Zadatak 161 (Goga, gimnazija)

Nađi x i y iz jednadžbe: $8^{\frac{x}{3}}-2^{x+1}\cdot\sin^2\frac{y}{2}-\sqrt{x-1}=4^{\frac{x}{2}}.$

Rješenje 161

Ponovimo!

$$\left(a^n\right)^m=a^{n\cdot m}.$$

$$8^{\frac{x}{3}}-2^{x+1}\cdot\sin^2\frac{y}{2}-\sqrt{x-1}=4^{\frac{x}{2}}\Rightarrow\left(2^3\right)^{\frac{x}{3}}-2^{x+1}\cdot\sin^2\frac{y}{2}-\sqrt{x-1}=\left(2^2\right)^{\frac{x}{2}}\Rightarrow$$

$$\Rightarrow 2^x-2^{x+1}\cdot\sin^2\frac{y}{2}-\sqrt{x-1}=2^x\Rightarrow 2^x-2^{x+1}\cdot\sin^2\frac{y}{2}-\sqrt{x-1}=2^x\Rightarrow -2^{x+1}\cdot\sin^2\frac{y}{2}-\sqrt{x-1}=0\Rightarrow$$

$$\Rightarrow -2^{x+1}\cdot\sin^2\frac{y}{2}=\sqrt{x-1}\ /\cdot(-1)\Rightarrow 2^{x+1}\cdot\sin^2\frac{y}{2}=-\sqrt{x-1}.$$

Budući da je lijeva strana ove jednadžbe nenegativna (pozitivna ili jednaka nuli), jednadžba ima smisla samo ako je

$$x-1=0\Rightarrow x=1.$$

Sada računamo y:

$$\left.\begin{matrix}x=1\\2^{x+1}\cdot\sin^2\frac{y}{2}=-\sqrt{x-1}\end{matrix}\right\}\Rightarrow 2^{1+1}\cdot\sin^2\frac{y}{2}=-\sqrt{1-1}\Rightarrow 2^2\cdot\sin^2\frac{y}{2}=0\Rightarrow 4\cdot\sin^2\frac{y}{2}=0\Rightarrow$$

$$\Rightarrow 4\cdot\sin^2\frac{y}{2}=0\ /:4\Rightarrow\sin^2\frac{y}{2}=0\ /\sqrt{\ }\Rightarrow\sin\frac{y}{2}=0\Rightarrow\frac{y}{2}=\sin^{-1}0\Rightarrow\frac{y}{2}=k\cdot\pi\ /\cdot 2\Rightarrow y=k\cdot 2\cdot\pi,\ k\in Z.$$

Vježba 161

Nađi x i y iz jednadžbe: $16^{\frac{x}{4}}-2^{x+1}\cdot\sin^2\frac{y}{2}-\sqrt{x-1}=8^{\frac{x}{3}}.$

Rezultat: $x=1\ ,\ y=k\cdot 2\cdot\pi,\ k\in Z.$

Zadatak 162 (Iva, gimnazija)

Riješi jednadžbu: $\log_4(x+12)\cdot\log_x 2=1\ ,\ x>0\ ,\ x\neq 1.$

Rješenje 162

Ponovimo!

$$\log_b a=\frac{\log_c a}{\log_c b}\quad,\quad\log_b a=\frac{\log a}{\log b}\quad,\quad\log_b a^n=n\cdot\log_b a\quad,\quad\log_b a=\frac{1}{\log_a b}.$$

$$\log_b b=1\quad,\quad\log_{b^n}a=\frac{1}{n}\cdot\log_b a\quad,\quad\log_b f(x)=\log_b g(x)\Rightarrow f(x)=g(x).$$

1.inačica

$$\log_4(x+12)\cdot\log_x 2=1\Rightarrow\log_4(x+12)\cdot\frac{1}{\log_2 x}=1\Rightarrow\log_4(x+12)\cdot\frac{1}{\log_2 x}=1\ /\cdot\log_2 x\Rightarrow$$

$$\Rightarrow\log_4(x+12)=\log_2 x\Rightarrow\log_{2^2}(x+12)=\log_2 x\Rightarrow\frac{1}{2}\cdot\log_2(x+12)=\log_2 x\Rightarrow$$

$$\Rightarrow\frac{1}{2}\cdot\log_2(x+12)=\log_2 x\ /\cdot 2\Rightarrow\log_2(x+12)=2\cdot\log_2 x\Rightarrow\log_2(x+12)=\log_2 x^2\Rightarrow$$

$$\Rightarrow x+12=x^2\Rightarrow -x^2+x+12=0\ /\cdot(-1)\Rightarrow x^2-x-12=0\Rightarrow\left.\begin{matrix}x^2-x-12=0\\a=1\ ,\ b=-1\ ,\ c=-12\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=1\ ,\ b=-1\ ,\ c=-12 \\ x_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1-4\cdot 1\cdot(-12)}}{2\cdot 1} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+48}}{2}\Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{1\pm\sqrt{49}}{2}\Rightarrow x_{1,2}=\frac{1\pm 7}{2}\Rightarrow \left.\begin{matrix}x_1=\frac{1+7}{2}\\ x_2=\frac{1-7}{2}\end{matrix}\right\}\Rightarrow \left.\begin{matrix}x_1=\frac{8}{2}\\ x_2=\frac{-6}{2}\end{matrix}\right\}\Rightarrow \left.\begin{matrix}x_1=4\\ x_2=-3\ \text{nema smisla}\end{matrix}\right\}\Rightarrow x=4.$$

2.inačica

$$\log_4(x+12)\cdot\log_x 2=1\Rightarrow \log_4(x+12)\cdot\frac{\log_4 2}{\log_4 x}=1\Rightarrow \log_4(x+12)\cdot\frac{\log_{2^2} 2}{\log_4 x}=1\Rightarrow$$

$$\Rightarrow \log_4(x+12)\cdot\frac{\frac{1}{2}\cdot\log_2 2}{\log_4 x}=1\Rightarrow \log_4(x+12)\cdot\frac{\log_2 2}{2\cdot\log_4 x}=1\Rightarrow \log_4(x+12)\cdot\frac{1}{2\cdot\log_4 x}=1\Rightarrow$$

$$\Rightarrow \log_4(x+12)\cdot\frac{1}{2\cdot\log_4 x}=1\ /\cdot\ 2\cdot\log_4 x\Rightarrow \log_4(x+12)=2\cdot\log_4 x\Rightarrow \log_4(x+12)=\log_4 x^2\Rightarrow$$

$$\Rightarrow x+12=x^2\Rightarrow -x^2+x+12=0\ /\cdot(-1)\Rightarrow x^2-x-12=0\Rightarrow \left.\begin{matrix}x^2-x-12=0\\ a=1\ ,\ b=-1\ ,\ c=-12\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=1\ ,\ b=-1\ ,\ c=-12 \\ x_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1-4\cdot 1\cdot(-12)}}{2\cdot 1} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+48}}{2}\Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{1\pm\sqrt{49}}{2}\Rightarrow x_{1,2}=\frac{1\pm 7}{2}\Rightarrow \left.\begin{matrix}x_1=\frac{1+7}{2}\\ x_2=\frac{1-7}{2}\end{matrix}\right\}\Rightarrow \left.\begin{matrix}x_1=\frac{8}{2}\\ x_2=\frac{-6}{2}\end{matrix}\right\}\Rightarrow \left.\begin{matrix}x_1=4\\ x_2=-3\ \text{nema smisla}\end{matrix}\right\}\Rightarrow x=4.$$

3.inačica

$$\log_4(x+12)\cdot\log_x 2=1\Rightarrow \frac{\log(x+12)}{\log 4}\cdot\frac{\log 2}{\log x}=1\Rightarrow \frac{\log(x+12)}{\log 2^2}\cdot\frac{\log 2}{\log x}=1\Rightarrow \frac{\log(x+12)}{2\cdot\log 2}\cdot\frac{\log 2}{\log x}=1\Rightarrow$$

$$\Rightarrow \frac{\log(x+12)}{2\cdot\log 2}\cdot\frac{\log 2}{\log x}=1\Rightarrow \frac{\log(x+12)}{2}\cdot\frac{1}{\log x}=1\Rightarrow \frac{\log(x+12)}{2\cdot\log x}=1\Rightarrow \frac{\log(x+12)}{2\cdot\log x}\ /\cdot\ 2\cdot\log x\Rightarrow$$

$$\Rightarrow \log(x+12)=2\cdot\log x\Rightarrow \log(x+12)=\log x^2\Rightarrow x+12=x^2\Rightarrow -x^2+x+12=0\ /\cdot(-1)\Rightarrow$$

$$\Rightarrow x^2-x-12=0\Rightarrow \left.\begin{matrix}x^2-x-12=0\\ a=1\ ,\ b=-1\ ,\ c=-12\end{matrix}\right\}\Rightarrow \left.\begin{matrix} a=1\ ,\ b=-1\ ,\ c=-12 \\ x_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{1\pm\sqrt{1-4\cdot 1\cdot(-12)}}{2\cdot 1}\Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+48}}{2}\Rightarrow x_{1,2}=\frac{1\pm\sqrt{49}}{2}\Rightarrow x_{1,2}=\frac{1\pm 7}{2}\Rightarrow \left.\begin{matrix}x_1=\frac{1+7}{2}\\ x_2=\frac{1-7}{2}\end{matrix}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{matrix}x_1=\frac{8}{2}\\ x_2=\frac{-6}{2}\end{matrix}\right\}\Rightarrow \left.\begin{matrix}x_1=4\\ x_2=-3\ \text{nema smisla}\end{matrix}\right\}\Rightarrow x=4.$$

Vježba 162

Riješi jednadžbu : $\log_4 (x+6) \cdot \log_x 2 = 1 \ , \ x > 0 \ , \ x \neq 1.$

Rezultat: 3.

Zadatak 163 (Hana, gimnazija)

Riješi jednadžbu : $\log_2 \sin x + \log_{\frac{1}{2}} (-\cos x) = 0$ u intervalu $\langle 0, 2 \cdot \pi \rangle.$

Rješenje 163

Ponovimo!

$$\log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y \quad , \quad \log_b 1 = 0 \quad , \quad \log_b a = c \ \Leftrightarrow \ b^c = a.$$

$$\log_b f(x) = \log_b g(x) \ \Rightarrow \ f(x) = g(x).$$

U jednadžbi:

$$\log_2 \sin x + \log_{\frac{1}{2}} (-\cos x) = 0$$

prvo provedemo diskusiju:

$$\left.\begin{matrix} \sin x > 0 \\ -\cos x > 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \sin x > 0 \\ -\cos x > 0 \ /\cdot (-1) \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \sin x > 0 \\ \cos x < 0 \end{matrix}\right\} \Rightarrow x \in \left\langle \frac{\pi}{2}, \pi \right\rangle \ \text{drugi kvadrant}$$

Sada je:

1.inačica

$$\log_2 \sin x + \log_{\frac{1}{2}} (-\cos x) = 0 \Rightarrow \log_2 \sin x + \log_{2^{-1}} (-\cos x) = 0 \Rightarrow \log_2 \sin x - \log_2 (-\cos x) = 0 \Rightarrow$$

$$\Rightarrow \log_2 \frac{\sin x}{-\cos x} = 0 \Rightarrow \log_2 (-tg\, x) = 0 \Rightarrow -tg\, x = 2^0 \Rightarrow -tg\, x = 1 \ /\cdot (-1) \Rightarrow tg\, x = -1 \Rightarrow x = \frac{3 \cdot \pi}{4}.$$

2.inačica

$$\log_2 \sin x + \log_{\frac{1}{2}} (-\cos x) = 0 \Rightarrow \log_2 \sin x + \log_{2^{-1}} (-\cos x) = 0 \Rightarrow \log_2 \sin x - \log_2 (-\cos x) = 0 \Rightarrow$$

$$\Rightarrow \log_2 \sin x = \log_2 (-\cos x) = 0 \Rightarrow \sin x = -\cos \ /\cdot \frac{1}{\cos x} \Rightarrow \frac{\sin x}{\cos x} = -1 \Rightarrow tg\, x = -1 \Rightarrow x = \frac{3 \cdot \pi}{4}.$$

Vježba 163

Riješi jednadžbu : $\log_2 \sin x + \log_{\frac{1}{2}} \cos x = 0$ u intervalu $\langle 0, \pi \rangle.$

Rezultat: $\frac{\pi}{4}.$

Zadatak 164 (Boris, gimnazija)

Dokaži nejednakost : $\frac{1}{\log_2 \pi} + \frac{1}{\log_5 \pi} > 2.$

Rješenje 164

Ponovimo!

$$\log_b a = \frac{1}{\log_a b} \quad , \quad \log_b (x \cdot y) = \log_b x + \log_b y \quad , \quad \log_b x > \log_b y \ \Rightarrow x > y \ , \ b > 1$$

$$\log_b a^n = n \cdot \log_b a \quad , \quad 10 > \pi^2 \quad , \quad \log_b b = 1.$$

$$\frac{1}{\log_2 \pi}+\frac{1}{\log_5 \pi}=\log_\pi 2+\log_\pi 5=\log_\pi (2\cdot 5)=\log_\pi 10>\log_\pi \pi^2=2\cdot\log_\pi \pi=2\cdot 1=2.$$

Vježba 164

Dokaži nejednakost: $\frac{1}{\log_2 \pi}+\frac{1}{\log_3 \pi}>1.$

Rezultat: Dokaz analogan.

Zadatak 165 (Kiki, maturantica gimnazije)

Riješi jednadžbu: $\log x+\log x^2+\log x^3+\log x^4+\ ...\ +\log x^{100}=5050\ ,\ x>0.$

Rješenje 165

Ponovimo!

$\log a^n=n\cdot\log a\quad,\quad \log(a\cdot b)=\log a+\log b\quad,\quad \log a=\log b\ \Rightarrow\ a=b\quad,\quad a^n\cdot a^m=a^{n+m}.$

$a^1=a\quad,\quad \log 10=1\quad,\quad 1+2+3+4+\ ...\ +n=\frac{n\cdot(n+1)}{2}.$

$$1+2+3+4+\ ...\ +100=\frac{100\cdot(100+1)}{2}=\frac{100\cdot 101}{2}=50\cdot 101=5050.$$

1.inačica

$$\log x+\log x^2+\log x^3+\log x^4+\ ...\ +\log x^{100}=5050\ \Rightarrow$$

$$\Rightarrow\ \log x+2\cdot\log x+3\cdot\log x+4\cdot\log x+\ ...\ +100\cdot\log x=5050\ \Rightarrow\ \log x\cdot(1+2+3+4+\ ...\ +100)=5050\ \Rightarrow$$

$$\Rightarrow\ \log x\cdot 5050=5050\ \Rightarrow\ \log x\cdot 5050=5050\ /:5050\ \Rightarrow\ \log x=1\ \Rightarrow\ \log x=\log 10\ \Rightarrow\ x=10.$$

2.inačica

$$\log x+\log x^2+\log x^3+\log x^4+\ ...\ +\log x^{100}=5050\ \Rightarrow$$

$$\log\left(x\cdot x^2\cdot x^3\cdot x^4\cdot\ ...\ \cdot x^{100}\right)=5050\ \Rightarrow\ \log\left(x^1\cdot x^2\cdot x^3\cdot x^4\cdot\ ...\ \cdot x^{100}\right)=5050\ \Rightarrow$$

$$\Rightarrow\ \log x^{1+2+3+4+...+100}=5050\ \Rightarrow\ \log x^{5050}=5050\ \Rightarrow\ 5050\cdot\log x=5050\ \Rightarrow$$

$$\Rightarrow\ 5050\cdot\log x=5050\ /:5050\ \Rightarrow\ \log x=1\ \Rightarrow\ \log x=\log 10\ \Rightarrow\ x=10.$$

Vježba 165

Riješi jednadžbu: $\log x+\log x^2+\log x^3+\log x^4+\ ...\ +\log x^{10}=55\ ,\ x>0.$

Rezultat: x = 10.

Zadatak 166 (Martin, gimnazija)

Nađi korijen jednadžbe: $\log_x 2+\log_x 3=\frac{1}{3}.$

Rješenje 166

Ponovimo!

$\log_b a+\log_b c=\log_b(a\cdot c)\quad,\quad \log_b a=c\ \Leftrightarrow\ b^c=a\quad,\quad \left(a^n\right)^m=a^{n\cdot m}.$

Najprije provedemo diskusiju jednadžbe

$$\log_x 2+\log_x 3=\frac{1}{3}.$$

$$x>0\ \text{ i }\ x\neq 1\ \Rightarrow\ x\in\langle 0,1\rangle\cup\langle 1,+\infty\rangle.$$

Sada rješavamo jednadžbu:

$$\log_x 2+\log_x 3=\frac{1}{3} \Rightarrow \log_x (2\cdot 3)=\frac{1}{3} \Rightarrow \log_x 6=\frac{1}{3} \Rightarrow x^{\frac{1}{3}}=6\ /^3 \Rightarrow \left(x^{\frac{1}{3}}\right)^3=6^3 \Rightarrow x=216.$$

Vježba 166

Nađi korijen jednadžbe: $\log_x 2+\log_x 3=\frac{1}{2}$.

Rezultat: 36.

Zadatak 167 (Martin, gimnazija)

Riješi jednadžbu: $x^2\cdot\log_2 x\cdot\log_x 4=x+3$.

Rješenje 167

Ponovimo!

$$\log_b a=\frac{1}{\log_a b} \quad , \quad \log_b a^n=n\cdot\log_b a.$$

Najprije provedemo diskusiju jednadžbe

$$x^2\cdot\log_2 x\cdot\log_x 4=x+3.$$

$$x>0 \text{ i } x\neq 1 \Rightarrow x\in\langle 0,1\rangle\cup\langle 1,+\infty\rangle.$$

Sada rješavamo jednadžbu:

$$x^2\cdot\log_2 x\cdot\log_x 4=x+3 \Rightarrow x^2\cdot\frac{1}{\log_x 2}\cdot\log_x 2^2=x+3 \Rightarrow x^2\cdot\frac{1}{\log_x 2}\cdot 2\cdot\log_x 2=x+3 \Rightarrow$$

$$\Rightarrow x^2\cdot\frac{1}{\log_x 2}\cdot 2\cdot\log_x 2=x+3 \Rightarrow 2\cdot x^2=x+3 \Rightarrow 2\cdot x^2-x-3=0 \Rightarrow \left.\begin{matrix} 2\cdot x^2-x-3=0 \\ a=2\ ,\ b=-1\ ,\ c=-3\end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=2\ ,\ b=-1\ ,\ c=-3 \\ x_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1-4\cdot 2\cdot(-3)}}{2\cdot 2} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+24}}{4} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{25}}{4} \Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{1\pm 5}{4} \Rightarrow \left.\begin{matrix} x_1=\frac{1+5}{4} \\ x_2=\frac{1-5}{4}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1=\frac{6}{4} \\ x_2=-\frac{4}{4}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1=\frac{3}{2} \text{ rješenje} \\ x_2=-1 \text{ nije rješenje zbog diskusije}\end{matrix}\right\}.$$

Vježba 167

Riješi jednadžbu: $x^2\cdot\log_5 x\cdot\log_x 25=x+3$.

Rezultat: $\frac{3}{2}$.

Zadatak 168 (Jelena, ekonomska škola)

Ako je $a=\log 5$, $b=\log 3$, koliko je $\log_{30} 8$?

Rješenje 168

Ponovimo!

$$\log_{10} a=\log a \quad , \quad \log_b a=\frac{\log a}{\log b} \quad , \quad \log(a\cdot b)=\log a+\log b \quad , \quad \log a^n=n\cdot\log a.$$

$$\log\frac{a}{b}=\log a-\log b \quad , \quad \log 10=1.$$

$$\log_{30} 8=\frac{\log 8}{\log 30}=\frac{\log 2^{3}}{\log(10\cdot 3)}=\frac{3\cdot\log 2}{\log 10+\log 3}=\frac{3\cdot\log\frac{10}{5}}{1+\log 3}=\frac{3\cdot(\log 10-\log 5)}{1+\log 3}=$$

$$=\frac{3\cdot(1-\log 5)}{1+\log 3}=\begin{bmatrix} a=\log 5 \\ b=\log 3\end{bmatrix}=\frac{3\cdot(1-a)}{1+b}.$$

Vježba 168

Ako je $a=\log 5$, $b=\log 3$, koliko je $\log_3 8$?

Rezultat: $\frac{3\cdot(1-a)}{b}.$

Zadatak 169 (Jelena, ekonomska škola)

Riješi jednadžbu: $\log_2 \log_5 (2-x)=0.$

Rješenje 169

Ponovimo!

$$\log_b a=c \Leftrightarrow b^{c}=a \quad , \quad a^{0}=1 \quad , \quad a^{1}=a.$$

Diskusija:

$$2-x>0 \Rightarrow -x>-2 \;/\cdot(-1) \Rightarrow x<2 \Rightarrow x\in\langle -\infty, 2\rangle.$$

Dakle, ako jednadžba ima rješene ono mora biti iz intervala

$$\langle -\infty, 2\rangle.$$

Sada rješavamo jednadžbu:

$$\log_2 \log_5 (2-x)=0 \Rightarrow 2^{0}=\log_5(2-x) \Rightarrow 1=\log_5(2-x) \Rightarrow \log_5(2-x)=1 \Rightarrow$$

$$\Rightarrow 5^{1}=2-x \Rightarrow 5=2-x \Rightarrow x=2-5 \Rightarrow x=-3.$$

To je rješenje jer je

$$-3\in\langle -\infty, 2\rangle.$$

Vježba 169

Riješi jednadžbu: $\log_2 \log_3 (1-x)=0.$

Rezultat: $-2.$

Zadatak 170 (Denis, ekonomska škola)

Riješi jednadžbu: $\log 10+\frac{1}{3}\cdot\log\left(757+3^{\sqrt{2\cdot x}}\right)=2.$

Rješenje 170

Ponovimo!

$$\log 10=1 \quad , \quad \log f(x)=\log g(x) \Rightarrow f(x)=g(x) \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$\log 1000=3.$$

$$\log 10+\frac{1}{3}\cdot\log\left(757+3^{\sqrt{2\cdot x}}\right)=2 \Rightarrow 1+\frac{1}{3}\cdot\log\left(757+3^{\sqrt{2\cdot x}}\right)=2 \Rightarrow \frac{1}{3}\cdot\log\left(757+3^{\sqrt{2\cdot x}}\right)=2-1 \Rightarrow$$

$$\Rightarrow \frac{1}{3}\cdot\log\left(757+3^{\sqrt{2\cdot x}}\right)=1 \; /\cdot 3 \;\Rightarrow\; \log\left(757+3^{\sqrt{2\cdot x}}\right)=3 \;\Rightarrow\; \log\left(757+3^{\sqrt{2\cdot x}}\right)=\log 1000 \;\Rightarrow$$

$$\Rightarrow\; 757+3^{\sqrt{2\cdot x}}=1000 \;\Rightarrow\; 3^{\sqrt{2\cdot x}}=1000-757 \;\Rightarrow\; 3^{\sqrt{2\cdot x}}=243 \;\Rightarrow\; 3^{\sqrt{2\cdot x}}=3^{5} \;\Rightarrow$$

$$\Rightarrow\; \sqrt{2\cdot x}=5 \; /^{2} \;\Rightarrow\; \left(\sqrt{2\cdot x}\right)^{2}=5^{2} \;\Rightarrow\; 2\cdot x=25 \; /:2 \;\Rightarrow\; x=\frac{25}{2} \;\Rightarrow\; x=12.5.$$

Vježba 170

Riješi jednadžbu: $\log 100+\frac{1}{2}\cdot\log\left(2^{x}-7\right)=2.$

Rezultat: 3.

Zadatak 171 (Denis, ekonomska škola)

Riješi nejednadžbu: $\log\left(x+2\right)\le 1.$

Rješenje 171

Ponovimo!

$$\log 10=1 \quad , \quad \log f\left(x\right)\le\log g\left(x\right) \;\Rightarrow\; f\left(x\right)\le g\left(x\right).$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f\left(x\right)=\log_{b} x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x\in\left\langle 0,+\infty\right\rangle.$$

$$\log_{10} x=\log x \;\; \left(\text{obični ili dekadski logaritam}\right)$$

Najprije provedemo diskusiju (tražimo domenu funkcije):

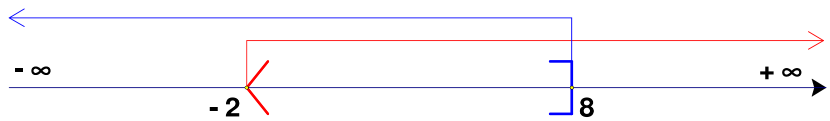
$$\log\left(x+2\right)\le 1 \;\Rightarrow\; x+2>0 \;\Rightarrow\; x>-2 \;\Rightarrow\; x\in\left\langle -2,+\infty\right\rangle.$$

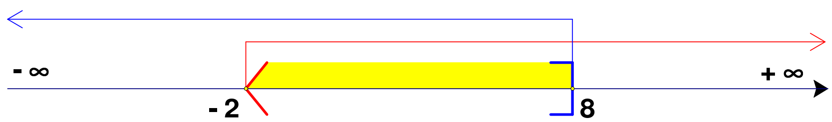
Sada riješimo nejednadžbu:

$$\log\left(x+2\right)\le 1 \;\Rightarrow\; \log\left(x+2\right)\le\log 10 \;\Rightarrow\; x+2\le 10 \;\Rightarrow\; x\le 10-2 \;\Rightarrow\; x\le 8 \;\Rightarrow\; x\in\left\langle -\infty,8\right].$$

Rješenje nejednadžbe dobije se kao presjek (zajednički dio) intervala

$$\left\langle -2,+\infty\right\rangle \text{ i } \left\langle -\infty,8\right].$$





$$x\in\left\langle -2,8\right].$$

Vježba 171

Riješi nejednadžbu: $\log\left(x-2\right)\le 1.$

Rezultat: $x\in\left\langle 2,12\right].$

Zadatak 172 (Luka, gimnazija)

Riješi jednadžbu: $3\cdot\log x+2\cdot\log\sqrt{\frac{1}{x}}=2.$

Rješenje 172

Ponovimo!

$$\log 1 = 0 \quad , \quad \log 10 = 1 \quad , \quad \log 100 = 2 \quad , \quad \log \sqrt{a} = \frac{1}{2} \cdot \log a \quad , \quad \log \frac{a}{b} = \log a - \log b.$$

$$\log a^n = n \cdot \log a \quad , \quad \left(\sqrt{a}\right)^2 = a \quad , \quad \log (a \cdot b) = \log a + \log b.$$

$$\log f(x) = \log g(x) \;\Rightarrow\; f(x) = g(x).$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x) = \log_b x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x \in \langle 0, +\infty \rangle.$$

$$\log_{10} x = \log x \;\; (\text{obični ili dekadski logaritam})$$

1.inačica

Najprije provedemo diskusiju (tražimo domenu funkcije):

$$f(x) = \log x \;\Rightarrow\; x > 0 \;\Rightarrow\; x \in \langle 0, +\infty \rangle.$$

Sada riješimo jednadžbu:

$$3 \cdot \log x + 2 \cdot \log \sqrt{\frac{1}{x}} = 2 \;\Rightarrow\; 3 \cdot \log x + 2 \cdot \frac{1}{2} \cdot \log \frac{1}{x} = 2 \;\Rightarrow\; 3 \cdot \log x + 2 \cdot \frac{1}{2} \cdot \log \frac{1}{x} = 2 \;\Rightarrow$$

$$\Rightarrow\; 3 \cdot \log x + \log \frac{1}{x} = 2 \;\Rightarrow\; 3 \cdot \log x + \log 1 - \log x = 2 \;\Rightarrow\; 3 \cdot \log x + 0 - \log x = 2 \;\Rightarrow\; 3 \cdot \log x - \log x = 2 \;\Rightarrow$$

$$\Rightarrow\; 2 \cdot \log x = 2 \;/:2 \;\Rightarrow\; \log x = 1 \;\Rightarrow\; \log x = \log 10 \;\Rightarrow\; x = 10.$$

2.inačica

Najprije provedemo diskusiju (tražimo domenu funkcije):

$$f(x) = \log x \;\Rightarrow\; x > 0 \;\Rightarrow\; x \in \langle 0, +\infty \rangle.$$

Sada riješimo jednadžbu:

$$3 \cdot \log x + 2 \cdot \log \sqrt{\frac{1}{x}} = 2 \;\Rightarrow\; \log x^3 + \log \left(\sqrt{\frac{1}{x}}\right)^2 = 2 \;\Rightarrow\; \log x^3 + \log \frac{1}{x} = 2 \;\Rightarrow$$

$$\Rightarrow\; \log \left(x^3 \cdot \frac{1}{x}\right) = 2 \;\Rightarrow\; \log x^2 = 2 \;\Rightarrow\; \log x^2 = \log 100 \;\Rightarrow\; x^2 = 100 \;\Rightarrow$$

$$\Rightarrow\; x^2 = 100 \;/\sqrt{\;} \;\Rightarrow\; x_{1,2} = \pm\sqrt{100} \;\Rightarrow\; \left.\begin{array}{l} x_1 = 10 \text{ je rješenje} \\ x_2 = -10 \text{ nije rješenje} \end{array}\right\}.$$

3.inačica

Najprije provedemo diskusiju (tražimo domenu funkcije):

$$f(x) = \log x \;\Rightarrow\; x > 0 \;\Rightarrow\; x \in \langle 0, +\infty \rangle.$$

Sada riješimo jednadžbu:

$$3 \cdot \log x + 2 \cdot \log \sqrt{\frac{1}{x}} = 2 \;\Rightarrow\; 3 \cdot \log x + \log \left(\sqrt{\frac{1}{x}}\right)^2 = 2 \;\Rightarrow\; 3 \cdot \log x + \log \frac{1}{x} = 2 \;\Rightarrow$$

$$\Rightarrow\; 3 \cdot \log x + \log x^{-1} = 2 \;\Rightarrow\; 3 \cdot \log x - 1 \cdot \log x = 2 \;\Rightarrow\; 2 \cdot \log x = 2 \;/:2 \;\Rightarrow$$

$$\Rightarrow \log x = 1 \Rightarrow \log x = \log 10 \Rightarrow x = 10.$$

Vježba 172

Riješi jednadžbu: $3 \cdot \log x + 2 \cdot \log \sqrt{\frac{1}{x}} = 4.$

Rezultat: 100.

Zadatak 173 (Cazim, srednja škola)

Riješi logaritamsku nejednadžbu: $\log_2 \left(1 + \log_{\frac{1}{9}} x\right) < 1.$

Rješenje 173

Ponovimo!

$$\log_b b = 1.$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x) = \log_b x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x \in \langle 0, +\infty \rangle.$$

Logaritamska funkcija je rastuća za bazu veću od 1:

$$\log_b f(x) < \log_b g(x) \Rightarrow f(x) < g(x) \ , \ b > 1.$$

Logaritamska funkcija je padajuća za bazu manju od 1:

$$\log_b f(x) < \log_b g(x) \Rightarrow f(x) > g(x) \ , \ 0 < b < 1.$$

Najprije moramo potražiti za koje je realne brojeve x definirana zadana funkcija (tražimo domenu). Obzirom da argument logaritamske funkcije mora biti pozitivan, to je

$$f(x) = \log_{\frac{1}{9}} x \Rightarrow x > 0 \ \text{domena}.$$

Uz taj uvjet slijedi:

$$\log_2 \left(1 + \log_{\frac{1}{9}} x\right) < 1 \Rightarrow \log_2 \left(1 + \log_{\frac{1}{9}} x\right) < \log_2 2 \Rightarrow \log_2 \left(1 + \log_{\frac{1}{9}} x\right) < \log_2 2 \Rightarrow$$

$$\Rightarrow 1 + \log_{\frac{1}{9}} x < 2 \Rightarrow \log_{\frac{1}{9}} x < 2 - 1 \Rightarrow \log_{\frac{1}{9}} x < 1 \Rightarrow \log_{\frac{1}{9}} x < \log_{\frac{1}{9}} \frac{1}{9} \Rightarrow \log_{\frac{1}{9}} x < \log_{\frac{1}{9}} \frac{1}{9} \Rightarrow$$

$$\Rightarrow x > \frac{1}{9}.$$

Zato rješenje nejednadžbe čine svi realni brojevi iz intervala:

$$\left.\begin{array}{l} x > 0 \ \text{domena} \\ x > \frac{1}{9} \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{gledamo presjek rješenja} \\ \text{ili zajednički dio} \end{array}\right] \Rightarrow x > \frac{1}{9} \Rightarrow x \in \left\langle \frac{1}{9}, +\infty \right\rangle.$$

Vježba 173

Riješi logaritamsku nejednadžbu: $\log_2 \left(1 - \log_9 x\right) < 1.$

Rezultat: $x > \frac{1}{9}.$

Zadatak 174 (Cazim, srednja škola)

Riješi logaritamsku nejednadžbu: $\left[\log_{0.2}(x-1)\right]^2 > 4.$

Rješenje 174

Ponovimo!

$$\log_b b^n = n \quad , \quad a^{-n} = \frac{1}{a^n}.$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x) = \log_b x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x \in \langle 0, +\infty \rangle.$$

Logaritamska funkcija je rastuća za bazu veću od 1:

$$\log_b f(x) < \log_b g(x) \Rightarrow f(x) < g(x) \;,\; b > 1.$$

Logaritamska funkcija je padajuća za bazu manju od 1:

$$\log_b f(x) < \log_b g(x) \Rightarrow f(x) > g(x) \;,\; 0 < b < 1.$$

Kvadratna nejednadžba oblika

$$x^2 > a, \; a > 0$$

ekvivalentna je s

$$|x| > \sqrt{a}$$

i ima kao rješenja

$$x < -\sqrt{a} \text{ i } x > \sqrt{a}$$

ili zapisano u obliku skupa

$$x \in \langle -\infty, -\sqrt{a} \rangle \cup \langle \sqrt{a}, +\infty \rangle.$$

Najprije moramo potražiti za koje je realne brojeve x definirana zadana funkcija (tražimo domenu). Obzirom da argument logaritamske funkcije mora biti pozitivan, to je

$$f(x) = \log_{0.2}(x-1) \Rightarrow x-1 > 0 \Rightarrow x > 1 \text{ domena}.$$

Uz taj uvjet slijedi:

$$\left[\log_{0.2}(x-1)\right]^2 > 4 \Rightarrow \left[\log_{0.2}(x-1)\right]^2 > 4 \; /\sqrt{\;} \Rightarrow \left|\log_{0.2}(x-1)\right| > 2 \Rightarrow \left.\begin{matrix} \log_{0.2}(x-1) < -2 \\ \log_{0.2}(x-1) > 2 \end{matrix}\right\}.$$

Dobili smo sustav od dvije logaritamske nejednadžbe. Svaku riješimo posebno pa ćemo dobiti rješenje prve nejednadžbe i rješenje druge nejednadžbe. Rezultat početne (zadane) nejednadžbe bit će unija tih rješenja.

Rješenje prve nejednadžbe

- $\log_{0.2}(x-1) < -2 \Rightarrow \log_{0.2}(x-1) < \log_{0.2} 0.2^{-2} \Rightarrow \log_{0.2}(x-1) < \log_{0.2} 0.2^{-2} \Rightarrow$

$$\Rightarrow x-1 > 0.2^{-2} \Rightarrow x-1 > \frac{1}{0.2^2} \Rightarrow x-1 > \frac{1}{0.04} \Rightarrow x-1 > 25 \Rightarrow x > 25+1 \Rightarrow x > 26 \Rightarrow$$

$$\Rightarrow x \in \langle 26, +\infty \rangle.$$

Zato rješenje prve nejednadžbe čine svi realni brojevi iz intervala:

$$\left.\begin{array}{l} x>1 \ \text{domena} \\ x>26 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{gledamo presjek rješenja} \\ \text{ili zajednički dio} \end{array}\right] \Rightarrow x>26 \Rightarrow x\in\left\langle 26, +\infty\right\rangle.$$

Rješenje druge nejednadžbe

- $\log_{0.2}(x-1)>2 \Rightarrow \log_{0.2}(x-1)>\log_{0.2}0.2^2 \Rightarrow \log_{0.2}(x-1)>\log_{0.2}0.2^2 \Rightarrow$

$$\Rightarrow x-1<0.2^2 \Rightarrow x-1<0.04 \Rightarrow x<0.04+1 \Rightarrow x<1.04 \Rightarrow x\in\left\langle -\infty, 1.04\right\rangle.$$

Zato rješenje druge nejednadžbe čine svi realni brojevi iz intervala:

$$\left.\begin{array}{l} x>1 \ \text{domena} \\ x<1.04 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{gledamo presjek rješenja} \\ \text{ili zajednički dio} \end{array}\right] \Rightarrow x\in\left\langle 1, 1.04\right\rangle.$$

Rješenje zadane logaritamske nejednadžbe je unija tih rezultata

$$x\in\left\langle 1, 1.04\right\rangle \cup \left\langle 26, +\infty\right\rangle.$$

Vježba 174

Riješi logaritamsku nejednadžbu: $\log_{\frac{1}{4}}\frac{1-2\cdot x}{4}\geq 0$

Rezultat: $x\in\left[-\frac{3}{2}, \frac{1}{2}\right\rangle.$

Zadatak 175 (Snješka, srednja škola)

Riješi logaritamsku nejednadžbu: $2\cdot\log_9 x-\log_3 5x>\log_{\frac{1}{3}}(x+3).$

Rješenje 175

Ponovimo!

$$\log_{b^n} a=\frac{1}{n}\cdot\log_b a \quad , \quad \log_{\frac{1}{b}} a=-\log_b a \quad , \quad \log_b x-\log_b y=\log_b\frac{x}{y}.$$

$$\log_b x+\log_b y=\log_b(x\cdot y) \quad , \quad \log_b 1=0.$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x)=\log_b x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x\in\left\langle 0, +\infty\right\rangle.$$

Logaritamska funkcija je rastuća za bazu veću od 1:

$$\log_b f(x)<\log_b g(x) \Rightarrow f(x)<g(x) \ , \ b>1.$$

Logaritamska funkcija je padajuća za bazu manju od 1:

$$\log_b f(x)<\log_b g(x) \Rightarrow f(x)>g(x) \ , \ 0<b<1.$$

Najprije moramo potražiti za koje je realne brojeve x definirana zadana funkcija (tražimo domenu). Obzirom da argument logaritamske funkcije mora biti pozitivan, to je

$$\left.\begin{array}{l} f(x)=\log_9 x \\ g(x)=\log_3 5x \\ h(x)=\log_{\frac{1}{3}}(x+3) \end{array}\right\} \Rightarrow \left.\begin{array}{l} x>0 \\ 5\cdot x>0 \\ x+3>0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x>0 \\ 5\cdot x>0 \ /:5 \\ x>-3 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x>0 \\ x>0 \\ x>-3 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{gledamo presjek rješenja} \\ \text{ili zajednički dio} \end{array}\right] \Rightarrow x>0 \ \text{domena}.$$

Uz taj uvjet slijedi:

$$2\cdot\log_9 x-\log_3 5x>\log_{\frac{1}{3}}(x+3) \Rightarrow 2\cdot\log_{3^2} x-\log_3 5x>\log_{3^{-1}}(x+3) \Rightarrow$$

$$\Rightarrow 2\cdot\frac{1}{2}\cdot\log_3 x-\log_3 5x>-\log_3(x+3) \Rightarrow 2\cdot\frac{1}{2}\cdot\log_3 x-\log_3 5x>-\log_3(x+3) \Rightarrow$$

$$\Rightarrow \log_3 x-\log_3 5x+\log_3(x+3)>0 \Rightarrow \log_3\frac{x}{5\cdot x}+\log_3(x+3)>0 \Rightarrow \log_3\frac{x}{5\cdot x}+\log_3(x+3)>0 \Rightarrow$$

$$\Rightarrow \log_3\frac{1}{5}+\log_3(x+3)>0 \Rightarrow \log_3\frac{x+3}{5}>0 \Rightarrow \log_3\frac{x+3}{5}>\log_3 1 \Rightarrow \log_3\frac{x+3}{5}>\log_3 1 \Rightarrow$$

$$\Rightarrow \frac{x+3}{5}>1 \Rightarrow \frac{x+3}{5}>1 \ /\cdot 5 \Rightarrow x+3>5 \Rightarrow x>5-3 \Rightarrow x>2 \Rightarrow x\in\langle 2,+\infty\rangle.$$

Zato rješenje nejednadžbe čine svi realni brojevi iz intervala:

$$\left.\begin{array}{l} x>0 \ \text{domena} \\ x>2 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{gledamo presjek rješenja} \\ \text{ili zajednički dio} \end{array}\right] \Rightarrow x>2 \Rightarrow x\in\langle 2,+\infty\rangle.$$

Vježba 175

Riješi logaritamsku nejednadžbu: $2\cdot\log_9 x-\log_{\frac{1}{3}}(x+3)>\log_3 5x.$

Rezultat: x > 2.

Zadatak 176 (Cazim, srednja škola)

Riješi jednadžbu: $\left|\log_x 3-\log_x 2\right|=2.$

Rješenje 176

Ponovimo!

$$\log_b a-\log_b c=\log_b\frac{a}{c} \quad , \quad \log_b a=c \Leftrightarrow b^c=a \quad , \quad \log_b b^n=n.$$

$$|x|=\begin{cases} x, & x\ge 0 \\ -x, & x<0 \end{cases} \quad , \quad |x|=a \ , \ a>0 \Rightarrow \left.\begin{array}{l} x_1=a \\ x_2=-a \end{array}\right\} \quad , \quad \sqrt{a}\cdot\sqrt{b}=\sqrt{a\cdot b} \quad , \quad \sqrt{\frac{a}{b}}=\frac{\sqrt{a}}{\sqrt{b}}.$$

$$\left(\sqrt{a}\right)^2=a \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad \log_b f(x)=\log_b g(x) \Rightarrow f(x)=g(x).$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x)=\log_b x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x\in\langle 0,+\infty\rangle.$$

1.inačica

$$\left|\log_x 3-\log_x 2\right|=2 \Rightarrow \left|\log_x \frac{3}{2}\right|=2 \Rightarrow \left.\begin{matrix}\log_x \frac{3}{2}=2\\ \log_x \frac{3}{2}=-2\end{matrix}\right\}.$$

- Rješavamo jednadžbu

$$\log_x \frac{3}{2}=2.$$

$$\log_x \frac{3}{2}=2 \Rightarrow x^2=\frac{3}{2} \Rightarrow x^2=\frac{3}{2}\ /\sqrt{\ } \Rightarrow x_{1,2}=\pm\sqrt{\frac{3}{2}} \Rightarrow x_{1,2}=\pm\frac{\sqrt{3}}{\sqrt{2}} \Rightarrow \begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix} \Rightarrow$$

$$\Rightarrow x_{1,2}=\pm\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \Rightarrow x_{1,2}=\pm\frac{\sqrt{6}}{\left(\sqrt{2}\right)^2} \Rightarrow x_{1,2}=\pm\frac{\sqrt{6}}{2} \Rightarrow \left.\begin{matrix}x_1=\frac{\sqrt{6}}{2}\\ x_2=-\frac{\sqrt{6}}{2}\ \text{nema smisla}\end{matrix}\right\}.$$

- Rješavamo jednadžbu

$$\log_x \frac{3}{2}=-2.$$

$$\log_x \frac{3}{2}=-2 \Rightarrow x^{-2}=\frac{3}{2} \Rightarrow \frac{1}{x^2}=\frac{3}{2} \Rightarrow 3\cdot x^2=2 \Rightarrow 3\cdot x^2=2\ /:3 \Rightarrow x^2=\frac{2}{3} \Rightarrow$$

$$\Rightarrow x^2=\frac{2}{3}\ /\sqrt{\ } \Rightarrow x_{3,4}=\pm\sqrt{\frac{2}{3}} \Rightarrow x_{3,4}=\pm\frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix} \Rightarrow$$

$$\Rightarrow x_{3,4}=\pm\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \Rightarrow x_{3,4}=\pm\frac{\sqrt{6}}{\left(\sqrt{3}\right)^2} \Rightarrow x_{3,4}=\pm\frac{\sqrt{6}}{3} \Rightarrow \left.\begin{matrix}x_3=\frac{\sqrt{6}}{3}\\ x_4=-\frac{\sqrt{6}}{3}\ \text{nema smisla}\end{matrix}\right\}.$$

2.inačica

$$\left|\log_x 3-\log_x 2\right|=2 \Rightarrow \left|\log_x \frac{3}{2}\right|=2 \Rightarrow \left.\begin{matrix}\log_x \frac{3}{2}=2\\ \log_x \frac{3}{2}=-2\end{matrix}\right\}.$$

- Rješavamo jednadžbu

$$\log_x \frac{3}{2}=2.$$

$$\log_x \frac{3}{2}=2 \Rightarrow \log_x \frac{3}{2}=\log_x x^2 \Rightarrow \frac{3}{2}=x^2 \Rightarrow x^2=\frac{3}{2}\ /\sqrt{\ } \Rightarrow x_{1,2}=\pm\sqrt{\frac{3}{2}} \Rightarrow x_{1,2}=\pm\frac{\sqrt{3}}{\sqrt{2}} \Rightarrow$$

$$\Rightarrow \begin{bmatrix}\text{racionalizacija}\\ \text{nazivnika}\end{bmatrix} \Rightarrow x_{1,2}=\pm\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \Rightarrow x_{1,2}=\pm\frac{\sqrt{6}}{\left(\sqrt{2}\right)^2} \Rightarrow$$

$$\Rightarrow x_{1,2}=\pm\frac{\sqrt{6}}{2} \Rightarrow \left.\begin{matrix} x_1=\frac{\sqrt{6}}{2} \\ x_2=-\frac{\sqrt{6}}{2} \text{ nema smisla} \end{matrix}\right\}.$$

- Rješavamo jednadžbu

$$\log_x \frac{3}{2}=-2.$$

$$\log_x \frac{3}{2}=-2 \Rightarrow \log_x \frac{3}{2}=\log_x x^{-2} \Rightarrow \frac{3}{2}=x^{-2} \Rightarrow \frac{3}{2}=\frac{1}{x^2} \Rightarrow 3\cdot x^2=2 \Rightarrow 3\cdot x^2=2 \;/:3 \Rightarrow$$

$$\Rightarrow x^2=\frac{2}{3} \Rightarrow x^2=\frac{2}{3} \;/\sqrt{\ } \Rightarrow x_{3,4}=\pm\sqrt{\frac{2}{3}} \Rightarrow x_{3,4}=\pm\frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \begin{bmatrix} \text{racionalizacija} \\ \text{nazivnika} \end{bmatrix} \Rightarrow$$

$$\Rightarrow x_{3,4}=\pm\frac{\sqrt{2}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} \Rightarrow x_{3,4}=\pm\frac{\sqrt{6}}{\left(\sqrt{3}\right)^2} \Rightarrow x_{3,4}=\pm\frac{\sqrt{6}}{3} \Rightarrow \left.\begin{matrix} x_3=\frac{\sqrt{6}}{3} \\ x_4=-\frac{\sqrt{6}}{3} \text{ nema smisla} \end{matrix}\right\}.$$

Vježba 176

Riješi jednadžbu: $\left|\log_x 3-\log_x 2\right|=1.$

Rezultat: $x_1=\frac{3}{2}\ ,\ x_2=\frac{2}{3}.$

Zadatak 177 (Cazim, srednja škola)

Riješi jednadžbu: $\log_{\frac{1}{2}}\left|1-x\right|=2.$

Rješenje 177

Ponovimo!

$$\log_b a=c \Leftrightarrow b^c=a \quad , \quad |x|=\begin{cases} x, & x\ge 0 \\ -x, & x<0 \end{cases} \quad , \quad |x|=a\ ,\ a>0 \Rightarrow \left.\begin{matrix} x_1=a \\ x_2=-a \end{matrix}\right\}.$$

$$\log_b b^n=n \quad , \quad \log_b f(x)=\log_b g(x) \Rightarrow f(x)=g(x).$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x)=\log_b x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x\in\left\langle 0, +\infty\right\rangle.$$

1.inačica

$$\log_{\frac{1}{2}}\left|1-x\right|=2 \Rightarrow \left(\frac{1}{2}\right)^2=\left|1-x\right| \Rightarrow \frac{1}{4}=\left|1-x\right| \Rightarrow \left|1-x\right|=\frac{1}{4} \Rightarrow \left.\begin{matrix} 1-x=\frac{1}{4} \\ 1-x=-\frac{1}{4} \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} -x=\frac{1}{4}-1 \\ -x=-\frac{1}{4}-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x=-\frac{3}{4} \\ -x=-\frac{5}{4} \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x=-\frac{3}{4} \; / \cdot (-1) \\ -x=-\frac{5}{4} \; / \cdot (-1) \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1=\frac{3}{4} \\ x_2=\frac{5}{4} \end{array}\right\}.$$

2.inačica

$$\log_{\frac{1}{2}} |1-x|=2 \Rightarrow \log_{\frac{1}{2}} |1-x|=\log_{\frac{1}{2}} \left(\frac{1}{2}\right)^2 \Rightarrow |1-x|=\left(\frac{1}{2}\right)^2 \Rightarrow |1-x|=\frac{1}{4} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 1-x=\frac{1}{4} \\ 1-x=-\frac{1}{4} \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x=\frac{1}{4}-1 \\ -x=-\frac{1}{4}-1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x=-\frac{3}{4} \\ -x=-\frac{5}{4} \end{array}\right\} \Rightarrow \left.\begin{array}{l} -x=-\frac{3}{4} \; / \cdot (-1) \\ -x=-\frac{5}{4} \; / \cdot (-1) \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1=\frac{3}{4} \\ x_2=\frac{5}{4} \end{array}\right\}.$$

Vježba 177

Riješi jednadžbu: $\log_{\frac{1}{2}} |1-x|=1.$

Rezultat: $x_1=\frac{1}{2}\ ,\ x_2=\frac{3}{2}.$

Zadatak 178 (Josip, srednja škola)

Riješi nejednadžbu: $\log_2 \left(\log_{\frac{1}{2}} x\right)>0.$

Rješenje 178

Ponovimo!

$$\log_b x>\log_b y\ ,\ b>1 \Rightarrow x>y\ ,\quad \log_b x>\log_b y\ ,\ 0<b<1 \Rightarrow x<y.$$

$$\log_b 1=0\quad ,\quad \log_b b=1.$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x)=\log_b x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x \in \langle 0, +\infty \rangle.$$

Obzirom da argument logaritamske funkcije mora biti pozitivan, to je

$$f(x)=\log_{\frac{1}{2}} x \Rightarrow x>0 \text{ domena}.$$

Uz taj uvjet slijedi:

$$\log_2 \left(\log_{\frac{1}{2}} x\right)>0 \Rightarrow \log_2 \left(\log_{\frac{1}{2}} x\right)>\log_2 1 \Rightarrow \log_2 \left(\log_{\frac{1}{2}} x\right)>\log_2 1 \Rightarrow$$

$$\Rightarrow \log_{\frac{1}{2}} x>1 \Rightarrow \log_{\frac{1}{2}} x>\log_{\frac{1}{2}} \frac{1}{2} \Rightarrow \log_{\frac{1}{2}} x>\log_{\frac{1}{2}} \frac{1}{2} \Rightarrow x<\frac{1}{2}.$$

Rješenje nejednadžbe čine svi realni brojevi iz intervala:

$$\left.\begin{array}{l} x>0 \ \text{domena} \\ x<\frac{1}{2} \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{gledamo presjek rješenja} \\ \text{ili zajednički dio} \end{array}\right] \Rightarrow 0<x<\frac{1}{2} \Rightarrow x \in \left\langle 0, \frac{1}{2} \right\rangle.$$

Vježba 178

Riješi nejednadžbu: $\log_2 \left(\log_2 x \right) > 0.$

Rezultat: x > 2.

Zadatak 179 (Neno, srednja škola)

Koliki je y, ako vrijedi $x^2 + x \cdot y = 74$ i $\log \sqrt{x} + \log \sqrt{y} = \frac{1}{2}$?

Rješenje 179

Ponovimo!

$$\log a + \log b = \log \left(a \cdot b \right) \quad , \quad \sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b} \quad , \quad \log \sqrt{a} = \frac{1}{2} \cdot \log a \quad , \quad \log 10 = 1.$$

$$\log f(x) = \log g(x) \Rightarrow f(x) = g(x).$$

Logaritamska funkcija s bazom b realna je funkcija oblika

$$f(x) = \log_b x,$$

gdje je b > 0 i b ≠ 1. Područje definicije (domena) logaritamske funkcije je interval pozitivnih realnih brojeva

$$x \in \left\langle 0, +\infty \right\rangle.$$

Obzirom da argument logaritamske funkcije mora biti pozitivan, to je

$$\left.\begin{array}{l} f(x) = \log \sqrt{x} \Rightarrow x>0 \\ g(x) = \log \sqrt{y} \Rightarrow y>0 \end{array}\right\} \text{domena}.$$

Uz taj uvjet riješimo sustav:

$$\left.\begin{array}{l} x^2 + x \cdot y = 74 \\ \log \sqrt{x} + \log \sqrt{y} = \frac{1}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + x \cdot y = 74 \\ \log \left(\sqrt{x} \cdot \sqrt{y} \right) = \frac{1}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + x \cdot y = 74 \\ \log \sqrt{x \cdot y} = \frac{1}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + x \cdot y = 74 \\ \frac{1}{2} \cdot \log \left(x \cdot y \right) = \frac{1}{2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x^2 + x \cdot y = 74 \\ \frac{1}{2} \cdot \log \left(x \cdot y \right) = \frac{1}{2} \ / \cdot 2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + x \cdot y = 74 \\ \log \left(x \cdot y \right) = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + x \cdot y = 74 \\ \log \left(x \cdot y \right) = \log 10 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2 + x \cdot y = 74 \\ x \cdot y = 10 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{metoda} \\ \text{supstitucije} \end{array}\right] \Rightarrow x^2 + 10 = 74 \Rightarrow x^2 = 74 - 10 \Rightarrow x^2 = 64 \ / \sqrt{\ } \Rightarrow x_{1,2} = \pm \sqrt{64} \Rightarrow$$

$$\Rightarrow x_{1,2} = \pm 8 \Rightarrow \left.\begin{array}{l} x_1 = -8 \ \text{nije rješenje} \\ x_2 = 8 \end{array}\right\} \Rightarrow x = 8 \Rightarrow \left.\begin{array}{l} x = 8 \\ x \cdot y = 10 \end{array}\right\} \Rightarrow 8 \cdot y = 10 \ /: 8 \Rightarrow$$

$$\Rightarrow y = \frac{10}{8} \Rightarrow y = \frac{5}{4} \Rightarrow y = 1.25.$$

Vježba 179

Koliki je x, ako vrijedi $x^2 + x \cdot y = 74$ i $\log x + \log y = 1$?

Rezultat: 8.

Zadatak 180 (Dijana, srednja škola)

Riješi jednadžbu: $\left(\frac{1}{2}\right)^{x}\cdot\left(\frac{2}{3}\right)^{x+1}\cdot\left(\frac{3}{4}\right)^{x+2}=6.$

Rješenje 180

Ponovimo!

$$a^{n}\cdot a^{m}=a^{n+m}\quad,\quad a^{n}\cdot b^{n}=(a\cdot b)^{n}\quad,\quad a^{1}=a\quad,\quad a^{-n}=\frac{1}{a^{n}}.$$

$$a^{f(x)}=a^{g(x)}\;\Rightarrow\;f(x)=g(x).$$

$$\left(\frac{1}{2}\right)^{x}\cdot\left(\frac{2}{3}\right)^{x+1}\cdot\left(\frac{3}{4}\right)^{x+2}=6\;\Rightarrow\;\left(\frac{1}{2}\right)^{x}\cdot\left(\frac{2}{3}\right)^{x}\cdot\left(\frac{2}{3}\right)^{1}\cdot\left(\frac{3}{4}\right)^{x}\cdot\left(\frac{3}{4}\right)^{2}=6\;\Rightarrow$$

$$\Rightarrow\left(\frac{1}{2}\right)^{x}\cdot\left(\frac{2}{3}\right)^{x}\cdot\frac{2}{3}\cdot\left(\frac{3}{4}\right)^{x}\cdot\frac{9}{16}=6\;\Rightarrow\;\left(\frac{1}{2}\right)^{x}\cdot\left(\frac{2}{3}\right)^{x}\cdot\left(\frac{3}{4}\right)^{x}\cdot\frac{2}{3}\cdot\frac{9}{16}=6\;\Rightarrow$$

$$\Rightarrow\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\right)^{x}\cdot\frac{2}{3}\cdot\frac{9}{16}=6\;\Rightarrow\;\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\right)^{x}\cdot\frac{1}{3}\cdot\frac{9}{8}=6\;\Rightarrow\;\left(\frac{1}{4}\right)^{x}\cdot\frac{1}{3}\cdot\frac{9}{8}=6\;\Rightarrow\;\left(\frac{1}{4}\right)^{x}\cdot\frac{1}{1}\cdot\frac{3}{8}=6\;\Rightarrow$$

$$\Rightarrow\left(\frac{1}{4}\right)^{x}\cdot\frac{3}{8}=6\;/\cdot\frac{8}{3}\;\Rightarrow\;\left(\frac{1}{4}\right)^{x}=16\;\Rightarrow\;4^{-x}=4^{2}\;\Rightarrow\;-x=2\;/\cdot(-1)\;\Rightarrow\;x=-2.$$

Vježba 180

Riješi jednadžbu: $\left(\frac{1}{2}\right)^{x}\cdot\left(\frac{2}{3}\right)^{x+1}=18.$

Rezultat: $-3.$