Zadatak 081 (Vedrana, gimnazija)

Riješite jednadžbu: $\frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \frac{1}{3}$.

Rješenje 081

Ponovimo!

$$a^n \cdot a^m = a^{n+m} \quad , \quad a^0 = 1 \quad , \quad a^{-n} = \frac{1}{a^n} \quad , \quad a^1 = a \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x).$$

1.inačica

$$\frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \frac{1}{3} \Rightarrow 3 \cdot \left(2^x - 2^{-x}\right) = 2^x + 2^{-x} \Rightarrow 3 \cdot \left(2^x - 2^{-x}\right) = 2^x + 2^{-x} \; / \cdot 2^x \Rightarrow$$

$$\Rightarrow 3 \cdot \left(2^x - 2^{-x}\right) \cdot 2^x = \left(2^x + 2^{-x}\right) \cdot 2^x \Rightarrow 3 \cdot \left(2^{2 \cdot x} - 1\right) = 2^{2 \cdot x} + 1 \Rightarrow 3 \cdot 2^{2 \cdot x} - 3 = 2^{2 \cdot x} + 1 \Rightarrow$$

$$\Rightarrow 3 \cdot 2^{2 \cdot x} - 2^{2 \cdot x} = 1 + 3 \Rightarrow 2 \cdot 2^{2 \cdot x} = 4 \; / : 2 \Rightarrow 2^{2 \cdot x} = 2 \Rightarrow 2^{2 \cdot x} = 2^1 \Rightarrow 2 \cdot x = 1 \; / : 2 \Rightarrow x = \frac{1}{2}.$$

2.inačica

$$\frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \frac{1}{3} \Rightarrow \left[\begin{array}{c} \text{supstitucija} \\ t = 2^x \Rightarrow \frac{1}{t} = 2^{-x} \end{array}\right] \Rightarrow \frac{t - \frac{1}{t}}{t + \frac{1}{t}} = \frac{1}{3} \Rightarrow 3 \cdot \left(t - \frac{1}{t}\right) = t + \frac{1}{t} \Rightarrow 3 \cdot t - 3 \cdot \frac{1}{t} = t + \frac{1}{t} \; / \cdot t \Rightarrow$$

$$\Rightarrow 3 \cdot t^2 - 3 = t^2 + 1 \Rightarrow 3 \cdot t^2 - t^2 = 1 + 3 \Rightarrow 2 \cdot t^2 = 4 \; / : 2 \Rightarrow t^2 = 2 \Rightarrow \left[\begin{array}{c} t = 2^x \\ t^2 = 2^{2 \cdot x} \end{array}\right] \Rightarrow 2^{2 \cdot x} = 2^1 \Rightarrow$$

$$\Rightarrow 2 \cdot x = 1 \; / : 2 \Rightarrow x = \frac{1}{2}.$$

3.inačica

$$\frac{2^x - 2^{-x}}{2^x + 2^{-x}} = \frac{1}{3} \Rightarrow 3 \cdot \left(2^x - 2^{-x}\right) = 2^x + 2^{-x} \Rightarrow 3 \cdot 2^x - 3 \cdot 2^{-x} = 2^x + 2^{-x} \Rightarrow 3 \cdot 2^x - 2^x = 2^{-x} + 3 \cdot 2^{-x} \Rightarrow$$

$$\Rightarrow 2 \cdot 2^x = 4 \cdot 2^{-x} \; / : 2 \Rightarrow 2^x = 2 \cdot 2^{-x} \Rightarrow 2^x = 2^{1-x} \Rightarrow x = 1 - x \Rightarrow x + x = 1 \Rightarrow 2 \cdot x = 1 \; / : 2 \Rightarrow x = \frac{1}{2}.$$

Vježba 081

Riješite jednadžbu: $\frac{2^x + 2^{-x}}{2^x - 2^{-x}} = 3$.

Rezultat: $\frac{1}{2}$.

Zadatak 082 (Vedrana, gimnazija)

Koliko ima uređenih parova (x, y) realnih brojeva koji su rješenja sustava jednadžbi:

$$\left\{ \log_x y = x^3 - 2 \cdot x^2 \quad , \quad \left(x^2 - 2 \cdot x\right) \cdot \log_y x = 1. \right.$$

Rješenje 082

Ponovimo!

$$\log_b a = \frac{1}{\log_a b} \; , \; a > 0 \; , \; a \neq 1 \; , \; b > 0 \; , \; b \neq 1.$$

$$\left.\begin{array}{l} \log_x y = x^3 - 2 \cdot x^2 \\ \left(x^2 - 2 \cdot x\right) \cdot \log_y x = 1 \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{uvjeti} \\ x > 0 \; , \; x \neq 1 \\ y > 0 \; , \; y \neq 1 \end{array}\right] \Rightarrow \left.\begin{array}{l} \log_x y = x^3 - 2 \cdot x^2 \\ \log_y x = \frac{1}{x^2 - 2 \cdot x} \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log_x y = x^3 - 2 \cdot x^2 \\ \frac{1}{\log_x y} = \frac{1}{x^2 - 2 \cdot x} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} \log_x y = x^3 - 2\cdot x^2 \\ \log_x y = x^2 - 2\cdot x \end{array}\right\} \Rightarrow x^3 - 2\cdot x^2 = x^2 - 2\cdot x \Rightarrow x^3 - 2\cdot x^2 - x^2 + 2\cdot x = 0 \Rightarrow x^3 - 3\cdot x^2 + 2\cdot x = 0 \Rightarrow$$

$$\Rightarrow x\cdot\left(x^2 - 3\cdot x + 2\right) = 0 \Rightarrow \left.\begin{array}{l} x = 0 \\ x^2 - 3\cdot x + 2 = 0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = 0 \\ x_{2,3} = \dfrac{-b \pm \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = 0 \\ x_{2,3} = \dfrac{3 \pm \sqrt{9-8}}{2} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x_1 = 0 \\ x_{2,3} = \dfrac{3 \pm 1}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = 0 \\ x_2 = \dfrac{3+1}{2} \\ x_3 = \dfrac{3-1}{2} \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = 0 \ \text{nije rješenje zbog uvjeta} \\ x_2 = 2 \\ x_3 = 1 \ \text{nije rješenje zbog uvjeta} \end{array}\right\} \Rightarrow x = 2 \Rightarrow \left.\begin{array}{l} x = 2 \\ \log_x y = x^2 - 2\cdot x \end{array}\right\} \Rightarrow$$

$$\Rightarrow \log_2 y = 2^2 - 2\cdot 2 \Rightarrow \log_2 y = 4 - 4 \Rightarrow \log_2 y = 0 \Rightarrow y = 2^0 \Rightarrow y = 1 \ \text{nije rješenje zbog uvjeta.}$$

Sustav jednadžbi nema rješenja.

Vježba 082

Koliko ima uređenih parova (x, y) realnih brojeva koji su rješenja sustava jednadžbi:

$$\left\{ \log_x y = x \quad , \quad \log_y x = \frac{1}{2}. \right.$$

Rezultat: (x, y) = (2, 4).

Zadatak 083 (Vedrana, gimnazija)

Riješite jednadžbu: $\left(\frac{1}{2}\right)^{\log_3 (5\cdot x)} = 16^{\log_9 \left(25\cdot x^2\right)}.$

Rješenje 083

Ponovimo!

$$a^{-n} = \frac{1}{a^n} \quad , \quad a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x) \quad , \quad \log_{b^n} a^n = \log_b a \quad , \quad \log_b a = c \Leftrightarrow b^c = a \quad , \quad a^0 = 1.$$

$$\left(\frac{1}{2}\right)^{\log_3 (5\cdot x)} = 16^{\log_9 \left(25\cdot x^2\right)} \Rightarrow \left(2^{-1}\right)^{\log_3 (5\cdot x)} = \left(2^4\right)^{\log_9 \left(25\cdot x^2\right)} \Rightarrow 2^{-\log_3 (5\cdot x)} = 2^{4\cdot\log_9 \left(25\cdot x^2\right)} \Rightarrow$$

$$\Rightarrow -\log_3 (5\cdot x) = 4\cdot\log_9 \left(25\cdot x^2\right) \Rightarrow -\log_3 (5\cdot x) = 4\cdot\log_{3^2} (5\cdot x)^2 \Rightarrow -\log_3 (5\cdot x) = 4\cdot\log_3 (5\cdot x) \Rightarrow$$

$$\Rightarrow -\log_3 (5\cdot x) - 4\cdot\log_3 (5\cdot x) = 0 \Rightarrow -5\cdot\log_3 (5\cdot x) = 0 \ /:(-5) \Rightarrow \log_3 (5\cdot x) = 0 \Rightarrow$$

$$\Rightarrow 5\cdot x = 3^0 \Rightarrow 5\cdot x = 1 \Rightarrow x = \frac{1}{5}.$$

Vježba 083

Riješite jednadžbu: $\left(\frac{1}{3}\right)^{\log_3 (5\cdot x)} = 81^{\log_9 \left(25\cdot x^2\right)}.$

Rezultat: $\frac{1}{5}.$

Zadatak 084 (Anamarija, gimnazija)

Nađite rješenje logaritamske jednadžbe $\log\left(-x^3 + 2\right) = 0$ u skupu prirodnih brojeva.

Rješenje 084

Ponovimo!

$$\log 1 = 0 \quad , \quad \log a = \log b \Rightarrow a = b \ \left[\text{injektivnost funkcije } f(x) = \log x\right].$$

Najprije provedimo diskusiju, tj. odredimo za koje vrijednosti od x jednadžba ima smisla:

$$-x^3+2>0 \Rightarrow -x^3>-2 \ /\cdot(-1) \Rightarrow x^3<2 \ /\sqrt[3]{\ } \Rightarrow x<\sqrt[3]{2}.$$

Tražimo rješenje jednadžbe:

$$\log\left(-x^3+2\right)=0 \Rightarrow \log\left(-x^3+2\right)=\log 1 \Rightarrow -x^3+2=1 \Rightarrow -x^3=1-2 \Rightarrow -x^3=-1 \ /\cdot(-1) \Rightarrow$$

$$\Rightarrow x^3=1 \ /\sqrt[3]{\ } \Rightarrow x=\sqrt[3]{1} \Rightarrow x=1.$$

Jedini rješenje je x = 1.

Vježba 084

Nađite rješenje logaritamske jednadžbe $\log\left(-x^3+1\right)=0$ u skupu cijelih brojeva.

Rezultat: x = 0.

Zadatak 085 (Marija, studentica PA)

Riješite jednadžbu: $\log_x 2\cdot\log_{2x} 2=\log_{4x} 2\cdot\log_{8x} 2.$

Rješenje 085

Ponovimo!

$$\log_b a=\frac{\log a}{\log b} \quad , \quad n\cdot\log a=\log a^n \quad , \quad \log(a\cdot b)=\log a+\log b \quad , \quad \log f(x)=\log g(x) \Rightarrow f(x)=g(x).$$

$$a^{-n}=\frac{1}{a^n} \quad , \quad \sqrt[n]{a^m}=a^{\frac{m}{n}}.$$

$$\log_x 2\cdot\log_{2x} 2=\log_{4x} 2\cdot\log_{8x} 2 \Rightarrow \frac{\log 2}{\log x}\cdot\frac{\log 2}{\log 2x}=\frac{\log 2}{\log 4x}\cdot\frac{\log 2}{\log 8x} \Rightarrow \frac{(\log 2)^2}{\log x\cdot\log 2x}=\frac{(\log 2)^2}{\log 4x\cdot\log 8x} \Rightarrow$$

$$\Rightarrow \frac{(\log 2)^2}{\log x\cdot\log 2x}=\frac{(\log 2)^2}{\log 4x\cdot\log 8x} \ /:(\log 2)^2 \Rightarrow \frac{1}{\log x\cdot\log 2x}=\frac{1}{\log 4x\cdot\log 8x} \Rightarrow \log 4x\cdot\log 8x=\log x\cdot\log 2x \Rightarrow$$

$$\Rightarrow (\log 4+\log x)\cdot(\log 8+\log x)=\log x\cdot(\log 2+\log x) \Rightarrow \left(\log 2^2+\log x\right)\cdot\left(\log 2^3+\log x\right)=\log x\cdot(\log 2+\log x) \Rightarrow$$

$$\Rightarrow (2\cdot\log 2+\log x)\cdot(3\cdot\log 2+\log x)=\log x\cdot(\log 2+\log x) \Rightarrow$$

$$\Rightarrow 6\cdot(\log 2)^2+2\cdot\log 2\cdot\log x+3\cdot\log 2\cdot\log x+(\log x)^2=\log 2\cdot\log x+(\log x)^2 \Rightarrow$$

$$\Rightarrow 2\cdot\log 2\cdot\log x+3\cdot\log 2\cdot\log x-\log 2\cdot\log x=-6\cdot(\log 2)^2 \Rightarrow 4\cdot\log 2\cdot\log x=-6\cdot(\log 2)^2 \ /\cdot\frac{1}{4\cdot\log 2} \Rightarrow$$

$$\Rightarrow \log x=\frac{-6\cdot(\log 2)^2}{4\cdot\log 2} \Rightarrow \log x=-\frac{3}{2}\cdot\log 2 \Rightarrow \log x=\log 2^{-\frac{3}{2}} \Rightarrow x=2^{-\frac{3}{2}} \Rightarrow x=\frac{1}{\sqrt{2^3}} \Rightarrow$$

$$\Rightarrow x=\frac{1}{\sqrt{2^2\cdot 2}} \Rightarrow x=\frac{1}{2\cdot\sqrt{2}} \Rightarrow x=\frac{1}{2\cdot\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \Rightarrow x=\frac{\sqrt{2}}{4}.$$

Vježba 085

Riješite jednadžbu: $\log_x 3=\log_{3x} 9.$

Rezultat: x = 3.

Zadatak 086 (Marija, studentica PA)

Riješite jednadžbu: $\log_{(x+1)} 3+\log_{2x}\frac{1}{3}=0.$

Rješenje 086

Ponovimo!

$$\log_b a = \frac{\log a}{\log b} \quad , \quad \log 1 = 0 \quad , \quad \log f(x) = \log g(x) \Rightarrow f(x) = g(x) \text{ (injektivnost)}.$$

Diskusija!

$$\left.\begin{matrix} x+1>0 \\ 2x>0 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x>-1 \\ x>0 \end{matrix}\right\} \Rightarrow x>0 \quad , \quad \left.\begin{matrix} x+1\neq 1 \\ 2x\neq 1 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x\neq 0 \\ x\neq \frac{1}{2} \end{matrix}\right\}.$$

Rješavamo jednadžbu:

$$\log_{(x+1)} 3 + \log_{2x} \frac{1}{3} = 0 \Rightarrow \frac{\log 3}{\log(x+1)} + \frac{\log \frac{1}{3}}{\log 2x} = 0 \Rightarrow \frac{\log 3}{\log(x+1)} + \frac{\log 1 - \log 3}{\log 2x} = 0 \Rightarrow$$

$$\Rightarrow \frac{\log 3}{\log(x+1)} + \frac{0 - \log 3}{\log 2x} = 0 \Rightarrow \frac{\log 3}{\log(x+1)} - \frac{\log 3}{\log 2x} = 0 \,/\cdot \frac{1}{\log 3} \Rightarrow \frac{1}{\log(x+1)} - \frac{1}{\log 2x} = 0 \Rightarrow$$

$$\Rightarrow \frac{1}{\log(x+1)} = \frac{1}{\log 2x} \Rightarrow \log 2x = \log(x+1) \Rightarrow 2x = x+1 \Rightarrow 2x - x = 1 \Rightarrow x = 1.$$

Vježba 086

Riješite jednadžbu: $\log_{(x+1)} 5 + \log_{2x} \frac{1}{5} = 0.$

Rezultat: x = 1.

Zadatak 087 (Marija, studentica PA)

Riješite jednadžbu: $\log x^3 \cdot \log(0.1 \cdot x) = 2 - \log x^2.$

Rješenje 087

Ponovimo!

$$\log a^n = n \cdot \log a \quad , \quad \log(a \cdot b) = \log a + \log b \quad , \quad \log_b a = c \Leftrightarrow b^c = a \quad , \quad a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}.$$

Rješavamo jednadžbu:

$$\log x^3 \cdot \log(0.1 \cdot x) = 2 - \log x^2 \Rightarrow 3 \cdot \log x \cdot (\log 0.1 + \log x) = 2 - 2 \cdot \log x \Rightarrow 3 \cdot \log x \cdot (-1 + \log x) = 2 - 2 \cdot \log x \Rightarrow$$

$$\Rightarrow -3 \cdot \log x + 3 \cdot (\log x)^2 = 2 - 2 \cdot \log x \Rightarrow -3 \cdot \log x + 3 \cdot (\log x)^2 - 2 + 2 \cdot \log x = 0 \Rightarrow 3 \cdot (\log x)^2 - \log x - 2 = 0 \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \text{supstitucija} \\ t = \log x \end{bmatrix} \Rightarrow 3 \cdot t^2 - t - 2 = 0 \Rightarrow \left.\begin{matrix} a = 3 \\ b = -1 \\ c = -2 \end{matrix}\right\} \Rightarrow t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1 - 4 \cdot 3 \cdot (-2)}}{2 \cdot 3} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{1 \pm \sqrt{1 + 24}}{6} \Rightarrow t_{1,2} = \frac{1 \pm \sqrt{25}}{6} \Rightarrow t_{1,2} = \frac{1 \pm 5}{6} \Rightarrow \left.\begin{matrix} t_1 = \frac{1+5}{6} \\ t_2 = \frac{1-5}{6} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1 = \frac{6}{6} = 1 \\ t_2 = -\frac{4}{6} = -\frac{2}{3} \end{matrix}\right\}.$$

Rješenja jednadžbe iznose:

$$\left.\begin{matrix} t = \log x \\ t = 1 \end{matrix}\right\} \Rightarrow \log x = 1 \Rightarrow 10^1 = x \Rightarrow x_1 = 10 \quad , \quad \left.\begin{matrix} t = \log x \\ t = -\frac{2}{3} \end{matrix}\right\} \Rightarrow \log x = -\frac{2}{3} \Rightarrow 10^{-\frac{2}{3}} = x \Rightarrow x_2 = \frac{1}{\sqrt[3]{10^2}}.$$

Vježba 087

Riješite jednadžbu: $\log(0.1 \cdot x) = 2 - \log x^2.$

Rezultat: x = 10.

Zadatak 088 (Izzy, Kiki, ekonomska škola)

Riješite jednadžbu: $\log_{16} x + \log_8 x + \log_2 x = \frac{19}{36}.$

Rješenje 088

Ponovimo!

$$\log_{b^k} a=\frac{1}{k}\cdot\log_b a \quad , \quad \log_b a=\frac{\log a}{\log b} \quad , \quad \log_b a=c \Leftrightarrow b^c=a \quad , \quad \log f(x)=\log g(x) \Rightarrow f(x)=g(x).$$

1.inačica

$$\log_{16} x+\log_8 x+\log_2 x=\frac{19}{36} \Rightarrow \log_{2^4} x+\log_{2^3} x+\log_2 x=\frac{19}{36} \Rightarrow \frac{1}{4}\cdot\log_2 x+\frac{1}{3}\cdot\log_2 x+\log_2 x=\frac{19}{36} \Rightarrow$$

$$\Rightarrow \log_2 x\cdot\left(\frac{1}{4}+\frac{1}{3}+1\right)=\frac{19}{36} \Rightarrow \log_2 x\cdot\frac{3+4+12}{12}=\frac{19}{36} \Rightarrow \log_2 x\cdot\frac{19}{12}=\frac{19}{36} \; /\cdot\frac{12}{19} \Rightarrow \log_2 x=\frac{1}{3} \Rightarrow$$

$$\Rightarrow 2^{\frac{1}{3}}=x \Rightarrow x=\sqrt[3]{2}.$$

2.inačica

$$\log_{16} x+\log_8 x+\log_2 x=\frac{19}{36} \Rightarrow \frac{\log x}{\log 16}+\frac{\log x}{\log 8}+\frac{\log x}{\log 2}=\frac{19}{36} \Rightarrow \log x\cdot\left(\frac{1}{\log 16}+\frac{1}{\log 8}+\frac{1}{\log 2}\right)=\frac{19}{36} \Rightarrow$$

$$\Rightarrow \log x\cdot\left(\frac{1}{\log 2^4}+\frac{1}{\log 2^3}+\frac{1}{\log 2}\right)=\frac{19}{36} \Rightarrow \log x\cdot\left(\frac{1}{4\cdot\log 2}+\frac{1}{3\cdot\log 2}+\frac{1}{\log 2}\right)=\frac{19}{36} \Rightarrow$$

$$\Rightarrow \log x\cdot\frac{3+4+12}{12\cdot\log 2}=\frac{19}{36} \Rightarrow \log x\cdot\frac{19}{12\cdot\log 2}=\frac{19}{36} \; /\cdot\frac{12\cdot\log 2}{19} \Rightarrow \log x=\frac{19}{36}\cdot\frac{12\cdot\log 2}{19} \Rightarrow \log x=\frac{1}{3}\cdot\log 2 \Rightarrow$$

$$\Rightarrow \log x=\log 2^{\frac{1}{3}} \Rightarrow x=2^{\frac{1}{3}} \Rightarrow x=\sqrt[3]{2}.$$

Vježba 088

Riješite jednadžbu: $\log_{16} x+\log_8 x+\log_2 x=\frac{19}{12}.$

Rezultat: x = 2.

Zadatak 089 (2A, TUPŠ)

Ako je $\log_a x=s$ i $\log_a y^2=t$, nađite $\log_a \frac{\sqrt{x}}{y}.$

Rješenje 089

Ponovimo!

$$\log_b \sqrt{a}=\frac{1}{2}\cdot\log_b a \quad , \quad \log_b \frac{a}{c}=\log_b a-\log_b c.$$

$$\left.\begin{array}{l}\log_a x=s \;,\; \log_a y^2=t \\ \log_a \frac{\sqrt{x}}{y}=\log_a \sqrt{x}-\log_a y\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_a x=s \;,\; 2\cdot\log_a y=t \\ \log_a \frac{\sqrt{x}}{y}=\frac{1}{2}\cdot\log_a x-\log_a y\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_a x=s \;,\; \log_a y=\frac{t}{2} \\ \log_a \frac{\sqrt{x}}{y}=\frac{1}{2}\cdot\log_a x-\log_a y\end{array}\right\} \Rightarrow$$

$$\Rightarrow \log_a \frac{\sqrt{x}}{y}=\frac{1}{2}\cdot s-\frac{t}{2}=\frac{s-t}{2}.$$

Vježba 089

Ako je $\log_a x=s$ i $\log_a y^3=t$, nađite $\log_a \frac{\sqrt{x}}{y}.$

Rezultat: $\frac{3\cdot s-2\cdot t}{6}.$

Zadatak 090 (Maturant, gimnazija)

Nađite rješenje nejednadžbe: $\log\left(\frac{x-3}{x+3}\right)\le 1.$

Rješenje 090

Ponovimo!

$$\log_b x \le \log_b y \ , \ b > 1 \ \Rightarrow \ x \le y.$$

Diskusija!

$$\log\left(\frac{x-3}{x+3}\right) \le 1 \ \Rightarrow \ \left[\begin{array}{l} \text{diskusija} \\ \frac{x-3}{x+3} > 0 \end{array}\right].$$

Prvi slučaj:

$$\frac{x-3}{x+3} > 0 \ \Rightarrow \ \left.\begin{array}{l} x-3>0 \\ x+3>0 \end{array}\right\} \ \Rightarrow \ \left.\begin{array}{l} x>3 \\ x>-3 \end{array}\right\} \ \Rightarrow \ x>3 \ \Rightarrow \ x \in \langle 3, +\infty \rangle.$$

Drugi slučaj:

$$\frac{x-3}{x+3} > 0 \ \Rightarrow \ \left.\begin{array}{l} x-3<0 \\ x+3<0 \end{array}\right\} \ \Rightarrow \ \left.\begin{array}{l} x<3 \\ x<-3 \end{array}\right\} \ \Rightarrow \ x<-3 \ \Rightarrow \ x \in \langle -\infty, -3 \rangle.$$

Rješavamo nejednadžbu:

$$\log\left(\frac{x-3}{x+3}\right) \le 1 \ \Rightarrow \ \log\left(\frac{x-3}{x+3}\right) \le \log 10 \ \Rightarrow \ \frac{x-3}{x+3} \le 10 \ \Rightarrow \ \frac{x-3}{x+3} - 10 \le 0 \ \Rightarrow \ \frac{x-3-10\cdot(x+3)}{x+3} \le 0 \ \Rightarrow$$

$$\Rightarrow \ \frac{x-3-10\cdot x-30}{x+3} \le 0 \ \Rightarrow \ \frac{-9\cdot x-33}{x+3} \le 0.$$

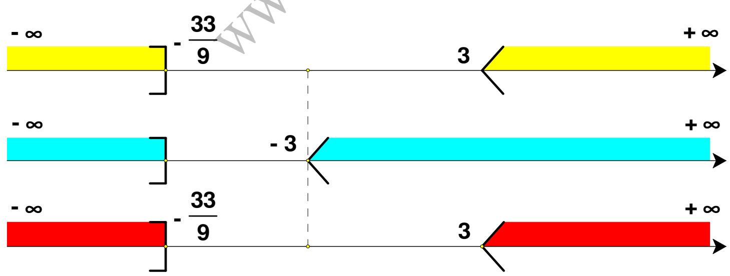
Prvi slučaj:

$$\frac{-9\cdot x-33}{x+3} \le 0 \ \Rightarrow \ \left.\begin{array}{l} -9\cdot x-33 \le 0 \\ x+3>0 \end{array}\right\} \ \Rightarrow \ \left.\begin{array}{l} -9\cdot x \le 33 \ /:(-9) \\ x>-3 \end{array}\right\} \ \Rightarrow \ \left.\begin{array}{l} x \ge -\frac{33}{9} \\ x>-3 \end{array}\right\} \ \Rightarrow \ x>-3 \ \Rightarrow \ x \in \langle -3, +\infty \rangle.$$

Drugi slučaj:

$$\frac{-9\cdot x-33}{x+3} \le 0 \ \Rightarrow \ \left.\begin{array}{l} -9\cdot x-33 \ge 0 \\ x+3<0 \end{array}\right\} \ \Rightarrow \ \left.\begin{array}{l} -9\cdot x \ge 33 \ /:(-9) \\ x<-3 \end{array}\right\} \ \Rightarrow \ \left.\begin{array}{l} x \le -\frac{33}{9} \\ x<-3 \end{array}\right\} \ \Rightarrow \ x \le -\frac{33}{9} \ \Rightarrow \ x \in \left\langle -\infty, -\frac{33}{9} \right\rangle.$$

Konačno rješenje:



$$x \in \left\langle -\infty, -\frac{33}{9} \right] \cup \langle 3, +\infty \rangle.$$

Vježba 090

Nađite rješenje nejednadžbe: $\log(x-2) \le 1.$

Rezultat: $x \in \langle 2, 12].$

Zadatak 091 (Vedrana, gimnazija)

Nađite zbroj rješenja jednadžbe: $6\cdot 9^{\frac{1}{x}} - 13\cdot 6^{\frac{1}{x}} + 6\cdot 4^{\frac{1}{x}} = 0.$

Rješenje 091

$$6\cdot 9^{\frac{1}{x}} - 13\cdot 6^{\frac{1}{x}} + 6\cdot 4^{\frac{1}{x}} = 0 \ \Rightarrow \ 6\cdot\left(3^2\right)^{\frac{1}{x}} - 13\cdot(2\cdot 3)^{\frac{1}{x}} + 6\cdot\left(2^2\right)^{\frac{1}{x}} = 0 \ \Rightarrow \ 6\cdot 3^{\frac{2}{x}} - 13\cdot 2^{\frac{1}{x}}\cdot 3^{\frac{1}{x}} + 6\cdot 2^{\frac{2}{x}} = 0 \ \Rightarrow$$

$$\Rightarrow 6\cdot 3^{\frac{2}{x}}-13\cdot 2^{\frac{1}{x}}\cdot 3^{\frac{1}{x}}+6\cdot 2^{\frac{2}{x}}=0\ /:2^{\frac{2}{x}} \Rightarrow 6\cdot\frac{3^{\frac{2}{x}}}{2^{\frac{2}{x}}}-13\cdot\frac{2^{\frac{1}{x}}\cdot 3^{\frac{1}{x}}}{2^{\frac{2}{x}}}+6\cdot\frac{2^{\frac{2}{x}}}{2^{\frac{2}{x}}}=0 \Rightarrow$$

$$\Rightarrow 6\cdot\left(\frac{3}{2}\right)^{\frac{2}{x}}-13\cdot\frac{3^{\frac{1}{x}}}{2^{\frac{1}{x}}}+6=0 \Rightarrow 6\cdot\left(\left(\frac{3}{2}\right)^{\frac{1}{x}}\right)^2-13\cdot\left(\frac{3}{2}\right)^{\frac{1}{x}}+6=0 \Rightarrow \left[\begin{matrix}\text{supstitucija}\\ \left(\frac{3}{2}\right)^{\frac{1}{x}}=t\end{matrix}\right] \Rightarrow 6\cdot t^2-13\cdot t+6=0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=6\ ,\ b=-13\ ,\ c=6 \\ t_{1,2}=\frac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\} \Rightarrow t_{1,2}=\frac{13\pm\sqrt{169-4\cdot 6\cdot 6}}{2\cdot 6} \Rightarrow t_{1,2}=\frac{13\pm\sqrt{169-144}}{12} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{13\pm\sqrt{25}}{12} \Rightarrow t_{1,2}=\frac{13\pm 5}{12} \Rightarrow \left.\begin{matrix} t_1=\frac{13+5}{12} \\ t_2=\frac{13-5}{12}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\frac{18}{12} \\ t_2=\frac{8}{12}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\frac{3}{2} \\ t_2=\frac{2}{3}\end{matrix}\right\}.$$

Rješenja jednadžbe iznose:

$$\left(\frac{3}{2}\right)^{\frac{1}{x}}=t \Rightarrow \left.\begin{matrix}\left(\frac{3}{2}\right)^{\frac{1}{x}}=\frac{3}{2} \\ \left(\frac{3}{2}\right)^{\frac{1}{x}}=\frac{2}{3}\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\left(\frac{3}{2}\right)^{\frac{1}{x}}=\left(\frac{3}{2}\right)^{1} \\ \left(\frac{3}{2}\right)^{\frac{1}{x}}=\left(\frac{3}{2}\right)^{-1}\end{matrix}\right\} \Rightarrow \left.\begin{matrix}\frac{1}{x}=1 \\ \frac{1}{x}=-1\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x_1=1 \\ x_2=-1\end{matrix}\right\} \Rightarrow x_1+x_2=1+(-1)=0.$$

Vježba 091

Nađite umnožak rješenja jednadžbe: $6\cdot 9^{\frac{1}{x}}-13\cdot 6^{\frac{1}{x}}+6\cdot 4^{\frac{1}{x}}=0.$

Rezultat: $-1.$

Zadatak 092 (Vedrana, gimnazija)

Riješite jednadžbu: $\log_2|2-2\cdot x|=3.$

Rješenje 092

Ponovimo!

$$a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad |x|=\begin{cases} x, & x\ge 0 \\ -x, & x<0\end{cases}.$$

$$\log_2|2-2\cdot x|=3 \Rightarrow \log_2|2-2\cdot x|=\log_2 2^3 \Rightarrow |2-2\cdot x|=2^3 \Rightarrow |2-2\cdot x|=8 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix}2-2\cdot x=8 \\ 2-2\cdot x=-8\end{matrix}\right\} \Rightarrow \left.\begin{matrix}-2\cdot x=8-2 \\ -2\cdot x=-8-2\end{matrix}\right\} \Rightarrow \left.\begin{matrix}-2\cdot x=6\ /:(-2) \\ -2\cdot x=-10\ /:(-2)\end{matrix}\right\} \Rightarrow \left.\begin{matrix}x_1=-3 \\ x_2=5\end{matrix}\right\}.$$

Vježba 092

Riješite jednadžbu: $\log_2|2-2\cdot x|=4.$

Rezultat: $x_1=-7\ ,\ x_2=9.$

Zadatak 093 (Vedrana, gimnazija)

Riješite jednadžbu: $\log_4(x+12)\cdot\log_x 2=1.$

Rješenje 093

Ponovimo!

$$\log_b a=\frac{1}{\log_a b} \quad , \quad \log_{b^n} a=\frac{1}{n}\cdot\log_b a \quad , \quad n\cdot\log_b a=\log_b a^n \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$\log_4(x+12)\cdot\log_x 2=1 \Rightarrow \left[\begin{array}{l}\text{Diskusija!}\\ \left.\begin{array}{l}x+12>0\\ x>0\ ,\ x\neq 1\end{array}\right\}\Rightarrow x>0\ ,\ x\neq 1\end{array}\right] \Rightarrow \log_4(x+12)\cdot\frac{1}{\log_2 x}=1\ /\cdot\log_2 x \Rightarrow$$

$$\Rightarrow \log_4(x+12)=\log_2 x \Rightarrow \log_{2^2}(x+12)=\log_2 x \Rightarrow \frac{1}{2}\cdot\log_2(x+12)=\log_2 x\ /\cdot 2 \Rightarrow$$

$$\Rightarrow \log_2(x+12)=2\cdot\log_2 x \Rightarrow \log_2(x+12)=\log_2 x^2 \Rightarrow x+12=x^2 \Rightarrow x^2-x-12=0 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}a=1\ ,\ b=-1\ ,\ c=-12\\ x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1-4\cdot 1\cdot(-12)}}{2\cdot 1} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{1+48}}{2} \Rightarrow x_{1,2}=\frac{1\pm\sqrt{49}}{2} \Rightarrow$$

$$\Rightarrow x_{1,2}=\frac{1\pm 7}{2} \Rightarrow \left.\begin{array}{l}x_1=\dfrac{1+7}{2}\\ x_2=\dfrac{1-7}{2}\end{array}\right\} \Rightarrow \left.\begin{array}{l}x_1=\dfrac{8}{2}\\ x_2=\dfrac{-6}{2}\end{array}\right\} \Rightarrow \left.\begin{array}{l}x_1=4\\ x_2=-3 \text{ nema smisla}\end{array}\right\}.$$

Vježba 093

Riješite jednadžbu: $\log_2(x+12)\cdot\log_x 2=2.$

Rezultat: x = 4.

Zadatak 094 (Ana, gimnazija)

Riješite jednadžbu: $x+\log_5\left(5^x-4\right)=1.$

Rješenje 094

Ponovimo!

$$\log_b a^n=n\cdot\log_b a\quad,\quad \log_b b^n=n\quad,\quad \log_b f(x)=\log_b g(x) \Rightarrow f(x)=g(x)$$

$$a^{-n}=\frac{1}{a^n}\quad,\quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x).$$

$$x+\log_5\left(5^x-4\right)=1 \Rightarrow \left[\begin{array}{l}\text{Diskusija!}\\ 5^x-4>0 \Rightarrow 5^x>4 \Rightarrow 5^x>4\ /\log \Rightarrow\\ \log 5^x>\log 4 \Rightarrow x\cdot\log 5>\log 4 \Rightarrow x>\dfrac{\log 4}{\log 5}\end{array}\right] \Rightarrow \log_5\left(5^x-4\right)=1-x \Rightarrow$$

$$\Rightarrow \log_5\left(5^x-4\right)=\log_5 5^{1-x} \Rightarrow 5^x-4=5^{1-x} \Rightarrow 5^x-4=5\cdot 5^{-x} \Rightarrow 5^x-4=5\cdot\frac{1}{5^x} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l}\text{supstitucija}\\ t=5^x\end{array}\right] \Rightarrow t-4=\frac{5}{t}\ /\cdot t \Rightarrow t^2-4\cdot t-5=0 \Rightarrow \left.\begin{array}{l}a=1\ ,\ b=-4\ ,\ c=-5\\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{array}\right\} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{4\pm\sqrt{16-4\cdot 1\cdot(-5)}}{2\cdot 1} \Rightarrow t_{1,2}=\frac{4\pm\sqrt{16+20}}{2} \Rightarrow t_{1,2}=\frac{4\pm\sqrt{36}}{2} \Rightarrow t_{1,2}=\frac{4\pm 6}{2} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}t_1=\dfrac{4+6}{2}\\ t_2=\dfrac{4-6}{2}\end{array}\right\} \Rightarrow \left.\begin{array}{l}t_1=\dfrac{10}{2}\\ t_2=\dfrac{-2}{2}\end{array}\right\} \Rightarrow \left.\begin{array}{l}t_1=5\\ t_2=-1\end{array}\right\}.$$

Vraćamo se na supstituciju:

- $\left.\begin{array}{l}t=5\\ 5^x=t\end{array}\right\} \Rightarrow 5^x=5 \Rightarrow x=1.$

- $\left.\begin{array}{l} t=-1 \\ 5^x=t \end{array}\right\} \Rightarrow 5^x=-1$ nema smisla.

Vježba 094

Riješite jednadžbu: $\log_5\left(5^x-4\right)=0.$

Rezultat: x = 1.

Zadatak 095 (Mira, TUPŠ)

Riješite nejednadžbu: $2<\log_{\frac{1}{3}}(1-x).$

Rješenje 095

Ponovimo!

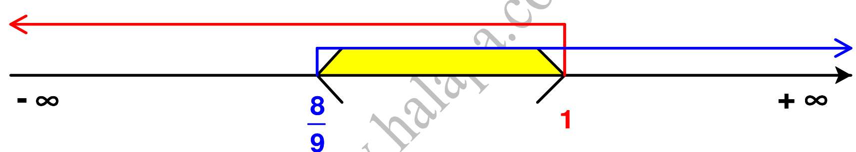
$$\log_b b^n=n \quad , \quad \log_b f(x)<\log_b g(x)\ ,\ 0<b<1 \Rightarrow f(x)>g(x).$$

$$2<\log_{\frac{1}{3}}(1-x) \Rightarrow \left[\begin{array}{c}\text{Diskusija!} \\ 1-x>0 \Rightarrow x<1\end{array}\right] \Rightarrow \log_{\frac{1}{3}}\left(\frac{1}{3}\right)^2<\log_{\frac{1}{3}}(1-x) \Rightarrow \left(\frac{1}{3}\right)^2>1-x \Rightarrow$$

$$\Rightarrow \frac{1}{9}>1-x \Rightarrow x>1-\frac{1}{9} \Rightarrow x>\frac{8}{9}.$$

Rješenje nejednadžbe iznosi:

$$\left.\begin{array}{l} x<1 \ \text{diskusija} \\ x>\frac{8}{9} \end{array}\right\} \Rightarrow x\in\left\langle\frac{8}{9},1\right\rangle.$$



Vježba 095

Riješite nejednadžbu: $\log(x-1)-1<\log 2.$

Rezultat: $x\in\langle 1, 21\rangle.$

Zadatak 096 (Mira, TUPŠ)

Riješite nejednadžbu: $3\cdot(0.5)^{x+1}-7>5.$

Rješenje 096

Ponovimo!

$$a^{-n}=\frac{1}{a^n} \quad , \quad a^{f(x)}>a^{g(x)}\ ,\ a>1 \Rightarrow f(x)>g(x) \quad , \quad a^{f(x)}>a^{g(x)}\ ,\ 0<a<1 \Rightarrow f(x)<g(x).$$

1.inačica

$$3\cdot(0.5)^{x+1}-7>5 \Rightarrow 3\cdot\left(\frac{5}{10}\right)^{x+1}>5+7 \Rightarrow 3\cdot\left(\frac{1}{2}\right)^{x+1}>12\ /:3 \Rightarrow \left(\frac{1}{2}\right)^{x+1}>4 \Rightarrow \left(\frac{1}{2}\right)^{x+1}>2^2 \Rightarrow$$

$$\Rightarrow \left(\frac{1}{2}\right)^{x+1}>\left(\frac{1}{2}\right)^{-2} \Rightarrow x+1<-2 \Rightarrow x<-3 \Rightarrow x\in\langle-\infty,-3\rangle.$$

2.inačica

$$3\cdot(0.5)^{x+1}-7>5 \Rightarrow 3\cdot\left(\frac{5}{10}\right)^{x+1}>5+7 \Rightarrow 3\cdot\left(\frac{1}{2}\right)^{x+1}>12\ /:3 \Rightarrow \left(\frac{1}{2}\right)^{x+1}>4 \Rightarrow \left(2^{-1}\right)^{x+1}>2^2 \Rightarrow$$

$$\Rightarrow 2^{-x-1}>2^2 \Rightarrow -x-1>2 \Rightarrow -x>2+1 \Rightarrow -x>3\ /\cdot(-1) \Rightarrow x<-3 \Rightarrow x\in\langle-\infty,-3\rangle.$$

Vježba 096

Riješite nejednadžbu: $\left(\frac{1}{2}\right)^x \le 2\cdot\sqrt{2}.$

Rezultat: $x \in \left[-\frac{3}{2}, +\infty\right\rangle.$

Zadatak 097 (Sany, TUPŠ)

Riješite jednadžbu: $\log_{16} x + \log_4 x + \log_2 x = 7.$

Rješenje 097

Ponovimo!

$$\log_{b^k} a = \frac{1}{k}\cdot\log_b a \quad , \quad \log_b a = \frac{\log a}{\log b} \quad , \quad \log_b a = c \Leftrightarrow b^c = a \quad , \quad \log f(x) = \log g(x) \Rightarrow f(x) = g(x)$$

$$n\cdot\log_b a = \log_b a^n.$$

1.inačica

$$\log_{16} x + \log_4 x + \log_2 x = 7 \Rightarrow \begin{bmatrix}\text{Diskusija!}\\ x>0\end{bmatrix} \Rightarrow \log_{2^4} x + \log_{2^2} x + \log_2 x = 7 \Rightarrow$$

$$\Rightarrow \frac{1}{4}\cdot\log_2 x + \frac{1}{2}\cdot\log_2 x + \log_2 x = 7 \;/\cdot 4 \Rightarrow \log_2 x + 2\cdot\log_2 x + 4\cdot\log_2 x = 28 \Rightarrow 7\cdot\log_2 x = 28 \;/:7 \Rightarrow$$

$$\Rightarrow \log_2 x = 4 \Rightarrow \log_2 x = \log_2 2^4 \Rightarrow \log_2 x = \log_2 16 \Rightarrow x = 16.$$

2.inačica

$$\log_{16} x + \log_4 x + \log_2 x = 7 \Rightarrow \begin{bmatrix}\text{Diskusija!}\\ x>0\end{bmatrix} \Rightarrow \frac{\log x}{\log 16} + \frac{\log x}{\log 4} + \frac{\log x}{\log 2} = 7 \Rightarrow \log x\cdot\left(\frac{1}{\log 16} + \frac{1}{\log 4} + \frac{1}{\log 2}\right) = 7 \Rightarrow$$

$$\Rightarrow \log x\cdot\left(\frac{1}{\log 2^4} + \frac{1}{\log 2^2} + \frac{1}{\log 2}\right) = 7 \Rightarrow \log x\cdot\left(\frac{1}{4\cdot\log 2} + \frac{1}{2\cdot\log 2} + \frac{1}{\log 2}\right) = 7 \Rightarrow$$

$$\Rightarrow \log x\cdot\frac{1+2+4}{4\cdot\log 2} = 7 \Rightarrow \log x\cdot\frac{7}{4\cdot\log 2} = 7 \;/\cdot\frac{4\cdot\log 2}{7} \Rightarrow \log x = 4\cdot\log 2 \Rightarrow \log x = \log 2^4 \Rightarrow$$

$$\Rightarrow \log x = \log 16 \Rightarrow x = 16.$$

Vježba 097

Riješite jednadžbu: $\log_{16} x + \log_8 x + \log_2 x = \frac{19}{12}.$

Rezultat: x = 2.

Zadatak 098 (Viky, maturantica)

Riješite jednadžbu: $\log_2^2 (x-1)^2 - \log_{0.5}(x-1) = 5.$

Rješenje 098

Ponovimo!

$$\log_b a^n = n\cdot\log_b a \quad , \quad \log_{b^n} a = \frac{1}{n}\cdot\log_b a \quad , \quad \log_b f(x) = \log_b g(x) \Rightarrow f(x) = g(x).$$

$$\log_2^2 (x-1)^2 - \log_{0.5}(x-1) = 5 \Rightarrow \begin{bmatrix}\text{Diskusija!}\\ x-1>0 \Rightarrow x>1\end{bmatrix} \Rightarrow \left(\log_2 (x-1)^2\right)^2 - \log_{\frac{5}{10}}(x-1) = 5 \Rightarrow$$

$$\Rightarrow \left(2\cdot\log_2 (x-1)\right)^2 - \log_{\frac{1}{2}}(x-1) = 5 \Rightarrow \left(2\cdot\log_2 (x-1)\right)^2 - \log_{2^{-1}}(x-1) = 5 \Rightarrow$$

$$\Rightarrow \left(2\cdot\log_2(x-1)^2\right)^2+\log_2(x-1)=5 \Rightarrow \begin{bmatrix}\text{supstitucija}\\ t=\log_2(x-1)\end{bmatrix} \Rightarrow (2\cdot t)^2+t=5 \Rightarrow 4\cdot t^2+t-5=0 \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} a=4\ ,\ b=1\ ,\ c=-5 \\ t_{1,2}=\dfrac{-b\pm\sqrt{b^2-4\cdot a\cdot c}}{2\cdot a}\end{matrix}\right\} \Rightarrow t_{1,2}=\frac{-1\pm\sqrt{1-4\cdot4\cdot(-5)}}{2\cdot4} \Rightarrow t_{1,2}=\frac{-1\pm\sqrt{1+80}}{8} \Rightarrow t_{1,2}=\frac{-1\pm\sqrt{81}}{8} \Rightarrow$$

$$\Rightarrow t_{1,2}=\frac{-1\pm9}{8} \Rightarrow \left.\begin{matrix} t_1=\dfrac{-1+9}{8}\\ t_2=\dfrac{-1-9}{8}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=\dfrac{8}{8}\\ t_2=\dfrac{-10}{8}\end{matrix}\right\} \Rightarrow \left.\begin{matrix} t_1=1\\ t_2=-\dfrac{5}{4}\end{matrix}\right\}.$$

Vraćamo se na supstituciju da bismo dobili konačna rješenja:

- $\left.\begin{matrix} t=1\\ \log_2(x-1)=t\end{matrix}\right\} \Rightarrow \log_2(x-1)=1 \Rightarrow \log_2(x-1)=\log_2 2 \Rightarrow x-1=2 \Rightarrow x_1=3.$
- $\left.\begin{matrix} t=-\dfrac{5}{4}\\ \log_2(x-1)=t\end{matrix}\right\} \Rightarrow \log_2(x-1)=-\dfrac{5}{4} \Rightarrow \log_2(x-1)=\log_2 2^{-\frac{5}{4}} \Rightarrow x-1=2^{-\frac{5}{4}} \Rightarrow x-1=\dfrac{1}{2^{\frac{5}{4}}} \Rightarrow$

$$\Rightarrow x-1=\frac{1}{\sqrt[4]{2^5}} \Rightarrow x_2=1+\frac{1}{\sqrt[4]{32}}.$$

Vježba 098

Riješite jednadžbu: $\log_2(x-1)-\log_{0.5}(x-1)=0.$

Rezultat: x = 2.

Zadatak 099 (Viky, maturantica)

Riješite jednadžbu: $\log_{\frac{1}{3}} x+4\cdot\log_{\sqrt{3}} x-\log_3 x=12.$

Rješenje 099

Ponovimo!

$$\log_b a=\frac{1}{\log_a b}\quad,\quad \log_{b^n} a=\frac{1}{n}\cdot\log_b a\quad,\quad \log_b f(x)=\log_b g(x) \Rightarrow f(x)=g(x).$$

$$\log_b a=\frac{\log_c a}{\log_c b}\quad,\quad \log_b a=c \Leftrightarrow b^c=a.$$

1.inačica

$$\log_{\frac{1}{3}} x+4\cdot\log_{\sqrt{3}} x-\log_3 x=12 \Rightarrow \begin{bmatrix}\text{Diskusija!}\\ x>0\end{bmatrix} \Rightarrow \log_{3^{-1}} x+4\cdot\log_{3^{\frac{1}{2}}} x-\log_3 x=12 \Rightarrow$$

$$\Rightarrow -\log_3 x+4\cdot2\cdot\log_3 x-\log_3 x=12 \Rightarrow -\log_3 x+8\cdot\log_3 x-\log_3 x=12 \Rightarrow 6\cdot\log_3 x=12\ /:6 \Rightarrow$$

$$\Rightarrow \log_3 x=2 \Rightarrow \log_3 x=\log_3 3^2 \Rightarrow x=3^2 \Rightarrow x=9.$$

2.inačica

$$\log_{\frac{1}{3}} x+4\cdot\log_{\sqrt{3}} x-\log_3 x=12 \Rightarrow \begin{bmatrix}\text{Diskusija!}\\ x>0\end{bmatrix} \Rightarrow \frac{\log x}{\log\frac{1}{3}}+4\cdot\frac{\log x}{\log\sqrt{3}}-\frac{\log x}{\log 3}=12 \Rightarrow$$

$$\Rightarrow \frac{\log x}{\log 1-\log 3}+4\cdot\frac{\log x}{\frac{1}{2}\cdot\log 3}-\frac{\log x}{\log 3}=12 \Rightarrow \frac{\log x}{0-\log 3}+8\cdot\frac{\log x}{\log 3}-\frac{\log x}{\log 3}=12 \Rightarrow -\frac{\log x}{\log 3}+8\cdot\frac{\log x}{\log 3}-\frac{\log x}{\log 3}=12 \Rightarrow$$

$$\Rightarrow 6\cdot\frac{\log x}{\log 3}=12\ /\cdot\frac{\log 3}{6} \Rightarrow \log x=2\cdot\log 3 \Rightarrow \log x=\log 3^2 \Rightarrow x=3^2 \Rightarrow x=9.$$

3.inačica

$$\log_{\frac{1}{3}} x+4\cdot\log_{\sqrt{3}} x-\log_3 x=12 \Rightarrow \begin{bmatrix}\text{Diskusija!}\\ x>0\end{bmatrix} \Rightarrow \frac{1}{\log_x \frac{1}{3}}+4\cdot\frac{1}{\log_x \sqrt{3}}-\frac{1}{\log_x 3}=12 \Rightarrow$$

$$\Rightarrow \frac{1}{\log_x 1-\log_x 3}+\frac{4}{\frac{1}{2}\cdot\log_x 3}-\frac{1}{\log_x 3}=12 \Rightarrow \frac{1}{0-\log_x 3}+\frac{8}{\log_x 3}-\frac{1}{\log_x 3}=12 \Rightarrow$$

$$\Rightarrow -\frac{1}{\log_x 3}+\frac{8}{\log_x 3}-\frac{1}{\log_x 3}=12 \Rightarrow \frac{-1+8-1}{\log_x 3}=12 \Rightarrow \frac{6}{\log_x 3}=12 \Rightarrow \log_x 3=\frac{6}{12} \Rightarrow$$

$$\Rightarrow \log_x 3=\frac{1}{2} \Rightarrow x^{\frac{1}{2}}=3 \Rightarrow \sqrt{x}=3\ /^2 \Rightarrow x=9.$$

Vježba 099

Riješite jednadžbu: $4\cdot\log_{\sqrt{3}} x-\log_3 x=7.$

Rezultat: x = 3.

Zadatak 100 (Viky, maturantica)

Riješite jednadžbu: $\log_{25}\left[\frac{1}{5}\cdot\log_3\left(2+\log_2 x\right)\right]=-\frac{1}{2}.$

Rješenje 100

Ponovimo!

$$\log_b a=c \Leftrightarrow b^c=a \quad , \quad \log_b f(x)=\log_b g(x) \Rightarrow f(x)=g(x).$$

$$\log_{25}\left[\frac{1}{5}\cdot\log_3\left(2+\log_2 x\right)\right]=-\frac{1}{2} \Rightarrow \begin{bmatrix}\text{Diskusija!}\\ x>0\end{bmatrix} \Rightarrow \log_{25}\left[\frac{1}{5}\cdot\log_3\left(2+\log_2 x\right)\right]=\log_{25} 25^{-\frac{1}{2}} \Rightarrow$$

$$\Rightarrow \frac{1}{5}\cdot\log_3\left(2+\log_2 x\right)=25^{-\frac{1}{2}} \Rightarrow \frac{1}{5}\cdot\log_3\left(2+\log_2 x\right)=\frac{1}{25^{\frac{1}{2}}} \Rightarrow \frac{1}{5}\cdot\log_3\left(2+\log_2 x\right)=\frac{1}{\sqrt{25}} \Rightarrow$$

$$\Rightarrow \frac{1}{5}\cdot\log_3\left(2+\log_2 x\right)=\frac{1}{5}\ /\cdot 5 \Rightarrow \log_3\left(2+\log_2 x\right)=1 \Rightarrow \log_3\left(2+\log_2 x\right)=\log_3 3 \Rightarrow$$

$$\Rightarrow 2+\log_2 x=3 \Rightarrow \log_2 x=3-2 \Rightarrow \log_2 x=1 \Rightarrow \log_2 x=\log_2 2 \Rightarrow x=2.$$

Vježba 100

Riješite jednadžbu: $\log_4\left(3+\log_3 x\right)=1.$

Rezultat: x = 3.