Zadatak 141 (Dino, maturant)

Riješite jednadžbu: $\sqrt[x-1]{b^{x+1}} \cdot \sqrt[x+3]{b^{x-1}} = \left(\frac{1}{b}\right)^{-2}$.

Rješenje 141

Ponovimo!

$$\sqrt[n]{a^m} = a^{\frac{m}{n}} \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

$$(a-b)^2 = a^2 - 2 \cdot a \cdot b + b^2.$$

$$\sqrt[x-1]{b^{x+1}} \cdot \sqrt[x+3]{b^{x-1}} = \left(\frac{1}{b}\right)^{-2} \Rightarrow b^{\frac{x+1}{x-1}} \cdot b^{\frac{x-1}{x+3}} = b^2 \Rightarrow b^{\frac{x+1}{x-1} + \frac{x-1}{x+3}} = b^2 \Rightarrow \frac{x+1}{x-1} + \frac{x-1}{x+3} = 2.$$

Rješavamo jednadžbu:

$$\frac{x+1}{x-1} + \frac{x-1}{x+3} = 2 \Rightarrow \begin{bmatrix} \text{Diskusija!} \\ x-1 \neq 0 \Rightarrow x \neq 1 \\ x+3 \neq 0 \Rightarrow x \neq -3 \end{bmatrix} \Rightarrow \frac{x+1}{x-1} + \frac{x-1}{x+3} = 2 \; / \cdot (x-1) \cdot (x+3) \Rightarrow$$

$$\Rightarrow (x+1) \cdot (x+3) + (x-1)^2 = 2 \cdot (x-1) \cdot (x+3) \Rightarrow x^2 + 3 \cdot x + x + 3 + x^2 - 2 \cdot x + 1 = 2 \cdot \left(x^2 + 3 \cdot x - x - 3\right) \Rightarrow$$

$$\Rightarrow x^2 + 3 \cdot x + x + 3 + x^2 - 2 \cdot x + 1 = 2 \cdot x^2 + 6 \cdot x - 2 \cdot x - 6 \Rightarrow 3 \cdot x + x + 3 - 2 \cdot x + 1 = 6 \cdot x - 2 \cdot x - 6 \Rightarrow$$

$$\Rightarrow 3 \cdot x + x + 3 - 2 \cdot x + 1 = 6 \cdot x - 2 \cdot x - 6 \Rightarrow 3 \cdot x + x + 3 + 1 = 6 \cdot x - 6 \Rightarrow 3 \cdot x + x - 6 \cdot x = -6 - 3 - 1 \Rightarrow$$

$$\Rightarrow -2 \cdot x = -10 \; / : (-2) \Rightarrow x = 5.$$

Vježba 141

Riješite jednadžbu: $\sqrt[x-1]{b^{x+1}} : \sqrt[x+3]{b^{x-1}} = b^0$.

Rezultat: $x = -\frac{1}{3}$.

Zadatak 142 (Ekonomistica, maturantica)

Ako je funkcija f(x) zadana izrazom f(x) = log x + 2 · log (2 · x) nađi vrijednost izraza $f(x) + f\left(\frac{1}{x}\right)$.

Rješenje 142

Ponovimo!

$$\log(a \cdot b) = \log a + \log b \quad , \quad \log \frac{a}{b} = \log a - \log b \quad , \quad \log a^n = n \cdot \log a \quad , \quad \log 1 = 0.$$

Računamo f(x):

$$f(x) = \log x + 2 \cdot \log(2 \cdot x) \Rightarrow f(x) = \log x + 2 \cdot (\log 2 + \log x) \Rightarrow f(x) = \log x + 2 \cdot \log 2 + 2 \cdot \log x \Rightarrow$$

$$\Rightarrow f(x) = 3 \cdot \log x + 2 \cdot \log 2 \Rightarrow f(x) = 3 \cdot \log x + \log 2^2 \Rightarrow f(x) = 3 \cdot \log x + \log 4.$$

Računamo $f\left(\frac{1}{x}\right)$:

$$f\left(\frac{1}{x}\right) = 3 \cdot \log \frac{1}{x} + \log 4 \Rightarrow f\left(\frac{1}{x}\right) = 3 \cdot (\log 1 - \log x) + \log 4 \Rightarrow f\left(\frac{1}{x}\right) = 3 \cdot (0 - \log x) + \log 4 \Rightarrow$$

$$\Rightarrow f\left(\frac{1}{x}\right) = -3 \cdot \log x + \log 4.$$

Konačno je:

$$\left.\begin{matrix} f(x) = 3 \cdot \log x + \log 4 \\ f\left(\frac{1}{x}\right) = -3 \cdot \log x + \log 4 \end{matrix}\right\} \Rightarrow f(x) + f\left(\frac{1}{x}\right) = 3 \cdot \log x + \log 4 - 3 \cdot \log x + \log 4 \Rightarrow$$

$$\Rightarrow f(x)+f\left(\frac{1}{x}\right)=2\cdot\log 4 \Rightarrow f(x)+f\left(\frac{1}{x}\right)=\log 4^{2} \Rightarrow f(x)+f\left(\frac{1}{x}\right)=\log 16.$$

Vježba 142

Ako je funkcija f(x) zadana izrazom f(x) = log x + 2 · log (2 · x) nađi vrijednost izraza $f(x)-f\left(\frac{1}{x}\right).$

Rezultat: $\log x^{6}.$

Zadatak 143 (Martin, srednja škola)

Riješi nejednadžbu: $\log_{\frac{1}{2}}(2\cdot x+1)<0.$

Rješenje 143

Ponovimo!

$$\log_b 1=0 \quad , \quad \left.\begin{matrix}\log_b x<\log_b y\\ 0<b<1\end{matrix}\right\} \Rightarrow x>y.$$

$$\left.\begin{matrix}\text{Neka je } b>0 \ , \ b\neq 1 \ , \ a>0. \text{ Tada je:}\\ \bullet \log_b a\le 0 \Leftrightarrow (b-1)\cdot(a-1)\le 0\\ \bullet \log_b a\ge 0 \Leftrightarrow (b-1)\cdot(a-1)\ge 0\end{matrix}\right\}. \qquad (♥)$$

1.inačica

$$\log_{\frac{1}{2}}(2\cdot x+1)<0.$$

Domenu rješenja dobit ćemo rješavanjem nejednadžbe:

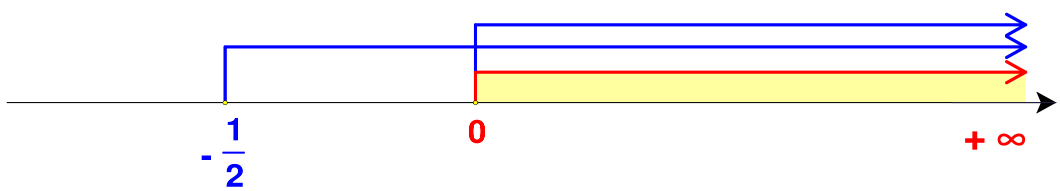
$$2\cdot x+1>0 \Rightarrow 2\cdot x>-1 \ /:2 \Rightarrow x>-\frac{1}{2} \Rightarrow x\in\left\langle -\frac{1}{2},+\infty\right\rangle.$$

Sada rješavamo zadanu nejednadžbu:

$$\log_{\frac{1}{2}}(2\cdot x+1)<0 \Rightarrow \log_{\frac{1}{2}}(2\cdot x+1)<\log_{\frac{1}{2}}1 \Rightarrow 2\cdot x+1>1 \Rightarrow 2\cdot x>1-1 \Rightarrow 2\cdot x>0 \ /:2 \Rightarrow x>0 \Rightarrow x\in\langle 0,+\infty\rangle.$$

Konačno rješenje je presjek skupova:

$$\left.\begin{matrix}x\in\left\langle -\frac{1}{2},+\infty\right\rangle\\ x\in\langle 0,+\infty\rangle\end{matrix}\right\} \Rightarrow \left[\begin{matrix}\text{presjek}\\ \text{skupova}\end{matrix}\right] \Rightarrow x\in\langle 0,+\infty\rangle.$$



2.inačica

$$\log_{\frac{1}{2}}(2\cdot x+1)<0.$$

Domenu rješenja dobit ćemo rješavanjem nejednadžbe:

$$2\cdot x+1>0 \Rightarrow 2\cdot x>-1 \ /:2 \Rightarrow x>-\frac{1}{2} \Rightarrow x\in\left\langle -\frac{1}{2},+\infty\right\rangle.$$

Uporabom svojstva (♥) dobije se:

$$\log_{\frac{1}{2}}(2\cdot x+1)<0 \Rightarrow \left(\frac{1}{2}-1\right)\cdot(2\cdot x+1-1)<0 \Rightarrow -\frac{1}{2}\cdot 2\cdot x<0 \Rightarrow -x<0 \ /\cdot(-1) \Rightarrow x>0 \Rightarrow x\in\langle 0,+\infty\rangle.$$

Konačno rješenje je presjek skupova:

$$\left.\begin{array}{l} x\in\left\langle -\frac{1}{2},+\infty\right\rangle \\ x\in\left\langle 0,+\infty\right\rangle \end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{presjek}\\ \text{skupova}\end{array}\right] \Rightarrow x\in\left\langle 0,+\infty\right\rangle .$$

Vježba 143

Riješi nejednadžbu: $\log_{\frac{1}{3}}(2\cdot x+1)<0.$

Rezultat: $x\in\left\langle 0,+\infty\right\rangle .$

Zadatak 144 (Krešo, maturant)

Riješite jednadžbu: $\log_3 x\cdot\log_9 x\cdot\log_{27} x=36.$

Rješenje 144

Ponovimo!

$$\log_b a=\frac{\log_c a}{\log_c b}\quad,\quad \log_b a^n=n\cdot\log_b a\quad,\quad \log_b b=1\quad,\quad \log_b a=c\Leftrightarrow b^c=a\quad,\quad \log_b a=\frac{1}{\log_a b}.$$

1.inačica

$$\log_3 x\cdot\log_9 x\cdot\log_{27} x=36.$$

Budući da su u jednadžbi logaritmi po različitim bazama (3, 9, 27), primjenom identiteta

$$\log_b a=\frac{\log_c a}{\log_c b}$$

mogu se svesti na jednu, istu bazu. Tada dobijemo ekvivalentnu jednadžbu:

$$\log_3 x\cdot\log_9 x\cdot\log_{27} x=36\Rightarrow \log_3 x\cdot\frac{\log_3 x}{\log_3 9}\cdot\frac{\log_3 x}{\log_3 27}=36\Rightarrow \log_3 x\cdot\frac{\log_3 x}{\log_3 3^2}\cdot\frac{\log_3 x}{\log_3 3^3}=36\Rightarrow$$

$$\Rightarrow \log_3 x\cdot\frac{\log_3 x}{2\cdot\log_3 3}\cdot\frac{\log_3 x}{3\cdot\log_3 3}=36\Rightarrow \log_3 x\cdot\frac{\log_3 x}{2\cdot 1}\cdot\frac{\log_3 x}{3\cdot 1}=36\Rightarrow \log_3 x\cdot\frac{\log_3 x}{2}\cdot\frac{\log_3 x}{3}=36\Rightarrow$$

$$\Rightarrow \frac{\log_3^3 x}{6}=36\ /\cdot 6\Rightarrow \log_3^3 x=216\Rightarrow \log_3^3 x=6^3\ /\sqrt[3]{}\Rightarrow \log_3 x=6\Rightarrow x=3^6\Rightarrow x=729.$$

2.inačica

$$\log_3 x\cdot\log_9 x\cdot\log_{27} x=36\Rightarrow \frac{1}{\log_x 3}\cdot\frac{1}{\log_x 9}\cdot\frac{1}{\log_x 27}=36\Rightarrow \frac{1}{\log_x 3}\cdot\frac{1}{\log_x 3^2}\cdot\frac{1}{\log_x 3^3}=36\Rightarrow$$

$$\Rightarrow \frac{1}{\log_x 3}\cdot\frac{1}{2\cdot\log_x 3}\cdot\frac{1}{3\cdot\log_x 3}=36\Rightarrow \frac{1}{6\cdot\log_x^3 3}=36\Rightarrow 6\cdot\log_x^3 3=\frac{1}{36}\ /\cdot\frac{1}{6}\Rightarrow \log_x^3 3=\frac{1}{216}\Rightarrow$$

$$\Rightarrow \log_x^3 3=\frac{1}{216}\ /\sqrt[3]{}\Rightarrow \log_x 3=\frac{1}{6}\Rightarrow x^{\frac{1}{6}}=3\ /^6\Rightarrow \left(x^{\frac{1}{6}}\right)^6=3^6\Rightarrow x=729.$$

Vježba 144

Riješite jednadžbu: $\log_3 x\cdot\log_9 x=2.$

Rezultat: x = 9.

Zadatak 144 (Danijel, srednja škola)

Riješite jednadžbu: $\left(7+4\cdot\sqrt{3}\right)^x+\left(7-4\cdot\sqrt{3}\right)^x=14.$

Rješenje 144

Ponovimo!

$$a^2-b^2=(a-b)\cdot(a+b) \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad \left(a\cdot\sqrt{b}\right)^n=a^n\cdot b \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$\left(a^m\right)^n=a^{m\cdot n} \quad , \quad a^1=a.$$

Najprije racionaliziramo izraz $7-4\cdot\sqrt{3}$:

$$7-4\cdot\sqrt{3}=\left(7-4\cdot\sqrt{3}\right)\cdot\frac{7+4\cdot\sqrt{3}}{7+4\cdot\sqrt{3}}=\frac{\left(7-4\cdot\sqrt{3}\right)\cdot\left(7+4\cdot\sqrt{3}\right)}{7+4\cdot\sqrt{3}}=\frac{7^2-\left(4\cdot\sqrt{3}\right)^2}{7+4\cdot\sqrt{3}}=$$

$$=\frac{49-16\cdot 3}{7+4\cdot\sqrt{3}}=\frac{49-48}{7+4\cdot\sqrt{3}}=\frac{1}{7+4\cdot\sqrt{3}}=\left(7+4\cdot\sqrt{3}\right)^{-1}.$$

Sada jednadžba glasi:

$$\left.\begin{array}{l}\left(7+4\cdot\sqrt{3}\right)^x+\left(7-4\cdot\sqrt{3}\right)^x=14\\ 7-4\cdot\sqrt{3}=\left(7+4\cdot\sqrt{3}\right)^{-1}\end{array}\right\}\Rightarrow \left(7+4\cdot\sqrt{3}\right)^x+\left(\left(7+4\cdot\sqrt{3}\right)^{-1}\right)^x=14\Rightarrow$$

$$\Rightarrow \left(7+4\cdot\sqrt{3}\right)^x+\left(\left(7+4\cdot\sqrt{3}\right)^x\right)^{-1}=14.$$

Uvedemo zamjenu (supstituciju)

$$t=\left(7+4\cdot\sqrt{3}\right)^x$$

i riješimo dobivenu kvadratnu jednadžbu:

$$t+t^{-1}=14 \Rightarrow t+\frac{1}{t}=14 \;/\cdot t \Rightarrow t^2+1=14\cdot t \Rightarrow t^2-14\cdot t+1=0 \Rightarrow \left.\begin{array}{l}t^2-14\cdot t+1=0\\ a=1\ ,\ b=-14\ ,\ c=1\end{array}\right\}\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}a=1\ ,\ b=-14\ ,\ c=1\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\}\Rightarrow t_{1,2}=\frac{14\pm\sqrt{196-4\cdot 1\cdot 1}}{2\cdot 1}\Rightarrow t_{1,2}=\frac{14\pm\sqrt{196-4}}{2}\Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{14\pm\sqrt{192}}{2}\Rightarrow t_{1,2}=\frac{14\pm\sqrt{64\cdot 3}}{2}\Rightarrow t_{1,2}=\frac{14\pm 8\cdot\sqrt{3}}{2}\Rightarrow t_{1,2}=\frac{14}{2}\pm\frac{8\cdot\sqrt{3}}{2}\Rightarrow t_{1,2}=7\pm 4\cdot\sqrt{3}\Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}t_1=7+4\cdot\sqrt{3}\\ t_2=7-4\cdot\sqrt{3}\end{array}\right\}\Rightarrow \left.\begin{array}{l}t_1=7+4\cdot\sqrt{3}\\ t_2=\left(7+4\cdot\sqrt{3}\right)^{-1}\end{array}\right\}.$$

Vraćamo se na supstituciju:

- $$\left.\begin{array}{l}t_1=7+4\cdot\sqrt{3}\\ t=\left(7+4\cdot\sqrt{3}\right)^x\end{array}\right\}\Rightarrow \left(7+4\cdot\sqrt{3}\right)^x=7+4\cdot\sqrt{3}\Rightarrow \left(7+4\cdot\sqrt{3}\right)^x=\left(7+4\cdot\sqrt{3}\right)^1\Rightarrow x_1=1.$$
- $$\left.\begin{array}{l}t_2=\left(7+4\cdot\sqrt{3}\right)^{-1}\\ t=\left(7+4\cdot\sqrt{3}\right)^x\end{array}\right\}\Rightarrow \left(7+4\cdot\sqrt{3}\right)^x=\left(7+4\cdot\sqrt{3}\right)^{-1}\Rightarrow x_2=-1.$$

Vježba 144

Riješite jednadžbu: $\left(7+4\cdot\sqrt{3}\right)^x-\left(7-4\cdot\sqrt{3}\right)^x=0.$

Rezultat: x = 0.

Zadatak 145 (Ante, Visoka škola za sigurnost)

Riješite jednadžbu: $2+3\cdot\log x=-1.$

Rješenje 145

Ponovimo!

$$\log_b a = c \Leftrightarrow b^c = a \quad , \quad \log_{10} a = \log a \quad , \quad a^{-n} = \frac{1}{a^n}.$$

Logaritamsku jednadžbu ima smisla promatrati samo tamo gdje je logaritamska funkcija f(x) = log x u njoj definirana. Zato mora biti

$$x > 0.$$

$$2 + 3 \cdot \log x = -1 \Rightarrow 3 \cdot \log x = -1 - 2 \Rightarrow 3 \cdot \log x = -3 \;/:3 \Rightarrow \log x = -1 \Rightarrow x = 10^{-1} \Rightarrow x = \frac{1}{10} \Rightarrow x = 0.1.$$

Vježba 145

Riješite jednadžbu: $2 + 3 \cdot \log x = -4.$

Rezultat: 0.01.

Zadatak 146 (Marija, Željka, Marijana, Luka, Vesna, Martina, TUPŠ)

Ako je $\log_a x = s$ i $\log_a y^2 = t$, koliko je $\log_a \frac{x}{y}$?

Rješenje 146

Ponovimo!

$$\log_b a^n = n \cdot \log_b a \quad , \quad \log_b \frac{x}{y} = \log_b x - \log_b y.$$

$$\log_a y^2 = t \Rightarrow 2 \cdot \log_a y = t \;/:2 \Rightarrow \log_a y = \frac{t}{2}.$$

Sada računamo $\log_a \frac{x}{y}$:

$$\left.\begin{array}{l} \log_a x = s \\ \log_a y = \frac{t}{2} \end{array}\right\} \Rightarrow \log_a \frac{x}{y} = \log_a x - \log_a y = s - \frac{t}{2}.$$

Vježba 146

Ako je $\log_a x = s$ i $\log_a y^2 = t$, koliko je $\log_a (x \cdot y)$?

Rezultat: $\frac{s \cdot t}{2}.$

Zadatak 147 (Marija, Željka, Marijana, Luka, Vesna, Martina, TUPŠ)

Broj bakterija B u nekoj populaciji mijenja se s vremenom t na sljedeći način $B(t) = 1000 \cdot 2^{3 \cdot t}$, gdje je t vrijeme u satima od početka mjerenja.

a) Koliko je bilo bakterija 40 minuta nakon početka mjerenja?

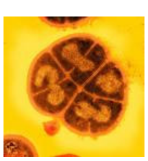
b) Koliko je bilo bakterija 1 sat prije početka mjerenja?

c) Nakon koliko je sati bilo 4096000 bakterija?

Rješenje 147

Ponovimo!

$$\left(a^n\right)^m = a^{n \cdot m} \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$



a)

$$t = 40 \text{ min} \Rightarrow t = \frac{40}{60}\, h \Rightarrow t = \frac{2}{3}\, h.$$

Broj bakterija 40 minuta nakon početka mjerenja iznosi:

$$\left.\begin{array}{l} t=\frac{2}{3}\ h \\ B(t)=1000\cdot 2^{3\cdot t}\end{array}\right\} \Rightarrow B\left(\frac{2}{3}\right)=1000\cdot 2^{3\cdot\frac{2}{3}} \Rightarrow B\left(\frac{2}{3}\right)=1000\cdot 2^{3\cdot\frac{2}{3}} \Rightarrow B\left(\frac{2}{3}\right)=1000\cdot 2^{2} \Rightarrow$$

$$\Rightarrow B\left(\frac{2}{3}\right)=1000\cdot 4 \Rightarrow B\left(\frac{2}{3}\right)=4000.$$

b)

Broj bakterija 1 sat prije početka mjerenja iznosi:

$$\left.\begin{array}{l} t=-1\ h \\ B(t)=1000\cdot 2^{3\cdot t}\end{array}\right\} \Rightarrow B(-1)=1000\cdot 2^{3\cdot(-1)} \Rightarrow B(-1)=1000\cdot 2^{-3} \Rightarrow B(-1)=1000\cdot\frac{1}{2^3} \Rightarrow$$

$$\Rightarrow B(-1)=1000\cdot\frac{1}{8} \Rightarrow B(-1)=125.$$

c)

Računamo broj sati nakon kojega je bilo 4096000 bakterija:

$$\left.\begin{array}{l} B(t)=4\,096\,000 \\ B(t)=1000\cdot 2^{3\cdot t}\end{array}\right\} \Rightarrow 1000\cdot 2^{3\cdot t}=4\,096\,000 \Rightarrow 1000\cdot 2^{3\cdot t}=4\,096\,000\ /:1000 \Rightarrow 2^{3\cdot t}=4\,096 \Rightarrow$$

$$\Rightarrow 2^{3\cdot t}=2^{12} \Rightarrow 3\cdot t=12 \Rightarrow 3\cdot t=12\ /:3 \Rightarrow t=4\ h.$$

Vježba 147

Broj bakterija B u nekoj populaciji mijenja se s vremenom t na sljedeći način $B(t)=1000\cdot 2^{3\cdot t}$, gdje je t vrijeme u satima od početka mjerenja. Koliko je bilo bakterija 20 minuta nakon početka mjerenja?

Rezultat: 2000.

Zadatak 148 (Valentina, maturantica)

Rješenje jednadžbe $0.25\cdot\sqrt[3]{4^{2\cdot x-1}}=8^{-\frac{2}{3}}$ pripada intervalu:

$A.\ \langle -4, -2\rangle \quad B.\ \langle -2, 0\rangle \quad C.\ \langle 0, 2\rangle \quad D.\ \langle 2, 4\rangle \quad E.\ \langle 4, 6\rangle$

Rješenje 148

Ponovimo!

$$\sqrt[n]{a^m}=a^{\frac{m}{n}} \quad , \quad \left(a^n\right)^m=a^{n\cdot m} \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^n\cdot a^m=a^{n+m}$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$0.25\cdot\sqrt[3]{4^{2\cdot x-1}}=8^{-\frac{2}{3}} \Rightarrow \frac{25}{100}\cdot 4^{\frac{2\cdot x-1}{3}}=8^{-\frac{2}{3}} \Rightarrow \frac{1}{4}\cdot\left(2^2\right)^{\frac{2\cdot x-1}{3}}=\left(2^3\right)^{-\frac{2}{3}} \Rightarrow$$

$$\Rightarrow \frac{1}{2^2}\cdot 2^{\frac{4\cdot x-2}{3}}=2^{-2} \Rightarrow 2^{-2}\cdot 2^{\frac{4\cdot x-2}{3}}=2^{-2} \Rightarrow 2^{-2+\frac{4\cdot x-2}{3}}=2^{-2} \Rightarrow$$

$$\Rightarrow -2+\frac{4\cdot x-2}{3}=-2 \Rightarrow -2+\frac{4\cdot x-2}{3}=-2 \Rightarrow \frac{4\cdot x-2}{3}=0 \Rightarrow \frac{4\cdot x-2}{3}=0\ /\cdot 3 \Rightarrow$$

$$\Rightarrow 4\cdot x-2=0 \Rightarrow 4\cdot x=2\ /:4 \Rightarrow x=\frac{2}{4} \Rightarrow x=\frac{1}{2} \Rightarrow \frac{1}{2}\in\langle 0, 2\rangle.$$ Odgovor pod C.

Vježba 148

Rješenje jednadžbe $\sqrt[3]{4^{2\cdot x-1}}=4\cdot 8^{-\frac{2}{3}}$ pripada intervalu:

$$A.\ \langle -4, -2\rangle \quad B.\ \langle -2, 0\rangle \quad C.\ \langle 0, 2\rangle \quad D.\ \langle 2, 4\rangle \quad E.\ \langle 4, 6\rangle$$

Rezultat: Odgovor pod C.

Zadatak 149 (Valentina, maturantica)

Koliko rješenja u skupu R ima jednadžba $\log_{x-1} 1 = 2$?

$$A.\ 0 \qquad B.\ 1 \qquad C.\ 2 \qquad D.\ 3 \qquad E.\ 4$$

Rješenje 149

Ponovimo!

$$\log_b a = c \Leftrightarrow b^c = a \ ,\ b > 0 \ ,\ b \neq 1 \ ,\ a > 0$$

$$a \cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

$$\log_{x-1} 1 = 2 \Rightarrow (x-1)^2 = 1 \Rightarrow (x-1)^2 = 1 \ /\sqrt{\ } \Rightarrow x-1 = \pm\sqrt{1} \Rightarrow x-1 = \pm 1 \Rightarrow \left.\begin{matrix} x-1 = 1 \\ x-1 = -1 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x = 1+1 \\ x = -1+1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1 = 2 \\ x_2 = 0 \end{matrix}\right\}.$$

Za x = 2 baza logaritma x – 1 iznosi 2 – 1 = 1. Za x = 0 baza logaritma x – 1 iznosi 0 – 1 = – 1.

Budući da baza logaritma mora biti pozitivan broj različit od 1, slijedi da jednadžba nema rješenja. Odgovor je pod A.

Vježba 149

Koliko rješenja u skupu R ima jednadžba $\log_{x-2} 4 = 2$?

$$A.\ 0 \qquad B.\ 1 \qquad C.\ 2 \qquad D.\ 3 \qquad E.\ 4$$

Rezultat: Odgovor je pod B.

Zadatak 150 (Hrvoje, gimnazija)

Riješite jednadžbu: $\log_2 \log_2 x = \log_4 \log_4 x.$

Rješenje 150

Ponovimo!

$$\log_b a = c \Leftrightarrow b^c = a \ ,\ b > 0 \ ,\ b \neq 1 \ ,\ a > 0 \quad , \quad \log_{b^k} a = \frac{1}{k} \cdot \log_b a \quad , \quad \log_b a^n = n \cdot \log_b a$$

$$\log_b \sqrt[n]{a} = \frac{1}{n} \cdot \log_b a \quad , \quad \log_b f(x) = \log_b g(x) \Rightarrow f(x) = g(x) \quad , \quad a^0 = 1 \quad , \quad \sqrt[n]{a^m} = a^{\frac{m}{n}}.$$

$$\log_b b = 1 \quad , \quad a \cdot b = 0 \Leftrightarrow a = 0 \text{ ili } b = 0 \text{ ili } a = b = 0.$$

Logaritamska funkcija definirana je za pozitivne realne brojeve:

$$f(x) = \log_b x \ ,\ x > 0.$$

1.inačica

$$\log_2 \log_2 x = \log_4 \log_4 x \Rightarrow \log_2 \log_2 x = \log_{2^2} \log_4 x \Rightarrow \log_2 \log_2 x = \frac{1}{2} \cdot \log_2 \log_4 x \Rightarrow$$

$$\Rightarrow \log_2 \log_2 x = \frac{1}{2} \cdot \log_2 \log_4 x \ / \cdot 2 \Rightarrow 2 \cdot \log_2 \log_2 x = \log_2 \log_4 x \Rightarrow$$

$$\Rightarrow \log_2 (\log_2 x)^2 = \log_2 \log_4 x \Rightarrow (\log_2 x)^2 = \log_4 x \Rightarrow (\log_2 x)^2 = \log_{2^2} x \Rightarrow$$

$$\Rightarrow (\log_2 x)^2 = \frac{1}{2} \cdot \log_2 x \Rightarrow (\log_2 x)^2 - \frac{1}{2} \cdot \log_2 x = 0 \Rightarrow \log_2 x \cdot \left(\log_2 x - \frac{1}{2}\right) = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \log_2 x = 0 \\ \log_2 x - \frac{1}{2} = 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_2 x = 0 \\ \log_2 x = \frac{1}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = 2^0 \\ x = 2^{\frac{1}{2}} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1 = 1 \text{ nije rješenje} \\ x_2 = \sqrt{2} \text{ je rješenje} \end{matrix}\right\}.$$

Za $x_1 = 1$ dobije se:

$$\log_2 \log_2 1 = \log_2 0 \text{ to nema smisla}.$$

Za $x_2 = \sqrt{2}$ dobije se:

$$\log_2 \log_2 \sqrt{2} = \log_4 \log_4 \sqrt{2} \Rightarrow \log_2\left(\frac{1}{2}\cdot\log_2 2\right) = \log_4\left(\frac{1}{2}\cdot\log_4 2\right) \Rightarrow$$

$$\Rightarrow \log_2\left(\frac{1}{2}\cdot 1\right) = \log_4\left(\frac{1}{2}\cdot\frac{1}{2}\right) \Rightarrow \log_2\left(\frac{1}{2}\right) = \log_4\left(\frac{1}{4}\right) \Rightarrow \log_2 2^{-1} = \log_4 4^{-1} \Rightarrow$$

$$\Rightarrow -1\cdot\log_2 2 = -1\cdot\log_4 4 \Rightarrow -1\cdot 1 = -1\cdot 1 \Rightarrow -1 = -1.$$

2.inačica

$$\log_2 \log_2 x = \log_4 \log_4 x \Rightarrow \log_2 \log_2 x = \log_4 \log_4 x \;/\cdot\frac{1}{2} \Rightarrow \frac{1}{2}\cdot\log_2 \log_2 x = \frac{1}{2}\cdot\log_4 \log_4 x \Rightarrow$$

$$\Rightarrow \log_{2^2} \log_2 x = \log_4 \sqrt{\log_4 x} \Rightarrow \log_4 \log_2 x = \log_4 \sqrt{\log_4 x} \Rightarrow \log_2 x = \sqrt{\log_4 x} \Rightarrow$$

$$\Rightarrow \log_2 x = \sqrt{\log_4 x} \;/^2 \Rightarrow \left(\log_2 x\right)^2 = \log_4 x \Rightarrow \left(\log_2 x\right)^2 = \log_{2^2} x \Rightarrow$$

$$\Rightarrow \left(\log_2 x\right)^2 = \frac{1}{2}\cdot\log_2 x \Rightarrow \left(\log_2 x\right)^2 - \frac{1}{2}\cdot\log_2 x = 0 \Rightarrow \log_2 x\cdot\left(\log_2 x - \frac{1}{2}\right) = 0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \log_2 x = 0 \\ \log_2 x - \frac{1}{2} = 0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_2 x = 0 \\ \log_2 x = \frac{1}{2} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x = 2^0 \\ x = 2^{\frac{1}{2}} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x_1 = 1 \text{ nije rješenje} \\ x_2 = \sqrt{2} \text{ je rješenje} \end{matrix}\right\}.$$

Vježba 150

Riješite jednadžbu: $\log_2 x = \log_4 x.$

Rezultat: x = 1.

Zadatak 151 (Hrvoje, gimnazija)

Riješite sustav jednadžbi : $\begin{cases} 2\cdot\log x - \log y = 2\cdot\log 2 + \log 3 \\ 2\cdot x^2 + y = 75. \end{cases}$

Rješenje 151

Ponovimo!

$$\log a^n = n\cdot\log a \quad , \quad \log\left(x\cdot y\right) = \log x + \log y \quad , \quad \log\frac{x}{y} = \log x - \log y.$$

$$\log f\left(x\right) = \log g\left(x\right) \Rightarrow f\left(x\right) = g\left(x\right).$$

Logaritamska funkcija definirana je za pozitivne realne brojeve:

$$f\left(x\right) = \log x \;,\; x > 0.$$

$$\left.\begin{matrix} 2\cdot\log x - \log y = 2\cdot\log 2 + \log 3 \\ 2\cdot x^2 + y = 75 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{Diskusija!} \\ x > 0 \;,\; y > 0 \end{matrix}\right] \Rightarrow \left.\begin{matrix} \log x^2 - \log y = \log 2^2 + \log 3 \\ 2\cdot x^2 + y = 75 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\log x^2 - \log y = \log 4 + \log 3 \\ 2\cdot x^2 + y = 75\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log \dfrac{x^2}{y} = \log(4\cdot 3) \\ 2\cdot x^2 + y = 75\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log \dfrac{x^2}{y} = \log 12 \\ 2\cdot x^2 + y = 75\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\dfrac{x^2}{y} = 12 \\ 2\cdot x^2 + y = 75\end{array}\right\} \Rightarrow \left.\begin{array}{l}\dfrac{x^2}{y} = 12 \; /\cdot y \\ 2\cdot x^2 + y = 75\end{array}\right\} \Rightarrow \left.\begin{array}{l}x^2 = 12\cdot y \\ 2\cdot x^2 + y = 75\end{array}\right\} \Rightarrow 2\cdot 12\cdot y + y = 75 \Rightarrow$$

$$\Rightarrow 24\cdot y + y = 75 \Rightarrow 25\cdot y = 75 \; /:25 \Rightarrow y = 3 \Rightarrow \left.\begin{array}{l}y = 3 \\ x^2 = 12\cdot y\end{array}\right\} \Rightarrow x^2 = 12\cdot 3 \Rightarrow$$

$$\Rightarrow x^2 = 36 \Rightarrow x^2 = 36 \; /\sqrt{\;} \Rightarrow x_{1,2} = \pm\sqrt{36} \Rightarrow \left.\begin{array}{l}x_1 = 6 \text{ je rješenje} \\ x_2 = -6 \text{ nije rješenje zbog } x > 0\end{array}\right\}.$$

Rješenje sustava glasi:

$$(x, y) = (6, 3).$$

Vježba 151

$$\text{Riješite sustav jednadžbi: } \left\{\begin{array}{l}2\cdot(\log x - \log 2) = \log y + \log 3 \\ 2\cdot x^2 + y = 75.\end{array}\right.$$

Rezultat: $(x, y) = (6, 3).$

Zadatak 152 (Matea, farmaceutska škola)

Riješi eksponencijalnu jednadžbu: $4^x + 4^{x+1} = 320.$

Rješenje 152

Ponovimo!

$$a^n\cdot a^m = a^{n+m} \quad , \quad a^1 = a \quad , \quad a\cdot x + a\cdot y = a\cdot(x+y) \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

$$4^x + 4^{x+1} = 320 \Rightarrow 4^x + 4^x\cdot 4^1 = 320 \Rightarrow 4^x + 4^x\cdot 4 = 320 \Rightarrow 4^x\cdot(1+4) = 320 \Rightarrow$$

$$\Rightarrow 4^x\cdot 5 = 320 \Rightarrow 5\cdot 4^x = 320 \; /:5 \Rightarrow 4^x = 64 \Rightarrow 4^x = 4^3 \Rightarrow x = 3.$$

Vježba 152

Riješi eksponencijalnu jednadžbu: $4^x + 4^{x+1} = 20.$

Rezultat: x = 1.

Zadatak 153 (Matea, farmaceutska škola)

Riješi logaritamsku jednadžbu: $\log \dfrac{x^2 - 1}{(2\cdot x + 1)^2} + 2\cdot\log \dfrac{2\cdot x + 1}{x - 1} - 1 = 0.$

Rješenje 153

Ponovimo!

$$\log\frac{a}{b} = \log a - \log b \quad , \quad \log a^n = n\cdot\log a \quad , \quad a^2 - b^2 = (a-b)\cdot(a+b) \quad , \quad \log(a\cdot b) = \log a + \log b.$$

$$\log a = c \Leftrightarrow a = 10^c \quad , \quad \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad , \quad \log_b b = 1 \quad , \quad \log f(x) = \log g(x) \Rightarrow f(x) = g(x).$$

1.inačica

Diskusija!

$$\left.\begin{matrix} 2\cdot x+1>0 \\ x-1>0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2\cdot x>-1 \\ x>1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2\cdot x>-1 \ /:2 \\ x>1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x>-\frac{1}{2} \\ x>1 \end{matrix}\right\} \Rightarrow x>1.$$

Sada rješavamo jednadžbu:

$$\log\frac{x^2-1}{(2\cdot x+1)^2}+2\cdot\log\frac{2\cdot x+1}{x-1}-1=0 \Rightarrow$$

$$\Rightarrow \log\left(x^2-1\right)-\log(2\cdot x+1)^2+2\cdot\left(\log(2\cdot x+1)-\log(x-1)\right)-1=0 \Rightarrow$$

$$\Rightarrow \log\left(x^2-1\right)-2\cdot\log(2\cdot x+1)+2\cdot\log(2\cdot x+1)-2\cdot\log(x-1)-1=0 \Rightarrow$$

$$\Rightarrow \log\left(x^2-1\right)-2\cdot\log(2\cdot x+1)+2\cdot\log(2\cdot x+1)-2\cdot\log(x-1)-1=0 \Rightarrow$$

$$\Rightarrow \log\left(x^2-1\right)-2\cdot\log(x-1)-1=0 \Rightarrow \log(x-1)\cdot(x+1)-2\cdot\log(x-1)-1=0 \Rightarrow$$

$$\Rightarrow \log(x-1)+\log(x+1)-2\cdot\log(x-1)-1=0 \Rightarrow \log(x+1)-\log(x-1)-1=0 \Rightarrow$$

$$\Rightarrow \log\frac{x+1}{x-1}-1=0 \Rightarrow \log\frac{x+1}{x-1}=1 \Rightarrow \log\frac{x+1}{x-1}=\log 10 \Rightarrow \frac{x+1}{x-1}=10 \ /\cdot(x-1) \Rightarrow$$

$$\Rightarrow x+1=10\cdot(x-1) \Rightarrow x+1=10\cdot x-10 \Rightarrow x-10\cdot x=-10-1 \Rightarrow -9\cdot x=-11 \ /:(-9) \Rightarrow$$

$$\Rightarrow x=\frac{11}{9} \Rightarrow \left[\begin{matrix} \text{to jest rješenje jer je} \\ x=\frac{11}{9}>1 \end{matrix}\right].$$

2.inačica

Diskusija!

$$\left.\begin{matrix} 2\cdot x+1>0 \\ x-1>0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2\cdot x>-1 \\ x>1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} 2\cdot x>-1 \ /:2 \\ x>1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x>-\frac{1}{2} \\ x>1 \end{matrix}\right\} \Rightarrow x>1.$$

Sada rješavamo jednadžbu:

$$\log\frac{x^2-1}{(2\cdot x+1)^2}+2\cdot\log\frac{2\cdot x+1}{x-1}-1=0 \Rightarrow \log\frac{x^2-1}{(2\cdot x+1)^2}+\log\left(\frac{2\cdot x+1}{x-1}\right)^2-1=0 \Rightarrow$$

$$\Rightarrow \log\frac{x^2-1}{(2\cdot x+1)^2}+\log\frac{(2\cdot x+1)^2}{(x-1)^2}-1=0 \Rightarrow \log\frac{x^2-1}{(2\cdot x+1)^2}\cdot\frac{(2\cdot x+1)^2}{(x-1)^2}-1=0 \Rightarrow$$

$$\Rightarrow \log\frac{x^2-1}{(2\cdot x+1)^2}\cdot\frac{(2\cdot x+1)^2}{(x-1)^2}-1=0 \Rightarrow \log\frac{x^2-1}{(x-1)^2}=1 \Rightarrow \log\frac{(x-1)\cdot(x+1)}{(x-1)\cdot(x-1)}=1 \Rightarrow$$

$$\Rightarrow \log\frac{(x-1)\cdot(x+1)}{(x-1)\cdot(x-1)}=1 \Rightarrow \log\frac{x+1}{x-1}=1 \Rightarrow \log\frac{x+1}{x-1}=\log 10 \Rightarrow \frac{x+1}{x-1}=10 \ /\cdot(x-1) \Rightarrow$$

$$\Rightarrow x+1=10\cdot(x-1) \Rightarrow x+1=10\cdot x-10 \Rightarrow x-10\cdot x=-10-1 \Rightarrow -9\cdot x=-11 \ /:(-9) \Rightarrow$$

$$\Rightarrow x=\frac{11}{9} \Rightarrow \left[\begin{matrix} \text{to jest rješenje jer je} \\ x=\frac{11}{9}>1 \end{matrix}\right].$$

Vježba 153

Riješi logaritamsku jednadžbu : $2\cdot\log\frac{2\cdot x+1}{x-1}-\log\frac{(2\cdot x+1)^2}{x^2-1}=1.$

Rezultat: $x=\frac{11}{9}.$

Zadatak 154 (Matea, farmaceutska škola)

Odredite parametre $A,\ K\in R,\ A\neq 0,\ K\neq 0,$ takve da vrijedi f(1) = 1, f(2) = 2, pri čemu je $f: R\to\langle 0, +\infty\rangle$ eksponencijalna funkcija zadana s $f(x)=A\cdot e^{K\cdot x}.$

Rješenje 154

Ponovimo!

$$\ln e=1\quad,\quad \ln e^a=a\quad,\quad e^{\ln a}=a\quad,\quad \frac{e^a}{e^b}=e^{a-b}.$$

Računamo parametar K:

$$\left.\begin{array}{l} f(1)=1\\ f(2)=2\\ f(x)=A\cdot e^{K\cdot x}\end{array}\right\}\Rightarrow\left.\begin{array}{l}A\cdot e^{K\cdot 1}=1\\ A\cdot e^{K\cdot 2}=2\end{array}\right\}\Rightarrow\left.\begin{array}{l}A\cdot e^{K}=1\\ A\cdot e^{2\cdot K}=2\end{array}\right\}\Rightarrow\left[\begin{array}{l}\text{podijelimo}\\ \text{jednadžbe}\end{array}\right]\Rightarrow\frac{A\cdot e^{2\cdot K}}{A\cdot e^{K}}=\frac{2}{1}\Rightarrow$$

$$\Rightarrow\frac{A\cdot e^{2\cdot K}}{A\cdot e^{K}}=2\Rightarrow\frac{e^{2\cdot K}}{e^{K}}=2\Rightarrow\left[\begin{array}{l}\text{dijelimo potencije}\\ \text{istih baza}\end{array}\right]\Rightarrow e^{2\cdot K-K}=2\Rightarrow e^{K}=2\Rightarrow$$

$$\Rightarrow\left[\begin{array}{l}\text{logaritmiramo}\\ \text{jednadžbu}\end{array}\right]\Rightarrow e^{K}=2\ /\ln\Rightarrow\ln e^{K}=\ln 2\Rightarrow K=\ln 2.$$

Računamo parametar A:

$$\left.\begin{array}{l} f(1)=1\\ f(x)=A\cdot e^{K\cdot x}\\ K=\ln 2\end{array}\right\}\Rightarrow\left.\begin{array}{l}A\cdot e^{K\cdot 1}=1\\ K=\ln 2\end{array}\right\}\Rightarrow\left.\begin{array}{l}A\cdot e^{K}=1\\ K=\ln 2\end{array}\right\}\Rightarrow A\cdot e^{\ln 2}=1\Rightarrow A\cdot 2=1\ /:2\Rightarrow A=\frac{1}{2}.$$

Vježba 154

Odredite parametre $A,\ K\in R,\ A\neq 0,\ K\neq 0,$ takve da vrijedi f(1) = 1, f(2) = 3, pri čemu je $f: R\to\langle 0, +\infty\rangle$ eksponencijalna funkcija zadana s $f(x)=A\cdot e^{K\cdot x}.$

Rezultat: $A=\frac{1}{3}\ ,\ K=\ln 3.$

Zadatak 155 (Boris, gimnazija)

Koliko je: $\log\frac{1}{2}+\log\frac{2}{3}+\log\frac{3}{4}+\log\frac{4}{5}+\ ...\ +\log\frac{n}{n+1}$?

Rješenje 155

Ponovimo!

$$\log\frac{a}{b}=\log a-\log b\quad,\quad\log(a\cdot b)=\log a+\log b\quad,\quad\log 1=0.$$

1.inačica

Uporabom pravila za logaritmiranje kvocijenta dobije se:

$$\log\frac{1}{2}+\log\frac{2}{3}+\log\frac{3}{4}+\log\frac{4}{5}+\ ...\ +\log\frac{n}{n+1}=$$

$$=\log 1-\log 2+\log 2-\log 3+\log 3-\log 4+\log 4-\log 5+\ ...\ +\log n-\log(n+1)=$$

$$=\log 1-\log 2+\log 2-\log 3+\log 3-\log 4+\log 4-\log 5+\ \dots\ +\log n-\log(n+1)=$$

$$=\log 1-\log(n+1)=0-\log(n+1)=-\log(n+1).$$

2.inačica

Uporabom pravila za logaritmiranje produkta dobije se:

$$\log\frac{1}{2}+\log\frac{2}{3}+\log\frac{3}{4}+\log\frac{4}{5}+\ \dots\ +\log\frac{n}{n+1}=\log\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\ \dots\ \cdot\frac{n}{n+1}\right)=\begin{bmatrix}\text{kratimo}\\ \text{razlomke}\end{bmatrix}=$$

$$=\log\left(\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}\cdot\frac{4}{5}\cdot\ \dots\ \cdot\frac{n}{n+1}\right)=\log\frac{1}{n+1}=\log 1-\log(n+1)=0-\log(n+1)=-\log(n+1).$$

Vježba 155

Koliko je: $\log\frac{1}{2}+\log\frac{2}{3}+\log\frac{3}{4}+\log\frac{4}{5}+\ \dots\ +\log\frac{9}{10}$?

Rezultat: -1.

Zadatak 156 (Nina, gimnazija)

Riješi jednadžbu: $2^{\cos 2x}=3\cdot 2^{\cos^2 x}-4.$

Rješenje 156

Ponovimo!

$$\cos 2\alpha=2\cdot\cos^2\alpha-1\quad,\quad\left(a^n\right)^m=a^{n\cdot m}\quad,\quad a^n\cdot a^m=a^{n+m}\quad,\quad a^{-n}=\frac{1}{a^n}\quad,\quad a^1=a.$$

$$2^{\cos 2x}=3\cdot 2^{\cos^2 x}-4\ \Rightarrow\ 2^{\cos 2x}-3\cdot 2^{\cos^2 x}+4=0\ \Rightarrow\ 2^{2\cdot\cos^2 x-1}-3\cdot 2^{\cos^2 x}+4=0\ \Rightarrow$$

$$\Rightarrow\ 2^{2\cdot\cos^2 x}\cdot 2^{-1}-3\cdot 2^{\cos^2 x}+4=0\ \Rightarrow\ 2^{2\cdot\cos^2 x}\cdot\frac{1}{2}-3\cdot 2^{\cos^2 x}+4=0\ /\cdot 2\ \Rightarrow$$

$$\Rightarrow\ 2^{2\cdot\cos^2 x}-6\cdot 2^{\cos^2 x}+8=0\ \Rightarrow\ \left(2^{\cos^2 x}\right)^2-6\cdot 2^{\cos^2 x}+8=0\ \Rightarrow\ \begin{bmatrix}\text{supstitucija}\\ t=2^{\cos^2 x}\end{bmatrix}\Rightarrow$$

$$\Rightarrow\ t^2-6\cdot t+8=0\ \Rightarrow\ \left.\begin{matrix}t^2-6\cdot t+8=0\\ a=1\ ,\ b=-6\ ,\ c=8\end{matrix}\right\}\Rightarrow\ \left.\begin{matrix}a=1\ ,\ b=-6\ ,\ c=8\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\}\Rightarrow\ t_{1,2}=\frac{6\pm\sqrt{36-4\cdot 1\cdot 8}}{2\cdot 1}\ \Rightarrow$$

$$\Rightarrow\ t_{1,2}=\frac{6\pm\sqrt{36-32}}{2}\ \Rightarrow\ t_{1,2}=\frac{6\pm\sqrt{4}}{2}\ \Rightarrow\ t_{1,2}=\frac{6\pm 2}{2}\ \Rightarrow\ \left.\begin{matrix}t_1=\dfrac{6+2}{2}\\ t_2=\dfrac{6-2}{2}\end{matrix}\right\}\Rightarrow\ \left.\begin{matrix}t_1=\dfrac{8}{2}\\ t_2=\dfrac{4}{2}\end{matrix}\right\}\Rightarrow\ \left.\begin{matrix}t_1=4\\ t_2=2\end{matrix}\right\}.$$

Vraćamo se supstituciji.

- $\left.\begin{matrix}t=2^{\cos^2 x}\\ t=4\end{matrix}\right\}\Rightarrow\ 2^{\cos^2 x}=4\ \Rightarrow\ 2^{\cos^2 x}=2^2\ \Rightarrow\ \cos^2 x=2\ /\sqrt{\ }\ \Rightarrow$

  $$\Rightarrow\ \cos x=\pm\sqrt{2}\ \text{ nema smisla jer je }\left|\cos x\right|\le 1.$$

- $\left.\begin{matrix}t=2^{\cos^2 x}\\ t=2\end{matrix}\right\}\Rightarrow\ 2^{\cos^2 x}=2\ \Rightarrow\ 2^{\cos^2 x}=2^1\ \Rightarrow\ \cos^2 x=1\ /\sqrt{\ }\ \Rightarrow$

  $$\Rightarrow\ \cos x=\pm\sqrt{1}\ \Rightarrow\ \cos x=\pm 1\ \Rightarrow\ x=k\cdot\pi\ ,\ k\in Z.$$

Vježba 156

Riješi jednadžbu: $2^{\cos x}-2=0.$

Rezultat: $x=k\cdot 2\cdot\pi\ ,\ k\in Z.$

Zadatak 157 (Pero, srednja škola)

Ako je $\begin{cases} 2^x\cdot3^y=108 \\ 3^x\cdot2^y=72 \end{cases}$, nađi x + y.

Rješenje 157

Ponovimo!

$$a^n\cdot b^n=(a\cdot b)^n \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad \left(\frac{a}{b}\right)^n=\frac{a^n}{b^n} \quad , \quad \left(\frac{a}{b}\right)^{-n}=\left(\frac{b}{a}\right)^n \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^1=a.$$

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad \log(a\cdot b)=\log a+\log b \quad , \quad \log a^n=n\cdot\log a.$$

$$\log_{10}a=\log a \quad , \quad \frac{\log a}{\log b}=\log_b a \quad , \quad \log_b a=c \Leftrightarrow b^c=a.$$

1.inačica

$$\left.\begin{array}{l} 2^x\cdot3^y=108 \\ 3^x\cdot2^y=72 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{pomnožimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow 2^x\cdot3^y\cdot3^x\cdot2^y=108\cdot72 \Rightarrow 2^x\cdot3^x\cdot3^y\cdot2^y=7776 \Rightarrow$$

$$\Rightarrow (2\cdot3)^x\cdot(3\cdot2)^y=6^5 \Rightarrow 6^x\cdot6^y=6^5 \Rightarrow 6^{x+y}=6^5 \Rightarrow x+y=5.$$

$$\begin{array}{r|l} 7776 & 6 \\ 1296 & 6 \\ 216 & 6 \\ 36 & 6 \\ 6 & 6 \\ 1 & \end{array}$$

2.inačica

- $2^x\cdot3^y=108 \Rightarrow 2^x\cdot3^y=2^2\cdot3^3 \Rightarrow \left.\begin{array}{l} x=2 \\ y=3 \end{array}\right\} \Rightarrow x+y=5.$

$$\begin{array}{r|l} 108 & 2 \\ 54 & 2 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

- $3^x\cdot2^y=72 \Rightarrow 3^x\cdot2^y=3^2\cdot3^3 \Rightarrow \left.\begin{array}{l} x=2 \\ y=3 \end{array}\right\} \Rightarrow x+y=5.$

$$\begin{array}{r|l} 72 & 2 \\ 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$$

Dakle,

x + y = 5.

3.inačica

$$\left.\begin{array}{l} 2^x\cdot3^y=108 \\ 3^x\cdot2^y=72 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{podijelimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow \frac{2^x\cdot3^y}{3^x\cdot2^y}=\frac{108}{72} \Rightarrow \frac{2^x}{3^x}\cdot\frac{3^y}{2^y}=\frac{3}{2} \Rightarrow$$

$$\Rightarrow \left(\frac{2}{3}\right)^{x} \cdot \left(\frac{3}{2}\right)^{y} = \frac{3}{2} \Rightarrow \left(\frac{3}{2}\right)^{-x} \cdot \left(\frac{3}{2}\right)^{y} = \left(\frac{3}{2}\right)^{1} \Rightarrow \left(\frac{3}{2}\right)^{-x+y} = \left(\frac{3}{2}\right)^{2} \Rightarrow -x+y=1 \Rightarrow y=y+1.$$

Sada računamo nepoznanice x i y:

$$\left.\begin{matrix} y=x+1 \\ 2^{x}\cdot 3^{y}=108 \end{matrix}\right\} \Rightarrow 2^{x}\cdot 3^{x+1}=108 \Rightarrow 2^{x}\cdot 3^{x}\cdot 3^{1}=108 \Rightarrow 2^{x}\cdot 3^{x}\cdot 3=108 \;/:3 \Rightarrow$$

$$\Rightarrow 2^{x}\cdot 3^{x}=36 \Rightarrow (2\cdot 3)^{x}=6^{2} \Rightarrow 6^{x}=6^{2} \Rightarrow x=2 \Rightarrow \left.\begin{matrix} x=2 \\ y=x+1 \end{matrix}\right\} \Rightarrow y=2+1 \Rightarrow y=3 \Rightarrow$$

$$\Rightarrow x+y=2+3 \Rightarrow x+y=5.$$

4.inačica

$$\left.\begin{matrix} 2^{x}\cdot 3^{y}=108 \\ 3^{x}\cdot 2^{y}=72 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{logaritmiramo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow \left.\begin{matrix} 2^{x}\cdot 3^{y}=108 \;/\log \\ 3^{x}\cdot 2^{y}=72 \;/\log \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log\left(2^{x}\cdot 3^{y}\right)=\log 108 \\ \log\left(3^{x}\cdot 2^{y}\right)=\log 72 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \log 2^{x}+\log 3^{y}=\log 108 \\ \log 3^{x}+\log 2^{y}=\log 72 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x\cdot\log 2+y\cdot\log 3=\log 108 \\ x\cdot\log 3+y\cdot\log 2=\log 72 \end{matrix}\right\} \Rightarrow \begin{bmatrix} \text{zbrojimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow$$

$$\Rightarrow x\cdot\log 2+y\cdot\log 3+x\cdot\log 3+y\cdot\log 2=\log 108+\log 72 \Rightarrow$$

$$\Rightarrow x\cdot\log 2+x\cdot\log 3+y\cdot\log 3+y\cdot\log 2=\log(108\cdot 72) \Rightarrow x\cdot(\log 2+\log 3)+y\cdot(\log 3+\log 2)=\log 7776 \Rightarrow$$

$$\Rightarrow x\cdot\log(2\cdot 3)+y\cdot\log(3\cdot 2)=\log 7776 \Rightarrow x\cdot\log 6+y\cdot\log 6=\log 7776 \Rightarrow (x+y)\cdot\log 6=\log 7776 \;/:\log 6 \Rightarrow$$

$$\Rightarrow x+y=\frac{\log 7776}{\log 6} \Rightarrow x+y=\log_{6} 7776 \Rightarrow x+y=5.$$

Vježba 157

Ako je $\begin{cases} 2^{x}\cdot 3^{y}=18 \\ 3^{x}\cdot 2^{y}=12 \end{cases}$, nađi x + y.

Rezultat: 3.

Zadatak 158 (Ksenija, gimnazija)

Izrazi $\log \sqrt[9]{5}$ kao funkciju broja a, ako je a = log 8.

Rješenje 158

Ponovimo!

$$\log \sqrt[n]{a}=\frac{1}{n}\cdot\log a \quad , \quad \log\frac{a}{b}=\log a-\log b \quad , \quad \log(a\cdot b)=\log a+\log b \quad , \quad \log 10=1.$$

Računamo:

$$\log \sqrt[9]{5}=\frac{1}{9}\cdot\log 5=\frac{1}{9}\cdot\log\frac{40}{8}=\frac{1}{9}\cdot(\log 40-\log 8)=[a=\log 8]=\frac{1}{9}\cdot(\log 40-a)=$$

$$=\frac{1}{9}\cdot(\log(4\cdot 10)-a)=\frac{1}{9}\cdot(\log 4+\log 10-a)=\frac{1}{9}\cdot(\log 4+1-a)=\frac{1}{9}\cdot\left(\log\frac{8}{2}+1-a\right)=$$

$$=\frac{1}{9}\cdot(\log 8-\log 2+1-a)=[a=\log 8]=\frac{1}{9}\cdot(a-\log 2+1-a)=\frac{1}{9}\cdot(a-\log 2+1-a)=$$

$$=\frac{1}{9}\cdot(-\log 2+1)=\frac{1}{9}\cdot(1-\log 2)=\frac{1}{9}\cdot\left(1-\log\sqrt[3]{8}\right)=\frac{1}{9}\cdot\left(1-\frac{1}{3}\cdot\log 8\right)=[a=\log 8]=$$

$$=\frac{1}{9}\cdot\left(1-\frac{1}{3}\cdot a\right)=\frac{1}{9}\cdot\frac{3-a}{3}=\frac{3-a}{27}.$$

Vježba 158

Izrazi log 8 kao funkciju broja a, ako je a = log 5.

Rezultat: 3 · (1 – a).

Zadatak 159 (Robert, gimnazija)

Izračunaj: $\log_4 8 \cdot \log_8 16 \cdot \log_{16} 32.$

Rješenje 159

Ponovimo!

$$\log_b a = c \Rightarrow b^c = a \quad , \quad \log_b a^n = n \cdot \log_b a \quad , \quad \log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log_{10} a = \log a.$$

$$\log_b a = \frac{\log a}{\log b} \quad , \quad \log_{b^n} a = \frac{1}{n} \cdot \log_b a \quad , \quad \log_{b^n} a^m = \frac{m}{n} \cdot \log_b a \quad , \quad \log_b b = 1.$$

1.inačica

Svedimo sve faktore na logaritam s bazom 10:

$$\log_4 8 \cdot \log_8 16 \cdot \log_{16} 32 = \frac{\log 8}{\log 4} \cdot \frac{\log 16}{\log 8} \cdot \frac{\log 32}{\log 16} = \frac{\log 8}{\log 4} \cdot \frac{\log 16}{\log 8} \cdot \frac{\log 32}{\log 16} =$$

$$= \frac{\log 32}{\log 4} = \frac{\log 2^5}{\log 2^2} = \frac{5 \cdot \log 2}{2 \cdot \log 2} = \frac{5 \cdot \log 2}{2 \cdot \log 2} = \frac{5}{2}.$$

2.inačica

Svedimo sve faktore na logaritam s bazom 2:

$$\log_4 8 \cdot \log_8 16 \cdot \log_{16} 32 = \frac{\log_2 8}{\log_2 4} \cdot \frac{\log_2 16}{\log_2 8} \cdot \frac{\log_2 32}{\log_2 16} = \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} = \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} = \frac{5}{2}.$$

3.inačica

$$\log_4 8 \cdot \log_8 16 \cdot \log_{16} 32 = \log_{2^2} 2^3 \cdot \log_{2^3} 2^4 \cdot \log_{2^4} 2^5 = \frac{3}{2} \cdot \log_2 2 \cdot \frac{4}{3} \cdot \log_2 2 \cdot \frac{5}{4} \cdot \log_2 2 =$$

$$= \frac{3}{2} \cdot 1 \cdot \frac{4}{3} \cdot 1 \cdot \frac{5}{4} \cdot 1 = \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} = \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} = \frac{5}{2}.$$

4.inačica

$$\log_4 8 \cdot \log_8 16 \cdot \log_{16} 32 = \log_{2^2} 8 \cdot \log_{2^3} 16 \cdot \log_{2^4} 32 = \frac{1}{2} \cdot \log_2 8 \cdot \frac{1}{3} \cdot \log_2 16 \cdot \frac{1}{4} \cdot \log_2 32 =$$

$$= \frac{1}{2} \cdot 3 \cdot \frac{1}{3} \cdot 4 \cdot \frac{1}{4} \cdot 5 = \frac{1}{2} \cdot 3 \cdot \frac{1}{3} \cdot 4 \cdot \frac{1}{4} \cdot 5 = \frac{5}{2}.$$

Vježba 159

Izračunaj: $\log_4 8 \cdot \log_8 16.$

Rezultat: 2.

Zadatak 160 (Darko, gimnazija)

Riješi jednadžbu $8^{\log x} + 3^{1 - \log x} \cdot 24^{\log 10x} = 73$, uz uvjet $x > 0$.

Rješenje 160

Ponovimo!

$$(a \cdot b)^n = a^n \cdot b^n \quad , \quad a^n \cdot a^m = a^{n+m} \quad , \quad a^0 = 1 \quad , \quad a^1 = a \quad , \quad \log 10 = 1 \quad , \quad \log 1 = 0.$$

$$\log(a \cdot b) = \log a + \log b \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

$$8^{\log x}+3^{1-\log x}\cdot 24^{\log 10x}=73 \Rightarrow 8^{\log x}+3^{1-\log x}\cdot (3\cdot 8)^{\log 10x}=73 \Rightarrow$$

$$\Rightarrow 8^{\log x}+3^{1-\log x}\cdot (3\cdot 8)^{\log 10+\log x}=73 \Rightarrow 8^{\log x}+3^{1-\log x}\cdot (3\cdot 8)^{1+\log x}=73 \Rightarrow$$

$$\Rightarrow 8^{\log x}+3^{1-\log x}\cdot 3^{1+\log x}\cdot 8^{1+\log x}=73 \Rightarrow 8^{\log x}+3^{1-\log x+1+\log x}\cdot 8^{1+\log x}=73 \Rightarrow$$

$$\Rightarrow 8^{\log x}+3^{1-\log x+1+\log x}\cdot 8^{1+\log x}=73 \Rightarrow 8^{\log x}+3^{2}\cdot 8^{1}\cdot 8^{\log x}=73 \Rightarrow$$

$$\Rightarrow 8^{\log x}+9\cdot 8\cdot 8^{\log x}=73 \Rightarrow 8^{\log x}+72\cdot 8^{\log x}=73 \Rightarrow (1+72)\cdot 8^{\log x}=73 \Rightarrow$$

$$\Rightarrow 73\cdot 8^{\log x}=73 \;/:73 \Rightarrow 8^{\log x}=1 \Rightarrow 8^{\log x}=8^{0} \Rightarrow \log x=0 \Rightarrow x=1.$$

Vježba 160

Riješi jednadžbu $8^{\log x}+3^{\log\frac{10}{x}}\cdot 24^{1+\log x}=73$, uz uvjet $x>0$.

Rezultat: x = 1.