Zadatak 221 (Klara, gimnazija)

Ako je $\log_5 2 = a$ i $\log_5 3 = b$, izračunaj $\log_{45} 100$.

Rješenje 221

Ponovimo!

$$\log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log_b a^n = n \cdot \log_b a \quad , \quad \log_b b = 1 \quad , \quad \log_b (x \cdot y) = \log_b x + \log_b y.$$

$$\log_{45} 100 = \frac{\log_5 100}{\log_5 45} = \frac{\log_5 (4 \cdot 25)}{\log_5 (5 \cdot 9)} = \frac{\log_5 4 + \log_5 25}{\log_5 5 + \log_5 9} = \frac{\log_5 2^2 + \log_5 5^2}{\log_5 5 + \log_5 3^2} =$$

$$= \frac{2 \cdot \log_5 2 + 2 \cdot \log_5 5}{\log_5 5 + 2 \cdot \log_5 3} = \frac{2 \cdot a + 2 \cdot 1}{1 + 2 \cdot b} = \frac{2 \cdot a + 2}{1 + 2 \cdot b}.$$

Vježba 221

Ako je $\log_5 2 = a$ i $\log_5 3 = b$, izračunaj $\log_{100} 45$.

Rezultat: $\frac{2 \cdot b + 1}{2 \cdot a + 2}.$

Zadatak 222 (Ana Maria, srednja škola)

Ako je $4^x + 4^{-x} = 34$, onda je $2^x + 2^{-x}$ jednako

$A)\ 4 \qquad B)\ 8 \qquad C)\ 6 \qquad D)\ 9$

Rješenje 222

Ponovimo!

$$\left(a^n\right)^m = \left(a^m\right)^n = a^{n \cdot m} \quad , \quad (a+b)^2 = a^2 + 2 \cdot a \cdot b + b^2 \quad , \quad \sqrt{a^2} = a \ , \ a \ge 0.$$

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^0 = 1.$$

Označimo

$$y = 2^x + 2^{-x}.$$

Kvadriranjem dobije se:

$$y = 2^x + 2^{-x} \Rightarrow y = 2^x + 2^{-x} \ /^2 \Rightarrow y^2 = \left(2^x + 2^{-x}\right)^2 \Rightarrow y^2 = \left(2^x\right)^2 + 2 \cdot 2^x \cdot 2^{-x} + \left(2^{-x}\right)^2 \Rightarrow$$

$$\Rightarrow y^2 = \left(2^2\right)^x + 2 \cdot 2^0 + \left(2^2\right)^{-x} \Rightarrow y^2 = 4^x + 2 \cdot 1 + 4^{-x} \Rightarrow y^2 = 4^x + 2 + 4^{-x} \Rightarrow$$

$$\Rightarrow \left[4^x + 4^{-x} = 34\right] \Rightarrow y^2 = 34 + 2 \Rightarrow y^2 = 36 \Rightarrow y^2 = 36 \ /\sqrt{\ } \Rightarrow y = \sqrt{36} \Rightarrow y = \sqrt{6^2} \Rightarrow y = 6.$$

Odgovor je pod C.

Vježba 222

Ako je $4^x + 4^{-x} = 47$, onda je $2^x + 2^{-x}$ jednako

$A)\ 5 \qquad B)\ 7 \qquad C)\ 9 \qquad D)\ 11$

Rezultat: B.

Zadatak 223 (Ana Maria, srednja škola)

Pojednostavni: $\log(a \cdot b) - \log(b \cdot c) - \log(c \cdot a)$.

$A)\ \log(a \cdot b \cdot c) \qquad B)\ -2 \cdot \log a \qquad C)\ -2 \cdot \log b \qquad D)\ -2 \cdot \log c$

Rješenje 223

Ponovimo!

$$\log\frac{x}{y}=\log x-\log y \quad , \quad \log(x\cdot y)=\log x+\log y \quad , \quad \log x^{n}=n\cdot\log x \quad , \quad x^{-n}=\frac{1}{x^{n}}.$$

$$\log 1=0 \quad , \quad \frac{n}{1}=n \quad , \quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}.$$

1.inačica

$$\log(a\cdot b)-\log(b\cdot c)-\log(c\cdot a)=(\log(a\cdot b)-\log(b\cdot c))-\log(c\cdot a)=\log\frac{a\cdot b}{b\cdot c}-\log(c\cdot a)=$$

$$=\log\frac{a\cdot b}{b\cdot c}-\log(c\cdot a)=\log\frac{a}{c}-\log(c\cdot a)=\log\frac{\frac{a}{c}}{c\cdot a}=\log\frac{\frac{a}{c}}{\frac{c\cdot a}{1}}=\log\frac{\frac{a}{c}}{\frac{c\cdot a}{1}}=\log\frac{\frac{1}{c}}{\frac{c}{1}}=\log\frac{1}{c^{2}}=$$

$$=\log c^{-2}=-2\cdot\log c.$$

Može i ovako:

$$\log\frac{1}{c^{2}}=\log 1-\log c^{2}=0-2\cdot\log c=-2\cdot\log c.$$

Odgovor je pod D.

2.inačica

$$\log(a\cdot b)-\log(b\cdot c)-\log(c\cdot a)=\log(a\cdot b)-(\log(b\cdot c)+\log(c\cdot a))=\log(a\cdot b)-\log(b\cdot c\cdot c\cdot a)=$$

$$=\log(a\cdot b)-\log\left(a\cdot b\cdot c^{2}\right)=\log\frac{a\cdot b}{a\cdot b\cdot c^{2}}=\log\frac{a\cdot b}{a\cdot b\cdot c^{2}}=\log\frac{1}{c^{2}}=\log c^{-2}=-2\cdot\log c.$$

Može i ovako:

$$\log\frac{1}{c^{2}}=\log 1-\log c^{2}=0-2\cdot\log c=-2\cdot\log c.$$

Odgovor je pod D.

3.inačica

$$\log(a\cdot b)-\log(b\cdot c)-\log(c\cdot a)=\log a+\log b-(\log b+\log c)-(\log c+\log a)=$$

$$=\log a+\log b-\log b-\log c-\log c-\log a=\log a+\log b-\log b-\log c-\log c-\log a=-2\cdot\log c.$$

Odgovor je pod D.

Vježba 223

Pojednostavni: $\log(b\cdot c)+\log(c\cdot a)-\log(a\cdot b)$.

$A)\ \log(a\cdot b\cdot c) \qquad B)\ 2\cdot\log a \qquad C)\ 2\cdot\log b \qquad D)\ 2\cdot\log c$

Rezultat: D.

Zadatak 224 (Vlado, gimnazija)

Ako je $\log_{2} 12=a$, $\log_{3} 2=b$, onda je $a-\frac{1}{b}$ jednako:

$A)\ 2 \qquad B)\ 3 \qquad C)\ 4 \qquad D)\ 5$

Rješenje 224

Ponovimo!

$$\log_b a=\frac{1}{\log_a b}\quad,\quad \log_b x-\log_b y=\log_b\frac{x}{y}\quad,\quad \log_b a^n=n\cdot\log_b a\quad,\quad \log_b b=1.$$

$$\log_b a=\frac{\log_c a}{\log_c b}\quad,\quad \log_{10} a=\log a\quad,\quad \log_b a=\frac{\log a}{\log b}\quad,\quad \frac{\frac{a}{b}}{\frac{c}{d}}=\frac{a\cdot d}{b\cdot c}\quad,\quad \frac{a}{n}-\frac{b}{n}=\frac{a-b}{n}.$$

$$\log_b x+\log_b y=\log_b(x\cdot y).$$

1.inačica

$$a-\frac{1}{b}=\log_2 12-\frac{1}{\log_3 2}=\log_2 12-\log_2 3=\log_2\frac{12}{3}=\log_2\frac{12}{3}=\log_2 4=\log_2 2^2=2\cdot\log_2 2=2\cdot1=2.$$

Odgovor je pod A.

2.inačica

$$a-\frac{1}{b}=\log_2 12-\frac{1}{\log_3 2}=\frac{\log 12}{\log 2}-\frac{1}{\frac{\log 2}{\log 3}}=\frac{\log 12}{\log 2}-\frac{\frac{1}{1}}{\frac{\log 2}{\log 3}}=\frac{\log 12}{\log 2}-\frac{\log 3}{\log 2}=\frac{\log 12-\log 3}{\log 2}=$$

$$=\frac{\log\frac{12}{3}}{\log 2}=\frac{\log\frac{12}{3}}{\log 2}=\frac{\log 4}{\log 2}=\frac{\log 2^2}{\log 2}=\frac{2\cdot\log 2}{\log 2}=\frac{2\cdot\log 2}{\log 2}=2.$$

Odgovor je pod A.

3.inačica

$$a-\frac{1}{b}=\log_2 12-\frac{1}{\log_3 2}=\frac{\log 12}{\log 2}-\frac{1}{\frac{\log 2}{\log 3}}=\frac{\log 12}{\log 2}-\frac{\frac{1}{1}}{\frac{\log 2}{\log 3}}=\frac{\log 12}{\log 2}-\frac{\log 3}{\log 2}=\frac{\log 12-\log 3}{\log 2}=$$

$$=\frac{\log(4\cdot3)-\log 3}{\log 2}=\frac{\log 4+\log 3-\log 3}{\log 2}=\frac{\log 4+\log 3-\log 3}{\log 2}=\frac{\log 4}{\log 2}=\frac{\log 2^2}{\log 2}=\frac{2\cdot\log 2}{\log 2}=\frac{2\cdot\log 2}{\log 2}=2.$$

Odgovor je pod A.

4.inačica

$$a-\frac{1}{b}=\log_2 12-\frac{1}{\log_3 2}=\frac{\log 12}{\log 2}-\frac{1}{\frac{\log 2}{\log 3}}=\frac{\log 12}{\log 2}-\frac{\frac{1}{1}}{\frac{\log 2}{\log 3}}=\frac{\log 12}{\log 2}-\frac{\log 3}{\log 2}=\frac{\log 12-\log 3}{\log 2}=$$

$$=\frac{\log 12-\log\frac{12}{4}}{\log 2}=\frac{\log 12-(\log 12-\log 4)}{\log 2}=\frac{\log 12-\log 12+\log 4}{\log 2}=\frac{\log 12-\log 12+\log 4}{\log 2}=$$

$$=\frac{\log 4}{\log 2}=\frac{\log 2^2}{\log 2}=\frac{2\cdot\log 2}{\log 2}=\frac{2\cdot\log 2}{\log 2}=2.$$

Odgovor je pod A.

Vježba 224

Ako je $\log_2 12=a$, $\log_3 2=b$, onda je $\frac{1}{b}-a$ jednako:

$A)\ -2\qquad B)\ -3\qquad C)\ -4\qquad D)\ -5$

Rezultat: A.

Zadatak 225 (Franjo, srednja škola)

Izračunaj: $5\cdot 9^{x+1}=15.$

Rješenje 225

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{1}=a \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad a^{n}\cdot a^{m}=a^{n+m}.$$

$$a^{0}=1 \quad , \quad a^{-n}=\frac{1}{a^{n}}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$5\cdot 9^{x+1}=15 \Rightarrow 5\cdot 9^{x+1}=15 \;/:5 \Rightarrow 9^{x+1}=3 \Rightarrow \left(3^{2}\right)^{x+1}=3 \Rightarrow 3^{2\cdot x+2}=3 \Rightarrow 3^{2\cdot x+2}=3^{1} \Rightarrow$$

$$\Rightarrow 2\cdot x+2=1 \Rightarrow 2\cdot x=1-2 \Rightarrow 2\cdot x=-1 \Rightarrow 2\cdot x=-1 \;/:2 \Rightarrow x=-\frac{1}{2}.$$

2.inačica

$$5\cdot 9^{x+1}=15 \Rightarrow 5\cdot 9^{x}\cdot 9^{1}=15 \Rightarrow 5\cdot 9^{x}\cdot 9=15 \Rightarrow 45\cdot 9^{x}=15 \Rightarrow 45\cdot 9^{x}=15 \;/:15 \Rightarrow$$

$$\Rightarrow 3\cdot 9^{x}=1 \Rightarrow 3\cdot\left(3^{2}\right)^{x}=1 \Rightarrow 3\cdot 3^{2\cdot x}=1 \Rightarrow 3^{1}\cdot 3^{2\cdot x}=1 \Rightarrow 3^{1+2\cdot x}=1 \Rightarrow 3^{1+2\cdot x}=3^{0} \Rightarrow$$

$$\Rightarrow 1+2\cdot x=0 \Rightarrow 2\cdot x=-1 \Rightarrow 2\cdot x=-1 \;/:2 \Rightarrow x=-\frac{1}{2}.$$

3.inačica

$$5\cdot 9^{x+1}=15 \Rightarrow 5\cdot 9^{x}\cdot 9^{1}=15 \Rightarrow 5\cdot 9^{x}\cdot 9=15 \Rightarrow 45\cdot 9^{x}=15 \Rightarrow 45\cdot 9^{x}=15 \;/:45 \Rightarrow$$

$$\Rightarrow 9^{x}=\frac{1}{3} \Rightarrow 9^{x}=\frac{1}{3^{1}} \Rightarrow 9^{x}=3^{-1} \Rightarrow \left(3^{2}\right)^{x}=3^{-1} \Rightarrow 3^{2\cdot x}=3^{-1} \Rightarrow 2\cdot x=-1 \Rightarrow$$

$$\Rightarrow 2\cdot x=-1 \;/:2 \Rightarrow x=-\frac{1}{2}.$$

Vježba 225

Izračunaj: $2\cdot 9^{x+1}=6.$

Rezultat: $x=-\frac{1}{2}.$

Zadatak 226 (Stipe, tehnička škola)

Riješi jednadžbu: $\log_{2}\left(4^{x+1}+4\right)\cdot\log_{2}\left(4^{x}+1\right)=\log_{2}8.$

Rješenje 226

Ponovimo!

$$a^{n}\cdot a^{m}=a^{n+m} \quad , \quad a^{1}=a \quad , \quad a^{0}=1 \quad , \quad \log_{b}a^{n}=n\cdot\log_{b}a \quad , \quad \log_{b}b=1.$$

$$\log_{b}(x\cdot y)=\log_{b}x+\log_{b}y \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad n=\frac{n}{1} \quad , \quad a^{-n}=\frac{1}{a^{n}}.$$

$$\frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d}.$$

Definicija:

$$\log_{b}a=c \;,\; b>0 \;,\; b\neq 1 \;,\; a>0 \Leftrightarrow b^{c}=a.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \;\rightarrow \qquad a = b^c$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

$$\log_2\left(4^{x+1}+4\right)\cdot\log_2\left(4^x+1\right)=\log_2 8 \Rightarrow \log_2\left(4^x\cdot4^1+4\right)\cdot\log_2\left(4^x+1\right)=\log_2 2^3 \Rightarrow$$

$$\Rightarrow \log_2\left(4^x\cdot4+4\right)\cdot\log_2\left(4^x+1\right)=3\cdot\log_2 2 \Rightarrow \log_2\left(4\cdot\left(4^x+1\right)\right)\cdot\log_2\left(4^x+1\right)=3\cdot1 \Rightarrow$$

$$\Rightarrow \left(\log_2 4+\log_2\left(4^x+1\right)\right)\cdot\log_2\left(4^x+1\right)=3 \Rightarrow \left(\log_2 2^2+\log_2\left(4^x+1\right)\right)\cdot\log_2\left(4^x+1\right)=3 \Rightarrow$$

$$\Rightarrow \left(2\cdot\log_2 2+\log_2\left(4^x+1\right)\right)\cdot\log_2\left(4^x+1\right)=3 \Rightarrow \left(2\cdot1+\log_2\left(4^x+1\right)\right)\cdot\log_2\left(4^x+1\right)=3 \Rightarrow$$

$$\Rightarrow \left(2+\log_2\left(4^x+1\right)\right)\cdot\log_2\left(4^x+1\right)=3 \Rightarrow 2\cdot\log_2\left(4^x+1\right)+\left(\log_2\left(4^x+1\right)\right)^2=3 \Rightarrow$$

$$\Rightarrow \left(\log_2\left(4^x+1\right)\right)^2+2\cdot\log_2\left(4^x+1\right)-3=0.$$

Uvodimo zamjenu (supstituciju).

$$t=\log_2\left(4^x+1\right).$$

$$\left.\begin{array}{l}\left(\log_2\left(4^x+1\right)\right)^2+2\cdot\log_2\left(4^x+1\right)-3=0\\ t=\log_2\left(4^x+1\right)\end{array}\right\}\Rightarrow t^2+2\cdot t-3=0 \Rightarrow \left.\begin{array}{l}t^2+2\cdot t-3=0\\ a=1\ ,\ b=2\ ,\ c=-3\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}a=1\ ,\ b=2\ ,\ c=-3\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\}\Rightarrow t_{1,2}=\frac{-2\pm\sqrt{4-4\cdot1\cdot(-3)}}{2\cdot1}\Rightarrow t_{1,2}=\frac{-2\pm\sqrt{4+12}}{2}\Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{-2\pm\sqrt{16}}{2}\Rightarrow t_{1,2}=\frac{-2\pm4}{2}\Rightarrow \left.\begin{array}{l}t_1=\dfrac{-2+4}{2}\\ t_2=\dfrac{-2-4}{2}\end{array}\right\}\Rightarrow \left.\begin{array}{l}t_1=\dfrac{2}{2}\\ t_2=-\dfrac{6}{2}\end{array}\right\}\Rightarrow \left.\begin{array}{l}t_1=1\\ t_2=-3\end{array}\right\}.$$

Vraćamo se supstituciji.

- $$\left.\begin{array}{l}t=\log_2\left(4^x+1\right)\\ t=1\end{array}\right\}\Rightarrow \log_2\left(4^x+1\right)=1 \Rightarrow 4^x+1=2^1 \Rightarrow 4^x+1=2 \Rightarrow 4^x=2-1 \Rightarrow$$

  $$\Rightarrow 4^x=1 \Rightarrow 4^x=4^0 \Rightarrow x=0.$$

- $$\left.\begin{array}{l}t=\log_2\left(4^x+1\right)\\ t=-3\end{array}\right\}\Rightarrow \log_2\left(4^x+1\right)=-3 \Rightarrow 4^x+1=2^{-3} \Rightarrow 4^x+1=\frac{1}{2^3} \Rightarrow 4^x+1=\frac{1}{8} \Rightarrow$$

  $$\Rightarrow 4^x=\frac{1}{8}-1 \Rightarrow 4^x=\frac{1}{8}-\frac{1}{1} \Rightarrow 4^x=\frac{1-8}{8} \Rightarrow 4^x=-\frac{7}{8}.$$

Nema smisla jer je 4^x za svaki realan broj x pozitivan.

$$x \in R \;\Rightarrow\; 4^x > 0.$$

Vježba 226

Riješi jednadžbu: $\frac{1}{3} \cdot \log_2\left(4^{x+1} + 4\right) \cdot \log_2\left(4^x + 1\right) = 1.$

Rezultat: x = 0.

Zadatak 227 (Mira, gimnazija)

Iracionalno rješenje jednadžbe $7 \cdot 2^x - 4^x = 12$ jednako je:

$A.\ \log_2 3 \qquad B.\ \log_3 2 \qquad C.\ \log_3 4 \qquad D.\ \log_4 3$

Rješenje 227

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^{f(x)} = a^{g(x)} \;\Rightarrow\; f(x) = g(x) \quad , \quad \log_b b = 1 \quad , \quad \log_b b^x = x.$$

Svaki se racionalni broj može zapisati u obliku razlomka kojemu je nazivnik prirodan broj. Svi brojevi koji nisu racionalni nazivaju se iracionalni.
Definicija:

$$\log_b a = c \;,\; b > 0 \;,\; b \neq 1 \;,\; a > 0 \;\Leftrightarrow\; b^c = a.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.
Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \;\rightarrow \qquad a = b^c$$

$$7 \cdot 2^x - 4^x = 12 \;\Rightarrow\; -4^x + 7 \cdot 2^x - 12 = 0 \;\Rightarrow\; -4^x + 7 \cdot 2^x - 12 = 0 \;/\cdot(-1) \;\Rightarrow\; 4^x - 7 \cdot 2^x + 12 = 0 \;\Rightarrow$$

$$\Rightarrow \left(2^2\right)^x - 7 \cdot 2^x + 12 = 0 \;\Rightarrow\; \left(2^x\right)^2 - 7 \cdot 2^x + 12 = 0 \;\Rightarrow\; \begin{bmatrix} \text{supstitucija} \\ t = 2^x \end{bmatrix} \;\Rightarrow\; t^2 - 7 \cdot t + 12 = 0 \;\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} t^2 - 7 \cdot t + 12 = 0 \\ a = 1 \;,\; b = -7 \;,\; c = 12 \end{matrix}\right\} \;\Rightarrow\; \left.\begin{matrix} a = 1 \;,\; b = -7 \;,\; c = 12 \\ t_{1,2} = \dfrac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \end{matrix}\right\} \;\Rightarrow\; t_{1,2} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot 12}}{2 \cdot 1} \;\Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{7 \pm \sqrt{49 - 48}}{2} \;\Rightarrow\; t_{1,2} = \frac{7 \pm \sqrt{1}}{2} \;\Rightarrow\; t_{1,2} = \frac{7 \pm 1}{2} \;\Rightarrow\; \left.\begin{matrix} t_1 = \dfrac{7+1}{2} \\ t_2 = \dfrac{7-1}{2} \end{matrix}\right\} \;\Rightarrow\; \left.\begin{matrix} t_1 = \dfrac{8}{2} \\ t_2 = \dfrac{6}{2} \end{matrix}\right\} \;\Rightarrow\; \left.\begin{matrix} t_1 = 4 \\ t_2 = 3 \end{matrix}\right\}.$$

Vraćamo se supstituciji.

- $\left.\begin{matrix} t = 2^x \\ t = 4 \end{matrix}\right\} \;\Rightarrow\; \begin{bmatrix} \text{metoda} \\ \text{komparacije} \end{bmatrix} \;\Rightarrow\; 2^x = 4 \;\Rightarrow\; 2^x = 2^2 \;\Rightarrow\; x = 2.$ rješenje je racionalan broj

- $\left.\begin{matrix} t = 2^x \\ t = 3 \end{matrix}\right\} \;\Rightarrow\; \begin{bmatrix} \text{metoda} \\ \text{komparacije} \end{bmatrix} \;\Rightarrow\; 2^x = 3 \;\Rightarrow\; \begin{bmatrix} \text{logaritmiramo} \\ \text{jednadžbu} \end{bmatrix} \;\Rightarrow\; 2^x = 3 \;/\log_2 \;\Rightarrow$

$$\Rightarrow \log_2 2^x = \log_2 3 \;\Rightarrow\; x = \log_2 3. \text{ rješenje je iracionalan broj}$$

Odgovor je pod A.

Vježba 227

Racionalno rješenje jednadžbe $7 \cdot 2^x - 4^x = 12$ jednako je:

$A.\ 2 \qquad B.\ 1 \qquad C.\ 3 \qquad D.\ 4$

Rezultat: A.

Zadatak 228 (Anamaria, gimnazija)

Riješi jednadžbu: $2^{x-1} + 2^{x-4} + 2^{x-2} = 6.5 + 3.25 + 1.625 + \ ...\ .$

Rješenje 228

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a \quad , \quad \frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}.$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

Zakon distribucije množenja prema zbrajanju

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Geometrijski red

$$\sum_{n=1}^{\infty} a_1 \cdot q^n = a_1 + a_1 \cdot q + a_1 \cdot q^2 + a_1 \cdot q^3 + \ ...$$

konvergentan je onda i samo onda ako vrijedi

$$|q| < 1.$$

Njegova je suma jednaka

$$s = \frac{a_1}{1 - q}.$$

$$2^{x-1} + 2^{x-4} + 2^{x-2} = 6.5 + 3.25 + 1.625 + \ ... \Rightarrow$$

$$\Rightarrow 2^x \cdot 2^{-1} + 2^x \cdot 2^{-4} + 2^x \cdot 2^{-2} = 6.5 + 3.25 + 1.625 + \ ... \Rightarrow$$

$$\Rightarrow 2^x \cdot \frac{1}{2^1} + 2^x \cdot \frac{1}{2^4} + 2^x \cdot \frac{1}{2^2} = 6.5 + 3.25 + 1.625 + \ ... \Rightarrow$$

$$\Rightarrow 2^x \cdot \frac{1}{2} + 2^x \cdot \frac{1}{16} + 2^x \cdot \frac{1}{4} = 6.5 + 3.25 + 1.625 + \ ... \Rightarrow 2^x \cdot \left(\frac{1}{2} + \frac{1}{16} + \frac{1}{4}\right) = 6.5 + 3.25 + 1.625 + \ ... \Rightarrow$$

$$\Rightarrow 2^x \cdot \frac{8 + 1 + 4}{16} = 6.5 + 3.25 + 1.625 + \ ... \Rightarrow 2^x \cdot \frac{13}{16} = 6.5 + 3.25 + 1.625 + \ ...\ .$$

Uočimo na desnoj strani jednadžbe beskonačan geometrijski red čiji količnik iznosi

$$q = \frac{3.25}{6.5} = \frac{1.625}{3.25} = \frac{1}{2}$$

pa je njegova suma jednaka:

$$s = \frac{6.5}{1 - \frac{1}{2}} \Rightarrow s = \frac{6.5}{\frac{1}{2}} \Rightarrow s = \frac{\frac{6.5}{1}}{\frac{1}{2}} \Rightarrow s = 13.$$

Sada računamo rješenje zadane jednadžbe.

$$2^x \cdot \frac{13}{16} = 13 \Rightarrow 2^x \cdot \frac{13}{16} = 13 \ / \cdot \frac{16}{13} \Rightarrow 2^x = 16 \Rightarrow 2^x = 2^4 \Rightarrow x = 4.$$

Vježba 228

Riješi jednadžbu: $2^x + 2^{x-3} + 2^{x-1} = 13 + 6.5 + 3.25 + \ ... \ .$

Rezultat: 4.

Zadatak 229 (Anamaria, gimnazija)

Riješi jednadžbu: $\log x + \log \sqrt[3]{x} + \log \sqrt[9]{x} + \ ... = 3.$

Rješenje 229

Ponovimo!

$$\log \sqrt[n]{x} = \frac{1}{n} \cdot \log x \quad , \quad \log_{10} x = \log x \quad , \quad \log 100 = 2 \quad , \quad \log f(x) = \log g(x) \Rightarrow f(x) = g(x).$$

Zakon distribucije množenja prema zbrajanju

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

Definicija:

$$\log_b a = c \ , \ b > 0 \ , \ b \neq 1 \ , \ a > 0 \Leftrightarrow b^c = a.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \ (\rightarrow) \qquad a = b^c$$

$$\log x + \log \sqrt[3]{x} + \log \sqrt[9]{x} + \ ... = 3 \Rightarrow \log x + \frac{1}{3} \cdot \log x + \frac{1}{9} \cdot \log x + \ ... = 3 \Rightarrow$$

$$\Rightarrow \log x \cdot \left(1 + \frac{1}{3} + \frac{1}{9} + \ ...\right) = 3.$$

Uočimo da je izraz u zagradi beskonačan geometrijski red čiji količnik iznosi

$$q = \frac{\frac{1}{3}}{1} = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1}{3}$$

pa je njegova suma jednaka:

$$s = \frac{1}{1 - \frac{1}{3}} \Rightarrow s = \frac{1}{\frac{2}{3}} \Rightarrow s = \frac{\frac{1}{1}}{\frac{2}{3}} \Rightarrow s = \frac{3}{2}.$$

Sada računamo rješenje zadane jednadžbe.

$$\log x \cdot \frac{3}{2} = 3 \Rightarrow \log x \cdot \frac{3}{2} = 3 \ / \cdot \frac{2}{3} \Rightarrow \log x = 2 \Rightarrow \log x = \log 100 \Rightarrow x = 100.$$

Ili

$$\log x \cdot \frac{3}{2} = 3 \Rightarrow \log x \cdot \frac{3}{2} = 3 \ / \cdot \frac{2}{3} \Rightarrow \log x = 2 \Rightarrow 10^2 = x \Rightarrow x = 10^2 \Rightarrow x = 100.$$

Vježba 229

Riješi jednadžbu: $\log x + \log \sqrt[3]{x} + \log \sqrt[9]{x} + \ ... = 6.$

Rezultat: 10000.

Zadatak 230 (Beky, gimnazija)

$$\text{Ako je } \log_a x = s \text{ i } \log_a y^2 = t, \text{ onda je } \log_a \frac{\sqrt{x}}{y} =$$

$$A.\ \frac{s-t}{2} \qquad B.\ \frac{s}{t} \qquad C.\ s-\frac{t}{2} \qquad D.\ \sqrt{s}-\frac{t}{2}$$

Rješenje 230

Ponovimo!

$$\log_b a^n = n \cdot \log_b a \quad , \quad \log_b \frac{x}{y} = \log_b x - \log_b y \quad , \quad \log_b \sqrt[n]{a} = \frac{1}{n} \cdot \log_b a.$$

Definicija:

$$\log_b a = c \ , \ b > 0 \ , \ b \neq 1 \ , \ a > 0 \ \Leftrightarrow \ b^c = a.$$

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_b a = c \qquad \log_b a = b^c \ \rightarrow \qquad a = b^c$$

1.inačica

Najprije transformiramo

$$\log_a y^2 = t \ \Rightarrow \ 2 \cdot \log_a y = t \ \Rightarrow \ 2 \cdot \log_a y = t \ /:2 \ \Rightarrow \ \log_a y = \frac{1}{2} \cdot t.$$

Sada računamo:

$$\log_a \frac{\sqrt{x}}{y} = \log_a \sqrt{x} - \log_a y = \frac{1}{2} \cdot \log_a x - \log_a y = \begin{bmatrix} \log_a x = s \\ \log_a y = \frac{1}{2} \cdot t \end{bmatrix} = \frac{1}{2} \cdot s - \frac{1}{2} \cdot t = \frac{s-t}{2}.$$

Odgovor je pod A

2.inačica

$$\log_a \frac{\sqrt{x}}{y} = \log_a \sqrt{x} - \log_a y = \frac{1}{2} \cdot \log_a x - 1 \cdot \log_a y = \frac{1}{2} \cdot \log_a x - \frac{2}{2} \cdot \log_a y =$$

$$= \frac{1}{2} \cdot \log_a x - \frac{1}{2} \cdot \left(2 \cdot \log_a y\right) = \frac{1}{2} \cdot \log_a x - \frac{1}{2} \cdot \log_a y^2 = \begin{bmatrix} \log_a x = s \\ \log_a y^2 = t \end{bmatrix} = \frac{1}{2} \cdot s - \frac{1}{2} \cdot t = \frac{s-t}{2}.$$

Odgovor je pod A

Vježba 230

$$\text{Ako je } \log_a x = s \text{ i } \log_a y = t, \text{ onda je } \log_a \frac{\sqrt{x}}{y} =$$

$$A.\ \frac{s-t}{2} \qquad B.\ \frac{s}{2}-t \qquad C.\ s-\frac{t}{2} \qquad D.\ \sqrt{s}-\frac{t}{2}$$

Rezultat: B.

Zadatak 231 (Boris, gimnazija)

$$\text{Izračunaj: } \frac{\log 2 - \log 3}{\log 225 - 2}.$$

Rješenje 231

Ponovimo!

$$\log 100 = 2 \quad , \quad \log\frac{x}{y} = \log x - \log y \quad , \quad \log a^{n} = n \cdot \log a \quad , \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}.$$

$$\frac{\log 2 - \log 3}{\log 225 - 2} = \frac{\log 2 - \log 3}{\log 225 - \log 100} = \frac{\log\frac{2}{3}}{\log\frac{225}{100}} = \frac{\log\frac{2}{3}}{\log\frac{225}{100}} = \frac{\log\frac{2}{3}}{\log\frac{9}{4}} = \frac{\log\frac{2}{3}}{\log\left(\frac{3}{2}\right)^{2}} = \frac{\log\frac{2}{3}}{2 \cdot \log\frac{3}{2}} =$$

$$= \frac{\log\left(\frac{3}{2}\right)^{-1}}{2 \cdot \log\frac{3}{2}} = \frac{-1 \cdot \log\frac{3}{2}}{2 \cdot \log\frac{3}{2}} = \frac{-1 \cdot \log\frac{3}{2}}{2 \cdot \log\frac{3}{2}} = -\frac{1}{2}.$$

Vježba 231

Izračunaj: $\dfrac{\log 3 - \log 2}{\log 225 - 2}$.

Rezultat: $\dfrac{1}{2}$.

Zadatak 232 (Boris, gimnazija)

Ako je $\log 64 = a$, koliko je $\log\sqrt[3]{25}$?

Rješenje 232

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_{b} a = c \qquad \log_{b} a = b^{c} \ \rightarrow \qquad a = b^{c}$$

$$\log_{10} x = \log x \quad , \quad \log a^{n} = n \cdot \log a \quad , \quad \log\frac{a}{b} = \log a - \log b \quad , \quad \log 10 = 1 \quad , \quad \log 100 = 2.$$

$$\log\sqrt[n]{a} = \frac{1}{n} \cdot \log a \quad , \quad \frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}.$$

Zakon distribucije množenja prema zbrajanju

$$a \cdot (b + c) = a \cdot b + a \cdot c \quad , \quad a \cdot b + a \cdot c = a \cdot (b + c).$$

1.inačica

$$\log 64 = a \ \Rightarrow \ \log 2^{6} = a \ \Rightarrow \ 6 \cdot \log 2 = a \ \Rightarrow \ 6 \cdot \log 2 = a \ /: 6 \ \Rightarrow \ \log 2 = \frac{a}{6}.$$

Sada je:

$$\log\sqrt[3]{25} = \frac{1}{3} \cdot \log 25 = \frac{1}{3} \cdot \log\frac{100}{4} = \frac{1}{3} \cdot (\log 100 - \log 4) = \frac{1}{3} \cdot \left(2 - \log 2^{2}\right) = \frac{1}{3} \cdot (2 - 2 \cdot \log 2) =$$

$$= \frac{1}{3} \cdot \left(2 - 2 \cdot \frac{a}{6}\right) = \frac{1}{3} \cdot \left(2 - 2 \cdot \frac{a}{6}\right) = \frac{1}{3} \cdot \left(2 - \frac{a}{3}\right) = \frac{2}{3} - \frac{a}{9} = \frac{6 - a}{9}.$$

2.inačica

$$\log 64 = a \ \Rightarrow \ \log 2^{6} = a \ \Rightarrow \ 6 \cdot \log 2 = a \ \Rightarrow \ 6 \cdot \log 2 = a \ /: 6 \ \Rightarrow \ \log 2 = \frac{a}{6}.$$

Sada je:

$$\log\sqrt[3]{25}=\frac{1}{3}\cdot\log 25=\frac{1}{3}\cdot\log 5^{2}=\frac{1}{3}\cdot 2\cdot\log 5=\frac{2}{3}\cdot\log 5=\frac{2}{3}\cdot\log\frac{10}{2}=\frac{2}{3}\cdot(\log 10-\log 2)=$$
$$=\frac{2}{3}\cdot\left(1-\frac{a}{6}\right)=\frac{2}{3}-\frac{2\cdot a}{18}=\frac{2}{3}-\frac{2\cdot a}{18}=\frac{2}{3}-\frac{a}{9}=\frac{6-a}{9}.$$

Vježba 232

Ako je $\log 4096=2\cdot a$, koliko je $\log\sqrt[3]{25}$?

Rezultat: $\frac{6-a}{9}.$

Zadatak 233 (Boris, gimnazija)

Riješi sustav jednadžbi: $\begin{cases} x+3^{y}=10 \\ y-2=\log_{3}x. \end{cases}$

Rješenje 233

Ponovimo!

Logaritam broja a po bazi b je broj c kojim treba potencirati bazu b da se dobije broj a.

Mnemotehničko pravilo za pamćenje osnovne veze eksponencijalne i logaritamske funkcije:

$$\log_{b}a=c \qquad \log_{b}a=b^{c} \rightarrow \qquad a=b^{c}$$

$$b^{\log_{b}a}=a \quad , \quad a^{n}\cdot a^{m}=a^{n+m} \quad , \quad \log_{b}1=0 \quad , \quad a^{-n}=\frac{1}{a^{n}}.$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad a^{0}=1 \quad , \quad n=\frac{n}{1} \quad , \quad \frac{a}{b}+\frac{c}{d}=\frac{a\cdot d+b\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c)=a\cdot b+a\cdot c \quad , \quad a\cdot b+a\cdot c=a\cdot(b+c).$$

1.inačica

$$\left.\begin{matrix} x+3^{y}=10 \\ y-2=\log_{3}x \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x+3^{y}=10 \\ y=\log_{3}x+2 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{supstitucije} \end{bmatrix} \Rightarrow x+3^{\log_{3}x+2}=10 \Rightarrow$$

$$\Rightarrow x+3^{\log_{3}x}\cdot 3^{2}=10 \Rightarrow x+x\cdot 3^{2}=10 \Rightarrow x+9\cdot x=10 \Rightarrow 10\cdot x=10 \Rightarrow 10\cdot x=10\ /:10 \Rightarrow x=1.$$

Računamo y.

$$\left.\begin{matrix} x=1 \\ y-2=\log_{3}x \end{matrix}\right\} \Rightarrow y-2=\log_{3}1 \Rightarrow y-2=0 \Rightarrow y=2.$$

Rješenje sustava je:

$$(x,\ y)=(1,\ 2).$$

2.inačica

$$\left.\begin{matrix} x+3^{y}=10 \\ y-2=\log_{3}x \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x+3^{y}=10 \\ 3^{y-2}=x \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{metoda} \\ \text{supstitucije} \end{bmatrix} \Rightarrow 3^{y-2}+3^{y}=10 \Rightarrow 3^{y}\cdot 3^{-2}+3^{y}=10 \Rightarrow$$

$$\Rightarrow 3^{y}\cdot\frac{1}{3^{2}}+3^{y}=10 \Rightarrow 3^{y}\cdot\left(\frac{1}{3^{2}}+1\right)=10 \Rightarrow 3^{y}\cdot\left(\frac{1}{9}+1\right)=10 \Rightarrow 3^{y}\cdot\left(\frac{1}{9}+\frac{1}{1}\right)=10 \Rightarrow$$

$$\Rightarrow 3^{y}\cdot\frac{1+9}{9}=10 \Rightarrow 3^{y}\cdot\frac{10}{9}=10 \Rightarrow 3^{y}\cdot\frac{10}{9}=10\ /\cdot\frac{9}{10} \Rightarrow 3^{y}=9 \Rightarrow 3^{y}=3^{2} \Rightarrow y=2.$$

Računamo x.

$$\left.\begin{array}{l} y=2 \\ x+3^{y}=10 \end{array}\right\} \Rightarrow x+3^{2}=10 \Rightarrow x+9=10 \Rightarrow x=10-9 \Rightarrow x=1.$$

Rješenje sustava je:

$$(x,\ y)=(1,\ 2).$$

Vježba 233

Riješi sustav jednadžbi: $\left\{\begin{array}{l} 3^{y}=10-x \\ y=\log_{3} x+2. \end{array}\right.$

Rezultat: $(x,\ y)=(1,\ 2).$

Zadatak 234 (Crna pantera, gimnazija)

Riješi jednadžbu: $25^{x+2}+5=6\cdot 5^{x+2}.$

Rješenje 234

Ponovimo!

$$\left(a^{n}\right)^{m}=a^{n\cdot m} \quad , \quad a^{1}=a \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad a^{0}=1.$$

$$a^{n}\cdot a^{m}=a^{n+m} \quad , \quad a^{-n}=\frac{1}{a^{n}} \quad , \quad a^{n}:a^{m}=a^{n-m}.$$

1.inačica

$$25^{x+2}+5=6\cdot 5^{x+2} \Rightarrow \left(5^{2}\right)^{x+2}+5=6\cdot 5^{x+2} \Rightarrow \left(5^{x+2}\right)^{2}+5=6\cdot 5^{x+2} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{supstitucija} \\ 5^{x+2}=t \end{array}\right] \Rightarrow t^{2}+5=6\cdot t \Rightarrow t^{2}-6\cdot t+5=0 \Rightarrow \left.\begin{array}{l} t^{2}-6\cdot t+5=0 \\ a=1 \ ,\ b=-6 \ ,\ c=5 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a=1 \ ,\ b=-6 \ ,\ c=5 \\ t_{1,2}=\dfrac{-b\pm\sqrt{b^{2}-4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow t_{1,2}=\frac{-(-6)\pm\sqrt{(-6)^{2}-4\cdot 1\cdot 5}}{2\cdot 1} \Rightarrow t_{1,2}=\frac{6\pm\sqrt{36-20}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{6\pm\sqrt{16}}{2} \Rightarrow t_{1,2}=\frac{6\pm 4}{2} \Rightarrow \left.\begin{array}{l} t_{1}=\dfrac{6+4}{2} \\ t_{2}=\dfrac{6-4}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_{1}=\dfrac{10}{2} \\ t_{2}=\dfrac{2}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} t_{1}=5 \\ t_{2}=1 \end{array}\right\}.$$

Vraćamo se supstituciji.

- $\left.\begin{array}{l} 5^{x+2}=t \\ t=5 \end{array}\right\} \Rightarrow 5^{x+2}=5 \Rightarrow 5^{x+2}=5^{1} \Rightarrow x+2=1 \Rightarrow x=1-2 \Rightarrow x_{1}=-1.$
- $\left.\begin{array}{l} 5^{x+2}=t \\ t=1 \end{array}\right\} \Rightarrow 5^{x+2}=1 \Rightarrow 5^{x+2}=5^{0} \Rightarrow x+2=0 \Rightarrow x_{2}=-2.$

2.inačica

$$25^{x+2}+5=6\cdot 5^{x+2} \Rightarrow 25^{x}\cdot 25^{2}+5=6\cdot 5^{x}\cdot 5^{2} \Rightarrow \left(5^{2}\right)^{x}\cdot\left(5^{2}\right)^{2}+5=6\cdot 5^{x}\cdot 5^{2} \Rightarrow$$

$$\Rightarrow \left(5^{x}\right)^{2}\cdot 5^{4}+5=6\cdot 5^{x}\cdot 5^{2} \Rightarrow \left(5^{x}\right)^{2}\cdot 5^{4}+5=6\cdot 5^{x}\cdot 5^{2} \ /:5 \Rightarrow \left(5^{x}\right)^{2}\cdot 5^{3}+1=6\cdot 5^{x}\cdot 5^{1} \Rightarrow$$

$$\Rightarrow 125\cdot\left(5^x\right)^2+1=6\cdot 5^x\cdot 5 \Rightarrow 125\cdot\left(5^x\right)^2+1=30\cdot 5^x \Rightarrow \begin{bmatrix}\text{supstitucija}\\ 5^x=t\end{bmatrix} \Rightarrow 125\cdot t^2+1=30\cdot t \Rightarrow$$

$$\Rightarrow 125\cdot t^2-30\cdot t+1=0 \Rightarrow \left.\begin{array}{l}125\cdot t^2-30\cdot t+1=0\\ a=125\ ,\ b=-30\ ,\ c=1\end{array}\right\} \Rightarrow \left.\begin{array}{l}a=125\ ,\ b=-30\ ,\ c=1\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{-(-30)\pm\sqrt{(-30)^2-4\cdot 125\cdot 1}}{2\cdot 125} \Rightarrow t_{1,2}=\frac{30\pm\sqrt{900-500}}{250} \Rightarrow t_{1,2}=\frac{30\pm\sqrt{400}}{250} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{30\pm 20}{250} \Rightarrow \left.\begin{array}{l}t_1=\dfrac{30+20}{250}\\ t_2=\dfrac{30-20}{250}\end{array}\right\} \Rightarrow \left.\begin{array}{l}t_1=\dfrac{50}{250}\\ t_2=\dfrac{10}{250}\end{array}\right\} \Rightarrow \left.\begin{array}{l}t_1=\dfrac{1}{5}\\ t_2=\dfrac{1}{25}\end{array}\right\} \Rightarrow \left.\begin{array}{l}t_1=\dfrac{1}{5^1}\\ t_2=\dfrac{1}{5^2}\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}t_1=5^{-1}\\ t_2=5^{-2}\end{array}\right\}.$$

Vraćamo se supstituciji.

- $\left.\begin{array}{l}5^x=t\\ t=5^{-1}\end{array}\right\} \Rightarrow 5^x=5^{-1} \Rightarrow x_1=-1.$
- $\left.\begin{array}{l}5^x=t\\ t=5^{-2}\end{array}\right\} \Rightarrow 5^x=5^{-2} \Rightarrow x_2=-2.$

Vježba 234

Riješi jednadžbu: $5^{2\cdot x+3}+1=6\cdot 5^{x+1}$.

Rezultat: $x_1=-1\ ,\ x_2=-2.$

Zadatak 235 (Paula, ekonomska škola)

Otapanje neke topljive tvari u vodi odvija se po zakonu $S=S_0\cdot\left(1-e^{-k\cdot t}\right)$ pri čemu je S količina tvari što se otopi u vremenu t, S_0 količina potrebna za zasićenost otopine, a k > 0 konstanta koja ovisi o vrsti tvari što se otapa. Ako se 20 g šećera otopi za 1 minutu, a 30 g za 2 minute, izračunaj količinu S_0 potrebnu da se postigne zasićenje otopine.

Rješenje 235

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^2-b^2=(a-b)\cdot(a+b)\quad,\quad \ln a^n=n\cdot\ln a\quad,\quad \ln e=1\quad,\quad \left.\begin{array}{l}a=b\\ c=d\end{array}\right\} \Rightarrow \frac{a}{c}=\frac{b}{d}.$$

Iz zadanih podataka S i t izračunamo konstantu k.

$$\left.\begin{array}{l}S=20\ ,\ t=1\\ S=30\ ,\ t=2\end{array}\right\} \Rightarrow \left[S=S_0\cdot\left(1-e^{-k\cdot t}\right)\right] \Rightarrow \left.\begin{array}{l}20=S_0\cdot\left(1-e^{-k\cdot 1}\right)\\ 30=S_0\cdot\left(1-e^{-k\cdot 2}\right)\end{array}\right\} \Rightarrow \left.\begin{array}{l}20=S_0\cdot\left(1-e^{-k}\right)\\ 30=S_0\cdot\left(1-e^{-2\cdot k}\right)\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} S_0\cdot\left(1-e^{-k}\right)=20 \\ S_0\cdot\left(1-e^{-2\cdot k}\right)=30 \end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{podijelimo}\\ \text{jednadžbe}\end{array}\right] \Rightarrow \frac{S_0\cdot\left(1-e^{-2\cdot k}\right)}{S_0\cdot\left(1-e^{-k}\right)}=\frac{30}{20} \Rightarrow \frac{S_0\cdot\left(1-e^{-2\cdot k}\right)}{S_0\cdot\left(1-e^{-k}\right)}=\frac{30}{20} \Rightarrow$$

$$\Rightarrow \frac{1-e^{-2\cdot k}}{1-e^{-k}}=\frac{3}{2} \Rightarrow \frac{1-e^{-2\cdot k}}{1-e^{-k}}=1.5 \Rightarrow \frac{1-\left(e^{-k}\right)^2}{1-e^{-k}}=1.5 \Rightarrow \frac{\left(1-e^{-k}\right)\cdot\left(1+e^{-k}\right)}{1-e^{-k}}=1.5 \Rightarrow$$

$$\Rightarrow \frac{\left(1-e^{-k}\right)\cdot\left(1+e^{-k}\right)}{1-e^{-k}}=1.5 \Rightarrow 1+e^{-k}=1.5 \Rightarrow e^{-k}=1.5-1 \Rightarrow e^{-k}=0.5 \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l}\text{logaritmiramo}\\ \text{jednadžbu}\end{array}\right] \Rightarrow e^{-k}=0.5 \;/\ln \Rightarrow \ln e^{-k}=\ln 0.5 \Rightarrow -k\cdot\ln e=\ln 0.5 \Rightarrow$$

$$\Rightarrow -k\cdot 1=\ln 0.5 \Rightarrow -k=\ln 0.5 \;/\cdot(-1) \Rightarrow k=-\ln 0.5 \Rightarrow k=0.693\,\frac{1}{\min} \Rightarrow$$

$$\Rightarrow k=0.693\,\frac{1}{60\ s} \Rightarrow k=0.01155\,\frac{1}{s}.$$

Sada računamo S_0.

- $$\left.\begin{array}{l} S=20\ ,\ t=1\ ,\ k=0.693 \\ S=S_0\cdot\left(1-e^{-k\cdot t}\right) \end{array}\right\} \Rightarrow \left.\begin{array}{l} S=20\ ,\ t=1\ ,\ k=0.693 \\ S_0=\dfrac{S}{1-e^{-k\cdot t}} \end{array}\right\} \Rightarrow S_0=\frac{20}{1-e^{-0.693\cdot 1}} \Rightarrow$$

$$\Rightarrow S_0=\frac{20}{1-e^{-0.693}} \Rightarrow S_0=\frac{20}{1-0.5007} \Rightarrow S_0=\frac{20}{0.4993} \Rightarrow S_0=40 \Rightarrow S_0=40\ g.$$

Ili

- $$\left.\begin{array}{l} S=30\ ,\ t=2\ ,\ k=0.693 \\ S=S_0\cdot\left(1-e^{-k\cdot t}\right) \end{array}\right\} \Rightarrow \left.\begin{array}{l} S=30\ ,\ t=2\ ,\ k=0.693 \\ S_0=\dfrac{S}{1-e^{-k\cdot t}} \end{array}\right\} \Rightarrow S_0=\frac{30}{1-e^{-0.693\cdot 2}} \Rightarrow$$

$$\Rightarrow S_0=\frac{30}{1-e^{-1.386}} \Rightarrow S_0=\frac{30}{1-0.25007} \Rightarrow S_0=\frac{30}{0.74993} \Rightarrow S_0=40 \Rightarrow S_0=40\ g.$$

Vježba 235

Otapanje neke topljive tvari u vodi odvija se po zakonu $S=S_0\cdot\left(1-e^{-k\cdot t}\right)$ pri čemu je S količina tvari što se otopi u vremenu t, S_0 količina potrebna za zasićenost otopine, a k > 0 konstanta koja ovisi o vrsti tvari što se otapa. Ako se 2 dag šećera otopi za 1 minutu, a 3 dag za 2 minute, izračunaj količinu S_0 potrebnu da se postigne zasićenje otopine.

Rezultat: $S_0 = 4$ dag.

Zadatak 236 (Nenad, maturant gimnazije)

Nađi vrijednost funkcije $f(x)=\frac{x+3}{x-3}$, ako je $x=3^{\log_9\left(3-2\cdot\sqrt{2}\right)}$.

Rješenje 236

Ponovimo!

$$(a-b)^2=a^2-2\cdot a\cdot b+b^2\quad,\quad\left(\sqrt{a}\right)^2=a\quad,\quad\log_b a^n=n\cdot\log_b a\quad,\quad\left(a^n\right)^m=a^{n\cdot m}.$$

$$b^{\log_b a} = a \quad , \quad a^2 - b^2 = (a-b)\cdot(a+b) \quad , \quad \frac{a}{b}\cdot\frac{c}{d} = \frac{a\cdot c}{b\cdot d}.$$

Zakon distribucije množenja prema zbrajanju

$$a\cdot(b+c) = a\cdot b + a\cdot c \quad , \quad a\cdot b + a\cdot c = a\cdot(b+c).$$

Množenje zagrada

$$(a+b)\cdot(c+d) = a\cdot c + a\cdot d + b\cdot c + b\cdot d.$$

Uočimo da se izraz $3-2\cdot\sqrt{2}$ može napisati kao kvadrat razlike (kvadrat binoma).

$$3-2\cdot\sqrt{2} = 2-2\cdot\sqrt{2}+1 = \left(\sqrt{2}\right)^2 - 2\cdot\sqrt{2}+1^2 = \left(\sqrt{2}-1\right)^2.$$

Sada vrijednost varijable x možemo transformirati na sljedeći način:

$$x = 3^{\log_9\left(3-2\cdot\sqrt{2}\right)} \Rightarrow x = 3^{\log_9\left(\sqrt{2}-1\right)^2} \Rightarrow x = 3^{2\cdot\log_9\left(\sqrt{2}-1\right)} \Rightarrow$$

$$\Rightarrow x = \left(3^2\right)^{\log_9\left(\sqrt{2}-1\right)} \Rightarrow x = 9^{\log_9\left(\sqrt{2}-1\right)} \Rightarrow x = \sqrt{2}-1.$$

Računamo vrijednost zadane funkcije:

$$\left.\begin{matrix} f(x) = \dfrac{x+3}{x-3} \\ x = \sqrt{2}-1 \end{matrix}\right\} \Rightarrow f(x) = \frac{\sqrt{2}-1+3}{\sqrt{2}-1-3} = \frac{\sqrt{2}+2}{\sqrt{2}-4} = \left[\begin{matrix}\text{racionalizacija} \\ \text{nazivnika}\end{matrix}\right] = \frac{\sqrt{2}+2}{\sqrt{2}-4}\cdot\frac{\sqrt{2}+4}{\sqrt{2}+4} =$$

$$= \frac{\left(\sqrt{2}+2\right)\cdot\left(\sqrt{2}+4\right)}{\left(\sqrt{2}-4\right)\cdot\left(\sqrt{2}+4\right)} = \frac{\left(\sqrt{2}\right)^2+4\cdot\sqrt{2}+2\cdot\sqrt{2}+8}{\left(\sqrt{2}\right)^2-4^2} = \frac{2+4\cdot\sqrt{2}+2\cdot\sqrt{2}+8}{2-16} = \frac{10+6\cdot\sqrt{2}}{-14} = -\frac{10+6\cdot\sqrt{2}}{14} =$$

$$= -\frac{2\cdot\left(5+3\cdot\sqrt{2}\right)}{14} = -\frac{2\cdot\left(5+3\cdot\sqrt{2}\right)}{-14} = -\frac{5+3\cdot\sqrt{2}}{7}.$$

Vježba 236

Nađi vrijednost funkcije $f(x) = \dfrac{x+3}{x-3}$, ako je $x = 2^{\log_4\left(3-2\cdot\sqrt{2}\right)}$.

Rezultat: $-\dfrac{5+3\cdot\sqrt{2}}{7}$.

Zadatak 237 (Andrea, strukovna škola)

Broj riba u ribnjaku raste u skladu s eksponencijalnim zakonom:

$$f(x) = 400\cdot 10^{0.02\cdot x},$$

gdje je x broj mjeseci proteklih od početka promatranja.

a) Koliki je bio broj riba na početku promatranja?
b) Koliki je bio broj riba u ribnjaku nakon godinu dana?
c) Za koliko će se vremena broj riba udvostručiti?

Rješenje 237

Ponovimo!

$$a^0 = 1 \quad , \quad \log_{10} x = \log x \quad , \quad \log 10 = 1 \quad , \quad \log a^n = n\cdot\log a \quad , \quad \log 10^n = n.$$

a) Računamo broj riba na početku promatranja

$$\left.\begin{array}{l} x=0 \\ f(x)=400\cdot 10^{0.02\cdot x} \end{array}\right\} \Rightarrow f(0)=400\cdot 10^{0.02\cdot 0} \Rightarrow f(0)=400\cdot 10^{0} \Rightarrow f(0)=400\cdot 1 \Rightarrow$$

$$\Rightarrow f(0)=400 \text{ riba.}$$

b) Računamo broj riba nakon godinu dana

$$\left.\begin{array}{l} x=1\ g=12\ mj \\ f(x)=400\cdot 10^{0.02\cdot x} \end{array}\right\} \Rightarrow f(12)=400\cdot 10^{0.02\cdot 12} \Rightarrow f(12)=400\cdot 10^{0.24} \Rightarrow$$

$$\Rightarrow f(12)=695.12\approx 695 \text{ riba.}$$

c) Računamo vrijeme za koje će se broj riba udvostručiti

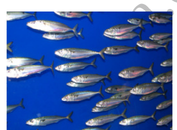
$$\left.\begin{array}{l} f(x)=400\cdot 10^{0.02\cdot x} \\ f(x)=2\cdot f(0) \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{komparacije} \end{array}\right] \Rightarrow 400\cdot 10^{0.02\cdot x}=2\cdot f(0) \Rightarrow$$

$$\Rightarrow 400\cdot 10^{0.02\cdot x}=2\cdot 400\cdot 10^{0.02\cdot 0} \Rightarrow 400\cdot 10^{0.02\cdot x}=2\cdot 400\cdot 10^{0} \Rightarrow 400\cdot 10^{0.02\cdot x}=2\cdot 400\cdot 1 \Rightarrow$$

$$\Rightarrow 400\cdot 10^{0.02\cdot x}=2\cdot 400 \Rightarrow 400\cdot 10^{0.02\cdot x}=2\cdot 400\ /:400 \Rightarrow 10^{0.02\cdot x}=2 \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{logaritmiramo} \\ \text{jednadžbu} \end{array}\right] \Rightarrow 10^{0.02\cdot x}=2\ /\log \Rightarrow \log 10^{0.02\cdot x}=\log 2 \Rightarrow 0.02\cdot x=\log 2 \Rightarrow$$

$$\Rightarrow 0.02\cdot x=\log 2\ /:0.02 \Rightarrow x=\frac{\log 2}{0.02} \Rightarrow x=15.05 \text{ mjeseci.}$$



Vježba 237

Broj riba u ribnjaku raste u skladu s eksponencijalnim zakonom:

$$f(x)=400\cdot 10^{0.02\cdot x},$$

gdje je x broj mjeseci proteklih od početka promatranja. Koliki je bio broj riba u ribnjaku nakon dvije godine?

Rezultat: 1207.

Zadatak 238 (Andrea, strukovna škola)

Vrijednost tvorničkog stroja mijenja se po eksponencijalnom zakonu

$$f(t)=40000\cdot 2^{-0.16\cdot t},$$

gdje je t broj godina uporabe, a f(t) vrijednost u kunama.

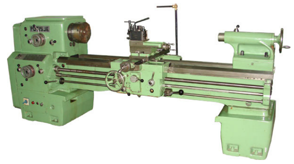
a) Kolika je bila vrijednost stroja pri kupnji?
b) Kolika je vrijednost stroja nakon 4 godine uporabe?
c) Za koliko godina će vrijednost pasti na 1000 kn?
d) Za koliko će se godina vrijednost dvostruko smanjiti od početne?

Rješenje 238

Ponovimo!

$$a^{0}=1 \quad , \quad \log_{10} x=\log x \quad , \quad \log 10=1 \quad , \quad \log a^{n}=n\cdot \log a \quad , \quad \log 10^{n}=n.$$

$$a^{-n}=\frac{1}{a^{n}} \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$



a) Računamo vrijednost stroja pri kupnji

$$\left.\begin{array}{l} t=0 \\ f(t)=40000\cdot 2^{-0.16\cdot t} \end{array}\right\} \Rightarrow f(0)=40000\cdot 2^{-0.16\cdot 0} \Rightarrow f(0)=40000\cdot 2^{0} \Rightarrow f(0)=40000\cdot 1 \Rightarrow$$

$$\Rightarrow f(0)=40000 \ kn.$$

b) Računamo vrijednost stroja nakon 4 godine uporabe

$$\left.\begin{array}{l} t=4 \\ f(t)=40000\cdot 2^{-0.16\cdot t} \end{array}\right\} \Rightarrow f(4)=40000\cdot 2^{-0.16\cdot 4} \Rightarrow f(4)=40000\cdot 2^{-0.64} \Rightarrow$$

$$\Rightarrow f(4)=\frac{40000}{2^{0.64}} \Rightarrow f(4)=25668.52 \ kn.$$

c) Računamo vrijeme za koje će vrijednost stroja pasti na 1000 kn

$$\left.\begin{array}{l} f(t)=40000\cdot 2^{-0.16\cdot t} \\ f(t)=1000 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{komparacije} \end{array}\right] \Rightarrow 40000\cdot 2^{-0.16\cdot t}=1000 \Rightarrow$$

$$\Rightarrow 40000\cdot 2^{-0.16\cdot t}=1000 \ /:40000 \Rightarrow 2^{-0.16\cdot t}=\frac{1000}{40000} \Rightarrow 2^{-0.16\cdot t}=\frac{1000}{40000} \Rightarrow$$

$$\Rightarrow 2^{-0.16\cdot t}=\frac{1}{40} \Rightarrow 2^{-0.16\cdot t}=40^{-1} \Rightarrow \left[\begin{array}{l} \text{logaritmiramo} \\ \text{jednadžbu} \end{array}\right] \Rightarrow 2^{-0.16\cdot t}=40^{-1} \ /\log \Rightarrow$$

$$\Rightarrow \log 2^{-0.16\cdot t}=\log 40^{-1} \Rightarrow -0.16\cdot t\cdot\log 2=-1\cdot\log 40 \Rightarrow -0.16\cdot t\cdot\log 2=-\log 40 \Rightarrow$$

$$\Rightarrow -0.16\cdot t\cdot\log 2=-\log 40 \ /\cdot\frac{-1}{0.16\cdot\log 2} \Rightarrow t=\frac{\log 40}{0.16\cdot\log 2} \Rightarrow t=33.26 \ godina.$$

d) Računamo vrijeme za koje će se vrijednost stroja dvostruko smanjiti od početne

$$\left.\begin{array}{l} f(t)=40000\cdot 2^{-0.16\cdot t} \\ f(t)=\frac{1}{2}\cdot f(0) \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{komparacije} \end{array}\right] \Rightarrow 40000\cdot 2^{-0.16\cdot t}=\frac{1}{2}\cdot f(0) \Rightarrow$$

$$\Rightarrow 40000\cdot 2^{-0.16\cdot t}=\frac{1}{2}\cdot 40000\cdot 2^{-0.16\cdot 0} \Rightarrow 40000\cdot 2^{-0.16\cdot t}=\frac{1}{2}\cdot 40000\cdot 2^{0} \Rightarrow$$

$$\Rightarrow 40000\cdot 2^{-0.16\cdot t}=\frac{1}{2}\cdot 40000\cdot 1 \Rightarrow 40000\cdot 2^{-0.16\cdot t}=\frac{1}{2}\cdot 40000 \Rightarrow$$

$$\Rightarrow 40000\cdot 2^{-0.16\cdot t}=\frac{1}{2}\cdot 40000 \ /:40000 \Rightarrow 2^{-0.16\cdot t}=\frac{1}{2} \Rightarrow 2^{-0.16\cdot t}=2^{-1} \Rightarrow$$

$$\Rightarrow -0.16\cdot t=-1 \Rightarrow -0.16\cdot t=-1 \ /\cdot\frac{-1}{0.16} \Rightarrow t=\frac{1}{0.16} \Rightarrow t=6.25 \ godina.$$

Vježba 238

Vrijednost tvorničkog stroja mijenja se po eksponencijalnom zakonu

$$f(t)=75000\cdot 2^{-0.16\cdot t},$$

gdje je t broj godina uporabe, a f(t) vrijednost u kunama. Kolika je bila vrijednost stroja pri kupnji?

Rezultat: 75000 kn.

Zadatak 239 (Malena, strukovna škola)

Izraz $\log_2 4a+\log_2 2a^2$ jednak je:

$A.\ 3+3\cdot\log_2 a \qquad B.\ 2\cdot a+2 \qquad C.\ 4+3\cdot\log_2 a \qquad D.\ 4\cdot a+3$

Rješenje 239

Ponovimo!

$$a^1=a \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \log_b b=1 \quad , \quad \log_b(x\cdot y)=\log_b x+\log_b y \quad , \quad \log_b a^n=n\cdot\log_b a.$$

1.inačica

$$\log_2 4a+\log_2 2a^2=\log_2 4+\log_2 a+\log_2 2+\log_2 a^2=\log_2 2^2+\log_2 a+\log_2 2+\log_2 a^2=$$

$$=2\cdot\log_2 2+\log_2 a+\log_2 2+2\cdot\log_2 a=2\cdot 1+\log_2 a+1+2\cdot\log_2 a=2+\log_2 a+1+2\cdot\log_2 a=$$

$$=3+3\cdot\log_2 a.$$

Odgovor je pod A.

2.inačica

$$\log_2 4a+\log_2 2a^2=\log_2\left(4a\cdot 2a^2\right)=\log_2\left(8\cdot a^3\right)=\log_2 8+\log_2 a^3=\log_2 2^3+\log_2 a^3=$$

$$=3\cdot\log_2 2+3\cdot\log_2 a=3\cdot 1+3\cdot\log_2 a=3+3\cdot\log_2 a.$$

Vježba 239

Izraz $\log_2 8a+\log_2 a^2$ jednak je:

$A.\ 3+3\cdot\log_2 a \qquad B.\ 2\cdot a+2 \qquad C.\ 4+3\cdot\log_2 a \qquad D.\ 4\cdot a+3$

Rezultat: A.

Zadatak 240 (Malena, strukovna škola)

Rješenje jednadžbe $5\cdot 9^{x+1}=15$ nalazi se u intervalu:

$A.\ \langle -\infty, -2\,] \qquad B.\ \langle -2, -1\,] \qquad C.\ \langle -1, 2\,] \qquad D.\ \langle 2, \infty\rangle$

Rješenje 240

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^1=a \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$n=\frac{n}{1} \quad , \quad \frac{a}{b}-\frac{c}{d}=\frac{a\cdot d-b\cdot c}{b\cdot d} \quad , \quad \sqrt{a}=a^{\frac{1}{2}}.$$

1.inačica

$$5\cdot 9^{x+1}=15 \Rightarrow 5\cdot 9^{x+1}=15\ /:5 \Rightarrow 9^{x+1}=3 \Rightarrow \left(3^2\right)^{x+1}=3^1 \Rightarrow 3^{2\cdot x+2}=3^1 \Rightarrow$$

$$2\cdot x+2=1 \Rightarrow 2\cdot x=1-2 \Rightarrow 2\cdot x=-1 \Rightarrow 2\cdot x=-1\ /:2 \Rightarrow x=-\frac{1}{2}.$$

Budući da je

$$-\frac{1}{2} \in \left\langle -1,\ 2 \right],$$

odgovor je pod C.

2.inačica

$$5 \cdot 9^{x+1} = 15 \ \Rightarrow\ 5 \cdot 9^{x+1} = 15 \ /:5 \ \Rightarrow\ 9^{x+1} = 3 \ \Rightarrow\ 9^{x+1} = \sqrt{9} \ \Rightarrow\ 9^{x+1} = 9^{\frac{1}{2}} \ \Rightarrow\ x+1 = \frac{1}{2} \ \Rightarrow$$

$$\Rightarrow\ x = \frac{1}{2} - 1 \ \Rightarrow\ x = -\frac{1}{2}.$$

Budući da je

$$-\frac{1}{2} \in \left\langle -1,\ 2 \right],$$

odgovor je pod C.

Vježba 240

Rješenje jednadžbe $7 \cdot 9^{x+1} = 21$ nalazi se u intervalu:

$A. \left\langle -\infty,\ -2 \right]$ $B. \left\langle -2,\ -1 \right]$ $C. \left\langle -1,\ 2 \right]$ $D. \left\langle 2,\ \infty \right\rangle$

Rezultat: C.