2 \\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow t_{1,2}=\frac{-(-5)\pm\sqrt{(-5)^2-4\cdot 3\cdot 2}}{2\cdot 3} \Rightarrow t_{1,2}=\frac{5\pm\sqrt{25-24}}{6} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{5\pm\sqrt{1}}{6} \Rightarrow t_{1,2}=\frac{5\pm 1}{6} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{5+1}{6} \\ t_2=\dfrac{5-1}{6} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=\dfrac{6}{6} \\ t_2=\dfrac{4}{6} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_1=1 \\ t_2=\dfrac{2}{3} \end{array}\right\}.$$

Vraćamo se supstituciji.

- $$\left.\begin{array}{l} t=\left(\dfrac{2}{3}\right)^x \\ t=1 \end{array}\right\} \Rightarrow \left(\frac{2}{3}\right)^x=1 \Rightarrow \left(\frac{2}{3}\right)^x=\left(\frac{2}{3}\right)^0 \Rightarrow x_1=0.$$
- $$\left.\begin{array}{l} t=\left(\dfrac{2}{3}\right)^x \\ t=\dfrac{2}{3} \end{array}\right\} \Rightarrow \left(\frac{2}{3}\right)^x=\frac{2}{3} \Rightarrow \left(\frac{2}{3}\right)^x=\left(\frac{2}{3}\right)^1 \Rightarrow x_2=1.$$

Zbroj rješenja jednadžbe iznosi:

$$x_1+x_2=0+1=1.$$

Vježba 244

Koliki je umnožak rješenja jednadžbe $3\cdot 4^x+2\cdot 9^x=5\cdot 6^x$.

Rezultat: 0.

Zadatak 245 (Građevinac, tehnička škola)

Izračunajte : $\log_2 5\cdot\log_{25} 8.$

Rješenje 245

Ponovimo!

$$\log_{b^n} a=\frac{1}{n}\cdot\log_b a \quad , \quad \log_b a^n=n\cdot\log_b a \quad , \quad \log_b a=\frac{1}{\log_a b} \quad , \quad \log_b a=\frac{\log_c a}{\log_c b}.$$

1.inačica

$$\log_2 5\cdot\log_{25} 8=\log_2 5\cdot\log_{5^2} 8=\log_2 5\cdot\frac{1}{2}\cdot\log_5 8=\log_2 5\cdot\frac{1}{2}\cdot\log_5 2^3=\log_2 5\cdot\frac{1}{2}\cdot 3\cdot\log_5 2=$$

$$=\frac{3}{2}\cdot\log_2 5\cdot\log_5 2=\frac{3}{2}\cdot\log_2 5\cdot\frac{1}{\log_2 5}=\frac{3}{2}\cdot\log_2 5\cdot\frac{1}{\log_2 5}=\frac{3}{2}.$$

2.inačica

$$\log_2 5\cdot\log_{25} 8=\frac{\log 5}{\log 2}\cdot\frac{\log 8}{\log 25}=\frac{\log 5}{\log 2}\cdot\frac{\log 2^3}{\log 5^2}=\frac{\log 5}{\log 2}\cdot\frac{3\cdot\log 2}{2\cdot\log 5}=\frac{\log 5}{\log 2}\cdot\frac{3\cdot\log 2}{2\cdot\log 5}=\frac{3}{2}.$$

Vježba 245

Izračunajte : $\log_2 3\cdot\log_9 8.$

Rezultat: $\frac{3}{2}.$

Zadatak 246 (Marija, srednja škola)

Ako je $\log_2 12 = a$, $\log_3 2 = b$, onda je $a - \frac{1}{b}$ jednako:

$A.\ 2 \qquad B.\ 3 \qquad C.\ 4 \qquad D.\ 5$

Rješenje 246

Ponovimo!

$$\log_b a = \frac{1}{\log_a b} \quad , \quad \log_b \frac{x}{y} = \log_b x - \log_b y \quad , \quad \log_b a^n = n \cdot \log_b a \quad , \quad \log_b b = 1.$$

$$\log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log_{10} a = \log a \quad , \quad \log_b a = \frac{\log a}{\log b} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c}.$$

$$n = \frac{n}{1} \quad , \quad \frac{a}{n} - \frac{b}{n} = \frac{a-b}{n} \quad , \quad \log_b (x \cdot y) = \log_b x + \log_b y.$$

1.inačica

$$\left.\begin{array}{l} \log_2 12 = a \\ \log_3 2 = b \end{array}\right\} \Rightarrow a - \frac{1}{b} = \log_2 12 - \frac{1}{\log_3 2} = \log_2 12 - \log_2 3 = \log_2 \frac{12}{3} = \log_2 4 =$$

$$= \log_2 2^2 = 2 \cdot \log_2 2 = 2 \cdot 1 = 2.$$

Odgovor je pod A.

2.inačica

$$\left.\begin{array}{l} \log_2 12 = a \\ \log_3 2 = b \end{array}\right\} \Rightarrow a - \frac{1}{b} = \log_2 12 - \frac{1}{\log_3 2} = \log_2 12 - \log_2 3 = \frac{\log 12}{\log 2} - \frac{\log 3}{\log 2} =$$

$$= \frac{\log 12 - \log 3}{\log 2} = \frac{\log \frac{12}{3}}{\log 2} = \frac{\log 4}{\log 2} = \frac{\log 2^2}{\log 2} = \frac{2 \cdot \log 2}{\log 2} = \frac{2 \cdot \log 2}{\log 2} = 2.$$

Odgovor je pod A.

3.inačica

$$\left.\begin{array}{l} \log_2 12 = a \\ \log_3 2 = b \end{array}\right\} \Rightarrow a - \frac{1}{b} = \log_2 12 - \frac{1}{\log_3 2} = \log_2 12 - \log_2 3 = \log_2 (4 \cdot 3) - \log_2 3 =$$

$$= \log_2 4 + \log_2 3 - \log_2 3 = \log_2 4 + \log_2 3 - \log_2 3 = \log_2 4 = \log_2 2^2 = 2 \cdot \log_2 2 = 2 \cdot 1 = 2.$$

Odgovor je pod A.

Vježba 246

Ako je $\log_2 12 = a$, $\log_3 2 = b$, onda je $\frac{1}{b} - a$ jednako:

$A.\ -2 \qquad B.\ -3 \qquad C.\ -4 \qquad D.\ -5$

Rezultat: A.

Zadatak 247 (Dizzy, gimnazija)

Riješi sustav: $\begin{cases} 2^x \cdot 3^y = 12 \\ 2^y \cdot 3^x = 18. \end{cases}$

Rješenje 247

Ponovimo!

$$\frac{a^m}{a^n} = a^{m-n} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad a^1 = a \quad , \quad a \cdot \frac{b}{c} = \frac{a \cdot b}{c}.$$

$$a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x) \quad , \quad a^n \cdot b^n = (a \cdot b)^n \quad , \quad a^n \cdot a^m = a^{n+m}.$$

Zakon distribucije množenja prema zbrajanju

$$a \cdot (b+c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b+c).$$

1.inačica

Podijelimo jednadžbe.

$$\left.\begin{matrix} 2^x \cdot 3^y = 12 \\ 2^y \cdot 3^x = 18 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{podijelimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow \frac{2^x \cdot 3^y}{2^y \cdot 3^x} = \frac{12}{18} \Rightarrow \frac{2^x \cdot 3^y}{2^y \cdot 3^x} = \frac{12}{18} \Rightarrow \frac{2^x \cdot 3^y}{2^y \cdot 3^x} = \frac{2}{3} \Rightarrow$$

$$\Rightarrow \frac{2^x}{2^y} \cdot \frac{3^y}{3^x} = \frac{2}{3} \Rightarrow 2^{x-y} \cdot 3^{y-x} = \frac{2}{3} \Rightarrow 2^{x-y} \cdot 3^{-(x-y)} = \frac{2}{3} \Rightarrow \frac{2^{x-y}}{3^{x-y}} = \frac{2}{3} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{3}\right)^{x-y} = \frac{2}{3} \Rightarrow \left(\frac{2}{3}\right)^{x-y} = \left(\frac{2}{3}\right)^1 \Rightarrow x - y = 1.$$

Iz sustava linearne i eksponencijalne jednadžbe izračunamo nepoznanice x i y.

$$\left.\begin{matrix} x - y = 1 \\ 2^x \cdot 3^y = 12 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = 1 + y \\ 2^x \cdot 3^y = 12 \end{matrix}\right\} \Rightarrow 2^{1+y} \cdot 3^y = 12 \Rightarrow 2^1 \cdot 2^y \cdot 3^y = 12 \Rightarrow 2 \cdot 2^y \cdot 3^y = 12 \Rightarrow$$

$$\Rightarrow 2 \cdot 2^y \cdot 3^y = 12 \; /:2 \Rightarrow 2^y \cdot 3^y = 6 \Rightarrow (2 \cdot 3)^y = 6 \Rightarrow 6^y = 6 \Rightarrow 6^y = 6^1 \Rightarrow y = 1 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} y = 1 \\ x = 1 + y \end{matrix}\right\} \Rightarrow x = 1 + 1 \Rightarrow x = 2.$$

Rješenje je

$$(x, \; y) = (2, 1).$$

2.inačica

Pomnožimo jednadžbe.

$$\left.\begin{matrix} 2^x \cdot 3^y = 12 \\ 2^y \cdot 3^x = 18 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{pomnožimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow 2^x \cdot 3^y \cdot 2^y \cdot 3^x = 12 \cdot 18 \Rightarrow 2^{x+y} \cdot 3^{x+y} = 12 \cdot 18 \Rightarrow$$

$$\Rightarrow (2 \cdot 3)^{x+y} = 2 \cdot 6 \cdot 3 \cdot 6 \Rightarrow 6^{x+y} = 6 \cdot 6 \cdot 6 \Rightarrow 6^{x+y} = 6^3 \Rightarrow x + y = 3.$$

Iz sustava linearne i eksponencijalne jednadžbe izračunamo nepoznanice x i y.

$$\left.\begin{matrix} x + y = 3 \\ 2^x \cdot 3^y = 12 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} y = 3 - x \\ 2^x \cdot 3^y = 12 \end{matrix}\right\} \Rightarrow 2^x \cdot 3^{3-x} = 12 \Rightarrow 2^x \cdot 3^3 \cdot 3^{-x} = 12 \Rightarrow 2^x \cdot 27 \cdot \frac{1}{3^x} = 12 \Rightarrow$$

$$\Rightarrow 27\cdot\frac{2^x}{3^x}=12 \Rightarrow 27\cdot\left(\frac{2}{3}\right)^x=12 \Rightarrow 27\cdot\left(\frac{2}{3}\right)^x=12 \;/:27 \Rightarrow \left(\frac{2}{3}\right)^x=\frac{12}{27} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{3}\right)^x=\frac{12}{27} \Rightarrow \left(\frac{2}{3}\right)^x=\frac{4}{9} \Rightarrow \left(\frac{2}{3}\right)^x=\left(\frac{2}{3}\right)^2 \Rightarrow x=2 \Rightarrow \left.\begin{matrix} x=2 \\ x+y=3 \end{matrix}\right\} \Rightarrow 2+y=3 \Rightarrow$$

$$\Rightarrow y=3-2 \Rightarrow y=1.$$

Rješenje je

$$(x,\ y)=(2,\ 1).$$

Vježba 247

Riješi sustav: $\begin{cases} 2^x\cdot 3^y=6 \\ 2^y\cdot 3^x=6. \end{cases}$

Rezultat: $(x,\ y)=(1,\ 1).$

Zadatak 248 (Ana, gimnazija)

Riješi sustav jednadžbi: $\begin{cases} \log_5(8\cdot x)=1+\log_5 4 \\ x^y=\frac{2}{5}. \end{cases}$

Rješenje 248

Ponovimo!

$$\log_b b=1 \quad , \quad \log_b(x\cdot y)=\log_b x+\log_b y \quad , \quad \log_b f(x)=\log_b g(x) \Rightarrow f(x)=g(x).$$

$$a^1=a \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad a^{-n}=\frac{1}{a^n}.$$

$$\log_b\frac{x}{y}=\log_b x-\log_b y \quad , \quad \log_b a^n=n\cdot\log_b a.$$

1.inačica

Iz prve jednadžbe sustava nađemo x.

$$\log_5(8\cdot x)=1+\log_5 4 \Rightarrow \log_5(8\cdot x)=\log_5 5+\log_5 4 \Rightarrow \log_5(8\cdot x)=\log_5(5\cdot 4) \Rightarrow$$

$$\Rightarrow \log_5(8\cdot x)=\log_5 20 \Rightarrow 8\cdot x=20 \Rightarrow 8\cdot x=20 \;/:8 \Rightarrow x=\frac{20}{8} \Rightarrow x=\frac{5}{2}.$$

Sada računamo y.

$$\left.\begin{matrix} x^y=\frac{2}{5} \\ x=\frac{5}{2} \end{matrix}\right\} \Rightarrow \left[\begin{matrix}\text{metoda} \\ \text{supstitucije}\end{matrix}\right] \Rightarrow \left(\frac{5}{2}\right)^y=\frac{2}{5} \Rightarrow \left(\frac{5}{2}\right)^y=\left(\frac{5}{2}\right)^{-1} \Rightarrow y=-1.$$

Rješenje sustava je

$$(x,\ y)=\left(\frac{5}{2},\ -1\right).$$

2.inačica

$$\left.\begin{matrix} \log_5(8\cdot x)=1+\log_5 4 \\ x^y=\frac{2}{5} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_5(8\cdot x)=\log_5 5+\log_5 4 \\ x^y=\frac{2}{5} \;/\log_5 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_5(8\cdot x)=\log_5(5\cdot 4) \\ \log_5 x^y=\log_5\frac{2}{5} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \log_5(8\cdot x)=\log_5 20 \\ y\cdot\log_5 x=\log_5\frac{2}{5}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_5 8+\log_5 x=\log_5 20 \\ y\cdot\log_5 x=\log_5\frac{2}{5}\ /\cdot\frac{1}{y}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_5 8+\log_5 x=\log_5 20 \\ \log_5 x=\frac{1}{y}\cdot\log_5\frac{2}{5}\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{metoda} \\ \text{supstitucije}\end{bmatrix} \Rightarrow \log_5 8+\frac{1}{y}\cdot\log_5\frac{2}{5}=\log_5 20 \Rightarrow \frac{1}{y}\cdot\log_5\frac{2}{5}=\log_5 20-\log_5 8 \Rightarrow$$

$$\Rightarrow \frac{1}{y}\cdot\log_5\frac{2}{5}=\log_5\frac{20}{8} \Rightarrow \frac{1}{y}\cdot\log_5\frac{2}{5}=\log_5\frac{5}{2} \Rightarrow \frac{1}{y}\cdot\log_5\frac{2}{5}=\log_5\left(\frac{2}{5}\right)^{-1} \Rightarrow$$

$$\Rightarrow \frac{1}{y}\cdot\log_5\frac{2}{5}=-1\cdot\log_5\frac{2}{5} \Rightarrow \frac{1}{y}\cdot\log_5\frac{2}{5}=-1\cdot\log_5\frac{2}{5}\ /\cdot\frac{1}{\log_5\frac{2}{5}} \Rightarrow \frac{1}{y}=-1 \Rightarrow y=-1.$$

Sada računamo x.

$$\left.\begin{matrix} x^y=\frac{2}{5} \\ y=-1\end{matrix}\right\} \Rightarrow \begin{bmatrix}\text{metoda} \\ \text{supstitucije}\end{bmatrix} \Rightarrow x^{-1}=\frac{2}{5} \Rightarrow \frac{1}{x}=\frac{2}{5} \Rightarrow 2\cdot x=5 \Rightarrow 2\cdot x=5\ /:2 \Rightarrow x=\frac{5}{2}.$$

Rješenje sustava je

$$(x,\ y)=\left(\frac{5}{2},\ -1\right).$$

Vježba 248

Riješi sustav jednadžbi: $\begin{cases} \log_5(8\cdot x)=1+\log_5 4 \\ x^y=\frac{5}{2}.\end{cases}$

Rezultat: $(x,\ y)=\left(\frac{5}{2},\ 1\right).$

Zadatak 249 (Marija, gimnazija)

Riješi sustav jednadžbi: $\begin{cases} \log x=1+\frac{1}{2}\cdot\log y \\ \log y=1+\log z \\ \log z=1-2\cdot\log x.\end{cases}$

Rješenje 249

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a=c \qquad \log_b a=b^c \ (\rightarrow) \qquad a=b^c$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\log_{10} x=\log x \quad , \quad \log 10=1 \quad , \quad \log 1=0 \quad , \quad \log\sqrt[n]{a}=\frac{1}{n}\cdot\log a \quad , \quad \log a^n=n\cdot\log a.$$

$$\log(a\cdot b)=\log a+\log b \quad , \quad \log\frac{a}{b}=\log a-\log b \quad , \quad \log f(x)=\log g(x) \Rightarrow f(x)=g(x).$$

$$\sqrt{a^2}=a\ ,\ a\ge 0 \quad , \quad a^0=1 \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}\cdot\frac{c}{d}=\frac{a\cdot c}{b\cdot d}.$$

1.inačica

$$\left.\begin{array}{l}\log x=1+\frac{1}{2}\cdot\log y\\ \log y=1+\log z\\ \log z=1-2\cdot\log x\end{array}\right\} \Rightarrow \begin{bmatrix}\text{metoda}\\ \text{supstitucije}\end{bmatrix} \Rightarrow \left.\begin{array}{l}\log x=1+\frac{1}{2}\cdot\log y\\ \log y=1+1-2\cdot\log x\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log x=1+\frac{1}{2}\cdot\log y\\ \log y=2-2\cdot\log x\end{array}\right\} \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{metoda}\\ \text{supstitucije}\end{bmatrix} \Rightarrow \log x=1+\frac{1}{2}\cdot(2-2\cdot\log x) \Rightarrow \log x=1+1-\log x \Rightarrow \log x+\log x=1+1 \Rightarrow$$

$$\Rightarrow 2\cdot\log x=2 \Rightarrow 2\cdot\log x=2\ /:2 \Rightarrow \log x=1 \Rightarrow \log x=\log 10 \Rightarrow x=10.$$

Računamo z.

$$\left.\begin{array}{l}\log z=1-2\cdot\log x\\ x=10\end{array}\right\} \Rightarrow \begin{bmatrix}\text{metoda}\\ \text{supstitucije}\end{bmatrix} \Rightarrow \log z=1-2\cdot\log 10 \Rightarrow \log z=1-2\cdot 1 \Rightarrow \log z=1-2 \Rightarrow$$

$$\Rightarrow \log z=-1 \Rightarrow \log z=\log 10^{-1} \Rightarrow z=10^{-1} \Rightarrow z=\frac{1}{10}.$$

Računamo y.

$$\left.\begin{array}{l}\log y=1+\log z\\ z=\frac{1}{10}\end{array}\right\} \Rightarrow \begin{bmatrix}\text{metoda}\\ \text{supstitucije}\end{bmatrix} \Rightarrow \log y=1+\log\frac{1}{10} \Rightarrow \log y=1+\log 10^{-1} \Rightarrow$$

$$\Rightarrow \log y=1-1\cdot\log 10 \Rightarrow \log y=1-1\cdot 1 \Rightarrow \log y=1-1 \Rightarrow \log y=0 \Rightarrow y=1.$$

Rješenje sustava je

$$(x,\ y,\ z)=\left(10,\ 1,\ \frac{1}{10}\right).$$

2.inačica

$$\left.\begin{array}{l}\log x=1+\frac{1}{2}\cdot\log y\\ \log y=1+\log z\\ \log z=1-2\cdot\log x\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log x=\log 10+\log\sqrt{y}\\ \log y=\log 10+\log z\\ \log z=\log 10-\log x^2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log x=\log\left(10\cdot\sqrt{y}\right)\\ \log y=\log(10\cdot z)\\ \log z=\log\frac{10}{x^2}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}x=10\cdot\sqrt{y}\\ y=10\cdot z\\ z=\frac{10}{x^2}\end{array}\right\} \Rightarrow \begin{bmatrix}\text{metoda}\\ \text{supstitucije}\end{bmatrix} \Rightarrow \left.\begin{array}{l}x=10\cdot\sqrt{y}\\ y=10\cdot\frac{10}{x^2}\end{array}\right\} \Rightarrow \left.\begin{array}{l}x=10\cdot\sqrt{y}\\ y=\frac{100}{x^2}\end{array}\right\} \Rightarrow \begin{bmatrix}\text{metoda}\\ \text{supstitucije}\end{bmatrix} \Rightarrow$$

$$\Rightarrow x=10\cdot\sqrt{\frac{100}{x^2}} \Rightarrow x=10\cdot\frac{10}{x} \Rightarrow x=\frac{100}{x} \Rightarrow x=\frac{100}{x}\ /\cdot x \Rightarrow x^2=100 \Rightarrow x^2=100\ /\sqrt{\ } \Rightarrow$$

$$\Rightarrow \; x=\sqrt{100} \; \Rightarrow \; x=10.$$

Računamo z.

$$\left.\begin{array}{l} z=\dfrac{10}{x^2} \\ x=10 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{supstitucije} \end{array}\right] \Rightarrow z=\frac{10}{10^2} \Rightarrow z=\frac{10}{10^2} \Rightarrow z=\frac{1}{10}.$$

Računamo y.

$$\left.\begin{array}{l} y=10\cdot z \\ z=\dfrac{1}{10} \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{supstitucije} \end{array}\right] \Rightarrow y=10\cdot\frac{1}{10} \Rightarrow y=10\cdot\frac{1}{10} \Rightarrow y=1.$$

Rješenje sustava je

$$(x,\; y,\; z)=\left(10,\; 1,\; \frac{1}{10}\right).$$

Vježba 249

Riješi sustav jednadžbi: $\begin{cases} \log x=\dfrac{1}{2}-\dfrac{1}{2}\cdot\log z \\ \log y=-2+2\cdot\log x \\ \log z=-1+\log y. \end{cases}$

Rezultat: $(x,\; y,\; z)=\left(10,\; 1,\; \dfrac{1}{10}\right).$

Zadatak 250 (Tony, gimnazija)

Dokaži: $\dfrac{\log_a x}{\log_{ab} x}=1+\log_a b,$ za sve pozitivne brojeve a, b i x.

Rješenje 250

Ponovimo!

$$\log_a b=\frac{1}{\log_b a} \quad , \quad \log_b (x\cdot y)=\log_b x+\log_b y \quad , \quad \log_{10} x=\log x \quad , \quad \log_b b=1.$$

$$\log_b a=\frac{\log_c a}{\log_c b} \quad , \quad \frac{a+b}{n}=\frac{a}{n}+\frac{b}{n} \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

1.inačica

$$\frac{\log_a x}{\log_{ab} x}=\frac{\dfrac{1}{\log_x a}}{\dfrac{1}{\log_x ab}}=\frac{\log_x ab}{\log_x a}=\log_a ab=\log_a a+\log_a b=1+\log_a b.$$

2.inačica

$$\frac{\log_a x}{\log_{ab} x}=\frac{\dfrac{1}{\log_x a}}{\dfrac{1}{\log_x ab}}=\frac{\log_x ab}{\log_x a}=\frac{\log_x a+\log_x b}{\log_x a}=\frac{\log_x a}{\log_x a}+\frac{\log_x b}{\log_x a}=\frac{\log_x a}{\log_x a}+\frac{\log_x b}{\log_x a}=$$

$$=1+\frac{\log_x b}{\log_x a}=1+\log_a b.$$

3.inačica

$$\frac{\log_a x}{\log_{ab} x}=\frac{\frac{\log x}{\log a}}{\frac{\log x}{\log ab}}=\frac{\frac{\log x}{\log a}}{\frac{\log x}{\log ab}}=\frac{\frac{1}{\log a}}{\frac{1}{\log ab}}=\frac{\log ab}{\log a}=\frac{\log a+\log b}{\log a}=\frac{\log a}{\log a}+\frac{\log b}{\log a}=\frac{\log a}{\log a}+\frac{\log b}{\log a}=$$

$$=1+\frac{\log b}{\log a}=1+\log_a b.$$

Vježba 250

Dokaži: $\frac{\log_a x}{\log_{ab} x}=\log_a ab,$ za sve pozitivne brojeve a, b i x.

Rezultat: Dokaz analogan.

Zadatak 251 (Branka, srednja škola)

Ako je $a=\log 5,\ b=\log 3,$ tada $\log_{30} 8$ iznosi:

$A.\ \frac{3\cdot b}{a+1}$ $B.\ \frac{3\cdot(1-a)}{1+b}$ $C.\ \frac{3\cdot a}{b-1}$ $D.\ \frac{b-1}{a+1}$

Rješenje 251

Ponovimo!

$$\log_{10} x=\log x \quad , \quad \log_b a=\frac{\log a}{\log b} \quad , \quad \log a^n=n\cdot\log a \quad , \quad \log(a\cdot b)=\log a+\log b.$$

$$\log\frac{a}{b}=\log a-\log b \quad , \quad \log 10=1.$$

$$\log_{30} 8=\frac{\log 8}{\log 30}=\frac{\log 2^3}{\log(10\cdot 3)}=\frac{3\cdot\log 2}{\log 10+\log 3}=\frac{3\cdot\log\frac{10}{5}}{\log 10+\log 3}=\frac{3\cdot(\log 10-\log 5)}{\log 10+\log 3}=$$

$$=\frac{3\cdot(1-a)}{1+b}.$$

Odgovor je pod B.

Vježba 251

Ako je $a=\log 5,\ b=\log 3,$ tada $\log_{30} 4$ iznosi:

$A.\ \frac{2\cdot(1-a)}{a+1}$ $B.\ \frac{2\cdot(1-a)}{1+b}$ $C.\ \frac{3\cdot b}{b+1}$ $D.\ \frac{b-1}{a+1}$

Rezultat: B.

Zadatak 252 (Ante, srednja škola)

Za rješenje x jednadžbe $0.5^{5-2\cdot x}-0.25^{3-x}=16$ vrijedi:

$A.\ 1\le x\le 3$ $B.\ 4\le x\le 6$ $C.\ 7\le x\le 9$ $D.\ 10\le x\le 12$

Rješenje 252

Ponovimo!

$$a^{-n}=\frac{1}{a^{n}} \quad , \quad \left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{1}=a \quad , \quad a^{n}\cdot a^{m}=a^{n+m}.$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$0.5^{5-2\cdot x}-0.25^{3-x}=16 \Rightarrow \left(\frac{5}{10}\right)^{5-2\cdot x}-\left(\frac{25}{100}\right)^{3-x}=16 \Rightarrow \left(\frac{1}{2}\right)^{5-2\cdot x}-\left(\frac{1}{4}\right)^{3-x}=16 \Rightarrow$$

$$\Rightarrow \left(\frac{1}{2}\right)^{5-2\cdot x}-\left(\frac{1}{2^{2}}\right)^{3-x}=16 \Rightarrow \left(2^{-1}\right)^{5-2\cdot x}-\left(2^{-2}\right)^{3-x}=16 \Rightarrow 2^{-5+2\cdot x}-2^{-6+2\cdot x}=16 \Rightarrow$$

$$\Rightarrow 2^{2\cdot x-5}-2^{2\cdot x-6}=16 \Rightarrow 2^{2\cdot x-6+1}-2^{2\cdot x-6}=16 \Rightarrow 2^{2\cdot x-6}\cdot 2^{1}-2^{2\cdot x-6}=16 \Rightarrow$$

$$\Rightarrow 2^{2\cdot x-6}\cdot\left(2^{1}-1\right)=16 \Rightarrow 2^{2\cdot x-6}\cdot(2-1)=16 \Rightarrow 2^{2\cdot x-6}\cdot 1=16 \Rightarrow 2^{2\cdot x-6}=16 \Rightarrow$$

$$\Rightarrow 2^{2\cdot x-6}=2^{4} \Rightarrow 2\cdot x-6=4 \Rightarrow 2\cdot x=4+6 \Rightarrow 2\cdot x=10 \Rightarrow 2\cdot x=10 \ /:2 \Rightarrow x=5.$$

Odgovor je pod B.

Vježba 252

Za rješenje x jednadžbe $0.5^{5-2\cdot x}-0.25^{3-x}=4$ vrijedi:

A. $1\le x\le 3$ *B.* $4\le x\le 6$ *C.* $7\le x\le 9$ *D.* $10\le x\le 12$

Rezultat: B.

Zadatak 253 (Matea, srednja škola)

Izraz $\log_4 8+\log_4 2$ jednak je:

A. 1 *B.* 2 *C.* $\log_4 6$ *D.* $\log_4 10$

Rješenje 253

Ponovimo!

$$\log_b x+\log_b y=\log_b(x\cdot y) \quad , \quad \log_b a^{n}=n\cdot\log_b a \quad , \quad \log_b b=1 \quad , \quad \log_b a=\frac{\log_c a}{\log_c b}.$$

$$\log_{10} a=\log a \quad , \quad \frac{a}{n}+\frac{b}{n}=\frac{a+b}{n} \quad , \quad \log_{b^{n}} a=\frac{1}{n}\cdot\log_b a \quad , \quad \log_{b^{n}} a^{m}=\frac{m}{n}\cdot\log_b a.$$

1.inačica

$$\log_4 8+\log_4 2=\log_4(8\cdot 2)=\log_4 16=\log_4 4^{2}=2\cdot\log_4 4=2\cdot 1=2.$$

Odgovor je pod B.

2.inačica

$$\log_4 8+\log_4 2=\frac{\log 8}{\log 4}+\frac{\log 2}{\log 4}=\frac{\log 8+\log 2}{\log 4}=\frac{\log(8\cdot 2)}{\log 4}=\frac{\log 16}{\log 4}=\frac{\log 4^{2}}{\log 4}=\frac{2\cdot\log 4}{\log 4}=\frac{2\cdot\log 4}{\log 4}=2.$$

Odgovor je pod B.

3.inačica

$$\log_4 8+\log_4 2=\frac{\log_2 8}{\log_2 4}+\frac{\log_2 2}{\log_2 4}=\frac{\log_2 8+\log_2 2}{\log_2 4}=\frac{\log_2 (8\cdot 2)}{\log_2 4}=\frac{\log_2 16}{\log_2 4}=\frac{\log_2 2^4}{\log_2 2^2}=$$

$$=\frac{4\cdot\log_2 2}{2\cdot\log_2 2}=\frac{4\cdot\log_2 2}{2\cdot\log_2 2}=2.$$

Odgovor je pod B.

4.inačica

$$\log_4 8+\log_4 2=\frac{\log_2 8}{\log_2 4}+\frac{\log_2 2}{\log_2 4}=\frac{\log_2 8+\log_2 2}{\log_2 4}=\frac{\log_2 (8\cdot 2)}{\log_2 4}=\frac{\log_2 16}{\log_2 4}=\frac{\log_2 4^2}{\log_2 4}=$$

$$=\frac{2\cdot\log_2 4}{\log_2 4}=\frac{2\cdot\log_2 4}{\log_2 4}=2.$$

Odgovor je pod B.

5.inačica

$$\log_4 8+\log_4 2=\frac{\log 8}{\log 4}+\frac{\log 2}{\log 4}=\frac{\log 8+\log 2}{\log 4}=\frac{\log 2^3+\log 2}{\log 2^2}=\frac{3\cdot\log 2+\log 2}{2\cdot\log 2}=\frac{4\cdot\log 2}{2\cdot\log 2}=$$

$$=\frac{4\cdot\log 2}{2\cdot\log 2}=\frac{4}{2}=2.$$

Odgovor je pod B.

6.inačica

$$\log_4 8+\log_4 2=\log_{2^2} 8+\log_{2^2} 2=\frac{1}{2}\cdot\log_2 8+\frac{1}{2}\cdot\log_2 2=\frac{1}{2}\cdot\log_2 2^3+\frac{1}{2}\cdot\log_2 2=$$

$$=\frac{3}{2}\cdot\log_2 2+\frac{1}{2}\cdot\log_2 2=\frac{3}{2}\cdot 1+\frac{1}{2}\cdot 1=\frac{3}{2}+\frac{1}{2}=\frac{3+1}{2}=\frac{4}{2}=2.$$

Odgovor je pod B.

7.inačica

$$\log_4 8+\log_4 2=\log_{2^2} 2^3+\log_{2^2} 2=\frac{3}{2}\cdot\log_2 2+\frac{1}{2}\cdot\log_2 2=\frac{3}{2}\cdot 1+\frac{1}{2}\cdot 1=\frac{3}{2}+\frac{1}{2}=\frac{3+1}{2}=\frac{4}{2}=2.$$

Odgovor je pod B.

8.inačica

$$\log_4 8+\log_4 2=\log_4 (8\cdot 2)=\log_4 16=\log_{2^2} 2^4=\frac{4}{2}\cdot\log_2 2=\frac{4}{2}\cdot 1=2\cdot 1=2.$$

Odgovor je pod B.

Vježba 253

Izraz $\log_4 8-\log_4 2$ jednak je:

A. 1 *B.* 2 *C.* $\log_4 6$ *D.* $\log_4 10$

Rezultat: A.

Zadatak 254 (Saša, strukovna škola)

Riješite nejednadžbu $0.1^{5\cdot x-3}\le 1$. Rješenje zapišite pomoću intervala.

Rješenje 254

Ponovimo!

$$a^{0}=1 \quad , \quad a^{f(x)}\le a^{g(x)} \;,\; 0<a<1 \;\Rightarrow\; f(x)\ge g(x).$$

$$a^{-1}=\frac{1}{a} \quad , \quad \left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{f(x)}\le a^{g(x)} \;,\; a>1 \;\Rightarrow\; f(x)\le g(x).$$

1.inačica

$$0.1^{5\cdot x-3}\le 1 \;\Rightarrow\; 0.1^{5\cdot x-3}\le 0.1^{0} \;\Rightarrow\; 5\cdot x-3\ge 0 \;\Rightarrow\; 5\cdot x\ge 3 \;\Rightarrow\; 5\cdot x\ge 3 \;/:5 \;\Rightarrow\; x\ge\frac{3}{5} \;\Rightarrow$$

$$\Rightarrow\; x\in\left[\frac{3}{5},\; +\infty\right\rangle.$$

2.inačica

$$0.1^{5\cdot x-3}\le 1 \;\Rightarrow\; \left(\frac{1}{10}\right)^{5\cdot x-3}\le 1 \;\Rightarrow\; \left(10^{-1}\right)^{5\cdot x-3}\le 10^{0} \;\Rightarrow\; 10^{-5\cdot x+3}\le 10^{0} \;\Rightarrow\; -5\cdot x+3\le 0 \;\Rightarrow$$

$$\Rightarrow\; -5\cdot x\le -3 \;\Rightarrow\; -5\cdot x\le -3 \;/:(-5) \;\Rightarrow\; x\ge\frac{3}{5} \;\Rightarrow\; x\in\left[\frac{3}{5},\; +\infty\right\rangle.$$

Vježba 254

Riješite nejednadžbu $0.1^{4\cdot x-3}\le 1$. Rješenje zapišite pomoću intervala.

Rezultat: $x\in\left[\frac{3}{4},\; +\infty\right\rangle.$

Zadatak 255 (Ljilja, srednja škola)

Riješite sustav jednadžbi: $\begin{cases} \log_5(8\cdot x)=1+\log_5 4 \\ x^{y}=\frac{2}{5} \end{cases}.$

Rješenje 255

Ponovimo!

$$\log_b b=1 \quad , \quad \log_b a+\log_b c=\log_b(a\cdot c) \quad , \quad \log_b f(x)=\log_b g(x) \;\Rightarrow\; f(x)=g(x).$$

$$a^{1}=a \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^{n} \quad , \quad a^{f(x)}=a^{g(x)} \;\Rightarrow\; f(x)=g(x).$$

$$\left.\begin{array}{l} \log_5(8\cdot x)=1+\log_5 4 \\ x^{y}=\frac{2}{5} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log_5(8\cdot x)=\log_5 5+\log_5 4 \\ x^{y}=\frac{2}{5} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log_5(8\cdot x)=\log_5(5\cdot 4) \\ x^{y}=\frac{2}{5} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} \log_5(8\cdot x)=\log_5 20 \\ x^{y}=\frac{2}{5} \end{array}\right\} \Rightarrow \left.\begin{array}{l} 8\cdot x=20 \\ x^{y}=\frac{2}{5} \end{array}\right\} \Rightarrow \left.\begin{array}{l} 8\cdot x=20 \;/:8 \\ x^{y}=\frac{2}{5} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=\frac{20}{8} \\ x^{y}=\frac{2}{5} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=\frac{20}{8} \\ x^{y}=\frac{2}{5} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x=\frac{5}{2} \\ x^{y}=\frac{2}{5} \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{supstitucije} \end{bmatrix} \Rightarrow \left(\frac{5}{2}\right)^{y}=\frac{2}{5} \Rightarrow \left(\frac{5}{2}\right)^{y}=\left(\frac{5}{2}\right)^{-1} \Rightarrow y=-1.$$

Rješenje sustava glasi:

$$(x,\ y)=\left(\frac{5}{2},\ -1\right).$$

Vježba 255

Riješite sustav jednadžbi: $\left\{\begin{matrix} \log_5(8\cdot x)=1+\log_5 4 \\ x^{y}=\frac{5}{2} \end{matrix}\right.$.

Rezultat: $(x,\ y)=\left(\frac{5}{2},\ 1\right).$

Zadatak 256 (Ivan, gimnazija)

Ako je $4^{x}=9$ i $9^{y}=256$, tada umnožak $x\cdot y$ iznosi:

$A.\ 58 \qquad B.\ 36 \qquad C.\ 10 \qquad D.\ 4$

Rješenje 256

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad \log_{10} a=\log a.$$

$$\log a^{n}=n\cdot\log a \quad , \quad \left.\begin{matrix} a=b \\ c=d \end{matrix}\right\} \Rightarrow a\cdot c=b\cdot d.$$

1.inačica

$$\left.\begin{matrix} 4^{x}=9 \\ 9^{y}=256 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \left(4^{x}\right)^{y}=9^{y} \\ 9^{y}=256 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 4^{x\cdot y}=9^{y} \\ 9^{y}=256 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{supstitucije} \end{bmatrix} \Rightarrow 4^{x\cdot y}=256 \Rightarrow$$

$$\Rightarrow 4^{x\cdot y}=4^{4} \Rightarrow x\cdot y=4.$$

Odgovor je pod D.

2.inačica

$$\left.\begin{matrix} 4^{x}=9 \\ 9^{y}=256 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{logaritmiramo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow \left.\begin{matrix} 4^{x}=9\ /\log \\ 9^{y}=256\ /\log \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log 4^{x}=\log 9 \\ \log 9^{y}=\log 256 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x\cdot\log 4=\log 9 \\ y\cdot\log 9=\log 256 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{pomnožimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow x\cdot\log 4\cdot y\cdot\log 9=\log 9\cdot\log 256 \Rightarrow$$

$$\Rightarrow x\cdot y\cdot\log 4\cdot\log 9=\log 9\cdot\log 4^{4} \Rightarrow x\cdot y\cdot\log 4\cdot\log 9=\log 9\cdot 4\cdot\log 4 \Rightarrow$$

$$\Rightarrow x\cdot y\cdot\log 4\cdot\log 9=4\cdot\log 9\cdot\log 4\ /\cdot\frac{1}{\log 4\cdot\log 9} \Rightarrow x\cdot y=4.$$

Odgovor je pod D.

Vježba 256

Ako je $4^{x}=9$ i $9^{y}=64$, tada umnožak $x\cdot y$ iznosi:

$A.\ 16 \qquad B.\ 8 \qquad C.\ 3 \qquad D.\ 2$

Rezultat: C.

Zadatak 257 (Ana, hotelijerska škola)

Riješi eksponencijalnu jednadžbu: $100^{2-5\cdot x}=\sqrt{1000}.$

Rješenje 257

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad \sqrt[n]{a^{m}}=a^{\frac{m}{n}} \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$100^{2-5\cdot x}=\sqrt{1000} \Rightarrow \left(10^{2}\right)^{2-5\cdot x}=\sqrt{10^{3}} \Rightarrow 10^{4-10\cdot x}=10^{\frac{3}{2}} \Rightarrow 4-10\cdot x=\frac{3}{2} \Rightarrow$$

$$\Rightarrow 4-10\cdot x=\frac{3}{2} \ /\cdot 2 \Rightarrow 8-20\cdot x=3 \Rightarrow -20\cdot x=3-8 \Rightarrow -20\cdot x=-5 \Rightarrow$$

$$\Rightarrow -20\cdot x=-5 \ /:(-20) \Rightarrow x=\frac{5}{20} \Rightarrow x=\frac{5}{20} \Rightarrow x=\frac{1}{4}.$$

Vježba 257

Riješi eksponencijalnu jednadžbu: $100^{2+5\cdot x}=\sqrt{1000}.$

Rezultat: $x=-\frac{1}{4}.$

Zadatak 258 (Cimos, hotelijerska škola)

Riješi eksponencijalnu jednadžbu: $10^{\frac{4-2\cdot x}{2}}=1.$

Rješenje 258

Ponovimo!

$$a^{0}=1 \ , \ a\neq 0 \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$10^{\frac{4-2\cdot x}{2}}=1 \Rightarrow 10^{\frac{4-2\cdot x}{2}}=10^{0} \Rightarrow \frac{4-2\cdot x}{2}=0 \Rightarrow \frac{4-2\cdot x}{2}=0 \ /\cdot 2 \Rightarrow$$

$$\Rightarrow 4-2\cdot x=0 \Rightarrow -2\cdot x=-4 \Rightarrow -2\cdot x=-4 \ /:(-2) \Rightarrow x=2.$$

Vježba 258

Riješi eksponencijalnu jednadžbu: $10^{\frac{6-3\cdot x}{2}}=1.$

Rezultat: x = 2.

Zadatak 259 (Tea, tehnička škola)

Riješi logaritamsku jednadžbu: $\log^{2} x+2\cdot\log(0.1\cdot x)=1.$

Rješenje 259

Ponovimo!

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

Definicija:

$$\log_{b} a=c \ , \ b>0 \ , \ b\neq 1 \ , \ a>0 \Leftrightarrow b^{c}=a.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \;\; \rightarrow \qquad a = b^c$$

$$\log_{10} x = \log x.$$

$$\log 10 = 1 \quad , \quad \log 0.1 = -1 \quad , \quad \log 0.001 = -3 \quad , \quad \log f(x) = \log g(x) \Rightarrow f(x) = g(x).$$

Diskusija!
Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x) = \log_b x,$$

gdje je $b > 0$ i $b \neq 1$. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x \in \langle 0, +\infty \rangle.$$

Prvo odredimo domenu funkcije:

$$f(x) = \log x \Rightarrow x > 0 \Rightarrow x \in \langle 0, +\infty \rangle.$$

Sada riješimo jednadžbu.

$$\log^2 x + 2 \cdot \log(0.1 \cdot x) = 1 \Rightarrow \log^2 x + 2 \cdot (\log 0.1 + \log x) = 1 \Rightarrow \log^2 x + 2 \cdot (-1 + \log x) = 1 \Rightarrow$$

$$\Rightarrow \log^2 x - 2 + 2 \cdot \log x = 1 \Rightarrow \log^2 x - 2 + 2 \cdot \log x - 1 = 0 \Rightarrow \log^2 x + 2 \cdot \log x - 3 = 0.$$

Uvedemo supstituciju (zamjenu)

$$t = \log x.$$

$$\left.\begin{matrix} \log^2 x + 2 \cdot \log x - 3 = 0 \\ t = \log x \end{matrix}\right\} \Rightarrow t^2 + 2 \cdot t - 3 = 0 \Rightarrow \left.\begin{matrix} t^2 + 2 \cdot t - 3 = 0 \\ a = 1 \;,\; b = 2 \;,\; c = -3 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a = 1 \;,\; b = 2 \;,\; c = -3 \\ t_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{matrix}\right\} \Rightarrow t_{1,2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot (-3)}}{2 \cdot 1} \Rightarrow t_{1,2} = \frac{-2 \pm \sqrt{4 + 12}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{-2 \pm \sqrt{16}}{2} \Rightarrow t_{1,2} = \frac{-2 \pm 4}{2} \Rightarrow \left.\begin{matrix} t_1 = \dfrac{-2+4}{2} \\ t_2 = \dfrac{-2-4}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = \dfrac{2}{2} \\ t_2 = \dfrac{-6}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = 1 \\ t_2 = -3 \end{matrix}\right\}.$$

Vraćamo se supstituciji (zamjeni).

- $\left.\begin{matrix} t = \log x \\ t = 1 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{komparacije} \end{bmatrix} \Rightarrow \log x = 1 \Rightarrow \log x = \log 10 \Rightarrow x_1 = 10.$
- $\left.\begin{matrix} t = \log x \\ t = -3 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{komparacije} \end{bmatrix} \Rightarrow \log x = -3 \Rightarrow \log x = \log 0.001 \Rightarrow x_2 = 0.001.$

Rješenja jednadžbe su: $x_1 = 10$, $x_2 = 0.001$.

Vježba 259

Riješi logaritamsku jednadžbu: $\log^2 x + 2 \cdot \log(0.1 \cdot x) - \log 10 = 0.$

Rezultat: $x_1 = 10$, $x_2 = 0.001$.

Zadatak 260 (Iva, srednja škola)

Vrijednost izraza $\frac{1-\log^2 5}{\log 50}$ jednaka je:

$A.\ \log 2 \qquad B.\ \log 4 \qquad C.\ 1 \qquad D.\ \log 5$

Rješenje 260

Ponovimo!

$$\log_{10} x = \log x \quad , \quad \log 10 = 1.$$

$$\log(a \cdot b) = \log a + \log b \quad , \quad \log\frac{a}{b} = \log a - \log b \quad , \quad a^2 - b^2 = (a-b)\cdot(a+b).$$

$$\frac{1-\log^2 5}{\log 50} = \frac{(1-\log 5)\cdot(1+\log 5)}{\log(5\cdot 10)} = \frac{(1-\log 5)\cdot(1+\log 5)}{\log 5 + \log 10} = \frac{(1-\log 5)\cdot(1+\log 5)}{\log 5 + 1} =$$

$$= \frac{(1-\log 5)\cdot(1+\log 5)}{1+\log 5} = \frac{(1-\log 5)\cdot(1+\log 5)}{1+\log 5} = 1-\log 5 = \begin{bmatrix}\text{dvije} \\ \text{inačice}\end{bmatrix} =$$

$$= \left.\begin{matrix}\log 10 - \log 5 \\ \log 10 - \log 5\end{matrix}\right\} = \left.\begin{matrix}\log\frac{10}{5} \\ \log(2\cdot 5) - \log 5\end{matrix}\right\} = \left.\begin{matrix}\log\frac{10}{5} \\ \log 2 + \log 5 - \log 5\end{matrix}\right\} = \left.\begin{matrix}\log 2 \\ \log 2 + \log 5 - \log 5\end{matrix}\right\} = \left.\begin{matrix}\log 2 \\ \log 2\end{matrix}\right\}.$$

Odgovor je pod A.

Vježba 260

Vrijednost izraza $\frac{1-\log^2 2}{\log 20}$ jednaka je:

$A.\ \log 2 \qquad B.\ \log 4 \qquad C.\ 1 \qquad D.\ \log 5$

Rezultat: D.