Zadatak 061 (4A, hotelijerska škola)

Koliko je $\log_{\frac{1}{2}} \frac{1}{27} \cdot \log_3 16$?

Rješenje 061

Ponovimo!

$$\frac{1}{a^n} = a^{-n} \quad , \quad \log_b a \cdot \log_a b = 1$$

$$\log_{\frac{1}{2}} \frac{1}{27} \cdot \log_3 16 = \log_{\frac{1}{2}} 3^{-3} \cdot \log_3 2^4 = \log_{\frac{1}{2}} 3^{-3} \cdot \log_3 \left(\frac{1}{2}\right)^{-4} = -3 \cdot \log_{\frac{1}{2}} 3 \cdot (-4) \cdot \log_3 \frac{1}{2} =$$

$$= 12 \cdot \log_{\frac{1}{2}} 3 \cdot \log_3 \frac{1}{2} = 12 \cdot 1 = 12.$$

Vježba 061

Koliko je $\log_2 \sqrt{3} \cdot \log_{\frac{1}{3}} \frac{1}{8}$?

Rezultat: $\frac{3}{2}$.

Zadatak 062 (4A, hotelijerska škola)

Riješite nejednadžbu $\log_5 (x+3) > 1$.

Rješenje 062

$$\left.\begin{matrix} \log_5 (x+3) > 1 \\ \underbrace{x+3>0}_{\text{diskusija}} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_5 (x+3) > \log_5 5 \\ x > -3 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x+3>5 \\ x>-3 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x>2 \\ x>-3 \end{matrix}\right\} \Rightarrow x > 2 \Rightarrow x \in \langle 2, +\infty \rangle.$$

Vježba 062

Riješite nejednadžbu $\log_3 (x+3) > 1$.

Rezultat: $x \in \langle 0, +\infty \rangle$.

Zadatak 063 (4A, hotelijerska škola)

Riješi jednadžbu: $16^{\frac{1}{x}} = 4^{\frac{x}{2}}$.

Rješenje 063

$$16^{\frac{1}{x}} = 4^{\frac{x}{2}} \Rightarrow \left(4^2\right)^{\frac{1}{x}} = 4^{\frac{x}{2}} \Rightarrow 4^{\frac{2}{x}} = 4^{\frac{x}{2}} \Rightarrow \frac{2}{x} = \frac{x}{2} \Rightarrow x^2 = 4 \,/\sqrt{\ } \Rightarrow \begin{cases} x_1 = -2 \\ x_2 = 2. \end{cases}$$

Vježba 063

Riješi jednadžbu: $9^{\frac{1}{x}} = 3^{\frac{x}{2}}$.

Rezultat: $x_1 = -2$, $x_2 = 2$.

Zadatak 064 (4A, hotelijerska škola)

Nađi rješenje jednadžbe $50^{\log x} \cdot 160^{\log x} = 400$.

Rješenje 064

Ponovimo!

$$a^n \cdot b^n = (a \cdot b)^n$$

$$50^{\log x}\cdot 160^{\log x}=400\Rightarrow(50\cdot 160)^{\log x}=400\Rightarrow 8000^{\log x}=400\Rightarrow\left(20^3\right)^{\log x}=20^2\Rightarrow 20^{3\cdot\log x}=20^2\Rightarrow$$

$$\Rightarrow 3\cdot\log x=2\Rightarrow\log x^3=2\Rightarrow\log x^3=\log 100\Rightarrow x^3=100\ /\sqrt[3]{\ }\Rightarrow x=\sqrt[3]{100}.$$

Vježba 064

Nađi rješenje jednadžbe $50^{\log x}\cdot 160^{\log x}=8000.$

Rezultat: x = 10.

Zadatak 065 (4A, hotelijerska škola)

Nađi zbroj rješenja jednadžbe: $\log_2\left(\log_x(5\cdot x-6)\right)=1.$

Rješenje 065

$$\log_2\left(\log_x(5\cdot x-6)\right)=1\Rightarrow\log_x(5\cdot x-6)=2^1\Rightarrow\log_x(5\cdot x-6)=2\Rightarrow 5\cdot x-6=x^2\Rightarrow x^2-5\cdot x+6=0\Rightarrow$$

$$\Rightarrow[\text{po Vièteovoj formuli}]\Rightarrow x_1+x_2=-(-5)=5.$$

Vježba 065

Nađi zbroj rješenja jednadžbe: $\log_2\left(\log_x(3\cdot x-5)\right)=1.$

Rezultat: $x_1+x_2=3.$

Zadatak 066 (Kristy, gimnazija)

Pojednostavnite izraz: $\left(\log_a x\right)^{-1}+\left(\log_{a^2}x\right)^{-1}+\left(\log_{a^3}x\right)^{-1}+\left(\log_{a^4}x\right)^{-1}.$

Rješenje 066

Ponovimo!

$$\log_b a=\left(\log_a b\right)^{-1}\Rightarrow\left(\log_b a\right)^{-1}=\log_a b$$

Zato je:

$$\left(\log_a x\right)^{-1}+\left(\log_{a^2}x\right)^{-1}+\left(\log_{a^3}x\right)^{-1}+\left(\log_{a^4}x\right)^{-1}=\log_x a+\log_x a^2+\log_x a^3+\log_x a^4=$$

$$=\log_x a+2\cdot\log_x a+3\cdot\log_x a+4\cdot\log_x a=10\cdot\log_x a.$$

Vježba 066

Pojednostavnite izraz $\left(\log_a x\right)^{-1}+\left(\log_{a^2}x\right)^{-1}+\left(\log_{a^3}x\right)^{-1}.$

Rezultat: $6\cdot\log_x a.$

Zadatak 067 (Vedrana, farmaceutska škola)

Ako je $\log_3 x\cdot\log_x 2x\cdot\log_{2x}y=\log_x x^2$, koliko iznosi y?

Rješenje 067

Ponovimo!

$$\log_a a^n=n\quad,\quad\log_b a=\frac{\log_c a}{\log_c b}$$

$$\log_3 x\cdot\log_x 2x\cdot\log_{2x}y=\log_x x^2\Rightarrow\log_3 x\cdot\frac{\log_3 2x}{\log_3 x}\cdot\frac{\log_3 y}{\log_3 2x}=2\Rightarrow\log_3 y=2\Rightarrow y=3^2\Rightarrow y=9.$$

Vježba 067

Ako je $\log_3 x\cdot\log_x 2x\cdot\log_{2x}y=\log_x x^3$, koliko iznosi y?

Rezultat: 27.

Zadatak 068 (Vedrana, gimnazija)

Ako je $\log_4 \log_2 \log_3 x = \log_3 \log_2 \log_4 y = 0$, koliko je $x + y$?

Rješenje 068

Ponovimo!

$$\log_b a = c \Leftrightarrow b^c = a$$

Riješimo sustav jednadžbi:

$$\left.\begin{array}{l}\log_4 \log_2 \log_3 x = 0\\ \log_3 \log_2 \log_4 y = 0\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_2 \log_3 x = 4^0\\ \log_2 \log_4 y = 3^0\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_2 \log_3 x = 1\\ \log_2 \log_4 y = 1\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_3 x = 2^1\\ \log_4 y = 2^1\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\log_3 x = 2\\ \log_4 y = 2\end{array}\right\} \Rightarrow \left.\begin{array}{l}x = 3^2\\ y = 4^2\end{array}\right\} \Rightarrow \left.\begin{array}{l}x = 9\\ y = 16\end{array}\right\} \Rightarrow x + y = 9 + 16 = 25.$$

Vježba 068

Ako je $\log_4 \log_2 \log_3 x = \log_3 \log_2 \log_4 y = 0$, koliko je $y - x$?

Rezultat: 7.

Zadatak 069 (Vedrana, gimnazija)

$$\text{Riješi sustav: } \left.\begin{array}{l}\log_2 x + \log_4 y + \log_4 z = 2\\ \log_3 y + \log_9 z + \log_9 x = 2\\ \log_4 z + \log_{16} x + \log_{16} y = 2\end{array}\right\}.$$

Rješenje 069

1.inačica

Ponovimo!

$$\log_b a = c \Leftrightarrow b^c = a \quad , \quad \log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log_b a^n = n \cdot \log_b a \quad , \quad \log_c a + \log_c b = \log_c (a \cdot b),$$

$$\log_{b^n} a^n = \log_b a$$

Prijeđemo na bazu 10 i dobiva se:

$$\left.\begin{array}{l}\log_2 x + \log_4 y + \log_4 z = 2\\ \log_3 y + \log_9 z + \log_9 x = 2\\ \log_4 z + \log_{16} x + \log_{16} y = 2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\frac{\log x}{\log 2} + \frac{\log y}{\log 4} + \frac{\log z}{\log 4} = 2\\ \frac{\log y}{\log 3} + \frac{\log z}{\log 9} + \frac{\log x}{\log 9} = 2\\ \frac{\log z}{\log 4} + \frac{\log x}{\log 16} + \frac{\log y}{\log 16} = 2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\frac{\log x}{\log 2} + \frac{\log y}{\log 2^2} + \frac{\log z}{\log 2^2} = 2\\ \frac{\log y}{\log 3} + \frac{\log z}{\log 3^2} + \frac{\log x}{\log 3^2} = 2\\ \frac{\log z}{\log 4} + \frac{\log x}{\log 4^2} + \frac{\log y}{\log 4^2} = 2\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\frac{\log x}{\log 2} + \frac{\log y}{2 \cdot \log 2} + \frac{\log z}{2 \cdot \log 2} = 2 \; / \cdot 2 \cdot \log 2\\ \frac{\log y}{\log 3} + \frac{\log z}{2 \cdot \log 3} + \frac{\log x}{2 \cdot \log 3} = 2 \; / \cdot 2 \cdot \log 3\\ \frac{\log z}{\log 4} + \frac{\log x}{2 \cdot \log 4} + \frac{\log y}{2 \cdot \log 4} = 2 \; / \cdot 2 \cdot \log 4\end{array}\right\} \Rightarrow \left.\begin{array}{l}2 \cdot \log x + \log y + \log z = 2 \cdot 2 \cdot \log 2\\ 2 \cdot \log y + \log z + \log x = 2 \cdot 2 \cdot \log 3\\ 2 \cdot \log z + \log x + \log y = 2 \cdot 2 \cdot \log 4\end{array}\right\} \Rightarrow$$

$$\left.\begin{array}{l}\log x^2+\log y+\log z=4\cdot\log 2\\ \Rightarrow \log y^2+\log z+\log x=4\cdot\log 3\\ \log z^2+\log x+\log y=4\cdot\log 4\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log\left(x^2\cdot y\cdot z\right)=\log 2^4\\ \log\left(y^2\cdot z\cdot x\right)=\log 3^4\\ \log\left(z^2\cdot x\cdot y\right)=\log 4^4\end{array}\right\} \Rightarrow \left.\begin{array}{l}x^2\cdot y\cdot z=2^4\\ y^2\cdot z\cdot x=3^4\\ z^2\cdot x\cdot y=4^4\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l}\text{pomnožimo}\\ \text{jednadžbe}\end{array}\right] \Rightarrow x^4\cdot y^4\cdot z^4=2^4\cdot 3^4\cdot 4^4 \Rightarrow (x\cdot y\cdot z)^4=(2\cdot 3\cdot 4)^4 \;/\sqrt[4]{} \Rightarrow$$

$$\Rightarrow x\cdot y\cdot z=2\cdot 3\cdot 4 \Rightarrow x\cdot y\cdot z=24.$$

Računamo nepoznanicu x:

$$\left.\begin{array}{l}x^2\cdot y\cdot z=2^4\\ x\cdot y\cdot z=24\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{podijelimo}\\ \text{jednadžbe}\end{array}\right] \Rightarrow \frac{x^2\cdot y\cdot z}{x\cdot y\cdot z}=\frac{2^4}{24} \Rightarrow x=\frac{16}{24}=\frac{2}{3}.$$

Računamo nepoznanicu y:

$$\left.\begin{array}{l}y^2\cdot z\cdot x=3^4\\ x\cdot y\cdot z=24\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{podijelimo}\\ \text{jednadžbe}\end{array}\right] \Rightarrow \frac{y^2\cdot z\cdot x}{x\cdot y\cdot z}=\frac{3^4}{24} \Rightarrow y=\frac{81}{24}=\frac{27}{8}.$$

Računamo nepoznanicu z:

$$\left.\begin{array}{l}z^2\cdot x\cdot y=4^4\\ x\cdot y\cdot z=24\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{podijelimo}\\ \text{jednadžbe}\end{array}\right] \Rightarrow \frac{z^2\cdot x\cdot y}{x\cdot y\cdot z}=\frac{4^4}{24} \Rightarrow z=\frac{256}{24}=\frac{32}{3}.$$

2.inačica

Prijeđemo u prvoj jednadžbi na bazu 2, u drugoj na bazu 3, a u trećoj jednadžbi na bazu 4:

$$\left.\begin{array}{l}\log_2 x+\log_4 y+\log_4 z=2\\ \log_3 y+\log_9 z+\log_9 x=2\\ \log_4 z+\log_{16} x+\log_{16} y=2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_2 x+\dfrac{\log_2 y}{\log_2 4}+\dfrac{\log_2 z}{\log_2 4}=2\\ \log_3 y+\dfrac{\log_3 z}{\log_3 9}+\dfrac{\log_3 x}{\log_3 9}=2\\ \log_4 z+\dfrac{\log_4 x}{\log_4 16}+\dfrac{\log_4 y}{\log_4 16}=2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_2 x+\dfrac{\log_2 y}{\log_2 2^2}+\dfrac{\log_2 z}{\log_2 2^2}=2\\ \log_3 y+\dfrac{\log_3 z}{\log_3 3^2}+\dfrac{\log_3 x}{\log_3 3^2}=2\\ \log_4 z+\dfrac{\log_4 x}{\log_4 4^2}+\dfrac{\log_4 y}{\log_4 4^2}=2\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\log_2 x+\dfrac{\log_2 y}{2\cdot\log_2 2}+\dfrac{\log_2 z}{2\cdot\log_2 2}=2\\ \log_3 y+\dfrac{\log_3 z}{2\cdot\log_3 3}+\dfrac{\log_3 x}{2\cdot\log_3 3}=2\\ \log_4 z+\dfrac{\log_4 x}{2\cdot\log_4 4}+\dfrac{\log_4 y}{2\cdot\log_4 4}=2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_2 x+\dfrac{\log_2 y}{2\cdot 1}+\dfrac{\log_2 z}{2\cdot 1}=2\\ \log_3 y+\dfrac{\log_3 z}{2\cdot 1}+\dfrac{\log_3 x}{2\cdot 1}=2\\ \log_4 z+\dfrac{\log_4 x}{2\cdot 1}+\dfrac{\log_4 y}{2\cdot 1}=2\end{array}\right\} \Rightarrow \left.\begin{array}{l}\log_2 x+\dfrac{\log_2 y}{2}+\dfrac{\log_2 z}{2}=2\\ \log_3 y+\dfrac{\log_3 z}{2}+\dfrac{\log_3 x}{2}=2\\ \log_4 z+\dfrac{\log_4 x}{2}+\dfrac{\log_4 y}{2}=2\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}\log_2 x+\dfrac{\log_2 y}{2}+\dfrac{\log_2 z}{2}=2\;/\cdot 2\\ \log_3 y+\dfrac{\log_3 z}{2}+\dfrac{\log_3 x}{2}=2\;/\cdot 2\\ \log_4 z+\dfrac{\log_4 x}{2}+\dfrac{\log_4 y}{2}=2\;/\cdot 2\end{array}\right\} \Rightarrow \left.\begin{array}{l}2\cdot\log_2 x+\log_2 y+\log_2 z=4\\ 2\cdot\log_3 y+\log_3 z+\log_3 x=4\\ 2\cdot\log_4 z+\log_4 x+\log_4 y=4\end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \log_2 x^2 + \log_2 y + \log_2 z = 4 \\ \log_3 y^2 + \log_3 z + \log_3 x = 4 \\ \log_4 z^2 + \log_4 x + \log_4 y = 4 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_2 \left(x^2 \cdot y \cdot z\right) = 4 \\ \log_3 \left(y^2 \cdot z \cdot x\right) = 4 \\ \log_4 \left(z^2 \cdot x \cdot y\right) = 4 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2 \cdot y \cdot z = 2^4 \\ y^2 \cdot z \cdot x = 3^4 \\ z^2 \cdot x \cdot y = 4^4 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left[\begin{matrix} \text{pomnožimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow x^4 \cdot y^4 \cdot z^4 = 2^4 \cdot 3^4 \cdot 4^4 \Rightarrow (x \cdot y \cdot z)^4 = (2 \cdot 3 \cdot 4)^4 \;/\sqrt[4]{\;} \Rightarrow$$

$$\Rightarrow x \cdot y \cdot z = 2 \cdot 3 \cdot 4 \Rightarrow x \cdot y \cdot z = 24.$$

Računamo nepoznanicu x:

$$\left.\begin{matrix} x^2 \cdot y \cdot z = 2^4 \\ x \cdot y \cdot z = 24 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{podijelimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow \frac{x^2 \cdot y \cdot z}{x \cdot y \cdot z} = \frac{2^4}{24} \Rightarrow x = \frac{16}{24} = \frac{2}{3}.$$

Računamo nepoznanicu y:

$$\left.\begin{matrix} y^2 \cdot z \cdot x = 3^4 \\ x \cdot y \cdot z = 24 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{podijelimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow \frac{y^2 \cdot z \cdot x}{x \cdot y \cdot z} = \frac{3^4}{24} \Rightarrow y = \frac{81}{24} = \frac{27}{8}.$$

Računamo nepoznanicu z:

$$\left.\begin{matrix} z^2 \cdot x \cdot y = 4^4 \\ x \cdot y \cdot z = 24 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{podijelimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow \frac{z^2 \cdot x \cdot y}{x \cdot y \cdot z} = \frac{4^4}{24} \Rightarrow z = \frac{256}{24} = \frac{32}{3}.$$

3.inačica

$$\left.\begin{matrix} \log_2 x + \log_4 y + \log_4 z = 2 \\ \log_3 y + \log_9 z + \log_9 x = 2 \\ \log_4 z + \log_{16} x + \log_{16} y = 2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_2 x + \log_4 (y \cdot z) = 2 \\ \log_3 y + \log_9 (z \cdot x) = 2 \\ \log_4 z + \log_{16} (x \cdot y) = 2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_{2^2} x^2 + \log_4 (y \cdot z) = 2 \\ \log_{3^2} y^2 + \log_9 (z \cdot x) = 2 \\ \log_{4^2} z^2 + \log_{16} (x \cdot y) = 2 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} \log_4 x^2 + \log_4 (y \cdot z) = 2 \\ \log_9 y^2 + \log_9 (z \cdot x) = 2 \\ \log_{16} z^2 + \log_{16} (x \cdot y) = 2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \log_4 \left(x^2 \cdot y \cdot z\right) = 2 \\ \log_9 \left(y^2 \cdot z \cdot x\right) = 2 \\ \log_{16} \left(z^2 \cdot x \cdot y\right) = 2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2 \cdot y \cdot z = 4^2 \\ y^2 \cdot z \cdot x = 9^2 \\ z^2 \cdot x \cdot y = 16^2 \end{matrix}\right\} \Rightarrow \left.\begin{matrix} x^2 \cdot y \cdot z = \left(2^2\right)^2 \\ y^2 \cdot z \cdot x = \left(3^2\right)^2 \\ z^2 \cdot x \cdot y = \left(4^2\right)^2 \end{matrix}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{matrix} x^2 \cdot y \cdot z = 2^4 \\ y^2 \cdot z \cdot x = 3^4 \\ z^2 \cdot x \cdot y = 4^4 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{pomnožimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow x^4 \cdot y^4 \cdot z^4 = 2^4 \cdot 3^4 \cdot 4^4 \Rightarrow (x \cdot y \cdot z)^4 = (2 \cdot 3 \cdot 4)^4 \;/\sqrt[4]{\;} \Rightarrow$$

$$\Rightarrow x \cdot y \cdot z = 2 \cdot 3 \cdot 4 \Rightarrow x \cdot y \cdot z = 24.$$

Računamo nepoznanicu x:

$$\left.\begin{matrix} x^2 \cdot y \cdot z = 2^4 \\ x \cdot y \cdot z = 24 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{podijelimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow \frac{x^2 \cdot y \cdot z}{x \cdot y \cdot z} = \frac{2^4}{24} \Rightarrow x = \frac{16}{24} = \frac{2}{3}.$$

Računamo nepoznanicu y:

$$\left.\begin{matrix} y^2 \cdot z \cdot x = 3^4 \\ x \cdot y \cdot z = 24 \end{matrix}\right\} \Rightarrow \left[\begin{matrix} \text{podijelimo} \\ \text{jednadžbe} \end{matrix}\right] \Rightarrow \frac{y^2 \cdot z \cdot x}{x \cdot y \cdot z} = \frac{3^4}{24} \Rightarrow y = \frac{81}{24} = \frac{27}{8}.$$

Računamo nepoznanicu z:

$$\left.\begin{array}{l} z^2 \cdot x \cdot y = 4^4 \\ x \cdot y \cdot z = 24 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{podijelimo} \\ \text{jednadžbe} \end{array}\right] \Rightarrow \frac{z^2 \cdot x \cdot y}{x \cdot y \cdot z} = \frac{4^4}{24} \Rightarrow z = \frac{256}{24} = \frac{32}{3}.$$

Vježba 069

Riješi sustav: $\left.\begin{array}{l} \log\left(x^2+y^2\right) = 1+\log 8 \\ \log(x+y) = \log 3 + \log(x-y) \end{array}\right\}.$

Rezultat:

$$\left.\begin{array}{l} \log\left(x^2+y^2\right) = 1+\log 8 \\ \log(x+y) = \log 3 + \log(x-y) \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log\left(x^2+y^2\right) = \log 10+\log 8 \\ \log(x+y) = \log 3 + \log(x-y) \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log\left(x^2+y^2\right) = \log 80 \\ \log(x+y) = \log 3 \cdot (x-y) \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} x^2+y^2 = 80 \\ x+y = 3\cdot(x-y) \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2+y^2 = 80 \\ x+y = 3\cdot x - 3\cdot y \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2+y^2 = 80 \\ 4\cdot y = 2\cdot x \ /:2 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x^2+y^2 = 80 \\ x = 2\cdot y \end{array}\right\} \Rightarrow (2\cdot y)^2 + y^2 = 80 \Rightarrow$$

$$\Rightarrow 5\cdot y^2 = 80 \ /:5 \Rightarrow y^2 = 16 \ /\sqrt{\ } \Rightarrow \left.\begin{array}{l} y_1 = 4 \\ y_2 = -4 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x_1 = 8 \\ x_2 = -8 \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{uvjeti} \\ x+y>0,\ x-y>0,\ x^2+y^2 \neq 0 \end{array}\right] \Rightarrow \left.\begin{array}{l} x = 8 \\ y = 4 \end{array}\right\}.$$

Zadatak 070 (Gregor, gimnazija)

Riješi jednadžbu: $\log_2\left(3-2^{-x}\right) = 1-x.$

Rješenje 070

Ponovimo!

$$\log_b a = c \Leftrightarrow b^c = a \quad , \quad a^{n-m} = a^n \cdot a^{-m} \quad , \quad a^0 = 1$$

$$\log_2\left(3-2^{-x}\right) = 1-x \Rightarrow 3-2^{-x} = 2^{1-x} \Rightarrow 3-2^{-x} = 2^1\cdot 2^{-x} \Rightarrow \left[\begin{array}{l} \text{supstitucija} \\ t = 2^{-x} \end{array}\right] \Rightarrow 3-t = 2\cdot t \Rightarrow$$

$$\Rightarrow 3\cdot t = 3 \ /:3 \Rightarrow t = 1 \Rightarrow 2^{-x} = 1 \Rightarrow 2^{-x} = 2^0 \Rightarrow -x = 0 \Rightarrow x = 0.$$

Vježba 070

Riješi jednadžbu: $\log_{\frac{1}{2}}\left(3-2^{-x}\right) = x-1.$

Rezultat: 0.

Zadatak 071 (Mira, gimnazija)

Ako je (x, y) rješenje sustava jednadžbi $\left\{\begin{array}{l} \log_y x - 3\cdot\log_x y = 2 \\ \log_2 x + \log_2 y = 4 \end{array}\right.$ nađi $\frac{x\cdot y}{x+4\cdot y}.$

Rješenje 071

Ponovimo!

$$\log_b a = \frac{\log_c a}{\log_c b} \quad , \quad \log_b a = c \Leftrightarrow b^c = a$$

$$\left.\begin{array}{l} \log_y x - 3\cdot\log_x y = 2 \\ \log_2 x + \log_2 y = 4 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{\log_2 x}{\log_2 y} - 3\cdot\frac{\log_2 y}{\log_2 x} = 2 \\ \log_2 x + \log_2 y = 4 \end{array}\right\} \Rightarrow \left[\begin{array}{c} \text{supstitucija} \\ u = \log_2 x \ , \ v = \log_2 y \end{array}\right] \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} \frac{u}{v}-3\cdot\frac{v}{u}=2 \\ u+v=4 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \frac{u}{v}-3\cdot\frac{v}{u}=2 \;/\cdot u\cdot v \\ u+v=4 \end{array}\right\} \Rightarrow \left.\begin{array}{l} u^2-3\cdot v^2=2\cdot u\cdot v \\ u+v=4 \end{array}\right\} \Rightarrow \left.\begin{array}{l} u^2-3\cdot v^2=2\cdot u\cdot v \\ u=4-v \end{array}\right\} \Rightarrow$$

$$\Rightarrow (4-v)^2-3\cdot v^2=2\cdot(4-v)\cdot v \Rightarrow 16-8\cdot v+v^2-3\cdot v^2=8\cdot v-2\cdot v^2 \Rightarrow$$

$$\Rightarrow -8\cdot v+v^2-3\cdot v^2-8\cdot v+2\cdot v^2=-16 \Rightarrow -16\cdot v=-16 \;/:(-16) \Rightarrow v=1 \Rightarrow \left.\begin{array}{l} u+v=4 \\ v=1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow u+1=4 \Rightarrow u=4-1=3.$$

Sada je:

$$\left.\begin{array}{l} u=3 \\ v=1 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \log_2 x=u \\ \log_2 y=v \end{array}\right] \Rightarrow \left.\begin{array}{l} \log_2 x=3 \\ \log_2 y=1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=2^3 \\ y=2^1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} x=8 \\ y=2 \end{array}\right\} \Rightarrow \frac{x\cdot y}{x+4\cdot y}=\frac{8\cdot 2}{8+4\cdot 2}=\frac{16}{16}=1.$$

Vježba 071

Ako je (x, y) rješenje sustava jednadžbi $\left\{\begin{array}{l} \log_y x-3\cdot\log_x y=2 \\ \log_2 x+\log_2 y=4 \end{array}\right.$ nađi $\frac{x}{4\cdot y}$.

Rezultat: 1.

Zadatak 072 (Vedrana, gimnazija)

Ako je funkcija f(x) zadana izrazom $f(x)=\log x+2\cdot\log(2\cdot x)$, koliko je $f(x)+f\left(\frac{1}{x}\right)$?

Rješenje 072

Ponovimo!

$$\log a+\log b=\log(a\cdot b) \quad , \quad \log a^n=n\cdot\log a \quad , \quad \log 1=0$$

Transformiramo funkciju f(x):

$$f(x)=\log x+2\cdot\log(2\cdot x)=\log x+2\cdot(\log 2+\log x)=\log x+2\cdot\log 2+2\cdot\log x=3\cdot\log x+\log 2^2=$$
$$=3\cdot\log x+\log 4.$$

Sada je:

$$f\left(\frac{1}{x}\right)=3\cdot\log\frac{1}{x}+\log 4=3\cdot(\log 1-\log x)+\log 4=3\cdot(0-\log x)+\log 4=-3\cdot\log x+\log 4.$$

Vrijednost zadanog izraza je:

$$f(x)+f\left(\frac{1}{x}\right)=3\cdot\log x+\log 4-3\cdot\log x+\log 4=2\cdot\log 4=\log 4^2=\log 16.$$

Vježba 072

Ako je funkcija f(x) zadana izrazom $f(x)=\log x+2\cdot\log(2\cdot x)$, koliko je $f(x)-f\left(\frac{1}{x}\right)$?

Rezultat: $\log x^6$.

Zadatak 073 (Vedrana, gimnazija)

Odredi skup brojeva kojem pripada rješenje jednadžbe: $\sqrt[x-1]{b^{x+1}}\cdot\sqrt[x+3]{b^{x-1}}=\left(\frac{1}{b}\right)^{-2}$.

Rješenje 073

Ponovimo!

$$\sqrt[n]{a^m}=a^{\frac{m}{n}} \quad , \quad a^n\cdot a^m=a^{n+m} \quad , \quad a^{-n}=\frac{1}{a^n} \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x)$$

$$\sqrt[x-1]{b^{x+1}} \cdot \sqrt[x+3]{b^{x-1}} = \left(\frac{1}{b}\right)^{-2} \Rightarrow b^{\frac{x+1}{x-1}} \cdot b^{\frac{x-1}{x+3}} = b^2 \Rightarrow b^{\frac{x+1}{x-1}+\frac{x-1}{x+3}} = b^2 \Rightarrow \frac{x+1}{x-1} + \frac{x-1}{x+3} = 2 \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \text{diskusija} \\ x \neq 1 \;,\; x \neq -3 \end{array}\right] \Rightarrow \frac{x+1}{x-1} + \frac{x-1}{x+3} = 2 \;/\cdot(x-1)\cdot(x+3) \Rightarrow (x+1)\cdot(x+3) + (x-1)^2 = 2\cdot(x-1)\cdot(x+3) \Rightarrow$$

$$\Rightarrow x^2 + 3\cdot x + x + 3 + x^2 - 2\cdot x + 1 = 2\cdot\left(x^2 + 3\cdot x - x - 3\right) \Rightarrow$$

$$\Rightarrow x^2 + 3\cdot x + x + 3 + x^2 - 2\cdot x + 1 = 2\cdot x^2 + 6\cdot x - 2\cdot x - 6 \Rightarrow 3\cdot x + x + 3 + 1 = 6\cdot x - 6 \Rightarrow$$

$$\Rightarrow 3\cdot x + x - 6\cdot x = -6 - 3 - 1 \Rightarrow -2\cdot x = -10 \;/:(-2)\; x = 5 \Rightarrow 5 \in N.$$

Vježba 073

Odredi skup brojeva kojem pripada rješenje jednadžbe: $\sqrt{b^{x+2}} = \left(\frac{1}{b}\right)^{-2}$.

Rezultat: $2 \in N.$

Zadatak 074 (Maja, između gimnazije i fakulteta)

Iz sustava jednadžbi izračunajte a · b.

$$\begin{cases} \log a + 3\cdot \log b = 3 \\ \log a - \log b = 1. \end{cases}$$

Rješenje 074

Ponovimo!

$$\log a^n = n\cdot \log a \quad , \quad \log a + \log b = \log a\cdot b \quad , \quad \log a - \log b = \log \frac{a}{b} \quad , \quad \log a = \log b \Rightarrow a = b.$$

1.inačica

$$\left.\begin{array}{l} \log a + 3\cdot \log b = 3 \\ \log a - \log b = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log a + \log b^3 = 3 \\ \log a - \log b = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log a\cdot b^3 = 3 \\ \log \frac{a}{b} = 1 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log a\cdot b^3 = \log 1000 \\ \log \frac{a}{b} = \log 10 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a\cdot b^3 = 1000 \\ \frac{a}{b} = 10 \;/\cdot b \end{array}\right\} \Rightarrow \left.\begin{array}{l} a\cdot b^3 = 1000 \\ a = 10\cdot b \end{array}\right\} \Rightarrow 10\cdot b\cdot b^3 = 1000 \Rightarrow 10\cdot b^4 = 1000 \;/:10 \Rightarrow b^4 = 100 \;/\sqrt{} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} a\cdot b^3 = 1000 \\ \frac{a}{b} = 10 \;/\cdot b \end{array}\right\} \Rightarrow \left.\begin{array}{l} a\cdot b^3 = 1000 \\ a = 10\cdot b \end{array}\right\} \Rightarrow 10\cdot b\cdot b^3 = 1000 \Rightarrow 10\cdot b^4 = 1000 \;/:10 \Rightarrow b^4 = 100 \;/\sqrt{} \Rightarrow b^2 = 10.$$

Umnožak a i b iznosi:

$$\left.\begin{array}{l} a = 10\cdot b \\ b^2 = 10 \end{array}\right\} \Rightarrow a\cdot b = 10\cdot b\cdot b = 10\cdot b^2 = 10\cdot 10 = 100.$$

2.inačica

$$\left.\begin{array}{l} \log a + 3\cdot \log b = 3 \\ \log a - \log b = 1 \end{array}\right\} \Rightarrow \left[\begin{array}{l} \text{metoda suprotnih} \\ \text{koeficijenata} \end{array}\right] \Rightarrow \left.\begin{array}{l} \log a + 3\cdot \log b = 3 \\ \log a - \log b = 1 \;/\cdot(-1) \end{array}\right\} \Rightarrow \left.\begin{array}{l} \log a + 3\cdot \log b = 3 \\ -\log a + \log b = -1 \end{array}\right\} \Rightarrow$$

$$\Rightarrow 4\cdot \log b = 2 \;/:4 \Rightarrow \log b = \frac{1}{2}.$$

Iz druge jednadžbe dobije se log a:

$$\log a - \log b = 1 \Rightarrow \log a = 1 + \log b \Rightarrow \log a = 1 + \frac{1}{2} \Rightarrow \log a = \frac{3}{2}.$$

Računamo umnožak a i b:

$$\log a + \log b = \frac{3}{2} + \frac{1}{2} \Rightarrow \log a + \log b = 2 \Rightarrow \log a\cdot b = 2 \Rightarrow \log a\cdot b = \log 100 \Rightarrow a\cdot b = 100.$$

Vježba 074

Iz sustava jednadžbi izračunajte a · b.

$$\begin{cases} \log a + 3 \cdot \log b = 2 \\ \log a - \log b = 1. \end{cases}$$

Rezultat: $10 \cdot \sqrt{10}.$

Zadatak 075 (Hrvoje, šumarska škola)

Ako je $4 \cdot \log_4 x = 0$, nađite vrijednost izraza $3 \cdot x + \log_2 \frac{1}{4}$.

Rješenje 075

Ponovimo!

$$\log_b 1 = 0 \quad , \quad \log_b b = 1 \quad , \quad \log_b b^n = n.$$

$4 \cdot \log_4 x = 0 \Rightarrow \log_4 x = 0 \Rightarrow x = 1.$

Vrijednost zadanog izraza iznosi:

$$\left.\begin{matrix} 3 \cdot x + \log_2 \frac{1}{4} \\ x = 1 \end{matrix}\right\} \Rightarrow 3 \cdot 1 + \log_2 \frac{1}{4} = 3 + \log_2 \frac{1}{2^2} = 3 + \log_2 2^{-2} = 3 - 2 \cdot \log_2 2 = 3 - 2 \cdot 1 = 3 - 2 = 1.$$

Vježba 075

Ako je $4 \cdot \log_4 x = 0$, nađite vrijednost izraza $2 \cdot x + \log_2 \frac{1}{4}$.

Rezultat: 0.

Zadatak 076 (Pčelica Maja, gimnazija)

Nađite zbroj kvadrata rješenja jednadžbe: $\log_2 \left(9^{x-1} + 7\right) = 2 + \log_2 \left(3^{x-1} + 1\right)$.

Rješenje 076

Ponovimo!

$$n = \log_b b^n \quad , \quad \log_b a + \log_b c = \log_b (a \cdot c) \quad , \quad \log_b f(x) = \log_b g(x) \Rightarrow f(x) = g(x) \quad , \quad \left(a^n\right)^m = a^{n \cdot m}$$

$$a^{f(x)} = a^{g(x)} \Rightarrow f(x) = g(x) \quad , \quad a^0 = 1.$$

$$\log_2 \left(9^{x-1} + 7\right) = 2 + \log_2 \left(3^{x-1} + 1\right) \Rightarrow \log_2 \left(9^{x-1} + 7\right) = \log_2 2^2 + \log_2 \left(3^{x-1} + 1\right) \Rightarrow$$

$$\Rightarrow \log_2 \left(9^{x-1} + 7\right) = \log_2 4 + \log_2 \left(3^{x-1} + 1\right) \Rightarrow \log_2 \left(9^{x-1} + 7\right) = \log_2 4 \cdot \left(3^{x-1} + 1\right) \Rightarrow$$

$$\Rightarrow 9^{x-1} + 7 = 4 \cdot \left(3^{x-1} + 1\right) \Rightarrow 9^{x-1} + 7 = 4 \cdot 3^{x-1} + 4 \Rightarrow 9^{x-1} - 4 \cdot 3^{x-1} + 7 - 4 = 0 \Rightarrow$$

$$\Rightarrow 9^{x-1} - 4 \cdot 3^{x-1} + 3 = 0 \Rightarrow \left(3^2\right)^{x-1} - 4 \cdot 3^{x-1} + 3 = 0 \Rightarrow \left(3^{x-1}\right)^2 - 4 \cdot 3^{x-1} + 3 = 0 \Rightarrow \begin{bmatrix} \text{supstitucija} \\ t = 3^{x-1} \end{bmatrix} \Rightarrow$$

$$\Rightarrow t^2 - 4 \cdot t + 3 = 0 \Rightarrow t_{1,2} = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a} \Rightarrow t_{1,2} = \frac{4 \pm \sqrt{16 - 4 \cdot 1 \cdot 3}}{2 \cdot 1} \Rightarrow t_{1,2} = \frac{4 \pm \sqrt{4}}{2} \Rightarrow$$

$$\Rightarrow t_{1,2} = \frac{4 \pm 2}{2} \Rightarrow \left.\begin{matrix} t_1 = \frac{4+2}{2} = \frac{6}{2} = 3 \\ t_2 = \frac{4-2}{2} = \frac{2}{2} = 1 \end{matrix}\right\}.$$

Rješenja jednadžbe su:

$$\left.\begin{array}{l}3^{x-1}=3\\3^{x-1}=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}3^{x-1}=3^1\\3^{x-1}=3^0\end{array}\right\}\Rightarrow\left.\begin{array}{l}x-1=1\\x-1=0\end{array}\right\}\Rightarrow\left.\begin{array}{l}x_1=2\\x_2=1\end{array}\right\}.$$

Zbroj kvadrata rješenja jednadžbe iznosi:

$$x_1^2+x_2^2=2^2+1^2=4+1=5.$$

Vježba 076

Nađite zbroj kubova rješenja jednadžbe: $\log_2\left(9^{x-1}+7\right)=2+\log_2\left(3^{x-1}+1\right).$

Rezultat: 9.

Zadatak 077 (Pčelica Maja, gimnazija)

Rješenje jednadžbe: $2^x+2^{x-1}+2^{x-2}=7^{x-1}+7^{x-2}+7^{x-3}$ nalazi se u:

A. $[-1,1]$ *B.* $[1,3]$ *C.* $[3,6\rangle$ *D.* $[6,8]$ *E.* $[8,11\rangle$

Rješenje 077

Ponovimo!

$$a^x\cdot a^y=a^{x+y}\quad,\quad a^{-n}=\frac{1}{a^n}\quad,\quad \frac{a^x}{b^x}=\left(\frac{a}{b}\right)^x.$$

$$2^x+2^{x-1}+2^{x-2}=7^{x-1}+7^{x-2}+7^{x-3}\Rightarrow 2^x+2^x\cdot2^{-1}+2^x\cdot2^{-2}=7^x\cdot7^{-1}+7^x\cdot7^{-2}+7^x\cdot7^{-3}\Rightarrow$$

$$\Rightarrow 2^x\cdot\left(1+2^{-1}+2^{-2}\right)=7^x\cdot\left(7^{-1}+7^{-2}+7^{-3}\right)\Rightarrow 2^x\cdot\left(1+\frac{1}{2}+\frac{1}{4}\right)=7^x\cdot\left(\frac{1}{7}+\frac{1}{49}+\frac{1}{343}\right)\Rightarrow$$

$$\Rightarrow 2^x\cdot\frac{4+2+1}{4}=7^x\cdot\frac{49+7+1}{343}\Rightarrow 2^x\cdot\frac{7}{4}=7^x\cdot\frac{57}{343}\Rightarrow 2^x\cdot\frac{7}{4}=7^x\cdot\frac{57}{343}\ /\cdot\frac{4}{7}\Rightarrow 2^x=7^x\cdot\frac{57}{343}\cdot\frac{4}{7}\Rightarrow$$

$$\Rightarrow 2^x=7^x\cdot\frac{228}{2401}\Rightarrow 2^x=7^x\cdot\frac{228}{2401}\ /\cdot\frac{1}{7^x}\Rightarrow\frac{2^x}{7^x}=\frac{228}{2401}\Rightarrow\left(\frac{2}{7}\right)^x=\frac{228}{2401}\ /\log\Rightarrow\log\left(\frac{2}{7}\right)^x=\log\frac{228}{2401}\Rightarrow$$

$$\Rightarrow x\cdot\log\frac{2}{7}=\log\frac{228}{2401}\Rightarrow x=\frac{\log\frac{228}{2401}}{\log\frac{2}{7}}\Rightarrow x=\frac{\log 228-\log 2401}{\log 2-\log 7}\Rightarrow x=1.88\in[1,3].$$

Odgovor je pod B.

Vježba 077

Rješenje jednadžbe: $2^x=7^x$ nalazi se u:

A. $[-1,1]$ *B.* $[1,3]$ *C.* $[3,6\rangle$ *D.* $[6,8]$ *E.* $[8,11\rangle$

Rezultat: x = 0. Odgovor je pod A.

Zadatak 078 (Anita, ekonomska škola)

Zbroj rješenja jednadžbe $\log_2\left(\log_y(5\cdot y-6)\right)=1$ je 5. Dokažite!

Rješenje 078

Ponovimo!

$$\log_b a=c\Leftrightarrow b^c=a\quad,\quad x^2+b\cdot x+c=0\Rightarrow x_1+x_2=-b\quad\text{(Viète-ova formula)}$$

Dokaz slijedi:

$$\log_2\left(\log_y(5\cdot y-6)\right)=1\Rightarrow 2^1=\log_y(5\cdot y-6)\Rightarrow\log_y(5\cdot y-6)=2\Rightarrow y^2=5\cdot y-6\Rightarrow$$

$$\Rightarrow y^2-5\cdot y+6=0\Rightarrow y_1+y_2=-(-5)\Rightarrow y_1+y_2=5.$$

Vježba 078

Umnožak rješenja jednadžbe $\log_2\left(\log_y(5\cdot y-6)\right)=1$ je 6. Dokažite!

Rezultat: Dokaz analogan.

Zadatak 079 (Jelena, gimnazija)

Odredite skup rješenja nejednadžbe: $0.7^{\frac{x^2+9}{x+3}}>1.$

Rješenje 079

Ponovimo!

$$0<a<1 \Rightarrow a^{f(x)}>a^{g(x)} \Rightarrow f(x)<g(x) \quad , \quad a^0=1$$

$$0.7^{\frac{x^2+9}{x+3}}>1 \Rightarrow 0.7^{\frac{x^2+9}{x+3}}>0.7^0 \Rightarrow \frac{x^2+9}{x+3}<0.$$

Budući da je brojnik pozitivan za svaki realan broj x $\left(x^2+9>0,\ \forall x\in R\right)$, razlomak će biti negativan ako je nazivnik negativan:

$$x+3<0 \Rightarrow x<-3 \Rightarrow x\in\left\langle-\infty,-3\right\rangle.$$

Vježba 079

Odredite skup rješenja nejednadžbe: $0.5^{\frac{x^2+7}{x+3}}>1.$

Rezultat: $x\in\left\langle-\infty,-3\right\rangle.$

Zadatak 080 (Branka, gimnazija)

Riješite sustav jednadžbi: $2^x\cdot(x+y)=10 \quad , \quad \sqrt[x]{x+y}=5.$

Rješenje 080

Ponovimo!

$$\sqrt[n]{a^n}=a \quad , \quad a^{f(x)}=a^{g(x)} \Rightarrow f(x)=g(x) \quad , \quad a^1=a \quad , \quad \sqrt[n]{a\cdot b}=\sqrt[n]{a}\cdot\sqrt[n]{b} \quad , \quad \sqrt[n]{a^m}=a^{\frac{m}{n}}$$

$$a^n\cdot a^m=a^{n+m}.$$

1.inačica

$$\left.\begin{array}{l}2^x\cdot(x+y)=10\\ \sqrt[x]{x+y}=5\end{array}\right\} \Rightarrow \left.\begin{array}{l}x+y=\frac{10}{2^x}\\ \sqrt[x]{x+y}=5\end{array}\right\} \Rightarrow \sqrt[x]{\frac{10}{2^x}}=5 \Rightarrow \frac{\sqrt[x]{10}}{\sqrt[x]{2^x}}=5 \Rightarrow \frac{\sqrt[x]{10}}{2}=5\ /\cdot 2 \Rightarrow \sqrt[x]{10}=10 \Rightarrow$$

$$\Rightarrow 10^{\frac{1}{x}}=10^1 \Rightarrow \frac{1}{x}=1 \Rightarrow x=1 \Rightarrow \left.\begin{array}{l}x+y=\frac{10}{2^x}\\ x=1\end{array}\right\} \Rightarrow 1+y=\frac{10}{2} \Rightarrow 1+y=5 \Rightarrow y=4.$$

2.inačica

$$\left.\begin{array}{l}2^x\cdot(x+y)=10\\ \sqrt[x]{x+y}=5\end{array}\right\} \Rightarrow \left.\begin{array}{l}2^x\cdot(x+y)=10\\ \sqrt[x]{x+y}=5\ /^x\end{array}\right\} \Rightarrow \left.\begin{array}{l}2^x\cdot(x+y)=10\\ \left(\sqrt[x]{x+y}\right)^x=5^x\end{array}\right\} \Rightarrow \left.\begin{array}{l}2^x\cdot(x+y)=10\\ x+y=5^x\end{array}\right\} \Rightarrow \left[\begin{array}{l}\text{podijelimo}\\ \text{jednadžbe}\end{array}\right] \Rightarrow$$

$$\Rightarrow \frac{2^x\cdot(x+y)}{x+y}=\frac{10}{5^x} \Rightarrow 2^x=\frac{10}{5^x}\ /\cdot 5^x \Rightarrow 2^x\cdot 5^x=10 \Rightarrow (2\cdot 5)^x=10 \Rightarrow 10^x=10^1 \Rightarrow x=1 \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l}2^x\cdot(x+y)=10\\ x=1\end{array}\right\} \Rightarrow 2^1\cdot(1+y)=10 \Rightarrow 2\cdot(1+y)=10\ /:2 \Rightarrow 1+y=5 \Rightarrow y=4.$$

3.inačica

$$\left.\begin{array}{l} 2^x \cdot (x+y) = 10 \\ \sqrt[x]{x+y} = 5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} 2^x \cdot (x+y) = 10 \ / \sqrt[x]{\ } \\ \sqrt[x]{x+y} = 5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \sqrt[x]{2^x \cdot (x+y)} = \sqrt[x]{10} \\ \sqrt[x]{x+y} = 5 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \sqrt[x]{2^x} \cdot \sqrt[x]{x+y} = \sqrt[x]{10} \\ \sqrt[x]{x+y} = 5 \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left.\begin{array}{l} 2 \cdot \sqrt[x]{x+y} = \sqrt[x]{10} \\ \sqrt[x]{x+y} = 5 \end{array}\right\} \Rightarrow \begin{bmatrix} \text{podijelimo} \\ \text{jednadžbe} \end{bmatrix} \Rightarrow \frac{2 \cdot \sqrt[x]{x+y}}{\sqrt[x]{x+y}} = \frac{\sqrt[x]{10}}{5} \Rightarrow 2 = \frac{\sqrt[x]{10}}{5} \ / \cdot 5 \Rightarrow 10 = \sqrt[x]{10} \Rightarrow$$

$$\Rightarrow 10^1 = 10^{\frac{1}{x}} \Rightarrow \frac{1}{x} = 1 \Rightarrow x = 1 \Rightarrow \left.\begin{array}{l} 2^x \cdot (x+y) = 10 \\ x = 1 \end{array}\right\} \Rightarrow 2^1 \cdot (1+y) = 10 \Rightarrow$$

$$\Rightarrow 2 \cdot (1+y) = 10 \ /:2 \Rightarrow 1+y = 5 \Rightarrow y = 4.$$

Vježba 080

Riješite sustav jednadžbi: $2^x \cdot (x+y) = 4 \quad , \quad \sqrt[x]{x+y} = 2.$

Rezultat: x = y = 1.