Zadatak 041 (4A, hotelijerska škola)

Riješite jednadžbu $\log_3 x + \log_{\sqrt{3}} x + \log_{\frac{1}{3}} x = 6.$

Rješenje 041

Diskusija! $x > 0.$

Budući da se u bazama logaritama pojavljuje broj 3, svedimo sve na bazu 3:

$$\log_3 x + \log_{\sqrt{3}} x + \log_{\frac{1}{3}} x = 6 \Rightarrow \log_3 x + \frac{\log_3 x}{\log_3 \sqrt{3}} + \frac{\log_3 x}{\log_3 \frac{1}{3}} = 6 \Rightarrow \log_3 x + \frac{\log_3 x}{\frac{1}{2} \cdot \log_3 3} + \frac{\log_3 x}{-1 \cdot \log_3 3} = 6 \Rightarrow$$

$$\Rightarrow \log_3 x + \frac{\log_3 x}{\frac{1}{2}} + \frac{\log_3 x}{-1} = 6 \Rightarrow \log_3 x + 2 \cdot \log_3 x - \log_3 x = 6 \Rightarrow 2 \cdot \log_3 x = 6 \ /:2 \Rightarrow \log_3 x = 3 \Rightarrow$$

$$\Rightarrow \log_3 x = \log_3 3^3 \Rightarrow x = 3^3 = 27.$$

Vježba 041

Riješite jednadžbu $\log_3 x + \log_{\sqrt{3}} x + \log_{\frac{1}{3}} x = 2.$

Rezultat: x = 3.

Zadatak 042 (4A, hotelijerska škola)

Riješite jednadžbu $\log_2 \left(\log_x (x+2) \right) = 1.$

Rješenje 042

Diskusija!

$$\log_x (x+2) > 0 \ , \ x > 0 \ , \ x \neq 1 \ , \ x+2 > 0 \Rightarrow x > -2$$

$$\log_2 \left(\log_x (x+2) \right) = 1 \Rightarrow \log_2 \left(\log_x (x+2) \right) = \log_2 2 \Rightarrow \log_x (x+2) = 2 \Rightarrow \log_x (x+2) = \log_x x^2 \Rightarrow$$

$$\Rightarrow x+2 = x^2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2} = \frac{1 \pm 3}{2} \Rightarrow \begin{cases} x_1 = 2 \\ x_2 = -1. \end{cases}$$

Sve uvjete zadatka zadovoljava samo $x_1 = 2$.

Vježba 042

Riješite jednadžbu $\log_2 \left(\log_x (2x+2) \right) = 0.$

Rezultat: Nema rješenja jer x = – 2 nije rješenje zbog uvjeta u zadatku.

Zadatak 043 (Anamarija, hotelijerska škola)

Riješite jednadžbu $\log_{\frac{1}{2}} \left(3 - 2^{-x} \right) = x - 1.$

Rješenje 043

Diskusija! $3 - 2^{-x} > 0 \Rightarrow 2^{-x} < 3.$

$$\log_{\frac{1}{2}} \left(3 - 2^{-x} \right) = x - 1 \Rightarrow \log_{\frac{1}{2}} \left(3 - 2^{-x} \right) = \log_{\frac{1}{2}} \left(\frac{1}{2} \right)^{x-1} \Rightarrow 3 - 2^{-x} = \left(\frac{1}{2} \right)^{x-1} \Rightarrow 3 - 2^{-x} = 2^{-x+1} \Rightarrow$$

$$\Rightarrow 3 - 2^{-x} = 2 \cdot 2^{-x} \Rightarrow 3 = 2 \cdot 2^{-x} + 2^{-x} \Rightarrow 3 \cdot 2^{-x} = 3 \ /:3 \Rightarrow 2^{-x} = 1 \Rightarrow 2^{-x} = 2^0 \Rightarrow -x = 0 \Rightarrow x = 0.$$

Vježba 043

Riješite jednadžbu $\log_2 \left(3 - 2^{-x} \right) = 1 - x.$

Rezultat: x = 0.

Zadatak 044 (Anamarija, hotelijerska škola)

Ako je log 5 – a = 0, nađite takav x da je $\log 40^{\frac{1}{x}}=1.$

Rješenje 044

$$\left.\begin{array}{l}\log 5-a=0\\ \log 40^{\frac{1}{x}}=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\log 5=a\\ \frac{1}{x}\cdot\log 40=1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\log 5=a\\ x=\log 40\end{array}\right\}\Rightarrow\left.\begin{array}{l}\log 5=a\\ x=\log(4\cdot 10)\end{array}\right\}\Rightarrow\left.\begin{array}{l}\log 5=a\\ x=\log 4+\log 10\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}\log 5=a\\ x=\log 2^2+1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\log 5=a\\ x=2\cdot\log 2+1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\log 5=a\\ x=2\cdot\log\frac{10}{5}+1\end{array}\right\}\Rightarrow\left.\begin{array}{l}\log 5=a\\ x=2\cdot(\log 10-\log 5)+1\end{array}\right\}\Rightarrow$$

$$\Rightarrow\left.\begin{array}{l}\log 5=a\\ x=2\cdot(1-\log 5)+1\end{array}\right\}\Rightarrow x=2\cdot(1-a)+1=2-2a+1=3-2a.$$

Vježba 044

Ako je log 5 – a = 0, nađite takav x da je $\log 40^{\frac{1}{x}}=2.$

Rezultat: $x=\frac{3}{2}-a.$

Zadatak 045 (Anamarija, hotelijerska škola)

Riješite jednadžbu $3\cdot\log x+2\cdot\log\sqrt{\frac{1}{x}}=2.$

Rješenje 045

Diskusija! $x>0.$

$$3\cdot\log x+2\cdot\log\sqrt{\frac{1}{x}}=2\Rightarrow 3\cdot\log x+2\cdot\log\left(\frac{1}{x}\right)^{\frac{1}{2}}=2\Rightarrow 3\cdot\log x+2\cdot\frac{1}{2}\cdot\log\frac{1}{x}=2\Rightarrow 3\cdot\log x+\log\frac{1}{x}=2\Rightarrow$$

$$\Rightarrow 3\cdot\log x+\log 1-\log x=2\Rightarrow 3\cdot\log x+0-\log x=2\Rightarrow 2\cdot\log x=2\ /:2\Rightarrow\log x=1\Rightarrow\log x=\log 10\Rightarrow x=10.$$

Vježba 045

Riješite jednadžbu $3\cdot\log x+2\cdot\log\sqrt{\frac{1}{x}}=4.$

Rezultat: x = 100.

Zadatak 046 (Anamarija, hotelijerska škola)

Nađite umnožak rješenja jednadžbe: $2\cdot 3^{|x|}+9^{\frac{|x|}{2}}-27^{\frac{|x|}{2}}=0.$

Rješenje 046

$$2\cdot 3^{|x|}+9^{\frac{|x|}{2}}-27^{\frac{|x|}{2}}=0\Rightarrow 2\cdot 3^{|x|}+\left(3^2\right)^{\frac{|x|}{2}}-\left(3^3\right)^{\frac{|x|}{2}}=0\Rightarrow 2\cdot 3^{|x|}+3^{|x|}-3^{\frac{3}{2}|x|}=0\Rightarrow$$

$$\Rightarrow 2\cdot 3^{|x|}+3^{|x|}=3^{\frac{3}{2}|x|}\Rightarrow 3\cdot 3^{|x|}=3^{\frac{3}{2}|x|}\Rightarrow 3^{1+|x|}=3^{\frac{3}{2}|x|}\Rightarrow$$

$$\Rightarrow\left[a^{f(x)}=a^{g(x)}\Rightarrow f(x)=g(x)\right]\Rightarrow 1+|x|=\frac{3}{2}\cdot|x|\ /\cdot 2\Rightarrow 2+2\cdot|x|=3\cdot|x|\Rightarrow|x|=2\Rightarrow x_{1,2}=\pm 2.$$

Umnožak je: $x_1\cdot x_2=-2\cdot 2=-4.$

Vježba 046

Nađite zbroj rješenja jednadžbe: $2\cdot 3^{|x|}+9^{\frac{|x|}{2}}-27^{\frac{|x|}{2}}=0.$

Rezultat: $x_1+x_2=-2+2=0.$

Zadatak 047 (4A, hotelijerska škola)

Nađi vezu između a, b i x ako je $30^a = 3,\ 30^b = 5,\ 30^{-x} = 8.$

Rješenje 047

$$30^a = 3 \;/\log \;\Rightarrow\; a\cdot\log 30 = \log 3 \;\Rightarrow\; a = \frac{\log 3}{\log 30},$$

$$30^b = 5 \;/\log \;\Rightarrow\; b\cdot\log 30 = \log 5 \;\Rightarrow\; b = \frac{\log 5}{\log 30},$$

$$30^{-x} = 8 \;/\log \;\Rightarrow\; -x\cdot\log 30 = \log 8 \;\Rightarrow\; x = -\frac{\log 8}{\log 30} = -\frac{\log 2^3}{\log 30} = -\frac{3\cdot\log 2}{\log 30} \;\Rightarrow\; -\frac{x}{3} = \frac{\log 2}{\log 30}.$$

Dalje je:

$$a+b = \frac{\log 3}{\log 30} + \frac{\log 5}{\log 30} = \frac{\log 3 + \log 5}{\log 30} = \frac{\log 15}{\log 30} = \frac{\log\frac{30}{2}}{\log 30} = \frac{\log 30 - \log 2}{\log 30} = \frac{\log 30}{\log 30} - \frac{\log 2}{\log 30} = 1 - \left(-\frac{x}{3}\right) = 1 + \frac{x}{3}.$$

Konačno dobije se:

$$a+b = 1 + \frac{x}{3} \;/\cdot 3 \;\Rightarrow\; 3\cdot(a+b) = 3 + x.$$

Vježba 047

Nađi vezu između a, b i x ako je $30^a = 3,\ 30^b = 5,\ 30^x = 8.$

Rezultat: 3(a + b) = 3 – x.

Zadatak 048 (4A, hotelijerska škola)

Izračunaj:

$$\frac{1}{\log 10^x\cdot\log 10^{3x}} + \frac{1}{\log 10^{3x}\cdot\log 10^{5x}} + \ldots + \frac{1}{\log 10^{99x}\cdot\log 10^{101x}}.$$

Rješenje 048

Ponovimo!

$$\log 10^n = n.$$

$$\frac{1}{\log 10^x\cdot\log 10^{3x}} + \frac{1}{\log 10^{3x}\cdot\log 10^{5x}} + \ldots + \frac{1}{\log 10^{99x}\cdot\log 10^{101x}} = \frac{1}{x\cdot 3x} + \frac{1}{3x\cdot 5x} + \ldots + \frac{1}{99x\cdot 101x} =$$

$$= \frac{1}{1\cdot 3x^2} + \frac{1}{3\cdot 5x^2} + \ldots + \frac{1}{99\cdot 101x^2} = \frac{1}{x^2}\cdot\left(\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \ldots + \frac{1}{99\cdot 101}\right).$$

Uočimo da su u nazivnicima faktori neparni brojevi pa razlomke možemo prikazati na sljedeći način:

$$\frac{1}{1\cdot 3} = \frac{1}{2}\cdot\left(\frac{1}{1} - \frac{1}{3}\right),\ \frac{1}{3\cdot 5} = \frac{1}{2}\cdot\left(\frac{1}{3} - \frac{1}{5}\right),\ \ldots,\ \frac{1}{99\cdot 101} = \frac{1}{2}\cdot\left(\frac{1}{99} - \frac{1}{101}\right).$$

U ovom rastavu poslužili smo se metodom neodređenih koeficijenata:

$$\frac{1}{(2n-1)\cdot(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1},$$ A i B su nepoznati koeficijenti koje moramo izračunati.

$$\frac{1}{(2n-1)\cdot(2n+1)} = \frac{A}{2n-1} + \frac{B}{2n+1} \;/\cdot(2n-1)\cdot(2n+1) \;\Rightarrow\; 1 = A\cdot(2n+1) + B\cdot(2n-1) \;\Rightarrow$$

$$1 = 2An + A + 2Bn - B \;\Rightarrow\; 1 = (2A+2B)\cdot n + (A-B) \;\Rightarrow\; \begin{cases} 2A+2B=0 \\ A-B=1 \end{cases} \;\Rightarrow\; \begin{cases} 2A+2B=0 \;/:2 \\ A-B=1 \end{cases} \;\Rightarrow$$

$$\Rightarrow \begin{cases} A+B=0 \\ A-B=1 \end{cases} \;\Rightarrow\; 2A=1 \;\Rightarrow\; A=\frac{1}{2} \;\Rightarrow\; B=-\frac{1}{2}.$$

Zato je:

$$\frac{1}{(2n-1)\cdot(2n+1)}=\frac{\frac{1}{2}}{2n-1}-\frac{\frac{1}{2}}{2n+1}=\frac{1}{2}\cdot\left(\frac{1}{2n-1}-\frac{1}{2n+1}\right).$$

Konačno se dobije:

$$\frac{1}{x^2}\cdot\left(\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\ ...\ +\frac{1}{99\cdot 101}\right)=\frac{1}{x^2}\cdot\left(\frac{1}{2}\cdot\left(\frac{1}{1}-\frac{1}{3}\right)+\frac{1}{2}\cdot\left(\frac{1}{3}-\frac{1}{5}\right)+\ ...\ +\frac{1}{2}\cdot\left(\frac{1}{99}-\frac{1}{101}\right)\right)=$$

$$=\frac{1}{x^2}\cdot\frac{1}{2}\cdot\left(\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}+\ ...\ +\frac{1}{99}-\frac{1}{101}\right)=\frac{1}{2\cdot x^2}\cdot\left(1-\frac{1}{101}\right)=\frac{1}{2\cdot x^2}\cdot\frac{100}{101}=\frac{50}{101\cdot x^2}.$$

Vježba 048

Izračunaj: $\frac{1}{\log 10^x\cdot\log 10^{3x}}+\frac{1}{\log 10^{3x}\cdot\log 10^{5x}}.$

Rezultat: $\frac{3}{5x^2}.$

Zadatak 049 (2A, hotelijerska škola)

Umnožak broja a i $\log\frac{1}{2}$ je pozitivan ako je a?

Rješenje 049

$$a\cdot\log\frac{1}{2}>0\Rightarrow\left[\text{zbog }\log\frac{1}{2}=-0.30103<0\right]\Rightarrow a<0.$$

Vježba 049

Umnožak broja a i $\log\frac{1}{2}$ je negativan ako je a?

Rezultat: a > 0.

Zadatak 050 (Anamarija, hotelijerska škola)

Ako je $4^x = 9$ i $9^y = 256$, koliki je umnožak x · y?

Rješenje 050

$$\left.\begin{array}{l}4^x=9\\ 9^y=256\end{array}\right\}\Rightarrow\left(4^x\right)^y=256\Rightarrow 4^{x\cdot y}=256\Rightarrow 4^{x\cdot y}=4^4\Rightarrow x\cdot y=4.$$

Vježba 050

Ako je $4^x = 9$ i $9^y = 64$, koliki je umnožak x · y?

Rezultat: 3.

Zadatak 051 (Natalija, hotelijerska škola)

Skup rješenja nejednadžbe $4\cdot 2^{1-x}<8$ je

$A)\ |x|<1$ $\qquad B)\ \langle-\infty, 0\rangle$ $\qquad C)\ \langle 0, +\infty\rangle$ $\qquad D)\ \langle 0, 1\rangle$

Rješenje 051

1.inačica

$$4\cdot 2^{1-x}<8\Rightarrow 2^2\cdot 2^{1-x}<2^3\Rightarrow 2^{2+1-x}<2^3\Rightarrow 2^{3-x}<2^3\Rightarrow 3-x<3\Rightarrow -x<3-3\Rightarrow$$

$$\Rightarrow -x<0\ /\cdot(-1)\Rightarrow x>0\Rightarrow x\in\langle 0, +\infty\rangle.$$

Odgovor je pod C.

2.inačica

$$4\cdot 2^{1-x}<8\ /:4\Rightarrow 2^{1-x}<2\Rightarrow 2^{1-x}<2^1\Rightarrow 1-x<1\Rightarrow -x<1-1\Rightarrow -x<0\ /\cdot(-1)\Rightarrow x>0\Rightarrow x\in\langle 0, +\infty\rangle.$$

Odgovor je pod C.

Vježba 051

Skup rješenja nejednadžbe $9 \cdot 3^{1-x} < 27$ je

$A)\ |x|<1 \qquad B)\ \langle -\infty, 0\rangle \qquad C)\ \langle 0, +\infty\rangle \qquad D)\ \langle 0, 1\rangle$

Rezultat: Odgovor je pod C.

Zadatak 052 (Anamarija, hotelijerska škola)

Riješite jednadžbu $\log_4 x + \log_8 x = 5.$

Rješenje 052

1.inačica

$\log_4 x + \log_8 x = 5.$ Diskusija: $x > 0.$

Uporabit ćemo pravilo: $\log_{b^n} a = \frac{1}{n} \cdot \log_b a.$

$$\log_4 x + \log_8 x = 5 \Rightarrow \log_{2^2} x + \log_{2^3} x = 5 \Rightarrow \frac{1}{2} \cdot \log_2 x + \frac{1}{3} \cdot \log_2 x = 5 \ / \cdot 6 \Rightarrow 3 \cdot \log_2 x + 2 \cdot \log_2 x = 30 \Rightarrow$$

$$\Rightarrow 5 \cdot \log_2 x = 30 \ / :5 \Rightarrow \log_2 x = 6 \Rightarrow \left[\log_b a = c \Leftrightarrow b^c = a\right] \Rightarrow x = 2^6 = 64.$$

2.inačica

$\log_4 x + \log_8 x = 5.$ Diskusija: $x > 0.$

Uporabit ćemo pravilo: $\log_b a = \frac{\log_c a}{\log_c b}.$

$$\log_4 x + \log_8 x = 5 \Rightarrow \frac{\log x}{\log 4} + \frac{\log x}{\log 8} = 5 \Rightarrow \frac{\log x}{\log 2^2} + \frac{\log x}{\log 2^3} = 5 \Rightarrow \frac{\log x}{2 \cdot \log 2} + \frac{\log x}{3 \cdot \log 2} = 5 \ / \cdot 6 \cdot \log 2 \Rightarrow$$

$$3 \cdot \log x + 2 \cdot \log x = 30 \cdot \log 2 \Rightarrow 5 \cdot \log x = 30 \cdot \log 2 \ / :5 \Rightarrow \log x = 6 \cdot \log 2 \Rightarrow \log x = \log 2^6 \Rightarrow$$

$$\Rightarrow \left[\log f(x) = \log g(x) \Rightarrow f(x) = g(x)\right] \Rightarrow x = 2^6 = 64.$$

Vježba 052

Riješite jednadžbu $\log_3 x + \log_{27} x = -4.$

Rezultat: $\frac{1}{27}.$

Zadatak 053 (Iva, komercijalna škola)

Ako je $a \cdot b = c^2$, koliko je $\log_{a \cdot b} c$?

Rješenje 053

1.inačica

$$\log_{a \cdot b} c = \left[\log_b a = \frac{1}{\log_a b}\right] = \frac{1}{\log_c (a \cdot b)} = \frac{1}{\log_c c^2} = \frac{1}{2 \cdot \log_c c} = \frac{1}{2 \cdot 1} = \frac{1}{2}.$$

2.inačica

Iz $a \cdot b = c^2$ odmah slijedi:

$$a \cdot b = c^2 \Rightarrow a \cdot b = c^2 \ / \log_{a \cdot b} \Rightarrow \log_{a \cdot b} (a \cdot b) = \log_{a \cdot b} c^2 \Rightarrow 1 = 2 \cdot \log_{a \cdot b} c \Rightarrow \log_{a \cdot b} c = \frac{1}{2}.$$

Vježba 053

Ako je $a \cdot b = c^4$, koliko je $\log_{a \cdot b} c$?

Rezultat: $\frac{1}{4}.$

Zadatak 054 (Mario, gimnazija)

Izračunaj: $\frac{1}{\log_a b}+\frac{1}{\log_{a^2} b}+\frac{1}{\log_{a^3} b}+\frac{1}{\log_{a^4} b}+\ ...\ +\frac{1}{\log_{a^n} b}.$

Rješenje 054

Uporabom relacije $\log_b a=\frac{1}{\log_a b}$, dobije se:

$$\frac{1}{\log_a b}+\frac{1}{\log_{a^2} b}+\frac{1}{\log_{a^3} b}+\frac{1}{\log_{a^4} b}+\ ...\ +\frac{1}{\log_{a^n} b}=\log_b a+\log_b a^2+\log_b a^3+\log_b a^4+\ ...\ +\log_b a^n=$$

$$=\log_b a+2\cdot\log_b a+3\cdot\log_b a+4\cdot\log_b a+\ ...\ +n\cdot\log_b a=(1+2+3+4+\ ...\ +n)\cdot\log_b a=$$

$$=\frac{n\cdot(n+1)}{2}\cdot\log_b a=\frac{n\cdot(n+1)}{2\cdot\log_a b}.$$

Vježba 054

Izračunaj: $\frac{1}{\log_a b}+\frac{1}{\log_{a^2} b}+\frac{1}{\log_{a^3} b}+\frac{1}{\log_{a^4} b}+\ ...\ +\frac{1}{\log_{a^{100}} b}.$

Rezultat: $\frac{5050}{\log_a b}.$

Zadatak 055 (Ivan, gimnazija)

Riješi jednadžbu: $\log_5 x+\log_{25}\frac{1}{x}=1.$

Rješenje 055

Koristeći formulu $\log_{b^k} a=\frac{1}{k}\cdot\log_b a$ jednadžba se može pisati u obliku:

$$\log_5 x+\log_{25}\frac{1}{x}=1\ \Rightarrow\ \log_5 x+\log_{25} x^{-1}=1\ \Rightarrow\ \log_5 x-\log_{25} x=1\ \Rightarrow\ \log_5 x-\log_{5^2} x=1\ \Rightarrow$$

$$\Rightarrow\ \log_5 x-\frac{1}{2}\cdot\log_5 x=1\ \Rightarrow\ \frac{1}{2}\cdot\log_5 x=1\ /\cdot 2\ \Rightarrow\ \log_5 x=2\ \Rightarrow\ x=5^2=25.$$

Uvrštavanjem u polaznu jednadžbu provjerimo da je to stvarno rješenje:

$$\left.\begin{array}{l}\log_5 x+\log_{25}\frac{1}{x}=1\\ x=25\end{array}\right\}\ \Rightarrow\ \log_5 25+\log_{25}\frac{1}{25}=1\ \Rightarrow\ \log_5 5^2+\log_{25} 25^{-1}=1\ \Rightarrow$$

$$\Rightarrow\ 2\cdot\log_5 5-1\cdot\log_{25} 25=1\ \Rightarrow\ \Rightarrow\ 2\cdot 1-1\cdot 1=1\ \Rightarrow\ 2-1=1\ \Rightarrow\ 1=1.$$

Vježba 055

Riješi jednadžbu: $\log_3 x+\log_9\frac{1}{x}=1.$

Rezultat: x = 9.

Zadatak 056 (Ivan, gimnazija)

Riješi jednadžbu: $1.04^{4\cdot x-1}=6.$

Rješenje 056

1.inačica

Podsjetimo se definicije logaritma kao eksponenta: $\log_b a=c\ \Leftrightarrow\ b^c=a.$ Zato je:

$$1.04^{4\cdot x-1}=6\ \Rightarrow\ \log_{1.04} 6=4\cdot x-1\ \Rightarrow\ -4\cdot x=-1-\log_{1.04} 6\ /:(-4)\ \Rightarrow\ x=\frac{1+\log_{1.04} 6}{4}=$$

$$\Rightarrow x=\frac{1}{4}\cdot\left(1+\log_{1.04}6\right)=\left[\log_b a=\frac{\log_c a}{\log_c b}\right]=\frac{1}{4}\cdot\left(1+\frac{\log 6}{\log 1.04}\right)\approx 11.67100261.$$

2.inačica

Logaritmiramo lijevu i desnu stranu jednadžbe (uzimamo dekadski logaritam lijeve i desne strane jednadžbe):

$$1.04^{4\cdot x-1}=6 \Rightarrow 1.04^{4\cdot x-1}=6 \;/\log \Rightarrow \log 1.04^{4\cdot x-1}=\log 6 \Rightarrow (4\cdot x-1)\cdot\log 1.04=\log 6 \;/:\log 1.04 \Rightarrow$$

$$\Rightarrow 4\cdot x-1=\frac{\log 6}{\log 1.04} \Rightarrow 4\cdot x=1+\frac{\log 6}{\log 1.04} \;/:4 \Rightarrow x=\frac{1}{4}\cdot\left(1+\frac{\log 6}{\log 1.04}\right)\approx 11.67100261.$$

Uvrštavanjem u polaznu jednadžbu provjerimo da je to stvarno rješenje:

$$\left.\begin{matrix}1.04^{4\cdot x-1}=6\\ x\approx 11.67100261\end{matrix}\right\} \Rightarrow 1.04^{4\cdot 11.67100261-1}=6 \Rightarrow 1.04^{45.68401044}=6 \Rightarrow 6=6.$$

Vježba 056

Riješi jednadžbu: $1.02^{4\cdot x-1}=5.$

Rezultat: $x\approx 20.5685.$

Zadatak 057 (Mario, gimnazija)

Riješi nejednadžbu: $\log_x 3\cdot\log_3(9\cdot x)>0.$

Rješenje 057

$$\log_x 3\cdot\log_3(9\cdot x)>0 \Rightarrow \left[\log_b a=\frac{\log_c a}{\log_c b}\right] \Rightarrow \frac{\log 3}{\log x}\cdot\frac{\log(9\cdot x)}{\log 3}>0 \Rightarrow \frac{\log(9\cdot x)}{\log x}>0 \Rightarrow \log_x(9\cdot x)>0 \Rightarrow$$

$$\Rightarrow \left[\begin{matrix}\text{uvjet}\\ x>0\\ x\neq 1\end{matrix}\right].$$

1.slučaj

$$\left.\begin{matrix}0<x<1\\ \log_x(9\cdot x)>0\end{matrix}\right\} \Rightarrow \log_x(9\cdot x)>\log_x 1 \Rightarrow 9\cdot x<1 \Rightarrow x<\frac{1}{9} \Rightarrow x\in\left\langle 0,\frac{1}{9}\right\rangle.$$

2.slučaj

$$\left.\begin{matrix}x>1\\ \log_x(9\cdot x)>0\end{matrix}\right\} \Rightarrow \log_x(9\cdot x)>\log_x 1 \Rightarrow 9\cdot x>1 \Rightarrow x>\frac{1}{9} \Rightarrow x\in\langle 1,+\infty\rangle.$$

Rješenje nejednadžbe iznosi: $x\in\left\langle 0,\frac{1}{9}\right\rangle\cup\langle 1,+\infty\rangle.$

Vježba 057

Riješi nejednadžbu: $\log_x 5\cdot\log_5(9\cdot x)>0.$

Rezultat: $x\in\left\langle 0,\frac{1}{9}\right\rangle\cup\langle 1,+\infty\rangle.$

Zadatak 058 (Ivan, gimnazija)

Riješi jednadžbu: $\log x^2+\log(-x)=3.$

Rješenje 058

$$\log x^2+\log(-x)=3 \Rightarrow \left[\begin{matrix}\text{uvjet}\\ -x>0 \Rightarrow x<0\end{matrix}\right] \Rightarrow \log\left(x^2\cdot(-x)\right)=3 \Rightarrow \log\left(-x^3\right)=3 \Rightarrow \log\left(-x^3\right)=\log 1000 \Rightarrow$$

$$\Rightarrow -x^3=1000 \;/\cdot(-1) \Rightarrow x^3=-10^3 \;/\sqrt[3]{\;} \Rightarrow x=-10.$$

Vježba 058

Riješi jednadžbu: $\log x^2 + \log(-x) = 6.$

Rezultat: $x = -100.$

Zadatak 059 (Ana, hotelijerska škola)

Ako je $\left(\log 5 - \sqrt{a}\right)^2 = 0$, izračunajte $\log \sqrt{32}.$

Rješenje 059

$$\left(\log 5 - \sqrt{a}\right)^2 = 0 \; / \sqrt{\;} \Rightarrow \log 5 - \sqrt{a} = 0 \Rightarrow \log 5 = \sqrt{a}.$$

Računamo $\log \sqrt{32}$:

$$\log \sqrt{32} = \log 32^{\frac{1}{2}} = \frac{1}{2} \cdot \log 32 = \frac{1}{2} \cdot \log 2^5 = \frac{5}{2} \cdot \log 2 = \frac{5}{2} \cdot \log \frac{10}{5} = \frac{5}{2} \cdot (\log 10 - \log 5) = \frac{5}{2} \cdot (1 - \log 5) = \frac{5}{2} \cdot \left(1 - \sqrt{a}\right).$$

Vježba 059

Ako je $\left(\log 5 - \sqrt{a}\right)^2 = 0$, izračunajte $\log \sqrt{8}.$

Rezultat: $\frac{3}{2} \cdot \left(1 - \sqrt{a}\right).$

Zadatak 060 (Tea, ekonomska škola)

Riješite jednadžbu $\log_{\frac{1}{2}} \log_{16} \log(0.1 \cdot x) = 2.$

Rješenje 060

Ponovino!

$$\log_b a = c \Leftrightarrow b^c = a, \quad \left(a^n\right)^m = a^{n \cdot m}$$

Diskusija! $x > 0.$

$$\log_{\frac{1}{2}} \log_{16} \log(0.1 \cdot x) = 2 \Rightarrow \left(\frac{1}{2}\right)^2 = \log_{16} \log(0.1 \cdot x) \Rightarrow \frac{1}{4} = \log_{16} \log(0.1 \cdot x) \Rightarrow \log_{16} \log(0.1 \cdot x) = \frac{1}{4} \Rightarrow$$

$$\Rightarrow 16^{\frac{1}{4}} = \log(0.1 \cdot x) \Rightarrow \left(2^4\right)^{\frac{1}{4}} = \log(0.1 \cdot x) \Rightarrow 2 = \log(0.1 \cdot x) \Rightarrow \log(0.1 \cdot x) = 2 \Rightarrow$$

$$\Rightarrow 10^2 = 0.1 \cdot x \Rightarrow 0.1 \cdot x = 100 \; / \cdot 10 \Rightarrow x = 1000 = 10^3.$$

Vježba 060

Riješite jednadžbu $\log_{\frac{1}{2}} \log_2 \log x = -1.$

Rezultat: $x = 10\,000 = 10^4.$